THE

ASTRONOMICAL JOURNAL

FOUNDED BY B. A. GOULD

EDITED BY

SETH C. CHANDLER

ASSOCIATE EDITORS

ASAPH HALL AND LEWIS BOSS

VOLUME XXIII

JANUARY 1903 TO JANUARY 1904

NUMBERS 529-552

WITH TWO PLATES.

BOSTON

1903

CONTENTS.

NUMBER 529.

NUMBER 530.

NUMBERS 531–532.

NUMBER 533.

NUMBER 534.

CONTENTS

NUMBER 535

NUMBER 536.

NUMBERS 537–538.

NUMBER 539.

NUMBERS 540–541.

NUMBERS 542–543.

NUMBER 544.

CONTENTS.

NUMBER 545.

NUMBER 546.

NUMBER 547.

NUMBER 548.

NUMBERS 549-550.

NUMBER 551.

NUMBER 552.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 529.

VOL. XXIII. BOSTON, 1903 JANUARY 10. NO. 1

THE PROBABLE VALUE OF THE CONSTANT OF ABERRATION.

By S. C. CHANDLER.

The general history of our knowledge of the value of the constant of aberration is familiar to astronomers. It has had some singular vicissitudes. A half a century ago this element was supposed to have been fixed by STRUVE with an accuracy which almost discouraged further investigation of it. NYRÉN, in 1883, thoroughly unsettled this belief; and some ten years later, after the discovery of the inconstancy in the position of the pole — and especially after DOOLITTLE's striking results had begun to inspire still greater distrust of STRUVE's value — all illusion of security vanished, the question was thrown wide open, and there was a renewal of active general investigation. The premature attempt of the Almanacs in 1896 to legislate upon the matter fortunately did not stifle, or even perceptibly retard this inquiry; and it is now manifest that the conventional correction to STRUVE's value then hastily adopted is entirely insufficient, being not over a third of the correction really required. This is not a matter of mere individual opinion, but is, I think, the conclusion to which any astronomer must arrive upon examination of the very extensive material now available. For whatever divergencies of opinion there may be as to the relative weights of the various determinations, the number of these is now so considerable that even wide differences of judgement as to these weights can exercise little influence on the mean result, which comes out about the same however the weights are disposed; namely, somewhere between 20″.52 and 20″.53. Moreover, the body of existing evidence is now so large that the addition of new determinations, as they from time to time appear in print, influences the mean result of all in a scarcely perceptible degree. So that it would seem that the time has nearly or quite arrived when a conventional value generally acceptable to astronomers can be decided upon, if desirable.

During the past ten years the writer has made the scrutiny of all existing evidence bearing on the aberration-constant a matter of much care. To present the details of this revision, with a full statement of the reasons for the exclusion of many determinations and for the modification of the printed results for others, as well as the method of assigning weights, would unduly extend this paper. Nevertheless some particulars on these points should be given, if only as a guide for others who may desire to review the ground for themselves, and exercise an independent judgement of the correctness of the present procedure, and how far differences of opinion as to details may affect the conclusions here reached as to the real value of the constant.

First, a list is given of the determinations, made from zenith-distances measured with meridian instruments, which I have not thought it allowable to employ in finding the mean value of this constant.

BESSEL (*Fund.*, p. 123): 20″.797: and PETERS (*Recherches*): 20″.522: from BRADLEY's zenith-sector observations of γ *Draconis*, 1754–55, at Greenwich.

AUWERS (*Monatsb., Berlin Ak.*, 1869): 20″.385: from MOLYNEUX's zenith-tube observations of γ *Draconis*, 1725–27, at Kew.

AUWERS (*Ibid.*): 20″.460: from BRADLEY's zenith-sector observations of thirteen stars, 1727–29, at Wanstead.

BUSCH (Oxford Memoir, 1838); from the same series as above, Kew, 20″.250, Wanstead, 20″.205.

BRINKLEY (Phil. Trans., 1821): 20″.372: from his observations with 8-ft. vertical circle at Dublin.

RICHARDSON (Mem. R. A. S., 1828): 20″.503: Greenwich Mural Circle.

HENDERSON (Mem. R.A.S., 1839): 20″.41: from zen. dist. of *Sirius* at Cape G. H.

HENDERSON (*Ibid.*, 1842): 20″.523: from altitudes α *Centauri* at Cape G. H.

MACLEAR (*Ibid.*, 1851): 20″.531: from altitudes α *Centauri* at Cape G. H.

MACLEAR (*Ibid.*, 1852): 20″.594: from altitudes β *Centauri* at Cape G. H.

MAIN (*Ibid.*, 1859–60); 20″.335: from Greenwich Reflex Zenith-Tube, 1852–59.

DOWNING (*Mon. Not.* XLII); 20″.378: from Greenwich Reflex Zenith-Tube, 1857–75.

THACKERAY (*Mon. Not.*, LIX, p. 350): 20″.67 U.C., 20″.30 L.C., obsns. of *Polaris*, Greenwich, 1851–93.

The principal reason for the exclusion of most of the above series is that the aberration is inherently and necessarily indeterminate from them, irrespective of the quality of the observations themselves; as I have shown in various places; see *A.J.* 287, p. 178; *A.J.* 427, p. 151; *Mon. Not.*, LXII, p. 122.

Secondly, I give a list of determinations from right-ascension observations which I have not employed.

BESSEL (*Fund.*, p. 118–121); 20″.643 from *Sirius*, *Procyon*, *Vega* and *Altair*; 20″.755 from *Polaris*, BRADLEY's observations.

STRUVE (Dorpat, 1822); 20″.349; and PETERS (*Recherches*); 20″.361: differences R.A. of six circumpolars observed by STRUVE.

PETERS (*Num. Const.*, p. 56); 20″.425, from STRUVE's and PREUSS's Dorpat obsns. of *Polaris*, 1822–38.

LINDENAU (*Abhandl.*, *Berlin Ak.*, 1841); 20″.449: R.A. of *Polaris* at various observatories, 1750–1816.

NEWCOMB (*Astron. Const.*, p. 138); 20″.55: R.A. of *Polaris*, Washington, 1866–67.

NEWCOMB (*Ibid.*, p. 139); 20″.39: R.A. of circumpolars, Greenwich.

THACKERAY (*Mon. Not.*, LIX, p. 347); 20″.42: R.A., *Polaris*, Greenwich, 1851–93.

The want of control of systematic diurnal changes in the instrument is the source of want of confidence in results for aberration found from right-ascensions. Whether this should go to the extent of exclusion of such results is of course a matter of opinion: although I presume no one would be inclined to give them anything but a small weight. At any rate the only series I have retained are those with the Pulkowa Transit, as hereinafter given.

Thirdly, the determinations by means of prime-vertical transits which have not been used are the two following:

HALL (*A.J.* 169, p. 1); NEWCOMB (*A.J.* 263): Washington, 1862–67.

WANACH (*A.N.* 3092): Pulkowa, 1890–91.

The reason, in the first case, has been already referred to (*A.J.* 287, p. 178); and in the second, is found in WANACH's own remarks thereupon.

Finally, of the determinations obtained by TALCOTT's method, the only ones not included are those for Waikiki (see *A.J.* 517, 518, 519), and Bethlehem for the year 1890.

It will thus be seen that the number of the existing determinations of the aberration here excluded is twenty-four as against the forty-three employed, hereinafter given. Such a wholesale rejection of so large a proportion of observation, numerically considered, may on its face seem startling, arbitrary, and possibly indefensible; and to require for its justification a specific statement, in each instance, of the grounds therefor. It has not been done carelessly, however, but at the cost of labor of intimate examination quite as great as has been expended on those determinations which have led to positive results, and which have been actually employed in obtaining my final mean of this constant. I am sometimes inclined to suspect that some former collocations of the material for the same purpose have been perhaps too cursorily made. Whether the dimensions of the limbo which I have constructed are too ample is a question that can not be decided off-hand; and I feel justified in throwing the burden of proof upon those who are inclined to think that a higher weight than zero should have been accorded to some of the values that have here been left out of the account. I will only take space for the general remark that the constant with which we are dealing is one in which known sources of constant error exist that tend to operate in one direction, and that the means are not now at hand for applying corrections for them with any degree of confidence; so that the safer road lies in the direction of exclusion of results in which such vitiation is to be feared, rather than towards seeking to enlarge the number of available results, in the hope of balancing errors.

We now come to the determinations employed in this investigation. These are given below, classified by the methods of observation, with such details as may be useful as to the data on which they rest, so far as they can be supplied. In the last column are the weights that I have been led to assign, according to the best judgement which I could exercise of all the circumstances bearing thereupon, among which the quoted probable errors are of course only subordinate. This was naturally a most delicate and difficult part of the task, as it always is in investigations of this sort at the point where computation has to be abandoned and mere estimate resorted to of the relative preponderance which should be allowed to the various collateral circumstances to be brought to this point of view. It is here that criticism finds a wide gateway to enter, and any dispute that may arise can with difficulty be rationally settled. Consequently it is fortunate that in the present case such differences of opinion can have little effect; for it will be found from experiment that very considerable departures, within reason, may be made from the relative weights here used without perceptible influence on the final mean.

Talcott's Method.

Observer	Station	Date	Groups	Pairs	Aberr'n	p.e.	Reference	Wt.
					″	″		
Küstner	Berlin	84.5–85.5	–	241	20.611	–	A.J. 429, 130, 133	1
Marcuse	Berlin	89.1–90.7	IX	1762	.490	±0.012	A.N. 3015	3
Marcuse	Berlin	91.1–93.0	IX	2831	.507	± .011	B.d. *Int. Erdm.*, 1899	6
Ristenpart	Karlsruhe	92.8–94.6	IV	1978	.499	–	VJS. 29	5
Weinek and Gruss	Prague	89.2–92.4	IX	3062	.504	± .027	A.J. 520	6
Becker	Strassburg	91.4–94.3	VII	386	.475	± .012	VJS. 30	2
Davidson	San Francisco	91.4–92.6	VIII	6768	.555	± .021	A.J. 520	10
Gill and Fearnley	Cape G. H.	92.1–94.2	VI	621	.575	± .008	Mon. Not., LVIII	4
Fergola	Naples	93.4–94.5	IV	2271	.533	–	Naples Acad., 1897	8
Schnauder and Hecker	Potsdam	93.9–98.0	X	4400	.498	± .016	A.J. 520	10
Doberck	Hongkong	–	–	–	.477	± .040	A.N. 3504	2
Stein	Leyden	99.4–00.5	X	1590	.541	± .011	Comm. géod. Néerl.	3
Kowalski	Kasan	92.4–93.5	IX	1223	.539	± .023	A.J. 520	3
Gratchew and Trozki	Kasan	93.6–95.0	IX	1684	.522	± .022	A.J. 520	3
Gratchew	Kasan	95.1–97.5	IX	2819	.594	± .018	A.J. 520	6
Rees, Jacoby and Davis,	New York	93.3–94.5	IV	1774	.457	± .013	A.J. 401	6
Rees and Davis	New York	94.5–96.0	IV	1081	.453	± .013	A.J. 451	5
Rees and Davis	New York	96.0–98.0	IV	1839	.469	± .011	A.J. 451	7
Rees and Davis	New York	98.0–00.0	IV	1824	.470	± .011	A.J. 474	7
Doolittle	Bethlehem	92.8–94.0	XI	2796	.551	± .009	Pub. Sayre Obs. 1902	4
Doolittle	Bethlehem	94.1–95.6	IV	2690	.537	–	Pub. Sayre Obs. 1901	7
Doolittle	Philadelphia	96.8–98.6	IV	3213	.580	± .009	A.J. 453, 490	10
Doolittle	Philadelphia	98.7–99.9	IV	1919	.540	± .010	A.J. 490, U.P. Pub. II	8
Doolittle	Philadelphia	00.4–01.7	IV	2657	.561	±0.008	A.J. 509, U.P. Pub. II	9
Intern'l Service	Six stations	99.8–02.0	XII	20302	20.512	–	A.N. 3808	16
			Mean		20.523	±0.005		151

Prime Vertical Transits.

Observer	Place	Date	Stars	Obsns.	Aberr'n	p.e.	Reference	Wt.
					″			
Struve	Pulkowa	40.3–42.8	7	285	20.50 :	–	A.J. 296, improved	3
Nyrén	Pulkowa	75.5–79.0	4	246	.547	–	A.J. 297, improved	6
Nyrén	Pulkowa	79.9–82.1	24	566	.521	–	A.J. 297, improved	15
			Mean		20.525			24

Meridian Zenith-Distances.

					″	″		
Pond	Greenwich	1812–19	*Pol.*	–	20.578	±0.043	A.J. 520	1
Pond	Greenwich	1825–36	8	2629	.512	± .019	A.J. 515	3
Struve and Preuss	Dorpat	1822–38	*Pol.*	1144	.551	± .043	Lundahl, 1842	2
Peters	Pulkowa	1842–44	*Pol.*	346	.510	± .021	A.J. 287	4
Peters	Pulkowa	1842–43	7	416	.467	–	A.J. 293	1
Gyldén	Pulkowa	1863–70	*Pol.*	195	.411 } .469 }	–	A.J. 293, 298	3
Gyldén and Nyrén	Pulkowa	1863–73	10	182	.520	–	A.J. 298	1
Nyrén	Pulkowa	1871–75	*Pol.*	163	.505	–	A.J. 293	4
Becker	Strassburg	1885–90	*Pol.*	558	.577	± .033	Ann. Strassb. Obs. I	2
Hall	Ann Arbor	1898–00	*Pol.*	290	.68	± .03	A.J. 518	1
			Mean		20.514			22

Right-Ascensions.

					″			
Schweizer	Pulkowa	1842–44	*Pol.*	396	20.562	–	A.J. 444, 462, A.N. 3562	2
Wagner (E & E)	Pulkowa	1861–72	*Pol.*	439	.502	–	" " " "	2
Wagner (Chron.)	Pulkowa	1861–72	*Pol.*	429	.522	–	" " " "	2
			Mean		20.53 :			6

Prismatic Apparatus.

Observer	Place	Date	Pairs	Obsns.	Aberr'n	p.e.	Reference	
					"	"		
Loewy and Puiseux	Paris	1890–91	2	109	20.417	±0.024	Compt. Rend. CXII	1
Comstock and Flint	Madison	1890–92	20	752 :	.413 / .499	±0.010	Publ. Washb. Obs. IX	4
				Mean	20.48 :			5

The double result quoted for the last (Comstock's) determination depends on the way the peculiar systematic personal difference is treated. An independent discussion of these observations has led me to the value 20″.49. The double result quoted for the *Polaris* vertical-circle observations of *Polaris*, 1863–70, arises from the curious contradiction between Nyrén's and my solutions, one of which I think must be affected with some purely numerical error.

Collecting and combining the results of the foregoing tables we have the following summary:

	No. Det.	Aberration	Wt.
		"	
Talcott's method,	25	20.523	151
Prime-vertical transits,	3	.525	24
Meridian zenith-distances,	10	.514	22
Right-ascensions,	3	.53 :	6
Prismatic apparatus,	2	.48 :	5
General mean,	43	20.521	208
Probable error,		± .005	

The brute mean of the forty-three individual results is 20″.523, differing only 0″.002 from the above weighted mean; the brute mean of the twenty-five results by Talcott's Method is 20″.522, differing but 0.001 from the above weighted mean; and the brute mean of the ten results by Meridian Zenith-Distances is 20″.534, which is 0″.020 greater than the weighted mean.

Various experiments with the weights varied within reasonable limits all give means between 20″.522 and 20″.526. In short, after considering the data in various ways, the impression on my own mind is strong that the real value of this much disputed constant is likely to be found near or slightly above 20″.52.

The above weighted mean, 20″.521, which may be regarded as the result of this investigation, corresponds to the solar parallax 8″.781.

It is interesting and apposite to remark that the discordance which formerly appeared to exist between the values of the aberration afforded by the three principal methods on which we must rely for its determination — namely, Talcott's Method, Prime Vertical Transits and Meridian Zenith-Distances — is now dissipated; and that the reasonably distrusted method of Right-Ascensions may, in at least a plausible way, be also brought into harmony with the others.

Nor can I neglect to emphasize the importance of the stride forward in our knowledge of this constant due to the suggestion and development by Küstner of his method, which has enriched our astronomy with a means of investigating the aberration less susceptible than any other to the disturbing influence of systematic error arising from known or imaginable causes, and which has been so prolific of result in its application.

From the point of view of the principles above adopted in the use of the data this investigation should stop here, and the task of demonstrating that improvement can be expected by utilizing some or all of the material here excluded should be left to those who would advocate that view. Nevertheless it seems appropriate and desirable to show how much the final result would have been affected by incorporating the determinations here excluded. I therefore give in the following tabulation, which is arranged in the same manner as the one already presented, the results of the twenty-four excluded series. They are used as given by the authors (rounded only to the nearest hundredth) without any attempt to apply corrections for variations of latitude, with which many of them are necessarily affected. An attempt has been made to assign relative weights such as I suppose would not be materially gainsaid by those who might be inclined to favor the employment of these determinations in their definitive mean. We thus have:

Talcott's Method.

	Aberr'n	Wt.
	"	
Waikiki,	20.43	½
Bethlehem, 1890,	.45	½
Mean,	20.44	1

Prime Vertical Transits.

Washington, 1862–67,	20.46	1
Wanach,	.40	1
Mean,	20.43	2

Meridian Zenith-Distances.

Peters (Bradley, 1754–55),	20.52	¼
Auwers (Kew),	.38	¼
Auwers (Wanstead),	.46	¼
Busch (same),	–	0
Brinkley,	.37	¼
Richardson,	.50	1
Henderson (*Sirius*),	.41	0
Henderson (*α Cent.*),	.52	¼
Maclear (*α Cent.*),	.53	¼
Maclear (*β Cent.*),	.59	¼
Main,	.34	¼
Downing,	.38	1
Thackeray,	.49	1
Mean,	20.46	5

RIGHT-ASCENSIONS.

	Aberr'n	Wt.
Bessel (Bradley),	20.70	¼
Peters (Dorpat, 6 st.),	.36	½
Peters (Dorpat, *Pol.*),	.42	1
Lindenau,	.45	¼
Newcomb (Wash.),	.55	1
Newcomb (Greenw.),	.39	1
Thackeray,	.42	1
Mean,	20.45	5

Whence we get the following summary of these excluded determinations:

	No. Det.	Aberr'n	Wt.
Talcott's Method,	2	20.44	1
Prime-vertical transits,	2	.43	2
Meridian Zenith-Distances,	13	.46	5
Right-ascensions,	7	.45	5
General mean,	24	20.450	13
The brute mean of the 24 values is		20.457	

Combining this result with the one previously found from forty-three accepted determinations we find:

	No. Det.	Aberr'n	Wt.
Mean from accepted values,	43	20.521	208
Mean from rejected values,	24	.450	13
General mean,	67	20.517	221

It thus appears that without excluding any of the data we should have a value (20″.517) only 0″.04 smaller than the one (20″.521) which has been derived as the definitive result of this investigation, by the exercise of a critical choice which it seems to me is demanded by the circumstances for the purpose of arriving at the most acceptable result. And here again it will be manifest upon trial that wide differences of opinion as to weights can be allowed full play without significant influence on the conclusion.

From the best point of view I can compass, it seems to me that we are driven by the facts either to 20″.52 if a rounded decimal is preferred for a conventional value of this constant, or to something slightly above it, if for any reason it is deemed that any slight practical advantage, or a possible approach to accuracy, is gained by splitting the hundredth.

The reduction to such a value of the aberration-constant can be effected by the addition of 0.0016 to the values of log C and log D, or of log h and log i, in the ephemerides where STRUVE's constant (20″.445) is employed in the tables for reduction to apparent place; or of 0.0011 in those where the value of the Paris Conference of 1896 (20″.47) is the basis.

NOTES ON SOME RECENTLY DISCOVERED VARIABLE STARS.

By A. STANLEY WILLIAMS.

The observations upon which the following notes are based were chiefly made with a 6½-inch reflector, though occasionally a 2¾-inch refractor was used. The results are necessarily at present of a somewhat provisional character, though as the observations have been carefully reduced it is not probable that any material alteration will have to be made in the stated times of maxima and minima. Light-scales were formed for most of the variables, either by means of the comparisons between the variable and the comparison-stars, or by means of sequences of steps specially made for that purpose. It should be added that cloud and fog have been abnormally prevalent during the past year, rendering it difficult to secure satisfactory and uninterrupted series of observations of *Algol*-type or very rapid variables.

562. V *Andromedae.*

The following are approximate photographic magnitudes of this star. 1899 Nov. 10, not visible, $< 11^{M}.5$; 1900 Dec. 19, not visible, $<12^{M}$; Dec. 21, not visible, $<12^{M}$; 1901 Jan. 15, not visible, $<12^{M}$; Dec. 10, 10^{M}; Dec. 18, 10^{M}; 1902 Jan. 30, $11\frac{1}{2}^{M}$.

1205. Y *Persei.*

Observations on 45 nights between 1901 Mar. 12 and 1902 Dec. 2 yield the following maxima and minima:

	Date	J.D.	Mag.
Maximum	1901 April 15	2415490	8.8
Minimum	Aug. 28	5625	9.6
Maximum	Dec. 21	5740	8.7
Minimum	1902 April 23	5863	9.8
Maximum	Sept. 12	6005	8.3

Dr. HARTWIG in 1901 found the period to be 236 days, but one slightly longer than this (251 days) better satisfies the above observations. The light variations are, however, by no means quite regular, and there are considerable differences in the form of the light-curve at different maxima. Thus, the curves of the first two maxima are rather flat, particularly that of the second one, whilst that of the third is comparatively sharply accentuated. The magnitudes given above are practically according to the scale of HAGEN's Second Chart and Catalogue for observing *Nova Persei*, but it should be mentioned that red stars usually appear decidedly fainter to me than they do to most observers, particularly as regards observations made

with the 2¾-inch refractor. The first two maxima and the two minima were observed with this instrument, and the third maximum with the 6½-inch reflector. Several observers have published observations made about the time of the first maximum, according to which the star must then have been about $8^M.3$. That is half a magnitude brighter than my observations make it. On four nights in 1901 both photographic and visual observations were made, the differences, visual — photographic, being −1.8, −1.8, −1.5 and −1.4. The mean difference is $-1^M.6$, and the photographic observations published in the *A.N.* 3698 agree very well with the later visual ones made with the 2¾-inch refractor when corrected by this amount; though in order to make them comparable with the reflector observations, and those of most other observers, the correction should be a full two magnitudes.

6685. *Y Lyrae.*

Four good series of observations of this rapid variable were secured in 1902. Below will be found the observed heliocentric Greenwich mean times of maximum, and the like observed times of T_0. The times T_0 are the times when the variable in its rapid rise from minimum attains to equality with the comparison-star 1 ($12^M.1$)*. The computed times according to the elements in the *Monthly Notices*, Vol. 62, p. 208, have been added for the purpose of comparison, together with the resulting residuals. The latter are not large, and it hardly seems necessary at present to attempt any revision of the elements.

Date	Computed T_0	Observed T_0	C — O
	h m	h m	M
1902 June 2	11 5.9	11 15.7	− 9.8
Aug. 22	9 29.0	9 32.2	− 3.2†
Sept. 7	11 32.8	11 50.6	−17.8
8	11 40.6	11 53.5	−12.9

Date	Computed maximum	Observed maximum	C — O
	h m	h m	M
1902 June 2	12 5.9	12 9.7	− 3.8
Aug. 22	10 29.0	10 44.2	−15.2
Sept. 7	12 32.8	12 44.6	−11.8
8	12 40.6	12 52.5	−11.9

6816. *Z Lyrae.*

In 1901 observations were made on 29 nights between May 21 and Oct. 6, and these indicate a pretty well defined maximum for July 1, mag. = 9.4 (equal to DM. +34°3385). Hartwig, from his own observations, makes it June 20 (see *A.N.* 3741, col. 370). In 1902 the star's brightness decreased from $11^M.2$ on May 27 to invisibility in a 6½-inch reflector (or below about 13^M) after July 25. The star was re-observed on Oct. 26, mag. = 12.3. By means of the light-curve of 1901 the date of maximum may be fixed for 1902 April 7 ±. Some photographic observations of 1900 and 1899 were published in the *A.N.* 3671. Those for 1900, with the help of the visual light-curve of 1901, fix the time of maximum pretty exactly for 1900 Sept. 2. The last two observations of 1899, with the help of the visual light-curve of 1901 and the photographic light-curve of 1900, indicate a maximum for 1899 Nov. 21 ±, on the assumption that this occurred between two observations*. Taking the mean of Hartwig's and my own determinations for 1901, we have the following four observed maxima:

Date	J.D.	C — O
		d
1899 Nov. 21 ±	241 4980 ±	+2
1900 Sept. 2	5265	+7
1901 June 26	5562	0
1902 Apr. 17 ±	5847 ±	+5

These yield the following approximate elements of variation:

$$\text{Maximum} = \text{J.D. } 241\,5562 + 290^d\ E.$$

The last column above contains the residuals according to these elements.

6827. *RT Lyrae.*

The star rose from $12^M.5$ on 1902 May 24 to a sharply defined maximum ($9^M.8$) on July 22, and then almost immediately declined rapidly and steadily to $12^M.7$ on Sept. 25. Observations were made on 16 nights between the above limiting dates. Some earlier photographic observations were published in the *A.N.* 3783. Those made in 1901 indicate a maximum about Nov. 11, but there is a good deal of uncertainty as to the exact date. The interval between the two maxima is 253 days; and assuming this to be period, maxima should have occurred on 1901 Mar. 3, 1900 June 23 and 1899 Oct. 13. The invisibility of the star on the photographs taken between 1900 Sept. 2 and Nov. 22 is in accordance with the first two maxima, but its faintness ($12^M.08$) on the plate of 1899 Sept. 28 shows that this period cannot be quite correct, since the computed maximum occurs only 15 days later. It is uncertain at present whether the period of 253 days is a little too short,

* See the diagram in the *Monthly Notices*, Vol. 62, p. 201, and in *Popular Astronomy*, 1902, p. 216. Dr. Hartwig has published the times of several maxima of this star observed by him in 1901 in the *Vierteljahrsschrift der Astr. Gesell.*, *Jahrgang* 36, p. 268. They were received too late for inclusion in the writer's discussion of the observations of this star.

† On August 22 the comparison-star 1 was very faint, and sometimes even invisible owing to moonlight, so that the time of T could not be satisfactorily observed.

* The first two observations of 1899 (Sept. 2 and 22) should be struck out. The plate of Sept. 2 is a trial one before the instrument was in adjustment, and the star images being drawn out into long trails it is difficult to identify the fainter stars, and it is doubtful if a faint trace is really due to this star. The plate of Sept. 22 is a poor one, and here also it is doubtful whether a faint mark is really due to the star.

or a little too long, though the probability is that it is too long. The next maximum should occur about 1903 Apr. 1.

6895. *RU Lyrae.*

This star rose from $12^M.7$ on 1902 May 27 to a sharply defined maximum (mag. = 9.9) on Aug. 21, and by Sept. 25 it had declined to $11^M.3$. Observations were made on 14 nights between the above limiting dates. Some earlier photographic observations were published in the *A.N.* 3796, and from these it was inferred that the period is almost exactly equal to a year, and that the next maximum would probably occur in August of the present year, as has actually happened. The period cannot be stated with greater exactness at present.

6915. *RV Lyrae.*

The following minimum of this *Algol*-variable has been observed and is in addition to those already published in the *A.N.* 3811.

1902 Sept. 3 $11^h 42^m$

Twelve observations were obtained between $8^h 44^m$ and $14^h 18^m$, but cloud interfered a good deal after 11^h, though the above result is on the whole a fairly satisfactory one. The observation is not reduced to the sun. The computed time from the elements in the *A.N.* 3811 is $11^h 36^m$.

6927. *U Sagittae.*

The undermentioned minima of this *Algol*-star have been observed, but the times are somewhat provisional, as owing to the unfavorable weather the observations are not sufficient to give a very satisfactory light curve. The heliocentric times of minimum according to EBELL's ephemeris in *A.N.* 3771 have been added for comparison with the differences O — C. The observed times are not, however, reduced to the sun.

Epoch	Date	Observed h m	Computed h m	O — C m
87	1902 Aug. 22	9 3	9 28	−25
90	Sept. 1	12 33	12 53	−20
92	8	6 56	7 9	−13
95	18	10 20	10 34	−14

Notes. E. 87, 12 observations between $8^h 29^m$ and $11^h 14^m$. E. 90, 9 observations between $8^h 27^m$ and $11^h 3^m$. E. 92, 7 observations between $8^h 11^m$ and $10^h 23^m$. E. 95, 11 observations between $8^h 43^m$ and $11^h 52^m$.

The star remains stationary at minimum for about $2^h 10^m$. Normally the variable is a white star, but during the stationary period at minimum, and for a few minutes before and after, it is of a decided reddish color.

7019. *TY Cygni.*

In 1901 the star rose from 12^M on Aug. 6 to a sharply lefined maximum on Nov. 11 (mag. = 9½). In 1902 the star rose from $12\frac{1}{2}^M$ on July 7 to a well defined maximum on Oct. 27 (mag. = 9). Three earlier photographic observations were published in the *A.N.* 3687. With the help of the visual light-curve these observations indicate a maximum for 1900 Nov. 22. From these three maxima the following elements have been derived:

$$\text{Maximum} = \text{J.D. } 241\,6051 + 352^d\ E$$

The star is not visible on a photograph taken by Prof. MAX WOLF on the nights of Sept. 25 and 30, 1891 (exp. = 12^h), and it cannot therefore have been photographically so bright as 14^M. According to the elements the star should have been almost exactly at minimum then, so that the invisibility of the star on this photograph is just what might be expected.

7318. *UW Cygni.*

The observations of the undermentioned minima of this *Algol*-variable were all more or less hindered by cloud or other causes. The last minimum is the only one at all properly observed during both the decreasing and increasing phases. The times, which are geocentric Greenwich mean times, have been derived by means of a provisional light-curve based on three minima observed last year.

	Date	Minimum h m	Quality
(a)	1902 July 18	10 12	Fair.
(b)	Aug. 25	9 22.7	Unsatisfactory.
(c)	Sept. 1	6 50	Somewhat approximate.
(d)	18	12 56	Approximate.
(e)	25	10 32	Pretty good.

Notes. (a) 15 observations between $10^h 48^m$ and $14^h 35^m$. (b) 9 observations between $8^h 18^m$ and $13^h 15^m$, much hindered by cloud. (c) 10 observations between $8^h 18^m$ and $13^h 15^m$. (d) 9 observations between $8^h 50^m$ and $11^h 21^m$. The decrease could not be followed further owing to the brightness of the moon. (e) 17 observations between $8^h 31^m$ and $12^h 5^m$; hazy between $10^h 27^m$ and $11^h 50^m$.

7505. *UX Cygni.*

This star has been invisible in the 6½-inch reflector, and consequently fainter than about $12\frac{1}{2}^M$, between 1902 July 11 and Nov. 17. Some photographic observations were published in the *A.N.* 3752, and to these may be added the following: — 1901 Nov. 15, mag. = 10.98. The observations of 1901 indicate a fairly definite maximum for 1901 Oct. 23; photographic mag. = 9.7. It is difficult at present to come to a satisfactory conclusion respecting the period of variation, though this is evidently long and the range of variation considerable.

7571*a*. *TW Cygni.*

Observations on 22 nights in 1901 indicate a well defined though somewhat flat maximum on Sept. 1. The star rose from $12^M.0$ on May 21 to $8^M.8$ at maximum, and by Nov. 3 had declined to $11^M.0$. In 1902 observations on 9 nights show a very definite maximum for Aug. 15, mag. = 9.6. Dr. HARTWIG, in 1901, suggested a period of 8½

months (see *A.N.* 3741), but this is evidently, however, somewhat longer. The photographic magnitude is 9.85 on two photographs taken on Oct. 6 and 9, 1899. Comparison of the brightness of the star upon three plates taken on three separate nights with corresponding visual observations, gives −1.5, −1.6 and −1.1 as the correction to the photographic magnitudes in order to reduce them to the visual ones. Means = $-1^M.4$. Applying this correction to the above mentioned observations of 1899, we get $8^M.15$ as the photographic reduced to the visual brightness, so that the star was probably at or very near a maximum on 1899 Oct. 7. The following elements satisfactorily satisfy the observations.

Maximum = J.D. 241 5977 + 347^d E.

It is noteworthy that there seems to have been a progressive diminution in the brightness of the star at maximum.

8610. *Z Pegasi.*

The following are the approximate photographic magnitudes of this variable. 1899 Nov. 6 and 10 not visible, $<12^M$; Nov. 29, $<11^M$; 1900 Nov. 18, $<12^M$; Nov. 22, $<11\frac{1}{2}^M$; Dec. 13, $<10\frac{1}{2}^M$; Dec. 15, $11\frac{1}{2}^M$; 1901 Jan. 14, 10^M.

20 Hove Park Villas, Hove, 1902 Dec. 20.

ON THE VARIABILITY OF DM. −1°1182.

By ZACCHEUS DANIEL.

The star DM. −1°1182, $9^M.3$, occurs in the list of suspected variables at the end of Chandler's *Third Catalogue of Variable Stars* (*A.J.* 379) where it has the number (2235), and the observed range is given as 8^M to $9^M.3$.

This star was observed nine times at Harvard College Observatory with the Meridian Photometer during the years 1886 to 1888. Of these observations, seven by Professor E. C. Pickering, from 1886 Feb. 16 to 1888 Feb. 28, yield a mean magnitude of 8.96. The mean deviation of each observation from this result is $0^M.19$, and the observed range is $0^M.8$. One observation by Pickering on 1886 Jan. 10, and one by Wendell on 1886 Jan. 26, which are not included in the mean, both give a residual of $-0^M.8$ from $8^M.96$. This makes a total observed range of $1^M.2$, or from $8^M.16$ to $9^M.36$.

These results are extracted from *H.C.O. Annals*, Volumes 23 and 24, where the star is designated M.P. 821.

I observed this star with the 10-inch Clark equatorial on four dates in 1901, from Jan. 19 to April 16, and on fourteen dates in 1902, from Jan. 3 to May 8. The mean result of the eighteen observations is $9^M.16$. The mean deviation of each observation from this value is $0^M.05$. The brightest observation is $9^M.07$; the faintest is $9^M.31$, which make an observed range of $0^M.24$.

All my observations agree in affording no evidence of variability. They are very accordant, and show no marked fluctuation.

The following are my comparison-stars. The magnitudes are the result of thirty-eight separate comparisons, and are adjusted closely to the DM. scale.

Star	Desig.	DM.	D	Grades
		M	M	
a =	DM. −1°1160	8.3	8.33	0.0
b =	−1°1170	9.3	8.93	6.0
c =	−1°1185	9.0	9.21	8.8
d =	−1°1177	9.5	9.61	12.8

Bucknell University, Lewisburg, Penna., 1902 Dec. 27.

OBSERVATION OF COMET *d* 1902 (*GIACOBINI*),

By E. E. BARNARD.

1902 90° time	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ	
h m s			m s	′ ″	h m s	° ′	s	s
Dec. 30 11 11 41	1	3	. . . .	−4 22.5	. . . .	+3 36.0	. .	0.747
11 26 28	1	18	+0 23.14	. . . .	7 3 33.1	. . . .	n9.049	. .
11 36 51	1	3	. . . .	−4 6.0	. . . .	+3 36.3	. .	0.745

*	α 1902	δ 1902	Red. to app. place α	Red. to app. place δ	Authority
	h m s	° ′	s	″	
1	7 3 5.1	+3 40.7	+5.07	−16.5	DM. +3°1563*

The position was measured with the large telescope. The comet was about 12^M, with elongated brightening at middle. There was a slight brushing out of the nebulosity following.

This object was observed on Dec. 3, with the 12-inch, but as there is no micrometer to that instrument, an accurate position could not be obtained. No other opportunity has offered to observe it with the large telescope. The catalogues here do not contain an accurate place of the comparison-star.*

* This star is Albany A.G. 2635; $\alpha = 7^h\,3^m\,13^s.20$, $\delta = +3°\,40'\,39''.1$ (1902.0). — Ed.

Yerkes Observatory, Williams Bay, Wis., 1903 June 3.

CONTENTS.

Published in Boston, Mass., semi-monthly, by S. C. Chandler. Address, Cambridge, Mass. Associate Editors, Asaph Hall and Lewis Boss.
Price, $5.00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. Entered at the Post Office, Boston, Mass., as second-class matter. *Closed Jan. 7.*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 530.

VOL. XXIII. BOSTON, 1903 JANUARY 31. NO. 2

OBSERVATIONS OF VARIABLE STARS OF LONG PERIOD.

BY W. C. BRENKE.

The following observations were made by the method of sequences with the aid of the twelve-inch refractor of the University of Illinois Observatory. The comparison-stars have been those of HAGEN'S "*Atlas Stellarum Variabilium*," except a few faint ones which were used during times of minima, and which have been connected by sequence of steps with HAGEN'S stars, making the scale of magnitudes uniformly that of the "*Atlas*." The magnitudes are given to two decimal places whenever two or more accordant observations were secured, while the first decimal only is given in cases of single observations, or of the mean of several estimates sufficiently discordant to make this decimal somewhat uncertain. The dates of maxima and minima were determined from curves, and the theoretical times have been added for comparison. Brackets are used to indicate those dates which are not well determined by the observations. The uncertainty in these cases is about $\pm 10^d$.

Fractions of a day in the observed dates indicate the approximate G.M.T. of the observations.

S Coronae.

Date	Mag.	Date	Mag.
1901 Apr. 8.7	8.0	1901 July 1.6	9.30
15.7	8.0	8.6	9.40
18.6	7.8	12.6	9.7
19.7	8.15	18.7	9.6
25.6	8.17	19.7	9.6
26.7	8.10	25.6	10.0
May 1.6	8.00	Aug. 5.6	10.1
4.6	8.33	11.7	11.2
13.7	8.32	Sept. 13.7	11.2
14.6	8.27	21.6	11.10
20.7	8.57	Oct. 2.6	12.03
28.6	8.37	4.6	12.27
30.6	8.50	14.6	12.02
June 6.6	8.68	Nov. 1.6	12.5
13.7	8.80	1902 Mar. 5.9	8.5
17.7	8.93	13.7	7.2
26.7	9.05	Apr. 2.7	8.1

R Herculis.

Date	Mag.	Date	Mag.
1901 May 30.7	9.86	1901 Sept. 13.7	invisible
June 6.6	10.05	21.6	..
13.7	10.67	Oct. 2.6	..
17.7	10.98	4.6	..
26.7	11.20	11.6	..
July 1.6	11.43	Nov. 1.6	..
8.6	11.60	1902 Feb. 17.9	8.65
12.6	11.57	Mar. 5.9	8.55
19.7	12.03	13.8	8.77
25.6	*invisible	Apr. 2.7	9.5
Aug. 5.6	12.20	13.7	9.75
15.6	12.6	June 5.7	11.5

* Bright moon.

U Herculis.

Date	Mag.	Date	Mag.
1901 May 28.7	10.5	1901 Sept. 13.7	10.07
30.7	10.5	21.6	10.05
June 6.7	10.15	Oct. 2.6	10.03
13.7	10.50	5.6	9.70
17.7	10.70	11.6	9.76
26.7	10.60	15.6	9.80
July 1.6	10.70	Nov. 1.6	9.10
8.6	11.03	1902 Feb. 17.9	7.55
12.6	10.62	Mar. 5.9	7.6
19.7	11.03	13.8	8.3
25.6	10.67	Apr. 2.9	8.1
Aug. 5.6	11.33	13.7	9.2
15.6	13.6		

W Herculis.

Date	Mag.	Date	Mag.
1901 May 28.7	12.02	1901 Sept. 13.7	8.05
30.7	11.9	21.6	8.0
June 6.7	12.06	Oct. 2.6	7.77
13.7	12.10	5.6	7.55
17.7	11.78	11.6	8.1
26.7	12.0	15.6	8.03
July 1.6	11.65	Nov. 1.6	8.37
8.6	11.25	1902 Feb. 17.9	12.2
12.6	10.95	Mar. 5.9	12.17
19.7	11.06	13.8	12.15
25.6	11.10	Apr. 2.9	11.9
Aug. 5.6	10.08	13.7	11.5
15.6	9.04	June 3.7	8.6

V Coronae.

Date	Mag.	Date	Mag.
1901 Apr. 8.7	8.67	1901 July 1.6	8.80
15.7	8.70	8.6	8.93
18.6	8.87	12.6	9.12
19.7	8.80	18.7	9.23
21.7	8.55	19.7	9.32
25.6	8.63	25.6	9.40
26.7	8.50	Aug. 5.6	9.70
May 1.6	8.70	11.7	10.30
4.6	8.56	Sept. 13.7	10.17
13.7	8.78	21.6	10.17
14.6	8.93	Oct. 2.6	11.00
20.7	8.63	4.6	11.00
28.6	8.63	14.6	11.34
30.6	8.68	Nov. 1.6	12.30
June 6.6	8.80	1902 Feb. 18.9	8.13
13.7	8.83	Mar. 5.9	8.35
17.7	8.86	13.7	8.35
26.7	8.97	Apr. 2.7	8.56

S Herculis.

Date	Mag.	Date	Mag.
1901 May 28.7	7.80	1901 Aug. 15.6	9.00
30.7	8.15	Sept. 13.7	10.05
June 6.7	8.37	21.6	10.08
13.7	8.20	Oct. 2.6	10.4
17.7	8.37	5.6	10.70
26.7	8.40	14.6	11.23
July 1.6	8.70	15.6	11.53
8.6	8.57	Nov. 1.6	12.20
12.6	8.63	1902 Feb. 17.9	8.60
19.7	8.65	Mar. 5.9	8.37
25.6	8.55	13.8	8.50
Aug. 5.6	8.70	Apr. 2.9	8.4

T Herculis.

Date	Mag.	Date	Mag.
1901 May 30.7	12.0	1901 Sept. 21.6	8.76
June 6.7	12.0	Oct. 2.6	9.18
13.7	11.9	5.6	9.25
17.7	11.13	14.6	9.92
26.7	10.50	15.6	9.92
July 1.6	10.13	Nov. 1.6	11.16
8.7	9.57	18.7	11.95
12.6	9.33	29.7	11.25
19.6	9.17	1902 Feb. 17.9	8.17
25.6	8.62	Mar. 5.9	8.45
Aug. 5.6	8.23	13.8	9.42
15.6	8.05	Apr. 3.9	10.3
Sept. 13.7	8.17	May 29.7	9.8

MAXIMA AND MINIMA.

Ch. No.	Star	Ph.	Mag.	Date Cal. [Ob.]	Julian [Ob.]	Comp'd
5504	*S Coronae*	Min.	<12.5	1901 Dec. 1]	5720]	5685
5675	*V Coronae*	Min.	<12	1901 Nov. 21]	5710]	5664
5770	*R Herculis*	Max.	8.5	1902 Feb. 27	5808	5798
5889	*U Herculis*	Min.	<13.5	1901 Aug. 18	5615	5689
5889	*U Herculis*	Max.	7.5	1902 Feb. 19	5800	5856
5950	*W Herculis*	Max.	7.5	1901 Oct. 5	5663	5669
5950	*W Herculis*	Min.	12.2	1902 Mar. 1]	5810]	5810
6044	*S Herculis*	Min.	<12	1901 Dec. 11]	5730]	5734
6512	*T Herculis*	Min.	12.0	1901 June 4	5540	5517
6512	*T Herculis*	Max.	8.0	1901 Aug. 28	5622	5595
6512	*T Herculis*	Min.	12.0	1901 Nov. 16	5705	5681
6512	*T Herculis*	Max.	8.0	1902 Feb. 9	5785	5759

University of Illinois Observatory, Urbana, Ill., 1902 December.

NOTE ON THE SECULAR PERTURBATIONS OF THE PLANETS.

By A. HALL.

About thirty years ago, at the request of Professors COFFIN and NEWCOMB, the Smithsonian examiners, I made some computations to test the results of STOCKWELL's memoir on the secular perturbations of the principal planets of our solar system. The numerical results proved to be very accurately deduced from the assumed data. STOCKWELL made calculations to show the effect of changes in the masses of the planets, and his memoir contains formulas for this purpose. It appeared to me that this might be done more directly, and LEVERRIER has pointed out a method. The arrangement of the planets is such that the equation of the eighth degree, whose roots furnish the coefficients, may, for the first approximation, be divided into two equations of the fourth degree; one of these equations belonging to the four interior planets, and the other to the four exterior planets. The general biquadratic can be written,

$$g^4 + p_1\, g^3 + p_2\, g^2 + p_3\, g + p_4 = 0.$$

The values of the coefficients in terms of the roots are,

$$\begin{aligned}
p_1 &= -g - g_1 - g_2 - g_3 \\
p_2 &= +gg_1 + gg_2 + gg_3 + g_1g_2 + g_1g_3 + g_2g_3 \\
p_3 &= -gg_1g_2 - gg_1g_3 - gg_2g_3 - g_1g_2g_3 \\
p_4 &= +gg_1g_2g_3
\end{aligned}$$

Differentiating these equations, and eliminating all the variations of g but one, be have,

$$\delta g = -\frac{\delta p_1 \cdot g^3 + \delta p_2 \cdot g^2 + \delta p_1 \cdot g + \delta p_4}{(g - g_1)(g - g_2)(g - g_3)}$$

This is LEVERRIER's result, but it does not appear to be a good form for computing, since small divisors may enter the denominator. STOCKWELL's equations seem to be safer, and to test them I have computed the roots from the masses adopted by G. W. HILL in his work on secular perturbations, and have compared these values with the roots found by HILL for the eccentricities and perihelia.

	Hill's $\frac{1}{m}$	log μ	Corr. to S.	Root g	Hill's value
			"	"	"
Mercury	10500000	9.729617n	+0.034496	+ 5.198299	+ 5.197704
Venus	408000	8.644612n	+0.040516	7.288943	7.283189
Earth	328000	9.093603	+0.302467	17.316840	17.322014
Mars	3093500	9.125356n	+0.219289	18.003715	18.003942
Jupiter	1047.355	6.699237	+0.020918	0.637602	0.634609
Saturn	3501.6	$-\infty$	−0.013135	2.714224	2.707114
Uranus	22869	8.919531	+0.014177	3.730781	3.722375
Neptune	19314	8.441669n	+0.028722	+22.189569	+22.418997

The form of STOCKWELL's mass was assumed to be $\frac{1+\mu}{m}$, and by comparing with HILL's values log μ was found.

The agreement is good except in the last case, where some error of calculation may exist. The solution may be completed by a method like that of NEWTON. We may, I think, consider the equation of the eighth degree as disposed of.

Goshen, Conn., 1902 Nov. 26.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A 4½-INCH REFRACTOR.

BY A. W. QUIMBY.

1902	Time	New Grs.	Total Grs.	Total Spots.	Fac. Grs.	Def.
July 12	4	1	1	1	1	good
13	5	–	–	–	1	fair
Aug. 16	4	1	1	2	1	fair
17	8	–	1	1	–	fair
18	9	–	1	1	–	fair
19	8	–	1	1	–	good
Sept. 13	8	–	–	–	1	fair
14	8	–	–	–	1	fair
17	8	–	–	–	1	fair
19	10	1	1	8	–	good
20	11	–	1	4	–	poor
22	9	2	3	10	2	good
23	8	–	3	10	3	good
24	7		2	11	2	poor
27	3	–	1	6	–	fair
28	5	–	1	4		poor
29	7		1	4		poor
30	9		1	3	–	poor
Oct. 1	11	–	1	2	1	poor
2	8	–	1	2	1	fair
3	8		1	1	1	fair
6	8	1	1	32	1	fair
Oct. 7	8	1	2	28	1	fair
8	8	–	2	24	–	poor
9	8	–	2	16		poor
10	9		1	11		fair
12	9	–	1	1	–	poor
13	7	–	1	1		poor
14	8	–	1	1	–	poor
15	8	–	1	2	1	fair
16	8	–	1	1	1	fair
17	10	–				poor
18	9					fair
19	2				1	poor
20	9		–		1	poor
21	8	1	1	1	5	fair
22	8	–			1	fair
23	9	1	1	9		fair
24	10	1	2	11	1	fair
25	8		2	30	1	poor
26	8		2	17		poor
27	3		2	18	1	fair
28	2		2	12	1	poor
29	8		2	4	1	poor
Oct. 30	8		2	2	2	poor
31	9				2	poor
Nov. 3	8				1	fair
4	9				1	fair
9	8		–		1	fair
10	8		–		1	fair
14	9	1	1	2		poor
15	10	2	3	11	3	fair
16	8	–	3	4	3	poor
17	8		1	2		poor
19	11		1	20		poor
20	8		1	26		fair
21	8		1	33		fair
22	9		1	24		fair
23	8		1	22		fair
24	10		1	16		poor
Dec. 9	3				1	good
17	8	1	1	2	1	fair
18	9	–	1	1	1	fair
20	12				1	good
22	8				1	good

Observations were made on 105 other days of the semester, beginning July 1, when neither spots nor faculae were seen. The sun was invisible on July 30; Sept. 21, 25, 26; Oct. 4, 5, 11; Nov. 11, 18, 25, 26; Dec. 11, 21, 29.

THE MISSING DURCHMUSTERUNG STAR +30° 583,

BY ZACCHEUS DANIEL.

In *A.J.* 430, page 179, Professor MARY W. WHITNEY states that no star was seen in the position for DM. +30° 583 on either 1897 Nov. 27, or 1898 Feb. 28.

The *Bonner Sternverzeichniss* gives the position,

$$\alpha = 3^h\ 43^m\ 12^s.7 \quad ; \quad \delta = +30^\circ\ 4'.6\ (1855)\ ;$$

and the magnitude, 9.5.

With a 4-inch refractor, I looked for this star on nine dates, from 1898 April 21, to 1899 April 5, inclusive, but I never could see any star brighter than the twelfth magnitude near the given position, although all other DM. stars near were always seen and identified. However, on 1898 Sept. 12, I saw a star of about the twelfth magnitude near the place. I also examined the region with the 10-inch refractor on nine dates, from 1900 Oct. 24, to 1902 Nov. 19, inclusive, with the same result. In good seeing, the 10-inch telescope always showed the twelfth-magnitude star and several fainter stars near it.

Bucknell University, Lewisburg, Penna., 1902 Dec. 22.

QUESTIONS RELATING TO STELLAR PARALLAX, ABERRATION AND KIMURA'S PHENOMENON.

BY S. C. CHANDLER.

1. In view of the narrow range within which it would appear, from *A.J.* 529, that we can now define the constant of aberration, the effect of stellar parallax on its determination by the KÜSTNER-TALCOTT method ought not to go unexamined. It has hitherto been neglected both by myself and others on the presumption that it is unimportant.

To obtain a convenient and sufficiently accurate formula for this correction let us take the expression for parallax in declination,

$$-\pi \sin\odot (\cos\epsilon \sin\delta \sin\alpha - \sin\epsilon \cos\delta) - \pi \cos\odot \sin\delta \cos\alpha$$

in which the earth-sun radius is treated as constant, and pursue an analogous transformation to that adopted for the aberration in *A.J.* 517 and 520. For a pair of stars of equal zenith-distance, north and south, in the same right-ascension, this becomes

$$(1) \qquad \pi \cos\zeta\, (m \sin\odot - n \sin\odot \sin\alpha - \sin q \cos\odot \cos\alpha)$$

where $m = \sin\epsilon \cos q \quad . \quad n = \cos\epsilon \sin q$

Introducing the apparent solar time of observation, $T = \alpha - \odot$, we get

$$(2) \qquad \pi \cos\zeta\, [m \sin\odot - n \cos T - \sin q\, (\cos\epsilon - 1)\, (\tfrac{1}{2} \sin 2\odot \sin T - \cos^2\odot \cos T)]$$

which is of the same form as eq. (2) for aberration, *A.J.* 520.

For our present purpose the term in $\sin q\,(\cos\epsilon - 1)$ is negligible. Using the subscripts 1 and 2 to designate evening and morning groups of observations we have, as the corrections of the latitude for error of assumed aberration and for parallax,

$$(3) \qquad \begin{aligned} &dk\,.\cos\zeta_1 (n \sin T_1 - m \cos\odot) + \pi \cos\zeta_1 (m \sin\odot - n \cos T_1) \\ &dk\,.\cos\zeta_2 (n \sin T_2 - m \cos\odot) + \pi \cos\zeta_2 (m \sin\odot - n \cos T_2) \end{aligned}$$

We can put without appreciable error in practice,

$$\cos\zeta_1 = \cos\zeta_2 = \cos\zeta_0$$

and the difference of these expressions for observations on the same night is therefore

$$(4) \qquad dk\,.\,n \cos\zeta_0 (\sin T_1 - \sin T_2) - \pi n \cos\zeta_0 (\cos T_1 - \cos T_2)$$

Then, in the determination of the aberration by KÜSTNER's method from the cyclical sum of all the group-combinations, denoting the coefficients of dk and π by A and B, respectively, and the absolute terms by ν, we have

$$\Sigma A\,.\,dk + \Sigma B\,.\,\pi + \Sigma\nu = 0$$

whence the aberration-correction

$$dk = -\frac{\Sigma\nu}{\Sigma A} - \pi \frac{\Sigma B}{\Sigma A}$$

But, since the average time of the observations will be about the same for all the group-combinations, we have

$$\frac{\Sigma B}{\Sigma A} = \frac{B}{A} = \frac{\cos T_1 - \cos T_2}{\sin T_1 - \sin T_2} = + \tan\tfrac{1}{2}(T_1 + T_2)$$

consequently, denoting by dk' the correction of the aberration-constant as ordinarily found by neglecting the effect of stellar parallax, the corrected value will be

$$dk = dk' - \pi \tan\tfrac{1}{2}(T_1 + T_2) \qquad (5)$$

From this it appears that the correction for stellar parallax is zero when the average apparent time of observation is 12^h, *i.e.*, when the evening and morning groups are symmetrically disposed as to apparent midnight. This is rarely practically the case. In most of the series for which aberration-determinations by this method are given on p. 3 of *A.J.* 529, this average falls before midnight, and the correction for parallax for most of them is therefore positive. Fortunately the printed data enable us to find these times approximately enough except for Strassburg and Hongkong. They are given in the following table, where the first column sufficiently designates the respective series, which are in the same order as in *A.J.* 529. Then follow the corrections, by eq. (5), of the aberration-determinations, expressed in terms of the unknown π; and in the last column their values in arc on the assumption $\pi = 0''.02$.

	T_1	T_2	Correction	
	h	h		$''$
Berlin,	—	—	—	+0.005
Berlin,	10.1	12.8	$+0.14\pi$	+ .003
Berlin,	10.1	12.8	+ .14	+ .003
Karlsruhe,	8.8	14.8	+ .05	+ .001
Prague,	10.1	12.8	+ .14	+ .003
Strassburg,	—	—	—	—
San Francisco,	11.0	13.9	— .12	— .002
Cape Good Hope,	8.3	16.4	— .09	— .002
Naples,	8.5	14.8	+ .09	+ .002
Potsdam,	9.3	11.7	+ .41	+ .008
Hongkong,	—	—	—	—
Leyden,	9.8	12.1	+ .28	+ .006
Kasan,	10.0	12.5	+ .19	+ .004
Kasan,	9.6	12.3	+ .28	+ .006
Kasan,	8.9	11.6	+ .49	+ .010
New York,	8.5	14.8	+ .09	+ .002
Bethlehem,	8.8	14.8	+ .05	+ .001
Philadelphia,	8.8	14.8	+ .05	+ .001
Int'l 6 stations,	10.0	12.0	+0.27	+0.005
	Weighted mean,		$+0.14\pi$	+0.003

The assumption $\pi = 0''.02$ is taken as a sort of measure of the superior limit which could with much probability, according to accepted notions, be assigned to this element for the class of stars employed. Some astronomers might

be inclined to reduce the estimate to one-half this quantity. From more than six hundred stars, actually used in five of the series, I find the average proper motion in declination to be about 0".05, or in arc of a great circle, total motion nearly 0".08; reduced to Boss's system about 0".09. The average magnitude is about the sixth. KAPTEYN's formulas would give for this case $\pi = 0''.017$. Prof. Boss, whom I consulted on the matter, is in favor of a decidedly lower value. The value 0".020 seems a fair estimate considered as a superior limit. But the main point is that the correction in question for the value of the aberration derived in *A.J.* 529 for these twenty-five series is essentially positive, so that their corrected mean would be 20".525 or 20".526 instead of 20".523 as there given. The general mean from the forty-three accepted series would therefore become 20".523, and I beg that this be regarded as the definitive mean of my investigation in that paper, corresponding to the value of the solar parallax 8".780, instead of the quantities there given. The change is of course trivial, but, being admittedly real, is necessary. So far as it goes it reinforces the likelihood that any rounded conventional value for this constant should be taken at least as high as 20".52.

It may be noted that the prime-vertical and meridian zenith-distance determinations given in the paper require no correction on this account, since the parallax was eliminated or simultaneously determined in the solutions.

2. The development of the foregoing formulas leads naturally to the suggestion that, in the KUSTNER-TALCOTT method, we should have a means of finding the average absolute parallax of a set of stars observed in common at a belt of stations in widely different longitudes, such as has been contrived and is now in successful operation for the determination of variations of latitude. The parallax so determined would be independent of errors in the star-declinations and of the latitude-variation. The high precision to which such observations have been brought, and the enormous mass of them, ought to make the method adequate for this purpose. I have therefore had the curiosity to develop and apply it, in the manner now to be shown.

Taking the mean of equations (3) we have

(6)
$$A.dk + B\pi = n\tfrac{1}{2}\cos\zeta\,[dk\,(\sin T_1 + \sin T_2) - \pi(\cos T_1 + \cos T_2)] - m\cos\zeta\,[dk\,.\cos\odot - \pi\sin\odot]$$

Now, with observations symmetrically disposed as to apparent midnight, the term in $\sin T$ will disappear from the mean observed on a given night; also the mean latitude for each station, deduced from a year's observations, will be affected by the constant value of the term in $\cos T$; so that the variations of latitude at a station in longitude λ (reckoned positive west from Greenwich), as found in the ordinary manner by subtracting this mean latitude from the observed values, may be expressed by

$$q - q_0 = x\sin\lambda - y\cos\lambda + z \tag{7}$$

where I have put

$$z = m\cos\zeta\,(dk\,.\cos\odot - \pi\sin\odot) \tag{8}$$

and the rectangular coordinates are reckoned, $+y$ towards Greenwich, $+x$ towards 90° east.

The values of x, y, z, can be determinately found from a belt of stations such as that of the International latitude-series for each date or group of dates. Then we may find dk and π from the equations of condition

$$m\cos\zeta\cos\odot\,.\,dk - m\cos\zeta\sin\odot\,.\,\pi = z \tag{9}$$

We therefore arrive at the curious result that the quantity z is nothing more than the empirical term, independent of the longitude of the station and varying with the time of year, which KIMURA has discovered and announced in *A.J.* 517, and which has been confirmed by ALBRECHT by means of the International latitude-series. It indubitably appears in the results for both 1900 and 1901.

Consequently, if the effect of stellar parallax furnishes the true explanation of this empirical term, we can find the value of π, or the average parallax of the observed stars, by introducing the values of z given by KIMURA and ALBRECHT as the absolute terms of equation (9). For the series 1900–1901 we take from *A.N.* 3808,

VALUES OF z.

	1899	1900	1901	1902	Mean	Computed
y	"	"	"	"	"	"
0.0	–	+.028	+.062	+.044	+.045	+.043
.1	–	+.021	+.055		+.038	+.036
.2		+.008	+.022		+.015	+.017
.3	–	–.011	–.005		–.009	–.008
.4	–	–.029	–.026	–	–.028	–.028
.5		–.033	–.036		–.034	–.037
.6	–	–.025	–.032	–	–.029	–.030
.7		–.008	–.016	–	–.012	–.011
.8		+.019	+.007	–	+.013	+.011
0.9	+.031	+.017	+.025		+.034	+.034

For the coefficients in eq. (9) we take $\cos\zeta = 0.98$, and $m = 0.309$ ($q = 39°\ 8'$); whence the observation-equations following, where w is an arbitrary constant to reduce the residual sum to zero, and the absolute terms are the mean observed values in the above table of z.

$$
\begin{array}{rrcrcr}
w & +.053\,dk & + .298\,\pi & = & +.045 \\
 & +.218 & +.213 & = & +.038 \\
 & +.300 & +.042 & = & +.015 \\
 & +.268 & -.142 & = & -.009 \\
 & +.133 & -.273 & = & -.028 \\
 & -.058 & -.298 & = & -.034 \\
 & -.218 & -.213 & = & -.029 \\
 & -.300 & -.042 & = & -.012 \\
 & -.268 & +.142 & = & +.013 \\
 & -.133 & +.273 & = & +.034
\end{array}
$$

The solution by equal weights gives

$$w = +0''.003 \quad , \quad dk = +0''.028 \quad , \quad \pi = +0''.128$$

from which we have the computed values in the last column of the table of z. The extraordinary accordance with the observed mean values must be largely fortuitous. If the result for parallax were reasonable this close agreement might be taken as an index of the efficiency of this method of finding parallax.

A similar computation for the data given by KIMURA for the other series in *A.J.* 517 gives us four other values of π, so that we have

1891–92	$\pi =$	$+0''.06$
95–96		$+ .02$
96–97		$+ .08$
98–99		$+ .14$
1900–01		$+ .13$
Mean	$\pi =$	$+0.086$

Now, it must be at once admitted that such parallaxes as these, for stars of the sixth magnitude and average proper motion of about $0''.08$ or $0''.09$, are inadmissible, according to orthodox notions. They are in flat contradiction of the inferences from the relations, apparently demonstrated by STUMPE and BOSS, between stellar proper motion and solar parallactic motion, taken in connection with the spectroscopic measurements of the sun's linear velocity. KAPTEYN's formulas would give for these stars an average parallax of not over $0''.017$.

If we abandon the idea of ascribing more than a moderate portion of KIMURA's phenomenon to the effect of stellar parallax we must seek the cause elsewhere for the principal part.

Let us see whether the observed phenomenon will correspond with a parallactic effect of another kind, namely, in a change of direction in our line of reference. Take the hypothesis that I have suggested in *A.J.* 524, that the earth's center of gravity may possibly be subject to an annual vibration along the line of the terrestrial axis. This does not seem to me intrinsically absurd. Let h be the linear semi-amplitude, expressed in feet, of such a vibration, and H the sun's longitude on the date corresponding to its southernmost point. The effect on measured latitudes will be the same for all longitudes, and will be

$$(10) \qquad \varphi - \varphi_0 = \frac{h}{\rho \sin 1''} \cos\varphi \cos(\odot - H)$$

where $\qquad \dfrac{1}{\rho \sin 1''} = 0''.01$

Solving for the constant w, and for h and H, using the observed values of z in the table we find

$$w = +0''.003 \quad , \quad h = 5.1 \text{ ft.} \quad , \quad H = 282^\circ.6 \text{ (January 2)}$$

the substitution of which gives us the same computed values in the last column of the z-table as before. By this hypothesis then there would be an oscillation of five feet from a mean position, the southernmost and northernmost points being reached on January 2 and July 2, respectively.

It is to be remarked that, as I have shown, stellar parallax is legitimately responsible for a part of the observed effect, so that the above numerical value of the linear semi-amplitude of the hypothetical oscillation would be correspondingly reduced, say to three or four feet.

Numerically, therefore, this hypothesis fits the facts, and on that account merely is perhaps worth suggesting for examination: but I presume that the required amount of shift is so great as to make the supposition unacceptable, as an explanation of the phenomenon. Remains, the possibility of anomalous refraction, already suggested by ALBRECHT; but this, by its nature, cannot be intelligibly formulated and tested at present. So as to the remote possibility that there is still lurking a weakness in the joints of the star-group combinations, not protected against by the existing scheme of observation. What seems to me certain is the desirability of enlarging and varying this program to meet and solve if possible this unforeseen dilemma. To dismiss it on the assumption that it is merely some form of purely subjective error for which no imaginable cause can be assigned would be to repeat a mistake that has confused some other astronomical questions at issue within recent memory.

There are three things that can be done, all of them, unfortunately, expensive and laborious. First, establishment of new equatorial and high northerly and southerly stations, either singly or in belts. Secondly and more feasibly accomplished, provision in the existing belt and at the same stations, or at a part of them, of a second observer; in order that the observations, instead of being as now confined to four hours of the night, can embrace the whole diurnal arc between sunset and sunrise, as nearly as possible. This could be accomplished by four-hour shifts for each observer on each night as at present, properly alternated on successive nights or pairs of nights; so as to eliminate personal differences as well as to cover practically the whole visible arcs. Thirdly, the easier but necessary undertaking of the reduction, of all the numerous series of observations made during the past twelve years by TALCOTT's method, to the correct value of the aberration-constant; so that by comparison of homogeneous results we may arrive at the best conclusions about this new and most interesting residual phenomenon, which, by the distinctness of the evidence that supports it, is most emphatic witness to the high precision to which astronomical measurement has been brought.

OBSERVATIONS OF COMET *b* 1902 (*PERRINE*),

MADE WITH THE 11-INCH EQUATORIAL AT THE SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.,

BY MARY E. BYRD.

1902 Greenwich M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Sept. 2 19 27 37	1	12 , 6	+0 20.36	−3 11.8	3 16 13.55	+35 21 32.3	*n*9.402	0.172	+3.97	+ 1.2
5 16 7 14	2	11 , 8	−0 20.51	+5 58.1	3 12 55.90	+36 37 10.5	*n*9.718	0.569	+1.10	+ 1.4
7 16 2 44	3	12 , 8	+0 9.68	−0 17.1	3 9 55.28	+37 36 16.3	*n*9.720	0.546	+1.23	+ 1.6
8 17 21 7	4	13 , 8	+2 18.70	−0 11.4	3 8 3.45	+38 9 51.3	*n*9.639	0.334	+1.31	+ 2.0
10 18 17 14	5	12 , 8	−1 15.63	+5 56.7	3 3 16.47	+39 20 21.7	*n*9.508	0.069	+1.13	+ 2.1
11 16 53 36	6	12 , 8	+0 21.22	+5 59.6	3 1 27.96	+39 55 28.4	*n*9.663	0.318	+1.51	+ 2.5
14 17 17 48	7	12 , 7	+1 31.22	+4 16.9	2 52 13.47	+42 0 51.2	*n*9.601	0.075	+1.76	+ 3.7
Oct. 8 13 57 12	8	12 , 8	−0 17.22	−4 5.3	19 49 6.49	+41 25 13.1	9.531	9.972	+2.38	+33.6
10 15 27 38	9	11 , 8	−1 6.88	+1 38.8	19 14 20.86	+31 3 16.2	9.691	0.557	+2.15	+30.5
14 14 3 43	11	12 , 8	−0 15.66	−3 27.5	18 30 22.95	+21 21 26.9	9.620	0.646	+2.02	+21.0
16 13 52 1	12	12 , 7	−2 8.04	+1 0.1	18 15 19.47	+16 6 55.8	9.619	0.695	+2.02	+21.5
20 12 51 40	13	12 , 8	+1 58.69	−2 49.4	17 53 31.37	+ 7 55 24.5	9.495	0.747	+2.00	+17.0
25 11 14 55	14	15 , 8	−0 1.15	−3 26.1	17 34 55.55	+ 0 45 28.1	9.542	0.767	+2.05	+13.2
31 11 8 16	15	12 , 9	−0 37.11	+6 36.5	17 18 27.47	− 5 7 15.2	9.549	0.793	+2.03	+10.1
Nov. 1 11 11 59	16	12 , 7	+1 15.71	−6 41.8	17 16 2.46	− 5 55 7.6	9.562	0.795	+2.02	+ 9.6
2 11 7 6	17	11 , 8	+1 10.55	+0 19.2	17 13 37.17	− 6 40 10.1	9.565	0.797	+2.02	+ 9.2

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 15 49.22	+35 24 42.9	Lund, A.G. 1731	10*	19 14 31.40	+33 57 28.7	Leiden, A.G. 7247
2	3 13 12.34	+36 31 11.0	Lund, A.G. 1712	11	18 30 36.59	+21 21 30.4	Berlin, B. A.G. 6545
3	3 9 44.37	+37 36 31.7	Lund, A.G. 1680	12	18 17 25.49	+16 5 34.2	Berlin, A. A.G. 6745
4	3 5 40.44	+38 10 3.7	Lund, A.G. 1643	13	17 51 30.68	+ 7 57 56.9	Leipzig II, A.G. 8176
5	3 4 57.67	+39 14 22.9	Lund, A.G. 1634	14	17 34 57.65	+ 0 48 41.3	Nicolajew, A.G. 4379
6	3 0 59.23	+39 49 26.3	Lund, A.G. 1595	15	17 19 2.55	5 14 1.8	Paris III, 22043
7	2 50 34.49	+41 56 0.6	Bonn, A.G. 2502	16	17 14 44.73	5 48 35.4	Paris III, 21949
8	19 49 21.33	+41 28 44.8	Bonn, A.G. 13492	17	17 11 54.90	6 40 38.5	Ottakring, A.G. Zones
9	19 15 25.59	+34 1 36.9	Leiden, A.G. 7258				

* Owing to fog, second measures for $\Delta\delta$ were made from *10 whose difference in declination from *9 was measured by micrometer.

EPHEMERIS OF COMET *d* 1902.

1903 Gr. M.T.	App. α	App. δ	log Δ	1903 Gr. M.T.	App. α	App. δ	log Δ
	h m s	° ′			h m s	° ′	
Feb. 1.5	6 41 31	+13 59.0	0.2852	Feb. 21.5	6 36 21	+20 19.6	0.3166
3.5	40 31	14 39.0		23.5	36 25	20 54.5	
5.5	39 43	15 18.6	0.2899	25.5	36 34	21 28.6	0.3248
7.5	38 56	15 57.9		27.5	36 51	22 2.0	
9.5	38 15	16 36.8	0.2954	Mar. 1.5	37 14	22 34.7	0.3335
11.5	37 41	17 15.4		3.5	37 44	23 6.7	
13.5	37 13	17 53.5	0.3018	5.5	38 21	23 37.8	0.3424
15.5	36 51	18 30.9		7.5	39 4	24 8.2	
17.5	36 34	19 7.8	0.3089	9.5	6 39 54	+24 37.9	0.3516
19.5	6 36 25	+19 41.0					

Computed from RISTENPART's elements, A.N. 3838. — ED.

COMET α 1903.

[From BIGCHILL'S Circular, No. 133, of January 27.]

A message received January 20, from Dr. KREUTZ at Kiel, via Harvard College Observatory, announced the discovery of a comet by GIACOBINI, at Nice, on January 15, together with a position secured at Nice on January 19. Captain C. M. CHESTER, Superintendent of the U. S. Naval Observatory, transmitted a position by Mr. DINWIDDIE of January 21, which was circulated by telegraph to American astronomers, and a third position has been received from Professor SEARES, Director of Laws Observatory, taken by himself. The latter was received via Harvard College Observatory. The positions and an orbit from Dr. KREUTZ are here given:

1903 Gr. M.T.	α	δ	Observer
	h m s	° ′ ″	
Jan. 19.2498	22 57 18	+2 12 27	Nice
21.1915	23 0 6.5	2 17 46	Dinwiddie
25.5643	23 1 38.3	+3 48 16	Seares

ELEMENTS.

T = 1903 March 14.84

$\pi - \Omega$ = 133 37 ⎫
Ω = 2 32 ⎬ Mean Eq. 1903.0
i = 30 30 ⎭

q = .4085

EPHEMERIS.

1903 Gr. Midnight	α	δ	Light
	h m s	° ′	
Jan. 25	23 1 36	+3 48	1.46
29	23 9 21	4 52	
Feb. 2	23 11 40	6 0	
6	23 20 20	+7 13	2.64

Computed from observations of January 19, 21 and 23. Light January 16 = 1.

ELEMENTS AND EPHEMERIS OF COMET α 1903 (GIACOBINI).

BY H. R. MORGAN AND ELEANOR A. LAMSON.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent].

The following elements were deduced from three observations at Nice, Jan. 19, and at Washington, Jan. 21 and Jan. 23:

ELEMENTS.

T = 1903 April 6.4289 Gr.M.T.

π = 127° 57′ 53″ ⎫
Ω = 359 39 10 ⎬ Ecliptic 1903.0
i = 37 36 38 ⎭

$\log q$ = 9.72151

Residuals (O—C): $d\lambda \cos\beta = -8.1''$
$d\beta = -1.6$

HELIOCENTRIC COORDINATES.

$$x = r[9.999997] \sin(218° \; 2' \; 13'' + v)$$
$$y = r[9.684735] \sin(127 \; 39 \; 12 + v)$$
$$z = r[9.912091] \sin(128 \; 9 \; 15 + v)$$

EPHEMERIS.

1903 Gr.M.T.	α	δ	log Δ	Light
	h m s	° ′		
Jan. 31.5	23 11 52	+5 22.4	0.3059	1.3
Feb. 4.5	23 17 14	6 30.9	0.2985	1.5
8.5	23 23 0	7 43.8	0.2901	1.7
12.5	23 29 12	9 1.2	0.2805	2.1
16.5	23 35 52	10 23.5	0.2698	2.4
20.5	23 43 5	+11 51.0	0.2577	2.9

Brightness on Jan. 19.5 is adopted as the unit.

No defined nucleus is seen as yet, but these elements indicate that the comet will be visible continuously east of the sun, becoming very much brighter, and passing the earth in May and June at about 0.5 of a unit's distance.

U. S. Naval Observatory, 1903 Jan. 27.

CONTENTS.

PUBLISHED IN BOSTON, MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS. ASSOCIATE EDITORS, ASAPH HALL AND LEWIS BOSS.
PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, LYNN, MASS. Entered at the Post Office, Boston, Mass., as second-class matter. *Closed Jan. 30.*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

Nos. 531-532.

VOL. XXIII. BOSTON, 1903 FEBRUARY 18. NOS. 3-4

POSITIONS AND MOTIONS OF 627 STANDARD STARS.

By LEWIS BOSS.

An effort to ascertain the positions of the principal standard stars with exactness finds its justification, even though a relatively small improvement in our knowledge is thereby attained, since the results have fundamental bearings upon a variety of the more important problems which engage the attention of practical astronomers.

In the derivation of the position of a single star from past observations it may not be of very great importance whether the systematic corrections employed conform to one, or another, of the various standard catalogues. But in computing a large number of positions and motions of stars, as the basis of any investigation requiring high precision, the selection of the standard catalogue from which the systematic corrections of the catalogues of observation are to be ascertained predetermines the final result in a marked degree.

Thus, if the position of the solar apex be derived from proper motions of the stars between +10° and −30° of declination, computed from the various catalogues of observation by the aid of systematic corrections derived from the Standard Catalogue, B, of this paper, its declination will come out more than 10° further north, than it will if systematic corrections in conformity with the standard catalogue of the *Berliner Jahrbuch*, A, be employed. It will thus be seen that the most probable position of the solar apex is far more a matter of the standard catalogue upon which it is based than it is of the mathematical method employed. Any attempt to improve our knowledge of the direction of the sun's way, therefore, involves in the first line an improvement in our knowledge of the positions and motions of the standard stars.

In like manner, any question relating to a supposed rotation of the celestial sphere, or of any part of it, must depend in a very important degree upon the conclusions to be drawn relative to the accuracy of our standard catalogues upon which such computations must ultimately rest.

No great refinement in the computation of orbits of bodies within the solar system, which range over wide limits in declination, is possible without attention to the systematic errors of the star-positions upon which such computations must be based.

Scope of this Work and Acknowledgements.

The catalogue of 627 principal standard stars here presented is the result of an attempt to provide an improved basis for computations relating to real, or apparent, systematic motions of the stars. It is to be regarded as the exhibit of an intermediate stage of an investigation which is in progress and which it is not advisable to complete until further important series of meridian observations provide the necessary basis for a more definitive computation. This is especially pertinent as to the southern hemisphere, in respect to which our knowledge derived from observation is still scanty to a lamentable degree. It is to be hoped that the next few years will witness very important additions to the testimony of observation as to the positions of the brighter stars at various epochs. The plan of observation with the new transit-circle of the Cape observatory may be expected to double, in the systematic sense, the weight of determination of stellar motion for the southern sky; and it is to be hoped that, in the near future, other contributions of a similar character may be secured in respect to that region of sky. Generally we may hope that there will soon be a notable increase of contributions in this line through the re-reduction of older series of observations, through the reduction of recent observations already made, and by further and more precise observations in this field. The stimulus of increased interest in problems relating to the sidereal system seems sufficient to warrant this expectation.

The positions of the present catalogue are the result of successive approximations founded, in the first instance, upon the right-ascensions tabulated in a five-year ephemeris at the end of "NEWCOMB'S *Standard and Zodiacal Stars*," N_1, and upon the declinations of the principal stars contained in the writer's work entitled, "*Declinations of Fixed Stars*" (also declinations of the *American Ephemeris*, 1881 to 1899), B. As the right-ascensions now stand they are

appears to represent essentially the equinox, N_1, of Newcomb's "*Equatorial Fundamental Stars*" in so far as terms in $\Delta\alpha_\alpha$ are concerned, but in respect to terms, $\Delta\alpha_\delta$, varying with the declination they present a new and independent system differing decidedly from those of Auwers and Newcomb. In declination a new system has resulted, though it is essentially identical with B as to terms in $\Delta\delta_\alpha$.

With the exception of Albany observations and the last six hours of the Paris Catalogues, the results of unpublished observations are not included. The results for the last six hours of the Paris Catalogue, for the stars in this catalogue and for many others, were communicated in the most prompt and obliging manner by Director Loewy of the National Observatory at Paris.

In this investigation were employed all star-catalogues found in the library of the Dudley Observatory which promise useful contributions to this purpose. Exceptions were made as to catalogues of annual results not yet compiled in the form of general catalogues and of some minor and partial catalogues, which were not included in the computations for the standard catalogue.

It may be well to remark that the positions of this catalogue have no dependence, in the systematic sense, upon any meridian observations of a date earlier than the Königsberg observations of 1820. It has also been assumed that in the computations for standard stars no special benefit (but some possible harm) could be anticipated from the employment, even in a differential sense, of observations like those of Mayer, Piazzi and Groombridge, however useful these may become in computations for the positions and motions of stars in general.

I am indebted to the courtesy of the Superintendent of the Naval Observatory, to Professor Pickering, Director of the Harvard College Observatory, and to Dr. Chandler, Editor of the *Astronomical Journal*, for the loan of important star-catalogues not found in the library of this observatory. It may have happened that a few star-catalogues, that might have proved useful, have been overlooked in this necessity of gathering material from so many sources. The present work must be regarded, however, merely as one of the approximations aiming at a more definitive result which cannot be reached with good advantage for some years to come. It is merely supposed that, in this approximation, a stage has been reached such that further amendments to the positions and motions will be small, or, at least, not abruptly different for adjacent regions of sky. Meanwhile the catalogue in its present state is offered as a possible improvement on what has gone before, and as that which will serve as the temporary basis for various works in progress at this Observatory.

Moreover, this is an attempt to produce a consistent system of standard star-positions and motions, by considering simultaneously all the material of observation from pole to pole in a homogeneous investigation, in which each series of observations is designed to exert its due influence upon the result with careful reference to its relations to other series of observations.

The preparation for this research has been in progress at odd times for several years. Within the last two years the time of the Observatory staff has been almost wholly given up to this purpose. Most efficient aid, in the more responsible parts of the work as well as in the details of computation, has been rendered by Assistants Arthur J. Roy and William B. Varnum.

Throughout this and related investigations in progress here the work has been efficiently aided by liberal appropriations from the Bache Fund of the National Academy of Sciences. Without such aid the work would not have been undertaken. Means for publication of the results in cooperation with the *Astronomical Journal* have also been accorded by the Directors of the Bache Fund. All these grants have been made with a sympathy and readiness of appreciation which have enhanced their value, and for which I express my warmest thanks.

Comparison with the Standard Catalogues of Newcomb and Auwers.

Perhaps the general result of the present research can most readily be defined by comparison with the "*Catalogue of Fundamental Stars for 1875 and 1900*," N_2, by Professor Newcomb, which is now serving as the basis for several astronomical ephemerides, and with the revised catalogues of Dr. Auwers, as they appear in "*Vorläufige Verbesserung des Fundamental-Catalogs*" (*A.N.*, Bd. 147, p. 49 ff.), A_n; "*Fundamental-Catalog für Zonen-Beobachtungen am Südhimmel*" (*A.N.*, Bd. 143, p. 361 ff.), A_s; and the revised positions of 303 stars, in an intermediate zone, as published in the *Berliner Jahrbuch* for 1901, A_i. The results of this comparison are presented in the following tables, in which the individual results were obtained by subtraction in the senses respectively indicated, and were then combined into regular groups with the use of weights printed in the catalogue. The epoch of the comparisons in R.A. is uniformly 1900.

Right-Ascension; $\Delta\alpha_s$ and 100 $\Delta\mu_s$.

Decl. +37°.5 to −22°.

	B − N_2			B − A_n		
α	No. ✱✱	$\Delta\alpha_s$	100 $\Delta\mu_s$	No. ✱✱	$\Delta\alpha_s$	$\Delta\mu_s$
h		s	s		s	s
0	25	−.005	−.005	15	.000	+.019
2	26	−.004	−.007	20	−.002	+.016
4	25	−.006	−.011	22	−.003	+.004
6	22	−.005	−.010	20	−.006	−.019
8	14	−.006	−.017	12	−.005	−.021
10	24	+.001	+.014	18	−.004	−.018
12	21	−.002	+.003	16	−.001	−.008
14	20	+.002	+.015	13	+.005	+.010
16	30	+.002	+.004	24	+.001	+.009
18	20	+.005	+.013	19	+.007	+.013
20	28	.000	+.004	21	+.005	−.004
22	31	−.002	+.005	25	.000	+.007

It should be remarked that the positions and motions of many stars were computed which are not included in the present catalogue, B, — especially in the sky south of −22°. These additional stars have been made use of in the preceding comparisons as well as in those which follow.

Following are the expressions for $\Delta\alpha_\delta$ and $\Delta\mu_\delta$ which result from the preceding table.

$$B - N_2: \quad \Delta\alpha_\delta = \overset{s}{0.000} \quad -\overset{s}{0.0039}\sin\alpha \quad -\overset{s}{0.0020}\cos\alpha$$
$$100\ \Delta\mu_\delta = 0.000 \quad -0.010 \quad -0.005$$

$$B - A_n: \quad \Delta\alpha_\delta = +0.0270 \quad -0.0051\sin\alpha \quad +0.0002\cos\alpha$$
$$100\ \Delta\mu_\delta = +0.088 \quad -0.010 \quad +0.010$$

The periodic terms in $\Delta\alpha_\delta$, though very small, are clearly indicated. The correction for A_n may safely be assumed to apply also to A_1 and A_2. The assumption of

$$+0^s.027 \ +0^s.088\ \frac{T'-1900}{100}$$

as the equinox-correction of A_n is somewhat arbitrary; but any defect in this assumption is compensated in the values of $\Delta\alpha$ and $\Delta\mu$ which appear in the tables of correction.

In general, the determination of the equinox-correction is beset with minor difficulties due to uncertainties in the errors which depend upon the declination. These uncertainties are also inherent in the original determinations of the successive positions of the equinox. In view of this it may be assumed that the equinox, N_1, has been preserved in the present computation with such accuracy as the state of the case would readily allow.

The system, B, appears to satisfy very well the mean of the best modern determinations of absolute right-ascension in respect to terms in $\Delta\alpha_\delta$, as will hereafter appear; so that the testimony of these determinations does not point to a correction of B in this respect which would account for any important part of the differences, $B - N_2$ and $B - A_n$.

Right-Ascension: $\Delta\alpha$ and 100 $\Delta\mu$.

$B - N_2$

δ	No. ✱✱	Δα	100 Δμ	δ	No. ✱✱	Δα	100 Δμ
+87	11	+.036	+.180	−5°	21	−.003	−.011
80	6	−.006	+.112	9	30	−.001	+.005
76	10	−.028	−.016	15	29	+.004	+.019
70	11	−.025	−.024	19	25	+.014	+.030
66	8	−.023	−.016	25	22	+.020	+.040
60	17	−.031	−.048	29	16	+.028	+.070
55	11	−.027	−.050	35	17	+.027	+.055
50	17	−.020	−.039	40	16	+.040	+.017
45	16	−.029	−.063	45	25	+.054	+.086
40	20	−.010	−.030	50	12	+.041	+.068
35	16	−.001	−.014	55	13	+.046	+.111
29	22	−.007	−.016	60	19	+.024	+.051
25	24	−.005	−.004	65	13	−.009	+.016
20	24	−.003	−.007	70	10	+.030	+.177
15	23	−.002	.000	75	4	+.019	+.137
10	27	−.002	−.001	79	9	+.055	+.214
+5	33	−.003	+.003	−87	15	−.018	−.113
0	17	−.001	+.006				

For the groups south of −20° the individual differences are very irregular, as will be seen by reference to the individual comparisons printed in connection with the Catalogue.

Right-Ascension: $\Delta\alpha$ and 100 $\Delta\mu$.

		$B - A_n$		$B -$ B.J.	
δ	No. ✱✱	Δα	100 Δμ	Δα	100 Δμ
+87	7	−.049	−.015	−.102	−.134
80	6	+.048	+.224	+.107	+.429
76	8	+.019	+.088	+.037	+.112
70	11	+.003	+.060	+.027	−.058
66	9	.000	+.045	+.017	+.116
60	17	−.018	−.001	−.010	−.079
55	10	−.031	−.031	−.021	−.016
50	17	−.026	−.021	−.031	−.036
45	16	−.029	−.026	−.028	−.017
40	20	−.019	−.001	−.019	+.014
35	16	−.021	−.012	−.024	−.014
29	20	−.017	−.022	−.023	−.033
25	22	−.015	−.023	−.009	−.005
20	18	−.007	−.030	−.007	−.014
15	22	−.002	−.005	.000	−.007
10	25	−.001	−.020	.000	−.011
+5	23	+.006	+.007	+.011	+.019
0	14	+.012	+.020	+.019	+.027
−4	14	+.006	+.024	+.017	+.032
9	21	+.010	+.018	+.006	+.016
15	22	+.013	+.022	+.018	+.063
19	12	+.011	+.036	+.001	+.017
25	13	+.016	+.041	+.025	+.046
−30	6	+.035	+.095	+.035	+.095

The meridians of right-ascension for 45° on either side of the equator, as defined by N_2 and A, seem to be inclined with reference to the meridian defined by B, both in the same direction and by nearly the same amounts. For both N_2 and A_n the discrepancy amounts to about

$$+0^s.015 \ +0^s.08\ \frac{T'-1900}{100}$$

in the neighborhood of −45° of declination; and to something like

$$-0^s.025 \ -0^s.02\ \frac{T'-1900}{100}$$

for A_n, and

$$-0^s.017 \ -0^s.04\ \frac{T'-1900}{100}$$

for N_2, at +45°. These discrepancies have received careful attention and, while the meridian south of the equator is still very uncertain, it does not seem probable that any considerable part of the differences is attributable to error in the meridian B, so far as weight of existing testimony is competent to decide.

The system of the "303 stars," A_1, and of the [illegible] stars, A_2, should be conformable in right-ascension with A_n. The determination of $\Delta\alpha$ and $\Delta\mu$ will therefore suffice for these

Right-Ascension: $\Delta\alpha$ and 100 $\Delta\mu$.

	$B-A_4$				$B-A_5$		
δ	No. **	$\Delta\alpha$	100 $\Delta\mu$	δ	No. **	$\Delta\alpha$	100 $\Delta\mu$
°		s	s	°		s	s
+2	21	+.009	+.013	−36	28	+.031	+.076
5	16	+.013	+.021	−40	23	+.011	+.103
10	26	+.009	+.035	−45	29	+.015	+.099
−15	21	+.009	+.023	−50	15	+.012	+.081
−20	22	+.013	+.030	−54	18	+.025	+.062
−24	12	+.018	+.049	−60	20	+.010	+.065
				−65	17	+.013	+.081
	$B-A_6$			−70	13	+.013	+.151
−22	9	+.016	+.030	−75	5	+.012	+.058
−24	21	+.020	+.047	−79	8	+.010	+.130
−29	20	+.037	+.089	−87	13	−.211	−.591

As in A_{II}, in order to have the differences actually found, $+0^s.027$ should be added to the above values of $\Delta\alpha$, and $+0^s.088$ to those of 100 $\Delta\mu$, — these corrections corresponding to the difference of equinoxes, B−A.

Comparison of Declinations.

The subjoined tables exhibit the results of comparison for the declinations, corresponding to the epoch 1900, except for B_5 (*Declination of Fixed Stars* and *American Ephemeris*, 1881–1899, and its extension southward from −20°, as published in *Ast. Jour.*, No. 450), for which the epoch of comparison is 1875.

The values of $\Delta\delta_\alpha$ and $\Delta\mu'_\alpha$ are first cleared from the effect of terms in $\Delta\delta_\delta$ and $\Delta\mu'_\delta$.

Declination: $\Delta\delta_\alpha$ and $\Delta\mu'_\alpha$.

	$B-N_2$	+80° to +40°			$B-A$	
α	No. **	$\Delta\delta_\alpha$	100 $\Delta\mu'_\alpha$	No. **	$\Delta\delta_\alpha$	100 $\Delta\mu'_\alpha$
h		"	"		"	"
0	8	+.05	+.09	8	+.15	+.19
2	10	−.07	−.08	9	−.01	−.05
4	10	−.03	+.06	10	−.03	+.03
6	5	+.02	+.08	5	+.03	+.22
8	4	.00	+.05	4	−.08	−.18
10	7	−.06	−.29	7	−.13	−.09
12	10	−.01	−.23	10	−.05	+.02
14	7	+.05	+.15	6	−.01	+.04
16	9	+.05	+.07	9	−.04	−.12
18	12	+.09	+.14	11	+.03	−.06
20	13	−.04	−.06	13	+.02	+.05
22	8	+.01	+.02	8	+.08	−.01
		+40° to −22°				
0	28	+.01	+.01	17	+.04	+.01
2	26	+.02	+.08	20	+.01	+.05
4	27	+.05	+.05	24	+.02	−.01
6	21	+.05	+.16	19	−.02	+.14
8	16	+.03	+.01	14	−.17	−.23
10	23	+.01	−.02	17	−.13	−.25
12	22	−.06	−.07	17	−.03	−.18
14	22	−.04	+.04	16	+.04	+.12
16	32	−.06	−.12	26	−.02	.00
18	21	−.01	−.06	20	+.01	+.06
20	28	−.05	−.12	22	+.06	−.03
22	32	+.04	+.10	25	+.13	+.19
		−22° to −70°				
0	13	+.16	+.16	11	+.17	+.65
2	11	+.15	+.28	15	+.18	+.56
4	16	−.10	−.09	19	−.06	−.03
6	15	+.01	−.08	19	−.06	−.32
8	13	−.11	−.28	11	−.10	−.11
10	16	+.29	+1.09	18	−.07	−.11
12	13	+.05	−.07	15	−.18	−.32
14	11	−.10	+.07	15	+.12	+.58
16	18	−.08	−.13	26	+.08	+.27
18	25	−.05	−.12	26	.00	−.48
20	11	−.09	−.57	10	−.17	−.98
22	10	−.08	−.25	13	+.11	+.26

The periodic terms result as follows:

Limits		$B-N_2$, $\Delta\delta_\alpha$		$B-N_2$, 100 $\Delta\mu'_\alpha$	
°	°	"	"	"	"
+80	+40	−.033 sin α	−.002 cos α	−.04 sin α	+.05 cos α
+40	−22	+.010	+.032	+.07	+.04
−22	−70	+.056	+.001	+.18	−.15

Limits		$B-A$, $\Delta\delta_\alpha$		$B-A$, 100 $\Delta\mu'_\alpha$	
°	°	"	"	"	"
+80	+40	−.030 sin α	+.073 cos α	+.02 sin α	+.06 cos α
+40	−22	−.048	+.074	−.06	+.11
−22	−70	−.032	+.080	+.08	+.12

The consistency of the comparisons, B−A, in the several zones is worthy of special note. It seems to offer satisfactory evidence that the numerical accuracy of the computations for the positions and motions of the individual stars in each catalogue are practically above reproach.

From these, omitting Newcomb's stars south of −22°, we may assume the definitive corrections to be:

$$B-N_2 \qquad \Delta\delta_\alpha = +0''.022 \sin\alpha \quad +0''.024 \cos\alpha$$
$$100\ \Delta\mu'_\alpha = +0.04 \qquad +0.04$$

$$B-A \qquad \Delta\delta_\alpha = -0''.041 \sin\alpha \quad +0''.075 \cos\alpha$$
$$100\ \Delta\mu'_\alpha = -0.01 \qquad +0.10$$

The values of $\Delta\delta_\alpha$ and 100 $\Delta\mu'_\alpha$ for the *Berliner Jahrbuch*, 1883–1900 may safely be assumed to be the same as for A_{II}; and for B_5, 1875, practically the same as for the catalogue of the present investigation. In view of the uncertainties relating to our present knowledge of the laws of variation of latitude at different epochs, no highly critical investigation of the systematic terms in $\Delta\delta_\delta$ for the present catalogue has been undertaken. It is believed that this can be much more effectively done at a later time. It is notable, however, that for the modern series of observed declinations the discrepancies from the standard following the order of right-ascension are comparatively minute, notwithstanding differences of epoch and of the longitude of the observatories concerned. It is proposed to treat this matter more in detail in a subsequent chapter.

It should also be noted that the system, B_s, with which comparison is made, though practically identical in the systematic sense with that of "*Declinations of Fixed Stars*," differs from it in that the individual positions and motions of 50 stars were first revised for the uses of the present computation.

Declination: Δδ and 100 Δμ′, for B—N₂ and B—Bₛ.

B—N_2				B—B_s, 1875			
δ	No.**	Δδ	100 Δμ′	δ	No.**	Δδ	100 Δμ′
°		″	″	°		″	″
+87	11	−.08	−.09	—	—	—	—
80	6	−.01	+.02	+78	11	+.12	+.34
76	10	+.06	+.28	—	—	—	—
70	11	−.01	−.06	69	19	+.15	+.35
66	9	.00	+.22	—	—	—	—
60	17	−.11	−.08	60	19	+.11	+.49
55	11	−.17	−.06	55	10	+.16	+.58
50	17	−.16	−.05	50	17	+.13	+.48
45	16	−.19	−.09	45	14	+.10	+.37
40	20	−.31	−.31	40	19	+.09	+.32
35	16	−.30	−.21	34	14	+.06	+.11
29	22	−.22	−.02	29	18	.00	+.17
25	24	−.25	−.21	25	18	−.02	+.14
20	24	−.22	−.15	20	16	−.04	+.13
15	23	−.27	−.37	15	19	−.07	+.11
10	27	−.22	+.01	10	23	−.05	+.26
+5	33	−.36	−.28	+5	21	−.04	+.25
0	16	−.36	−.23	0	11	−.06	+.14
−5	21	−.31	−.27	−6	13	.00	+.26
9	30	−.31	−.17	10	15	.00	+.39
15	29	−.35	−.16	14	18	+.03	+.45
19	25	−.29	−.16	19	13	+.04	+.54
25	23	−.26	+.05	24	21	.00	+.18
29	18	−.01	+.10	27	17	+.01	+.58
35	21	−.17	−.22	32	17	.00	+.31
40	17	−.03	+.36	40	15	−.04	+.42
45	25	+.08	+.67	44	29	−.01	+1.16
50	12	+.07	+.69	48	25	−.04	+1.44
55	13	−.02	+.55	51	21	+.01	+1.15
60	19	+.01	+.78	59	19	−.02	+1.34
65	13	+.27	+1.11	63	22	−.02	+1.12
70	10	+.17	+.87	66	15	.00	+1.15
75	1	−.10	−.28	70	11	−.05	+1.61
79	9	+.13	+.22	−78	12	+.02	.00
−87	15	−.06	−.16	—	—	—	

Declination: Δδ and 100 Δμ′, for B—Aₙ and B—B.J.

		B—A_n		B—B.J.	
δ	No.**	Δδ	100 Δμ′	Δδ	100 Δμ′
+87	7	−.05	−.05	+.04	+ .19
80	6	+.02	+.41	−.03	+ .39
76	8	+.21	+.94	+.39	+1.22
70	11	+.16	+.77	+.18	+ .82
66	9	+.22	+.62	+.23	+ .67
60	17	+.15	+.43	+.16	+ .48
55	10	+.13	+.28	+.15	+ .23
50	17	+.08	−.10	+.14	+ .17
45	16	+.18	.00	+.21	.00
40	20	−.12	−.99	−.32	1.51
35	16	−.28	−1.38	−.52	−2.16
29	20	−.19	−1.05	−.06	− .54
25	22	−.23	−1.17	−.22	1.04
20	18	−.30	−1.38	−.28	−1.34
15	22	−.18	−1.20	−.14	1.12
10	25	−.17	−1.18	−.15	−1.16
+5	23	−.12	−1.01	−.17	−1.07
0	14	−.04	−.84	+.05	− .68
−5	11	.00	−.92	−.10	−1.09
9	21	+.07	−.99	+.02	−1.11
15	22	+.22	−1.05	+.24	−1.00
19	12	+.38	−.91	+.44	.74
25	13	+.64	−.63	+.70	− .46
−30	6	+.75	−.82	+.65	− .99

Down to the limit of stars which can be observed with advantage in high northern latitudes the systematic differences, B—N_2, for the proper motions, are very small, and are chiefly due to the weight which Professor Newcomb attributed to the results of planetary observation (*Ast. Pap. Am. Eph.*, Vol. VIII, Pt. II, p. 191). As to the region south of −30° the values of Δμ′, are intrinsically very uncertain on account of the small totality of weight of the observed declinations for the south polar regions; and also, in some degree, because of the defects of the Mural Circle at the Cape, used by Henderson and Maclear, — a matter which will be treated in some detail in a subsequent chapter.

There is, of course, a close general resemblance between the numbers, B—A_n and B—B.J. But the latter, as might have been expected, show very much more clearly the systematic distortions produced by the dependence of the proper motions upon Bradley's declinations. In fact, the zones should be much less than five degrees wide in order properly to show the irregularities in the region +25° to +15°, wherein the defects of Bradley's quadrant are undoubtedly very great. In the tables of systematic differences, further on, it will be assumed that the systematic correction for B.J. (1883–1900) is the same as that for A_n, with the reservation that it would be difficult to assign any very exact corrections to the proper motions of the former in the zone of +25° to +15°.

Declinations: Δδ, and 100 Δμ′ for B—A. and B—Aₛ.

B—A.				B—A_s			
δ	No.**	Δδ	100 Δμ′	δ	No.**	Δδ	100 Δμ′
°		″	″	°		″	″
+2	24	−.20		−29	20	−.24	.65
−5	16	−.21		35	28	.02	+ .09
−10	26	−.21		40	23	−.07	.48
−15	21	−.16		45	29	+.09	+ .33
−20	22	−.06		50	15	+.06	+ .33
−24	12	+.15		54	18	+.01	+ .14
				60	20	+.20	+ .77
				65	17	+.33	+1.02
				70	13	+.10	+ .28
B—A_s				75	8	+.10	+ .52
−22	9	−.26	−.66	79	8	+.47	+ .28
−24	21	.15	−.34	87	13	.05	.25

Taking into account the smallness of the weights involved, the individual differences which make up the groups in the preceding table agree very well.

Tables of Systematic Correction for N_2 and A.

The results of the foregoing comparisons have been utilized to form tables of systematic corrections for N_2, A_n, A_I and A_s. In right-ascension no distinction is necessary between the various catalogues published by Dr. AUWERS, beginning with the *Fundamental-Catalog*; but in declination the distinction between the northern, intermediate, and southern catalogues must be preserved, so far as $\Delta\delta_\delta$ is concerned. Through the use of these tables the positions and motions of many stars not included in the present catalogue can be brought into systematic harmony with it, and apparently without materially less accuracy for the individual stars than could be reached by special computations for these stars in conformity with the system of B. This is especially true of the star-places computed by Dr. AUWERS in the catalogues, A_I and A_s. As will be seen by reference to the catalogue the positions and motions of south polar stars taken from N_2 agree better with the results of this investigation than do those taken from A_s, which, in turn, are quoted from the *Cape Catalogue for 1890*.

SYSTEMATIC CORRECTIONS: ORDER OF DECLINATIONS.

RIGHT-ASCENSIONS; CORRECTIONS, $\Delta\alpha_\delta$ AND 100 $\Delta\mu_\delta$.

	B — N_2		B — A	
	$\Delta\alpha_\delta$	100 $\Delta\mu_\delta$	$\Delta\alpha_\delta$	100 $\Delta\mu_\delta$
	s	s	s	s
+85	0.000	+0.068	+0.029	+0.119
80	−0.017	+0.040	+0.026	+0.127
75	−0.025	+0.001	+0.017	+0.092
70	−0.028	−0.018	+0.007	+0.061
65	−0.027	−0.031	−0.005	+0.031
+60	−0.028	−0.040	−0.021	−0.002
55	−0.026	−0.047	−0.028	−0.019
50	−0.023	−0.048	−0.029	−0.022
45	−0.019	−0.042	−0.026	−0.020
40	−0.013	−0.033	−0.023	−0.016
+35	−0.007	−0.021	−0.020	−0.015
30	−0.005	−0.013	−0.017	−0.018
25	−0.004	−0.006	−0.014	−0.022
20	−0.003	−0.003	−0.010	−0.022
15	−0.002	0.000	−0.005	−0.018
+10	−0.002	0.000	0.000	−0.009
+ 5	−0.002	0.000	+0.005	+0.001
0	−0.002	−0.001	+0.009	+0.016
− 5	−0.002	0.000	+0.010	+0.023
10	0.000	+0.005	+0.009	+0.025
−15	+0.005	+0.014	+0.010	+0.028
20	+0.013	+0.031	+0.013	+0.036
25	+0.020	+0.041	+0.021	+0.054
30	+0.025	+0.054	+0.031	+0.077
35	+0.032	+0.063	+0.037	+0.090
−40	+0.040	+0.070	+0.042	+0.097
45	+0.045	+0.076	+0.043	+0.096
50	+0.045	+0.081	+0.038	+0.081
55	+0.037	+0.087	+0.026	+0.071
60	+0.019	+0.094	+0.016	+0.068
−65	+0.009	+0.097	+0.011	+0.080
70	+0.016	+0.096	+0.012	+0.093
75	+0.035	+0.092	+0.012	+0.100
80	+0.032	+0.069	+0.006	+0.086
−85	0.000	0.000	−0.034	−0.034

DECLINATIONS; CORRECTIONS, $\Delta\delta_\delta$ AND 100 $\Delta\mu'_\delta$.

	B — N_2		B — A_n	
δ	$\Delta\delta_\delta$	100 $\Delta\mu'_\delta$	$\Delta\delta_\delta$	100 $\Delta\mu'_\delta$
	"	"	"	"
+90	0.00	0.00	0.00	0.00
85	0.00	0.00	0.00	0.00
80	0.00	0.00	+0.07	+0.44
75	0.00	+0.10	+0.14	+0.84
70	0.00	+0.14	+0.17	+0.79
+65	−0.01	+0.10	+0.18	+0.61
60	−0.09	+0.01	+0.17	+0.43
55	−0.14	−0.05	+0.15	+0.23
50	−0.19	−0.10	+0.12	−0.03
45	−0.23	−0.16	+0.04	−0.38
+40	−0.27	−0.20	−0.09	−0.83
35	−0.27	−0.20	−0.19	−1.12
30	−0.25	−0.17	−0.22	−1.20
25	−0.23	−0.16	−0.24	−1.22
20	−0.23	−0.18	−0.24	−1.24
+15	−0.25	−0.20	−0.21	−1.23
10	−0.28	−0.22	−0.17	−1.13
+ 5	−0.31	−0.23	−0.13	−0.99
0	−0.33	−0.24	−0.07	−0.89
− 5	−0.34	−0.23	+0.01	−0.92
−10	−0.34	−0.20	+0.11	−0.99
15	−0.33	−0.16	+0.26	−0.97
20	−0.28	−0.09	+0.43	−0.88
25	−0.19	+0.06	+0.61	−0.79
30	−0.12	+0.10	+0.83	−0.71
−35	−0.06	+0.16	–	–
40	0.00	+0.36	–	–
45	+0.03	+0.55		–
50	+0.04	+0.63	–	–
55	+0.03	+0.71	–	–
−60	+0.10	+0.93	–	–
65	+0.15	+0.97	–	–
70	+0.13	+0.67	–	–
75	+0.06	+0.18	–	–
80	0.00	0.00	–	–
−85	0.00	0.00	–	–
−90	0.00	0.00	–	–

	B — A_I	
δ	$\Delta\delta_\delta$	100 $\Delta\delta'_\delta$
°	"	"
+ 5	−0.21	−0.99
0	−0.21	−0.89
− 5	−0.21	−0.92
10	−0.21	−0.99
15	−0.16	−0.97
20	−0.05	−0.88
−25	+0.17	−0.79

	B — A_s	
δ	$\Delta\delta_\delta$	100 $\Delta\delta'_\delta$
−20	−0.22	−0.73
25	−0.20	−0.55
30	−0.15	−0.35
35	−0.08	−0.15
40	−0.03	0.00
45	+0.03	+0.17
50	+0.05	+0.27
55	+0.10	+0.43
60	+0.20	+0.71
65	+0.33	+0.91
70	+0.40	+0.83
75	+0.40	+0.53
80	+0.29	+0.22
85	+0.10	−0.08
−90	0.00	−0.30

SYSTEMATIC CORRECTIONS: ORDER OF R.A.

	Right-Ascension				Declination			
	B — N_2		B — A		B — N_2		B — A	
	$\Delta\alpha_s$	$100\,\Delta\mu_s$	$\Delta\alpha_s$	$100\,\Delta\mu_s$	$\Delta\delta_s$	$100\,\Delta\mu'_s$	$\Delta\delta_s$	$100\,\Delta\mu'_s$
h	s	s	s	s	″	″	″	″
0	−.002	−.005	+.027	+.098	+.02	+.01	+.08	+.10
1	−.003	−.007	+.026	+.095	+.03	+.05	+.06	+.09
2	−.004	−.009	+.025	+.092	+.03	+.05	+.04	+.08
3	−.004	−.011	+.024	+.088	+.03	+.06	+.02	+.06
4	−.004	−.011	+.023	+.084	+.03	+.05	.00	+.04
5	−.004	−.011	+.022	+.081	+.03	+.05	−.02	+.02
6	−.004	−.010	+.022	+.078	+.02	+.04	−.04	−.01
7	−.003	−.008	+.022	+.076	+.02	+.03	−.06	−.04
8	−.002	−.006	+.022	+.074	+.01	+.01	−.07	−.06
9	−.001	−.004	+.023	+.074	.00	.00	−.08	−.08
10	.000	−.001	+.024	+.074	−.01	−.01	−.09	−.09
11	+.001	+.002	+.026	+.076	−.02	−.03	−.08	−.10
12	+.002	+.905	+.027	+.078	−.02	−.04	−.08	−.10
13	+.003	+.007	+.028	+.081	−.03	−.05	−.06	−.09
14	+.004	+.009	+.029	+.084	−.03	−.05	−.04	−.08
15	+.004	+.011	+.030	+.088	−.03	−.06	−.02	−.06
16	+.004	+.011	+.031	+.092	−.03	−.05	.00	−.04
17	+.004	+.011	+.032	+.095	−.03	−.05	+.02	−.02
18	+.004	+.010	+.032	+.098	−.02	−.04	+.04	+.01
19	+.003	+.008	+.032	+.100	−.02	−.03	+.06	+.04
20	+.002	+.006	+.032	+.102	−.01	−.01	+.07	+.06
21	+.001	+.004	+.031	+.102	.00	.00	+.08	+.08
22	.000	+.001	+.030	+.102	+.01	+.01	+.09	+.09
23	−.001	−.002	+.028	+.100	+.02	+.03	+.08	+.10
24	−.002	−.005	+.027	+.098	+.02	+.01	+.08	+.10

Notes Relating to the Catalogue.

The subjoined catalogue of 627 stars is divided into three sections, the limits of which are indicated in the respective captions. The selection of these stars was made with the idea that these would be best adapted to serve as a connecting link between the various catalogues of observation. In general, they are the stars whose positions can be computed for the early part of the nineteenth century with the greatest certainty. Suitability for this purpose rather than distribution, or previous use as standard stars, governed the choice. At the lower limit of precision it is doubtless true that other stars might have been introduced that would have been better adapted to the intended use than some stars which have been admitted. But it is believed that the number of these is not very great. Between the limits of −22° and −37°, however, some stars, otherwise suitable, have been omitted pending the definitive reduction of the Albany right-ascensions for 1898.

When the epoch of the position in the catalogue is much earlier than the general mean, it is an indication that a great improvement in the star as a standard would be effected by repeated modern observations; so that, by means of such observations, some of the stars for which the mean date of observation is now in the sixties could easily be placed in a relatively higher class than they now occupy.

One of the most essential qualifications of a standard star is the certainty with which its position can be predicted for future epochs. The observations needed for the present epoch are within the control of astronomers; but those of past epochs can only be improved thro[illegible] of the older catalogues. It would therefore seem to be the part of wisdom to select, for the increase of our list of standards, those stars which have been well observed in the first sixty years of the nineteenth century, irrespective of the attention which they have received since that time.

The force of this is all the greater on account of the prevalent idea in regard to the supposed advantages, more imaginary than real, in adherence for a long term of years to the use of some one standard catalogue. If our standard catalogues were to be revised as often as they should be, the necessity for high weight as to the adopted proper motions, though still important, would not be of such vital consequence.

The names of several well known stars will be missed from the present collection. Nearly all of these are open to proof, or at least to well grounded suspicion, of periodic variation in proper motion. The duplicity of some of these stars also constitutes an objection to their use as standard stars. Among them are: *η Cassiopeae*, *α Can. Majoris*, *α Geminorum*, *α Can. min.*, *γ Virginis*, *ζ Bootis*, *ζ Herculis*, *61 Cygni*, *α Crucis*, and *α Centauri*. It is scarcely necessary to urge that these stars should still be included in observing lists where absolute determinations are intended, and also in those wherein differential determinations for the bright stars is the object in view.

From −20° to −40° of declination there is a rapid falling off in the precision with which the places of the principal stars are known; so that the mean weight of μ and μ' for far southern stars is scarcely one-fifth that for the northern. For this reason it appeared advisable to separate these southern from the northern stars in the catalogue.

It might also be remarked that the computed mean weight of $100\,\mu$ in the catalogue is never less than 0.22; so that, where the weight 0.2 is assigned the mean weight of $100\,\mu$ is about 0.23 for such stars.

It remains to explain the numbers printed in the catalogue, so far as this seems to be required.

No. The stars in the three divisions of the catalogue are numbered together according to their order in right-ascension.

Magnitude. The magnitudes are adopted from the Harvard Photometry. For the northern stars the magnitudes are mostly copied from Newcomb's *Fundamental Catalogue*, these having also been taken from the Harvard Photometry.

Sec. Var. The secular variations are computed from Professor Newcomb's constants contained in his recent work upon the *Precessional Constant*. They are practically identical with those computed from the constants of Struve and Peters. The secular variations in R.A. are given to the fourth decimal place, and in declination to the third.

μ *and* μ'. The values of μ and μ' correspond strictly to the epoch 1900, and are for R.A. in units of the fourth decimal; for declination in units of the third.

The catalogue was first constructed with the use of Struve's precessions throughout. Since these secular variations are virtually identical with those computed from Newcomb's constants, the result for annual variation has been the same as it would have been if Newcomb's precessions had been used from the first. Accordingly, precessions computed from Newcomb's constants have been subtracted from the annual variations as printed in the catalogue, resulting in the values of μ and μ' there given. This course was decided on, in view of the intended investigations of which this catalogue forms a part; and because the precessions of Newcomb offer a more consistent basis for correction than that which is afforded by the use of the so-called Struve constants. Moreover, from the results of my "*Tentative Researches upon Precession*," etc. (*A.J.* 501), making all due allowances for the uncertainty due to the imperfection and fragmentary nature of the material of observation employed, it seems to the writer probable that, in the interests of further and more comprehensive computations relating to the solar motion, preliminary values of the proper motions corresponding to Newcomb's precessions would offer a more convenient and consistent basis.

100 $\Delta\mu$ and 100 $\Delta\mu'$ express, respectively, the computed change of the proper motion in R.A. and declination for one century, under the assumption that stellar proper motion is uniform in the arc of a great circle. The unit is

CATALOGUE OF 627 STANDARD STARS.

First Section — (Declination, $+82°$ to $-21° 50'$).

No.	Name	Magnitude	R.A. 1900	Ann. V.	Sec. V.	μ	100 $\Delta\mu$	Ep. and Wt. T	p_α	p_δ	B—N $\Delta\alpha$	B—N $\Delta\mu$	B—A $\Delta\alpha$	B—A $\Delta\mu$
					.0001	.0001	.0001				.001	.0001	.001	.0001
		M	h m s	s	s	s	s				s	s	s	s
1	33 Piscium	4.6	0 0 13.010	+3.0709	— 14	— 13	0	70	32	1.3	—29	— 7	–	–
2	α Andromedae	2.1	0 3 13.022	3.0931	+ 185	+ 106	+ 1	66	160	7.1	— 8	— 2	+ 8	+ 7
3	β Cassiopeae	2.4	0 3 50.291	3.1765	+ 543	+ 675	—11	69	78	3.8	—51	— 4	+ 9	+ 9
5	γ Pegasi	2.9	0 8 5.123	3.0816	+ 102	0	0	68	170	6.9	—12	— 3	+20	+ 8
7	ι Ceti	3.8	0 11 19.981	3.0572	— 22	— 13	0	75	82	2.2	— 7	— 1	+32	+11
12	12 Ceti	6.2	0 24 56.127	+3.0612	+ 9	+ 3	0	75	95	1.9	—13	— 8	+43	+15
14	κ Cassiopeae	4.2	0 27 18.741	3.3777	+ 712	+ 17	0	67	61	2.4	—25	— 1	+27	+14
15	13 Ceti	5.2	0 30 6.043	3.0870	+ 14	+ 273	0	73	34	1.4	+ 7	+ 1	–	–
16	ζ Cassiopeae	3.8	0 31 23.796	3.3200	+ 197	+ 24	0	73	59	2.0	—61	—13	— 9	+ 5
17	π Andromedae	1.4	0 31 32.280	3.1939	+ 241	+ 17	0	76	41	1.3	— 5	— 2	+11	+ 9
18	ε Andromedae	4.6	0 33 16.166	+3.1612	+ 208	— 173	— 1	73	55	1.6	—15	— 1	+16	+10
19	δ Andromedae	3.5	0 33 58.726	3.1985	+ 224	+ 107	+ 1	75	41	1.4	—11	— 3	+15	+ 8
20	α Cassiopeae	2.4	0 34 49.739	3.3783	+ 561	+ 61	+ 1	66	122	5.5	—18	— 2	—11	+ 5
21	β Ceti	2.2	0 38 34.215	3.0133	— 54	+ 160	— 1	69	121	3.6	—11	0	+40	+16
22	ζ Andromedae	4.3	0 42 2.191	3.1722	+ 179	— 73	0	75	45	1.4	— 4	+ 1	+16	+ 9
23	δ Piscium	4.6	0 43 29.603	+3.1090	+ 80	+ 55	0	74	69	1.9	— 5	+ 1	+36	+15
24	20 Ceti	4.9	0 47 53.796	3.0638	+ 36	— 4	0	71	36	1.6	+ 8	+ 1	–	–
25	γ Cassiopeae	2.3	0 50 40.139	3.5876	+ 723	+ 41	+ 1	73	83	2.7	— 7	+ 5	+ 9	+10
26	μ Andromedae	3.9	0 51 12.016	3.3161	+ 309	+ 128	+ 1	75	82	1.7	—19	— 4	+16	+14
29	ε Piscium	4.5	0 57 45.115	3.1099	+ 88	— 54	0	71	141	3.1	— 4	0	+41	+13
30	μ Cassiopeae	5.2	1 1 36.828	+3.9608	+ 661	+3921	+39	63	34	2.0	+36	+ 9	–	–
32	80 Piscium	5.7	1 3 13.049	3.0868	+ 78	— 182	— 1	65	26	1.0	+ 5	— 2	–	–
33	η Ceti	3.6	1 3 33.548	3.0172	0	+ 141	0	73	35	1.1	—17	— 3	+53	+16
34	β Andromedae	2.1	1 4 7.837	3.3465	+ 289	+ 148	+ 1	75	118	3.9	+ 6	+ 1	— 3	+ 6
35	ζ Piscium	5.4	1 8 30.321	3.1299	+ 91	+ 89	0	71	35	1.4	—44	— 6	–	–
37	θ' Ceti	3.8	1 19 1.492	+2.9978	+ 18	— 54	0	70	136	4.0	+ 8	+ 3	+30	+11
38	δ Cassiopeae	2.8	1 19 16.186	3.8881	+ 793	+ 399	+ 6	67	63	3.7	—34	— 6	+20	+10
41	μ Piscium	5.2	1 24 56.668	3.1394	+ 91	+ 194	0	62	28	1.3	—19	— 5	–	–
42	η Piscium	3.7	1 26 7.861	3.2040	+ 142	+ 18	0	74	128	2.7	+ 9	+ 3	+26	+12
44	υ Persei	3.7	1 31 51.041	3.6600	+ 486	+ 61	0	70	67	2.9	—20	— 3	+ 3	+ 8

the fourth decimal for 100 $\Delta\mu$, and the third decimal for 100 $\Delta\mu'$.

Ep. and Wt. T is the mean epoch by weight of all the observations of the star, and p_α and p_δ the weights, respectively, of the R.A. and declination at those epochs. p_μ and p_μ' are, respectively, the weights of the computed centennial motions, 100μ and $100\mu'$. The probable error of the unit of weight is intended to be, $\pm0''.30 \sec\delta$, and $\pm0''.30$, in R.A. and declination respectively. If the weight, p', be desired for any epoch, T', we shall have:

$$p_\alpha' = \frac{p_\alpha \times p_\mu\left(\frac{100}{T'-T}\right)^2}{p_\alpha + p_\mu\left(\frac{100}{T'-T}\right)^2}$$

and correspondingly for the declinations.

B – N and B – A. These signify, respectively, the individual comparisons with the catalogues of Newcomb and Auwers, from which the foregoing tables of comparisons have been constructed. The unit for $\Delta\alpha$ is the third decimal; for $\Delta\mu$, the fourth; for $\Delta\delta$, the second; and for $\Delta\mu'$, the third. In the first section of the catalogue the comparisons are invariably with A_n; and south of $-22°$ always with A_s.

Explanation of the manner in which the right-ascensions and declinations of the catalogue were formed, together with tables of adopted weights and systematic corrections for the catalogues of observation, are to appear in later sections of this paper.

CATALOGUE OF 627 STANDARD STARS.

First Section. — (Declination, +82° to –21° 50′).

No.	Decl. 1900			Ann. V. and Sec. V.		μ' and 100 $\Delta\mu'$		Ep. and Wt.			B — N		B — A	
					.001	.001	.001	T	p_δ	$p_{\mu'}$	$\Delta\delta$ .01	$\Delta\mu'$ .001	$\Delta\delta$ .01	$\Delta\mu'$ .001
	°	′	″	″	″	″	″							
1	– 6	16	1.22	+20.137	– 9	+ 90	0	67	35	1.9	– 13	0	–	–
2	+28	32	17.86	19.884	– 15	– 161	0	64	169	7.6	– 15	+ 2	–14	– 9
3	+58	35	53.45	19.863	– 17	– 181	0	69	80	4.8	– 17	– 1	+25	+ 6
5	+14	37	39.16	20.021	– 24	– 13	0	66	165	7.1	– 31	– 4	–24	–14
7	– 9	22	12.03	19.976	– 36	– 32	0	77	71	2.4	– 28	– 2	+ 6	–11
12	– 4	30	35.61	+19.921	– 57	– 7	0	76	85	2.0	– 27	– 7	– 3	–10
14	+62	22	47.67	19.905	– 67	0	0	69	57	3.2	+ 5	+ 3	+30	+ 5
15	– 4	8	36.16	19.855	– 68	– 19	0	71	34	1.5	– 35	– 2	–	–
16	+53	20	47.65	19.852	– 75	– 7	0	72	54	2.3	– 5	0	+17	+ 3
17	+33	10	7.61	19.818	– 72	– 9	0	75	43	1.6	– 37	–10	–18	–19
18	+28	46	7.60	+19.588	– 75	– 248	0	71	11	1.1	– 15	+ 6	– 3	– 4
19	+30	18	49.51	19.741	– 78	86	0	73	46	1.9	+ 27	+11	– 11	–11
20	+55	59	20.15	19.785	– 83	– 31	0	63	140	7.6	– 5	+ 1	+23	+ 2
21	–18	32	7.79	19.803	– 82	+ 39	0	68	100	3.4	– 37	– 2	+37	– 8
22	+23	43	23.35	19.631	– 92	– 80	0	73	56	2.1	– 17	– 1	–17	–13
23	+ 7	2	27.00	+19.643	– 94	– 41	0	75	75	2.2	– 19	0	– 4	– 8
24	– 1	41	14.41	19.595	–101	– 16	0	66	41	2.0	– 79	–13	–	
25	+60	10	31.00	19.557	–123	– 2	0	69	78	3.2	0	+ 3	+42	+11
26	+37	57	24.85	19.576	–116	+ 27	0	71	69	2.1	– 28	– 3	–44	–19
29	+ 7	21	6.33	19.443	–121	+ 29	0	72	134	3.8	12	+ 3	–16	–12
30	+54	25	17.33	+17.771	–185	1556	–21	63	27	1.5	50	4		–
32	+ 5	7	11.19	19.106	–1 0	– 183	+ 1	62	37	1.4	93	12		
33	–10	42	11.15	19.118	–129	133	– 1	77	37	1.8	31	– 7	+22	– 41
34	+35	5	25.40	19.152	141	115	– 1	74	95	4.3	11	+ 2	38	–15
36	+ 7	2	17.50	19.106	–143	52	0	65	38	2.0	27	0		
37	– 8	11	57.79	+18.654	–156	213	0	69	119	4.0	25	+ 2	– 1	7
38	+59	12	56.03	18.811	–202	46	2	63	52	3.4	54	9	+17	5
41	+ 5	37	41.94	18.611	175	11	0	59	37	1.7	111	–18		
42	+14	49	18.99	18.638	–180	10	0	72	118	3.5	37	7	46	13
44	+18	7	17.80	18.346	–218	112	0	72	77	3.5	2	+ 7	+20	1

FIRST SECTION (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900			Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B − N		B − A	
							.0001	.0001	.0001	T	p_1	p_2	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		M.	h	m	s	s	s	s	s				s	s	s	s
46	ν Piscium	4.7	1	36	13.587	+3.4484	+ 91	14	0	75	100	2.2	− 5	0	+30	+10
47	φ Persei	4.2	1	37	23.346	3.7361	+ 532	+ 28	0	72	50	1.8	22	3	− 2	+ 8
48	τ Ceti	3.7	1	39	25.348	2.7868	+ 9	1195	+ 6	73	33	1.4	+24	+ 4	+37	+15
49	ο Piscium	4.5	1	40	6.713	3.1630	+ 112	+ 46	0	76	115	2.6	11	3	+25	+ 7
52	ζ Ceti	3.9	1	46	31.450	2.9604	+ 24	+ 24	0	73	46	1.6	15	+ 4	+35	+10
53	ε Cassiopeae	3.5	1	47	14.751	+4.2699	+1004	+ 58	+ 1	68	69	3.6	9	+ 6	+22	+11
54	α Trianguli	3.6	1	47	22.733	3.4093	+ 249	+ 12	1	75	44	1.7	13	3	+10	+ 8
55	β Arietis	2.7	1	49	6.845	3.3059	+ 183	+ 67	0	73	134	3.6	+ 9	+ 3	+19	+ 9
57	50 Cassiopeae	4.4	1	54	53.214	5.0333	+1894	83	2	71	84	3.0	+14	+ 8	+39	+17
59	α Piscium	3.9	1	56	52.339	3.1014	+ 84	+ 28	0	67	33	1.5	+26	+ 4	–	–
60	γ Andromedae	2.2	1	57	45.481	+3.6649	+ 394	+ 42	0	70	92	3.9	−12	− 4	+ 4	+10
61	α Arietis	2.2	2	1	32.048	3.3728	+ 201	+ 137	0	67	173	7.2	10	2	+11	+ 8
62	β Trianguli	3.1	2	3	35.138	3.5566	+ 305	+ 123	0	73	56	2.3	−16	− 3	+ 7	+10
63	65 Ceti	4.5	2	7	41.905	3.4747	+ 416	− 17	0	74	53	1.5	10	− 5	–	–
65	ο Ceti	Var.	2	14	17.649	3.0280	+ 63	0	− 1	74	57	1.8	−13	− 2	+41	+12
68	ι Cassiopeae	4.6	2	20	49.279	+4.8828	+1322	− 5	0	70	76	1.7	−27	− 2	+36	+17
69	ξ Ceti	4.3	2	22	50.461	3.4817	+ 116	+ 26	0	75	122	2.9	0	0	+25	+ 9
70	δ Ceti	4.0	2	34	21.356	3.0715	+ 82	+ 7	0	76	72	2.0	−17	− 4	+33	+12
71	θ Persei	4.3	2	37	21.963	4.0744	+ 513	+ 341	+ 2	72	61	2.4	−38	−12	−10	+ 3
73	γ Ceti	3.6	2	38	7.087	3.1044	+ 93	− 98	− 1	68	111	3.0	+ 1	− 1	+32	+11
74	μ Ceti	4.4	2	39	32.096	+3.2375	+ 125	+ 188	0	74	57	1.9	− 2	0	+26	+11
75	η Persei	3.9	2	43	23.880	4.3460	+ 678	+ 27	0	75	52	1.1	−61	−14	− 7	+ 3
76	41 Arietis	3.7	2	44	5.730	3.5213	+ 227	+ 49	0	74	63	1.9	− 5	− 1	+ 5	+ 6
78	η Eridani	4.1	2	51	32.514	2.9289	+ 50	+ 54	− 1	75	59	2.3	−10	− 6	+48	+14
79	ϵ^m Arietis	4.6	2	53	29.529	3.4225	+ 184	− 11	0	72	46	1.8	+ 5	− 1	–	–
81	α Ceti	2.8	2	57	3.057	+3.1317	+ 97	− 9	0	68	163	6.6	−12	0	+34	+11
82	γ Persei	3.1	2	57	32.992	4.3183	+ 593	+ 2	0	75	52	1.4	−33	− 8	− 1	+ 5
84	ρ Persei	Var.	2	58	45.943	3.8301	+ 331	+ 114	0	73	50	1.4	−11	− 2	+14	+12
85	β Persei	Var.	3	1	39.571	3.8878	+ 355	+ 5	0	71	73	2.9	− 4	− 3	+ 4	+ 9
86	ι Persei	4.2	3	1	50.797	4.3062	+ 516	+1290	+10	72	43	1.4	−34	−11	− 1	+ 8
88	δ Arietis	4.6	3	5	54.552	+3.4232	+ 171	+ 106	0	72	84	2.6	−14	− 4	+19	+ 8
90	ζ Arietis	5.0	3	9	9.113	3.4412	+ 176	− 17	0	72	38	1.4	+ 5	+ 2	–	–
92	α Persei	1.9	3	17	10.800	4.2615	+ 482	+ 27	0	67	123	5.4	−15	− 3	+ 8	+ 9
93	ο Tauri	3.8	3	19	25.846	3.2239	+ 114	− 44	0	75	75	1.9	+ 2	+ 2	+27	+ 8
94	ξ Tauri	3.8	3	21	44.904	3.2466	+ 116	+ 40	0	71	50	1.5	−14	0	+31	+11
95	5 Tauri	4.3	3	25	21.047	+3.3066	+ 129	+ 11	0	76	71	1.8	18	− 5	+26	+10
96	ε Eridani	3.8	3	28	13.128	2.8247	+ 56	− 657	+ 1	76	95	2.7	+ 9	+ 3	+13	+15
99	δ Persei	3.2	3	35	48.117	4.2533	+ 414	+ 31	0	70	84	3.6	−11	− 4	− 5	+ 6
100	δ Eridani	3.7	3	38	27.439	2.8749	+ 62	− 63	+ 4	73	56	1.9	−21	− 2	+50	+15
101	17 Tauri	3.8	3	38	56.131	3.5550	+ 177	+ 14	0	69	46	1.3	− 6	− 2	+ 9	+ 6
103	η Tauri	3.0	3	41	32.303	+3.5588	+ 175	+ 14	0	70	134	4.2	−12	− 2	+13	+ 7
105	27 Tauri	3.8	3	43	12.866	3.5601	+ 174	+ 14	0	69	49	1.7	− 4	+ 2	+13	+ 8
107	ζ Persei	3.0	3	47	50.650	3.7621	+ 220	+ 9	0	75	66	1.8	+ 4	− 2	+ 2	+ 5
110	ε Persei	3.0	3	51	8.459	4.0140	+ 286	+ 22	0	71	51	2.4	−32	− 9	0	+ 8
111	γ^1 Eridani	3.3	3	53	21.806	2.7976	+ 45	+ 45	− 1	71	109	3.0	−15	− 2	+40	+11
112	λ Tauri	3.5	3	55	8.332	+3.3193	+ 114	− 3	0	75	59	1.6	−17	− 5	+25	+ 8
114	ν Tauri	4.0	3	57	50.148	3.1877	+ 91	+ 2	0	80	50	1.2	−19	− 6	+24	+ 6
115	37 Tauri	4.5	3	58	46.908	3.5407	+ 151	+ 67	0	73	44	1.5	−12	− 3	–	–
116	48 Persei	4.0	4	1	23.959	4.3406	+ 362	+ 32	0	75	49	1.4	−25	−11	0	+ 4
118	ο Eridani	4.1	4	6	59.019	2.9264	+ 59	+ 6	0	74	78	1.9	−12	0	+28	+ 8
119	40 Eridani	4.5	4	10	40.167	+2.7641	+ 16	−1484	−20	70	36	1.6	+ 7	− 1	–	–
123	γ Tauri	3.9	4	14	6.084	3.4097	+ 114	+ 82	0	74	96	2.6	− 9	− 2	+18	+ 7
124	δ^1 Tauri	3.9	4	17	9.998	3.4553	+ 118	+ 77	0	75	59	2.0	− 3	+ 2	+19	+ 8
127	ε Tauri	3.7	4	22	16.581	3.4988	+ 119	+ 81	0	72	115	3.5	− 6	− 1	+16	+ 7
128	α Tauri	1.1	4	30	10.893	3.4385	+ 102	+ 48	− 1	66	178	7.5	+ 3	+ 1	+25	+ 7

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V.	μ′	100 Δμ′	T	p	p_δ	B — N Δδ	B — N Δα	B — A Δδ	B — A Δα
	° ′ ″	″	.001	″ .001	.001				.01	.001	.01	.001
46	+ 4 58 53.61	+18.307	−193	+ 1	0	73	91	2.9	34	− 2	7	10
47	+50 11 5.96	18.248	−233	− 16	0	71	63	2.3	25	1	+19	0
48	−16 27 50.72	19.046	−171	+856	+ 8	74	36	1.4	33	2	+18	5
49	+ 8 39 16.09	18.215	−203	+ 50	0	74	105	3.2	10	+ 6	11	9
52	−10 49 14.63	17.889	−204	− 31	0	77	36	1.4	31	4	−14	11
53	+63 10 39.51	+17.877	−289	− 17	0	66	61	3.5	17	− 2	+ 9	+ 5
54	+29 5 30.08	17.654	−232	−232	0	74	48	1.8	23	− 2	9	11
55	+20 19 9.46	17.706	−229	−111	0	73	125	3.8	20	1	30	13
57	+71 56 14.85	17.603	−359	+ 23	+ 1	75	78	2.8	+ 3	+ 3	+20	+ 6
59	+ 2 16 50.66	17.490	−228	− 6	0	67	33	2.0	18	0	—	—
60	+41 50 59.62	+17.406	−270	− 52	0	68	89	4.8	30	2	9	9
61	+22 59 22.60	17.148	−257	−146	− 1	66	177	7.7	22	− 2	13	−10
62	+34 30 51.47	17.156	−275	− 46	− 1	73	56	2.0	23	− 2	18	13
63	+ 8 22 39.64	17.008	−252	− 7	0	70	57	2.3	+ 16	+10	—	—
65	− 3 25 54.03	16.466	−251	−237	0	76	48	4.5	− 51	8	+ 5	−12
68	+66 57 10.57	+16.394	−117	+ 14	0	69	66	2.0	+ 10	+ 4	+27	+10
69	+ 8 0 42.79	16.274	−278	− 1	0	75	112	3.3	20	+ 3	1	11
70	− 0 6 10.43	15.670	−286	+ 1	0	79	57	2.1	34	4	+ 6	8
71	+48 18 19.94	15.115	−387	− 89	− 3	70	68	4.2	29	2	+ 2	3
73	+ 2 48 51.79	15.312	−294	−150	+ 1	64	103	3.7	30	+ 4	20	11
74	+ 9 44 31.10	+15.356	−311	27	− 2	72	60	2.5	35	2	21	12
75	+55 28 49.84	15.151	−421	13	0	75	50	2.3	28	1	−10	+ 3
76	+26 50 51.12	15.011	−344	−113	0	74	64	2.0	32	2	10	12
78	− 9 17 45.92	14.474	−298	215	− 1	75	51	2.5	17	− 2	−25	5
79	+20 56 25.16	14.564	−349	− 8	0	70	48	2.1	16	+ 2		—
81	+ 3 44 50.84	+14.279	−325	− 78	0	67	159	6.7	20	+ 1	6	10
82	+53 6 53.50	14.320	−447	− 6	0	76	58	2.8	42	− 2	− 9	− 4
84	+38 27 10.22	14.143	−400	−108	− 1	79	46	1.4	+ 2	+ 6	28	20
85	+40 34 13.50	14.067	−410	− 5	0	68	75	3.6	32	3	+12	3
86	+49 13 52.69	13.980	−468	− 80	−13	75	53	2.4	+ 62	+18	+13	+ 1
88	+19 20 51.73	+13.799	−369	6	1	71	89	2.8	29	− 7	36	19
90	+20 40 26.05	13.523	−374	75	0	68	51	2.2	+ 2	+ 7		—
92	+49 30 19.18	13.046	477	28	0	65	150	7.9	− 18	0	+ 9	− 2
93	+ 8 40 37.08	12.846	364	78	0	74	62	2.2	32	− 1	10	−12
94	+ 9 23 2.57	12.727	−371	− 41	0	71	53	1.8	+ 1	+ 5	5	− 7
95	+12 35 38.43	+12.520	382	4	0	74	64	2.2	29	− 6	+ 2	− 7
96	− 9 47 48.19	12.340	−323	+ 13	+ 8	77	81	2.4	46	−12	+16	−12
99	+47 28 4.36	11.764	−507	33	0	70	96	4.9	11	+ 3	+32	+ 4
100	−10 6 6.67	12.352	346	+743	+ 1	70	43	1.9	+ 16	+13	10	−22
101	+23 47 56.22	11.525	428	49	0	70	45	1.9	16	+ 1	25	−14
103	+23 47 15.44	+11.340	432	48	0	67	131	5.0	15	+ 2	3	6
105	+23 44 51.52	11.217	434	50	0	70	46	1.4	28	2	19	13
107	+31 35 12.02	10.913	465	17	0	74	63	2.0	32	2	26	15
110	+39 13 15.43	10.656	500	31	0	74	47	2.0	30	4	14	14
111	−13 47 34.66	10.410	352	112	1	70	114	1.5	25	2	−15	14
112	+12 42 28.12	+10.375	−448	14	0	76	54	1.5	24	3	4	10
114	+ 5 42 12.80	10.180	405	7	0	84	41	1.1	21	2	4	5
115	+24 48 31.23	10.052	451	63	1	69	43	2.1	40	6		—
116	+47 26 44.06	9.886	555	34	0	75	42	1.8	8	+ 1	+18	+ 2
118	− 7 5 53.89	9.571	380	− 84	0	75	63	1.6	40	5	2	10
119	7 48 30.67	+ 5.768	342	3437	+19	71	43	1.7	32	2		
123	+15 23 10.26	8.910	−450	27	1	72	93	3.4	22	1	4	9
124	+17 18 28.76	8.662	−458	34	1	73	54	2.3	29	4	21	9
127	+18 57 31.22	8.213	−470	38	1	74	106	3.0	19	4	11	9
128	+16 18 29.77	7.466	467	194	1	64	184	7.7	26	2	17	12

FIRST SECTION (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900			Ann. V. and Sec. V.		μ and 100 $\Delta\mu$		Ep. and Wt.			B − N		B − A	
							.0001	.0001	.0001	T	p_α	p_δ	$\Delta\alpha$ .001	$\Delta\mu$ .0001	$\Delta\alpha$ .001	$\Delta\mu$ .0001
		M	h	m	s	s		s	s				s	s	s	s
130	τ Tauri	4.3	4	36	14.523	+3.5965	+ 120	+ 1	0	74	53	2.0	− 6	− 3	+15	+ 7
133	μ Eridani	4.1	4	40	30.111	2.9981	+ 55	+ 13	0	75	76	2.0	7	+ 1	+32	+ 9
134	α Camelop.	4.4	4	44	6.327	5.9352	+ 677	+ 10	0	72	77	1.9	−66	−28	+36	+16
135	ι Aurigae	2.9	4	50	28.808	3.9016	+ 141	+ 7	0	74	102	2.2	+ 2	1	− 9	+ 2
136	β Camelop.	4.2	4	54	31.222	5.3204	+ 107	+ 3	0	75	64	1.4	+39	+ 8	+13	+ 5
137	ε Aurigae	3.2	4	54	17.487	+4.2975	+ 192	+ 4	0	66	54	2.8	−34	− 8	+ 5	+ 7
138	ζ Aurigae	3.9	4	55	29.189	4.1866	+ 172	+ 10	0	73	14	1.7	−10	− 1	+ 7	+10
139	ι Tauri	4.7	4	57	7.075	3.5829	+ 92	+ 52	0	68	37	1.7	− 2	1	+31	+ 8
140	η Aurigae	3.3	4	59	30.044	4.2004	+ 163	+ 26	1	69	57	2.3	29	−13	+ 4	+ 3
142	β Eridani	2.9	5	2	55.999	2.9482	+ 43	− 59	1	77	56	2.0	−17	3	+39	+12
143	λ Eridani	4.3	5	4	21.630	+2.8698	+ 40	+ 2	0	73	37	1.1	7	− 4	+33	+12
144	19 H Camelop.	5.4	5	6	4.177	9.8036	+2022	− 274	− 9	78	40	1.1	−55	− 1	+98	+51
145	α Aurigae	0.2	5	9	18.012	4.4259	+ 157	+ 82	− 5	64	129	6.3	−23	− 5	− 8	+ 6
146	β Orionis	0.3	5	9	13.897	2.8817	+ 39	+ 1	0	67	169	7.4	0	0	+30	+ 8
147	γ Orionis	1.7	5	19	16.028	3.2163	+ 16	− 1	0	74	58	2.8	0	+ 1	+30	+ 9
148	β Tauri	1.8	5	19	58.189	+3.7900	+ 77	+ 24	− 1	66	164	7.4	− 3	− 1	+ 3	+ 6
149	Gr. 966	6.3	5	26	20.993	7.9978	+ 704	− 10	+ 1	74	76	1.6	+ 3	− 7	+54	+15
150	δ Orionis	2.5	5	26	53.844	3.0638	+ 37	+ 1	0	69	145	4.9	− 5	+ 1	+11	+11
152	α Leporis	2.7	5	28	19.173	2.6451	+ 29	+ 2	0	68	80	2.4	−10	− 2	+25	+ 7
154	ε Orionis	1.7	5	31	8.335	3.0430	+ 34	0	0	69	129	4.4	− 4	0	+39	+ 9
155	ζ Tauri	3.0	5	31	40.074	+3.5839	+ 51	+ 3	0	72	58	2.0	− 8	− 3	+16	+ 3
156	ζ Orionis	1.9	5	35	42.767	3.0265	+ 31	+ 5	0	66	57	3.3	− 5	0	–	–
158	κ Orionis	2.2	5	43	0.811	2.8445	+ 26	+ 1	0	74	72	2.6	−15	0	+29	+ 8
160	α Orionis	0.9	5	49	45.463	3.2474	+ 26	+ 19	0	66	176	7.6	− 9	− 1	+31	+ 8
161	β Aurigae	2.1	5	52	11.607	4.4007	+ 36	− 44	0	69	74	2.8	−30	− 6	− 3	+ 5
162	θ Aurigae	2.7	5	52	54.122	+4.0909	+ 28	+ 44	− 1	75	55	1.6	− 8	− 3	− 4	+ 4
165	η Geminorum	3.5	6	8	50.189	3.6221	+ 5	− 44	0	71	86	2.3	−24	− 5	+ 6	+ 3
168	μ Geminorum	3.2	6	16	54.659	3.6306	− 7	+ 44	− 1	69	128	4.1	− 4	− 2	+ 8	+ 2
169	β Can. Maj.	2.0	6	18	17.747	2.6415	+ 16	− 4	0	73	58	1.9	− 3	+ 2	+24	+ 9
171	ν Geminorum	4.1	6	23	1.540	3.5630	− 11	− 6	0	77	40	1.4	+ 6	− 1	–	–
173	γ Geminorum	1.9	6	31	56.119	+3.4671	− 17	+ 31	0	74	136	3.3	− 3	− 2	+20	+ 6
175	ε Geminorum	3.2	6	37	46.812	3.6935	− 38	0	0	72	71	3.1	+ 5	+ 1	+12	+ 6
176	ξ Geminorum	3.4	6	39	40.630	3.3686	− 21	− 77	− 1	77	63	2.1	− 1	− 1	+16	+ 5
182	ζ Geminorum	4.0	6	58	10.711	3.5613	− 52	− 3	0	73	91	2.7	0	− 1	+11	+ 3
183	γ Can. Maj.	4.1	6	59	14.060	2.7144	+ 4	0	0	71	73	1.6	− 9	− 3	+11	0
186	λ Geminorum	3.6	7	12	20.797	+3.4507	− 57	− 33	0	77	79	2.6	−14	− 4	+16	+ 4
188	δ Geminorum	3.6	7	14	9.097	3.5873	− 75	− 13	0	69	133	3.6	−10	− 3	+15	+ 4
190	ι Geminorum	3.9	7	19	31.006	3.7319	− 104	− 86	0	73	61	2.6	− 4	− 1	+ 5	+ 4
192	Gr. 1308	5.8	7	20	28.648	6.2855	− 855	− 5	− 2	72	61	1.2	−94	−24	+ 7	+ 5
193	β Can. min.	3.1	7	21	43.693	3.2559	43	− 34	0	76	98	2.6	− 9	− 2	+19	+ 5
195	κ Geminorum	3.7	7	38	24.696	+3.6278	− 111	− 18	0	74	54	1.5	−16	− 4	+18	+ 7
196	β Geminorum	1.2	7	39	11.869	3.6776	− 128	− 471	+ 1	66	167	7.7	+ 2	− 1	+15	+ 5
199	φ Geminorum	5.0	7	47	22.708	3.6785	− 132	− 21	0	68	30	1.4	−10	− 1	–	–
202	χ Geminorum	5.1	7	57	22.664	3.6921	− 150	− 16	0	75	76	1.6	−23	− 5	0	+ 3
206	β Cancri	3.7	8	11	5.565	3.2568	− 72	− 35	0	76	112	2.7	+ 1	0	+18	+ 3
209	ο Ursae Maj.	3.5	8	21	57.602	+5.0230	− 768	− 167	− 2	70	78	3.6	−36	− 7	+18	+10
210	η Cancri	5.5	8	26	55.635	3.4762	− 132	− 26	0	74	85	1.5	+ 5	− 1	+ 6	+ 2
212	γ Cancri	4.8	8	37	30.025	3.4788	− 143	− 73	0	72	51	1.9	− 7	− 2	–	–
213	δ Cancri	4.2	8	39	0.194	3.4151	− 128	− 12	− 1	72	75	2.6	−13	− 3	+10	+ 5
214	ε Hydrae	3.5	8	41	28.871	3.1808	− 71	− 127	0	72	138	3.9	− 6	0	+33	+ 8
216	ι Ursae Maj.	3.1	8	52	21.821	+4.1292	− 445	− 437	+ 1	67	96	3.4	− 2	− 2	+ 2	+ 5
217	α² Cancri	4.3	8	53	1.148	3.2861	− 98	+ 25	0	73	82	2.4	+ 3	+ 1	+28	+ 8
219	κ Ursae Maj.	3.7	8	56	48.012	4.1169	− 434	− 30	0	73	60	1.8	−26	− 3	− 1	+ 8
220	σ² Ursae Maj.	4.8	9	1	35.984	5.3420	−1334	− 4	− 2	71	41	1.6	−35	− 1	+23	+17
221	κ Cancri	5.2	9	2	19.913	3.2539	− 94	− 14	0	75	79	2.0	− 5	− 2	–	–

FIRST SECTION — Declination, +82° to −21° 50′.

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. T	Wt. p	Wt. p_α	B − N Δδ .01	B − N Δα .001	B − A Δδ .01	B − A Δα .001	
130	+22 45 54.31	+ 7.141	−493	− 23	0	72	54	2.4	− 28	4	− 13	10	
133	− 3 26 16.50	6.805	−414	10	0	78	60	1.6	− 29	1	+17	− 4	
134	+66 10 22.51	6.523	−821	+ 5	0	72	96	3.4	6	0	+ 9	− 3	F 49
135	+33 0 27.84	5.961	−546	27	0	72	92	2.6	− 41	− 6	23	13	
136	+60 17 46.19	5.637	−746	13	0	72	58	2.3	− 42	2	+17	+ 4	F 10
137	+43 10 31.33	+ 5.612	−604	15	0	64	58	3.1	21	2	+ 4	− 3	
138	+40 55 47.52	5.539	−589	− 30	0	71	38	1.7	17	8	25	−13	
139	+21 26 49.70	5.383	−506	19	1	67	36	1.7	− 4	0	− 3	13	
140	+44 5 57.36	5.456	−594	75	0	68	61	2.6	44	3	−44	−10	
142	− 5 12 56.44	4.861	448	79	+ 1	80	58	2.2	30	5	+13	6	
143	− 8 52 56.39	+ 4.811	−408	8	0	74	30	1.1	23	0	7	11	
144	+79 6 59.13	4.828	−1388	+ 154	+ 4	78	15	1.2	2	0	+ 7	+ 8	
145	+45 53 16.95	3.970	633	429	− 1	63	151	7.7	14	0	+24	1	
146	− 8 49 1.67	4.361	−412	− 1	0	67	150	6.7	27	+ 1	+ 8	− 9	
147	+ 6 15 32.76	3.482	−463	− 19	0	75	65	3.2	25	− 2	+ 9	− 6	
148	+28 31 22.91	+ 3.307	−546	− 177	0	64	163	7.0	18	0	14	− 9	
149	+74 58 40.00	2.955	−1155	+ 22	0	76	61	1.4	+ 5	+ 5	+31	+12	74 B Cam
150	− 0 22 23.36	2.882	−413	− 3	0	67	128	4.8	30	− 1	10	−11	
152	−17 53 37.76	2.765	−383	+ 3	0	66	64	2.5	11	+ 3	+41	− 8	
154	− 4 15 56.76	2.516	−441	2	0	69	106	3.9	37	− 3	+ 5	− 4	
155	+21 4 53.63	+ 2.444	−520	28	0	71	59	2.3	1	+ 4	−10	− 7	
156	− 1 59 43.72	2.143	−440	7	0	68	51	2.9	7	+ 7	−	−	
158	− 9 42 18.48	1.479	−414	− 5	0	78	64	2.1	24	− 2	− 5	−10	
160	+ 7 23 18.41	0.904	−474	+ 8	0	64	169	7.2	21	− 1	−48	−12	
161	+44 56 14.48	0.678	−641	− 5	0	67	92	4.9	− 16	+ 1	+44	+ 1	
162	+37 12 20.49	+ 0.531	−597	90	0	73	56	2.3	22	+ 1	19	10	
165	+22 32 9.01	− 0.790	−527	− 17	0	70	79	2.8	9	0	−38	−14	
168	+22 33 53.90	1.591	−528	113	0	67	128	4.6	23	+ 1	−47	− 9	
169	−17 54 22.18	1.599	−383	0	0	71	41	1.6	47	− 4	−18	−10	
174	+20 16 34.57	2.032	516	21	0	73	38	2.1	− 38	− 6		−	
173	+16 29 4.78	− 2.831	500	47	0	72	127	4.2	16	+ 1	−22	−10	
175	+25 13 48.18	3.310	530	20	0	69	78	3.7	47	− 2	36	13	
176	+13 0 12.40	3.653	481	200	+ 1	77	59	1.6	34	− 7	− 9	8	
182	+20 43 1.27	5.042	500	8	0	69	87	3.1	21	0	35	13	
183	−15 29 7.77	5.438	−380	14	0	73	49	1.4	30	5	−44	−12	
186	+16 43 14.83	− 6.272	475	18	0	75	78	2.9	23	− 3	30	15	
188	+22 9 59.39	6.390	493	16	0	65	127	4.5	27	2	37	15	
190	+27 59 18.65	6.907	508	90	+ 1	73	70	2.8	26	3	46	17	
192	+68 40 12.23	6.939	858	43	0	76	63	1.5	+ 14	+ 2	+24	+ 8	145 B Cam
193	+ 8 29 27.19	7.040	444	12	0	76	93	2.7	14	+ 5	23	13	
195	+24 38 16.07	8.408	477	62	0	70	65	2.6	22	2	39	16	
196	+28 46 4.04	8.466	477	58	+ 6	64	165	7.5	20	2	23	12	
199	+27 1 28.75	9.089	474	36	0	66	38	1.6	13	10			
202	+28 4 29.04	9.876	465	52	0	75	70	1.8	15	+ 1	34	15	6 Cancri
206	+ 9 29 37.51	10.906	394	54	0	74	104	3.3	33	2	25	12	
209	+61 3 9.21	11.752	589	144	+ 2	70	90	1.5	18	1	+ 3	+ 3	
210	+20 46 51.26	12.044	402	54	0	72	74	2.2	20	0	40	15	
212	+24 49 41.23	12.768	385	50	+ 1	70	53	2.3	13	7			
213	+18 34 18.84	13.058	377	239	0	67	75	3.2	20	+ 1	38	15	
214	+ 6 47 8.64	13.038	346	53	+ 1	74	130	4.0	38	1	16	9	
216	+48 26 3.76	−13.944	429	− 249	+ 5	66	117	5.6	− 15	0	+ 3	3	
217	+12 14 41.57	13.777	343	40	0	73	78	2.6	9	+ 2	25	15	
219	+47 33 7.50	14.042	424	66	0	74	69	2.9	15	+ 1	1	4	
220	+67 32 26.09	14.344	−544	70	0	70	44	1.7	0	4	1	+ 4	
221	+44 4 14.54	14.330	−326	44	0	74	75	2.7	5	+ 2			

FIRST SECTION (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V.	Sec. V. .0001	μ .0001	100 Δμ .0001	Ep. T	p₁	p₂	B — N Δα .001	B — N Δμ .0001	B — A Δα .001	B — A Δμ .0001
		m	h m s	s	s	s	s				s	s	s	s
223	θ Hydrae	3.8	9 9 9.738	+3.1244	60	+ 87	1	76	68	2.3	11	− 1	+ 35	+ 7
226	83 Cancri	6.6	9 13 24.092	3.3553	135	78	0	74	81	1.5	11	− 3	+ 19	+ 7
228	α Lyncis	3.4	9 14 57.897	3.6676	265	176	+ 1	72	64	2.6	+ 1	+ 1	+ 11	+11
231	α Hydrae	2.2	9 22 40.117	2.9488	13	14	0	67	171	7.0	5	0	+ 29	+ 8
232	4 H Draconis	4.5	9 22 54.276	8.9179	7755	13	3	73	89	2.9	+20	+16	+121	+47
233	θ Ursae Maj.	3.3	9 26 10.340	+4.0388	550	1025	+ 5	68	96	3.8	+ 2	+ 2	+ 8	+ 7
235	ο Leonis	3.8	9 35 48.869	3.2061	92	98	0	72	98	2.9	6	− 3	+ 26	+ 5
236	ε Leonis	3.4	9 40 10.590	3.4138	179	32	0	70	136	3.8	+ 3	+ 2	+ 10	+ 6
237	υ Ursae Maj.	3.9	9 43 52.979	4.3046	809	379	+ 3	71	69	2.5	0	+ 4	+ 20	+10
239	μ Leonis	4.1	9 47 4.673	3.4208	195	163	0	73	96	2.9	+30	+ 8	+ 14	+ 6
240	ν Leonis	5.2	9 52 50.676	+3.2318	105	21	0	67	21	1.2	+51	+ 8	–	–
242	π Leonis	4.8	9 54 55.787	3.1739	80	24	0	74	99	2.7	+ 4	+ 5	+ 20	+ 6
243	η Leonis	3.6	10 1 52.916	3.2768	129	1	0	75	57	2.1	+70	+22	+ 22	+ 8
244	α Leonis	1.3	10 3 2.838	3.1997	100	− 168	0	67	180	7.7	3	0	+ 19	+ 5
245	λ Hydrae	3.9	10 5 42.782	2.9244	+ 14	137	0	72	43	1.7	4	− 1	+ 26	+ 7
247	λ Ursae Maj.	3.6	10 11 4.094	+3.6361	− 381	148	+ 1	70	71	3.0	33	− 7	+ 6	+ 8
249	γ¹ Leonis	2.3	10 14 27.628	3.3141	− 150	+ 215	1	70	116	3.8	+ 3	+ 3	–	–
250	μ Ursae Maj.	3.2	10 16 22.431	3.5908	− 358	73	+ 1	74	68	2.2	−15	− 5	+ 4	+ 5
252	μ Hydrae	4.1	10 21 15.233	2.9000	+ 40	88	0	75	56	1.6	+ 1	+ 1	+ 30	+ 7
253	β Leonis min.	4.4	10 22 6.175	3.4830	− 295	99	0	72	32	1.8	−25	− 5	+ 2	+ 9
256	9 H Draconis	5.0	10 26 36.313	+5.2255	2725	79	1	73	70	1.9	1	+ 4	+ 80	+25
257	ρ Leonis	3.8	10 27 32.795	3.1627	− 79	4	0	72	121	2.7	− 8	− 1	+ 27	+ 6
258	34 Sextantis	6.6	10 37 27.679	3.1002	− 44	59	0	69	35	1.3	18	0	–	–
262	53 Leonis	5.3	10 44 0.116	3.1574	− 80	− 1	0	75	101	2.2	2	− 2	+ 27	+ 6
263	ν Hydrae	3.3	10 44 41.422	2.9578	+ 54	+ 65	+ 1	70	37	1.6	+15	+ 4	+ 32	+10
264	46 Leonis min.	3.9	10 47 43.269	+3.3673	− 257	+ 73	1	74	55	1.6	+ 4	− 1	+ 2	+ 4
266	α Crateris	4.1	10 54 54.104	2.9197	+ 67	− 326	0	77	25	1.0	+11	0	–	–
267	58 Leonis	5.0	10 55 23.794	3.0999	− 37	+ 6	0	74	47	1.4	+ 4	+ 2	–	–
268	β Ursae Maj.	2.4	10 55 48.598	3.6493	− 624	+ 98	− 1	72	83	3.1	−37	− 7	− 11	+ 2
269	α Ursae Maj.	2.0	10 57 33.628	3.7402	− 804	− 169	+ 3	66	123	5.7	−30	− 4	+ 25	+12
270	χ Leonis	4.7	10 59 51.557	+3.0970	− 55	− 233	0	73	92	2.7	− 8	+ 1	+ 20	+ 6
271	ψ Ursae Maj.	3.2	11 4 2.623	3.3904	− 364	− 55	0	71	86	3.8	−28	− 2	+ 4	+11
273	δ Leonis	2.6	11 8 47.474	3.1970	− 132	+ 106	− 1	70	142	4.5	−16	− 2	+ 12	+ 4
274	θ Leonis	3.4	11 8 59.601	3.1523	− 98	− 45	0	77	44	1.2	+10	+ 4	+ 16	+ 5
275	ν Ursae Maj.	3.7	11 13 4.746	3.2513	− 225	− 18	0	72	46	2.1	−11	0	+ 1	+ 6
276	δ Crateris	3.8	11 14 20.439	+2.9967	+ 65	− 85	0	69	117	3.4	+12	+ 3	+ 44	+13
277	σ Leonis	4.1	11 15 58.835	3.0955	− 40	− 63	0	74	81	2.6	− 6	− 1	+ 29	+ 9
279	ι Leonis	4.0	11 18 42.709	3.1297	− 64	+ 105	0	72	57	2.2	+11	+ 1	+ 22	+ 5
280	γ Crateris	4.2	11 19 53.121	2.9931	+ 83	− 75	0	73	25	1.0	− 6	− 7	+ 27	+ 3
281	83 Leonis	6.1	11 21 41.645	3.0380	20	− 482	0	69	27	1.5	+14	+11	–	–
282	τ Leonis	5.2	11 22 47.704	+3.0866	− 20	+ 14	0	71	65	1.9	+ 8	+ 6	–	–
283	λ Draconis	4.1	11 25 28.345	3.6126	−1093	− 74	+ 2	70	93	3.8	−26	− 1	+ 37	+19
286	υ Leonis	4.5	11 31 49.728	3.0717	+ 4	+ 1	0	74	108	2.3	+ 3	+ 2	+ 41	+10
287	ν Virginis	4.2	11 40 43.189	3.0851	− 30	− 12	0	62	24	1.3	− 8	+ 2	–	–
288	χ Ursae Maj.	3.8	11 40 46.309	3.1845	− 352	− 137	+ 1	71	73	2.9	−40	− 8	0	+ 5
289	β Leonis	2.2	11 43 57.574	+3.0634	− 71	− 342	+ 1	67	169	7.2	−10	− 1	+ 26	+ 8
290	β Virginis	3.8	11 45 29.182	3.1253	− 1	+ 495	0	70	121	5.5	− 1	+ 1	+ 40	+11
291	γ Ursae Maj.	2.5	11 48 34.376	3.1756	− 431	+ 107	− 2	66	125	5.6	−40	− 8	− 5	+ 5
292	π Virginis	4.6	11 55 44.932	3.0752	− 21	− 3	0	71	58	1.7	+16	+ 6	–	–
293	ο Virginis	4.2	12 0 6.938	3.0575	− 30	− 147	0	76	95	2.6	+ 3	0	+ 30	+ 7
298	4 H Draconis	5.1	12 7 31.107	+2.8662	1209	+ 32	− 1	71	76	2.6	− 1	+ 6	+ 53	+16
300	δ Ursae Maj.	3.4	12 10 28.746	2.9899	− 420	+ 138	− 2	69	67	2.8	−83	−12	− 2	+ 5
301	γ Corvi	2.7	12 10 39.738	3.0800	+ 116	− 113	0	72	42	1.4	+ 9	+ 1	+ 29	+10
304	η Virginis	4.0	12 14 47.375	3.0684	+ 28	− 41	0	73	141	3.4	−17	− 5	+ 40	+10
305	16 Virginis	5.1	12 15 16.234	3.0463	+ 8	− 200	0	69	15	0.8	–	–	–	–

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V.	μ'	100 $\Delta\mu'$	Ep. T	Wt. p	p	B−N $\Delta\delta$	B−N $\Delta\mu'$	B−A $\Delta\delta$	B−A $\Delta\mu$
	° ′ ″	″	″ .001	.001	.001				″ .01	.001	″ .01	.001
223	+ 2 41 10.39	−15.012	−301	312	1	78	73	2.5	32	0	20	11
226	+18 7 45.82	15.112	318	132	+ 1	73	71	1.5	+ 6	4	−26	14
228	+34 18 55.57	15.060	314	+ 10	+ 2	71	66	2.2	31	2	33	13
231	− 8 13 30.38	15.175	266	+ 31	0	66	163	7.0	37	2	+11	6
232	+81 16 6.91	15.541	816	− 25	0	80	96	2.6	0	+ 2	+ 1	+ 1
233	+52 7 59.46	16.247	−351	549	+ 9	68	103	5.0	27	1	+ 9	0
235	+10 20 50.27	16.248	−267	− 39	+ 1	71	100	1.1	18	6	29	11
236	+24 11 1.93	16.454	−278	− 21	0	69	139	1.9	18	2	−32	15
237	+59 30 33.01	16.773	−341	159	+ 3	61	74	3.7	9	1	6	3
239	+26 28 40.55	16.832	−265	− 63	+ 1	71	88	2.4	31	9	50	18
240	+12 55 18.31	17.069	−242	29	0	61	29	1.4	10	2	—	
242	+ 8 31 26.50	17.162	233	− 27	0	74	92	3.1	28	0	34	17
243	+17 15 1.00	17.455	−228	12	0	71	65	3.0	− 17	8	− 16	16
244	+12 27 21.16	17.496	219	3	+ 1	66	193	8.8	26	2	21	13
245	−11 51 35.26	17.699	195	93	+ 1	76	40	1.2	41	6	10	17
247	+43 21 19.69	17.870	−234	− 45	+ 1	67	80	3.8	− 28	7	+13	1
249	+20 20 50.63	18.111	−208	153	− 1	68	112	4.4	25	1		
250	+42 0 8.77	18.012	221	+ 20	0	72	71	3.1	− 38	7	−13	7
252	−16 19 32.82	18.299	−168	− 81	+ 1	75	43	1.3	− 39	5	+31	7
253	+37 13 10.63	18.358	202	112	+ 1	71	41	1.7	16	0	−44	18
256	+76 13 41.43	−18.415	−293	− 10	0	77	75	1.9	8	0	5	+ 4
257	+ 9 19 16.35	18.444	−173	6	0	70	117	3.7	33	3	27	−15
258	+ 1 6 20.37	18.738	−151	+ 23	0	67	32	1.3	31	5	—	—
262	+11 1 27.56	18.989	−112	− 31	0	75	95	2.3	10	− 1	34	15
263	−15 40 13.30	18.781	−131	+ 193	0	71	32	1.2	− 80	18	+ 1	15
264	+31 45 14.23	−19.348	141	290	0	73	61	1.8	18	7	−62	23
266	−17 45 58.56	19.124	110	+ 119	+ 1	71	21	1.3	+ 23	+12		
267	+ 4 9 15.83	19.277	−117	− 21	0	72	45	2.0	− 15	+ 1		
268	+56 55 6.48	19.238	−139	+ 28	0	62	85	4.1	15	+ 2	0	+ 7
269	+62 17 27.18	19.381	138	− 71	+ 1	62	138	6.8	15	3	+11	6
270	+ 7 52 35.91	−19.407	108	17	+ 4	72	90	3.6	15	6	21	11
271	+15 2 27.88	19.490	111	38	0	71	91	4.0	33	5	+ 3	2
273	+21 4 17.57	19.693	95	115	0	68	117	5.6	29	1	−13	19
274	+15 58 34.05	19.638	93	86	0	75	58	2.2	23	1	−27	15
275	+33 38 23.53	19.613	88	+ 15	0	71	48	2.1	57	11	−39	16
276	14 11 11.58	19.455	78	+ 195	0	70	113	3.7	24	0	+23	10
277	+ 6 31 38.18	19.693	78	15	0	71	80	3.2	17	2	23	12
279	+11 4 18.15	19.807	74	85	0	76	64	2.4	32	1	16	9
280	17 8 5.25	19.744	67	4	0	73	22	1.0	14	− 2	+18	7
281	+ 3 33 28.93	19.594	61	+ 173	+ 1	67	29	1.6	78	13		
282	+ 3 24 25.01	19.802	61	19	0	69	60	2.5	35	2		
283	+69 52 58.80	19.844	70	24	0	72	112	4.8	14	4	− 5	5
286	0 16 18.10	19.861	46	+ 35	0	72	113	3.4	39	4	19	10
287	+ 7 5 23.02	20.163	29	187	0	62	31	1.8	30	1		
288	+18 20 1.82	19.960	30	+ 16	0	72	83	3.6	22	4	− 5	2
289	+15 7 51.62	20.121	22	123	0	65	166	7.5	31	5	37	16
290	+ 2 19 11.51	20.286	21	279	0	68	120	5.9	16	5	21	14
291	+51 15 2.72	20.019	15	+ 3	0	64	139	7.5	7	1	+16	+ 4
292	+ 7 10 18.80	20.076	0	33	0	71	51	2.4	20	1	—	
293	+ 9 17 18.01	20.009	+ 9	+ 38	0	76	103	3.3	17	+ 6	19	14
298	+78 10 18.82	20.018	+ 22	+ 18	0	77	87	2.4	14	1	1	+ 4
300	+57 35 17.61	20.023	+ 29	+ 3	0	67	85	1.7	20	2	+ 5	+ 1
301	−16 59 12.30	20.014	+ 29	+ 11	0	70	38	1.9	57	5	10	17
304	0 6 40.02	20.030	+ 37	25	0	73	138	4.4	24	+ 2	11	10
305	+ 3 52 9.60	20.080	+ 38	78	0	62	23	1.6				

First Section (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B−N		B−A	
					.0001	.0001	.0001	1	p_α	p_δ	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		m	h m s	s	s	s	s				s	s	s	s
307	γ Comae Ber.	4.6	12 21 57.344	+2.9957	121	63	0	72	21	0.9	0	6		–
308	δ² Corvi	3.1	12 21 44.318	3.0990	+ 119	141	0	72	70	1.8	10	1	+33	+ 9
310	β Canum Ven.	4.3	12 28 59.717	2.8582	193	628	+ 5	71	36	1.4	11	−11	+ 2	+ 4
312	κ Draconis	3.9	12 29 13.002	2.5849	529	118	+ 3	71	88	2.7	15	− 6	+26	+13
316	ε Ursae Maj.	1.8	12 49 37.886	2.6521	272	+ 140	− 2	71	71	3.1	1	+ 1	+ 6	+ 8
317	δ Virginis	3.7	12 50 33.958	+3.0205	+ 27	317	0	71	99	2.9	0	+ 1	+32	+ 7
318	α Canum Ven.	2.8	12 51 21.070	2.8132	147	199	+ 1	68	110	3.8	+10	+ 4	+15	+12
320	ε Virginis	3.0	12 57 11.911	2.9866	− 5	186	0	77	110	2.6	− 2	0	+27	+ 9
321	θ Virginis	4.4	13 4 16.299	3.1026	+ 79	24	0	74	115	2.7	+10	+ 6	+41	+10
322	53 Virginis	5.1	13 6 41.158	3.1862	+ 141	+ 63	+ 1	69	17	0.9	–	–	–	–
323	β Comae Ber.	4.3	13 7 12.149	+2.8032	− 76	− 604	+ 1	76	60	2.0	−18	− 5	+ 8	+ 5
324	61 Virginis	4.8	13 13 10.348	3.1313	+ 156	− 754	0	71	24	1.3	− 4	− 2	–	–
326	ζ Ursae Maj.	2.1	13 19 54.022	2.4242	− 172	+ 149	2	71	68	3.0	−32	− 5	+ 1	+ 9
327	α Virginis	1.2	13 19 55.426	3.1553	+ 116	− 28	0	66	174	7.6	− 7	0	+38	+11
330	ζ Virginis	3.4	13 29 35.826	3.0540	+ 61	191	0	74	146	3.7	+11	+ 1	+42	+13
332	82 Virginis	5.3	13 36 21.752	+3.1440	+ 108	− 69	0	75	60	1.6	+ 8	+ 4	–	–
333	τ Bootis	4.5	13 42 30.612	2.8510	− 5	340	+ 1	75	71	1.9	+ 6	+ 1	+24	+10
334	η Ursae Maj.	1.9	13 43 36.059	2.3690	− 100	122	+ 1	67	133	5.8	−16	− 4	+ 4	+ 7
336	η Bootis	2.8	13 49 55.396	2.8567	3	45	+ 1	71	153	4.7	− 7	− 1	+26	+ 7
337	τ Virginis	4.3	13 56 33.106	3.0506	+ 65	+ 13	0	74	112	2.8	+ 3	+ 3	+41	+11
340	α Draconis	3.6	14 1 10.899	+1.6228	+ 50	80	+ 1	69	107	4.4	−45	− 8	+21	+12
341	κ Virginis	4.3	14 7 33.618	3.1949	+ 123	+ 5	0	72	86	2.8	− 5	− 1	+41	+10
342	ι Virginis	4.2	14 10 46.194	3.1409	+ 106	− 12	+ 2	76	52	1.8	+12	0	+63	+12
344	α Bootis	0.3	14 11 5.999	2.7352	+ 25	781	+11	66	176	7.7	− 1	+ 1	+33	+ 8
345	λ Bootis	4.3	14 12 31.962	2.2833	− 49	− 177	0	73	53	2.0	−25	− 6	+ 5	+ 8
346	λ Virginis	4.5	14 13 41.853	+3.2396	+ 111	− 15	0	71	55	1.7	+19	+ 9	–	–
347	θ Bootis	4.2	14 21 47.566	2.0430	− 11	− 260	+ 7	72	78	3.0	−26	− 5	+ 2	+ 6
348	ρ Bootis	3.7	14 27 31.229	+2.5864	− 16	− 76	0	75	99	2.5	− 7	− 3	+14	+ 8
349	5 Ursae min.	4.4	14 27 13.907	−0.1791	+1171	+ 34	− 2	70	15	2.1	+33	+11	–	–
350	γ Bootis	3.0	14 28 3.088	+2.4172	− 28	− 95	0	73	58	2.3	−17	− 4	+ 9	+11
353	μ Virginis	3.9	14 37 17.359	+3.1571	+ 107	+ 71	+ 1	77	61	2.0	− 6	+ 1	+49	+15
355	ε Bootis	2.6	14 40 37.193	2.6202	+ 1	− 36	0	69	126	3.9	+ 3	0	–	–
356	8 Librae	5.3	14 45 9.261	3.3112	+ 155	− 70	0	66	66	3.0	+ 8	+ 3	+41	+13
357	α Librae	2.9	14 45 20.701	+3.3118	+ 155	− 74	0	67	150	5.7	+13	+ 4	+38	+12
358	β Ursae min.	2.2	14 50 59.588	−0.2205	+1005	− 74	+ 2	66	127	5.6	−61	− 9	+29	+16
359	ξ² Librae	5.7	14 51 20.446	+3.2489	+ 130	− 2	0	72	48	1.8	+ 9	+ 4	–	–
362	δ Librae	4.9	14 55 37.707	3.2001	+ 116	45	0	71	20	1.0	+22	+ 6	–	–
363	β Bootis	3.6	14 58 10.750	2.2597	+ 1	− 39	+ 1	74	82	2.0	−12	− 3	+15	+10
366	ψ Bootis	4.6	15 0 9.630	2.5700	+ 12	− 134	+ 1	74	66	1.5	−10	− 2	+ 4	+ 5
369	ι¹ Librae	4.7	15 6 31.196	3.4121	+ 171	− 26	0	72	49	1.4	+25	+ 5	+57	+17
371	δ Bootis	3.5	15 11 28.269	+2.4188	+ 11	+ 71	0	75	60	1.8	−10	− 3	+ 2	+ 5
372	β Librae	2.8	15 11 37.482	3.2230	+ 118	− 67	0	70	146	4.4	− 2	− 1	+33	+ 8
373	o² Librae	6.8	15 17 27.053	3.3397	+ 112	− 2	0	69	23	0.9	− 1	+ 3	–	–
374	η Coron. Bor.	5.0	15 19 4.398	2.4779	+ 18	+ 101	+ 1	71	21	1.4	+ 5	+ 4	–	–
376	μ¹ Bootis	4.3	15 20 12.717	2.2656	+ 11	− 126	0	68	54	1.8	−22	− 5	+ 8	+ 7
377	γ Ursae min.	3.1	15 20 53.046	−0.1273	+ 738	− 25	0	67	94	4.1	−62	− 6	+31	+16
378	ζ¹ Librae	6.0	15 22 36.948	+3.3770	+ 118	+ 11	0	74	55	1.5	+16	+ 5	–	–
379	ι Draconis	3.4	15 22 12.245	1.3293	+ 132	7	0	74	56	1.2	−95	−21	−12	+ 3
380	β Coron. Bor.	3.7	15 23 12.356	2.4732	+ 19	− 133	0	76	48	1.5	−14	− 3	+15	+ 9
383	37 Librae	4.9	15 28 42.690	3.2735	+ 119	+ 203	+ 1	70	22	1.2	–	–	–	–
384	γ Librae	4.1	15 29 55.874	+3.3503	+ 136	+ 47	0	72	47	1.5	− 7	0	+47	+13
385	α Coron. Bor.	2.3	15 30 27.223	2.5391	+ 25	+ 90	0	67	164	7.0	− 1	0	+18	+ 7
386	ζ Coron. Bor.	4.6	15 35 36.748	2.2594	+ 22	− 4	0	71	32	0.8	+ 7	+ 1	+37	+16
387	κ Librae	5.0	15 36 11.018	3.4492	+ 157	31	+ 1	73	28	1.4	+18	+ 4	–	–
388	η Librae	5.5	15 38 26.789	3.3689	+ 137	− 26	0	67	21	1.1	–	–	–	–

FIRST SECTION — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V. and Sec. V.	.001	μ' and 100 $\Delta\mu'$ .001	.001	Ep. and Wt. T	p	p_c	B − N $\Delta\delta$ .01	$\Delta\mu$ .001	B − A $\Delta\delta$ .01	$\Delta\mu$ .001	
307	+28 49 27.30	−20.012	+ 50	− 87	0	71	29	1.5	− 23	+ 1	−		
308	−15 57 31.46	20.074	+ 57	− 143	0	76	62	2.0	− 24	+ 5	+25	− 8	
310	+41 54 2.87	19.607	+ 60	+ 280	− 1	71	41	1.7	− 18	0	+ 5	− 1	F 8
312	+70 20 21.95	19.878	+ 56	+ 6	0	66	95	4.8	− 8	− 4	+11	+ 7	
316	+56 30 9.29	19.591	+ 92	− 12	0	71	74	3.4	− 9	0	+24	+ 7	
317	+ 3 56 26.80	−19.625	+101	− 64	− 1	72	86	3.5	− 40	− 4	− 7	− 9	
318	+38 51 29.86	19.503	+ 99	+ 43	− 1	68	122	4.4	− 43	− 6	−32	−14	F12
320	+11 29 47.55	19.409	+115	+ 17	− 1	76	95	3.2	− 23	+ 2	−19	−10	
321	− 5 0 18.83	19.294	+134	− 42	0	73	114	4.4	− 33	− 2	− 4	− 9	
322	−15 39 33.16	19.504	+142	− 301	0	69	20	0.9	−	−	−	−	
323	+28 23 5.71	−18.316	+124	+ 875	− 3	75	61	2.0	− 46	− 4	−27	−10	F 13
324	−17 45 18.49	20.118	+148	−1084	− 3	72	23	1.1	− 70	−14	−	−	
326	+55 26 51.03	18.871	+130	− 30	+ 1	65	67	3.9	− 26	0	− 6	− 1	
327	−10 38 22.04	18.876	+165	− 36	0	65	166	7.5	− 44	− 4	+ 7	−10	
330	− 0 5 5.11	18.500	+177	+ 34	− 1	73	126	3.8	− 50	− 5	−16	− 9	
332	− 8 44 54.50	−18.265	+191	+ 36	0	72	49	2.3	− 22	+ 3	−	−	m
333	+17 57 18.06	18.050	+185	+ 25	− 2	75	61	2.0	− 28	− 2	−27	−14	
334	+49 48 44.24	18.051	+158	− 21	− 1	65	160	8.3	− 5	+ 2	+11	+ 1	
336	+18 53 55.90	18.152	+200	− 367	0	70	152	5.2	− 30	− 1	−36	−15	
337	+ 2 1 42.10	17.535	+224	− 25	0	75	88	2.1	− 29	+ 4	+10	− 3	
340	+64 51 13.52	−17.272	+127	+ 15	− 1	70	125	5.1	0	+ 4	+24	+10	
341	− 9 48 30.13	16.891	+253	+ 130	0	71	80	3.0	− 29	− 2	− 4	−15	
342	− 5 31 24.44	17.298	+255	− 127	0	75	49	1.9	− 33	0	+10	− 6	
344	+19 12 10.43	18.858	+217	−2003	− 6	65	186	8.5	− 23	0	−29	−12	
345	+46 32 50.57	16.634	+188	+ 151	− 1	74	66	3.2	− 20	0	+ 9	− 1	
346	−12 51 39.13	−16.708	+267	+ 23	0	68	52	2.2	− 12	+ 3	−	−	
347	+52 18 16.32	16.737	+178	− 106	− 2	73	97	4.6	− 43	− 4	− 6	− 4	
348	+30 18 36.73	15.925	+233	+ 110	− 1	76	98	2.6	− 33	− 3	−32	−14	
349	+76 8 26.02	16.007	− 9	+ 17	0	65	54	2.7	− 18	− 4	−	−	
350	+38 41 43.92	15.861	+218	+ 143	− 1	68	62	3.1	− 33	− 2	− 9	− 4	
353	− 5 13 24.80	−15.802	+300	− 322	+ 4	77	51	1.8	− 24	0	+ 7	− 5	
355	+27 29 44.25	15.314	+253	+ 8	0	66	127	5.1	− 27	− 2			
356	−15 34 53.67	15.111	+324	− 78	− 1	68	44	2.1	− 49	− 4	+46	− 6	α^1
357	−15 37 31.85	15.129	+325	− 77	− 1	67	134	5.8	− 36	0	+38	− 7	α^2
358	+74 33 51.10	14.716	− 16	+ 5	− 1	65	161	7.7	+ 5	+ 2	+26	+11	
359	−11 0 22.34	−14.703	+328	− 2	0	70	41	2.1	− 27	4		−	
362	− 8 7 19.97	14.454	+329	− 14	0	69	23	1.1	− 15	+ 4			
363	+10 17 5.37	14.330	+237	− 43	0	73	82	2.4	− 30	− 3	+ 3	− 3	
366	+27 20 14.54	14.184	+270	− 19	− 1	73	61	1.7	− 34	− 5	−29	−13	
369	−19 24 18.10	13.815	+367	− 49	0	71	40	1.7	+ 8	+ 4	+10	−10	
371	+33 11 15.66	−13.575	+268	− 127	+ 1	74	67	1.9	− 29	− 4	25	−12	
372	− 9 0 50.76	13.468	+354	− 30	− 1	70	152	5.9	− 38	− 6	+ 3	−10	
373	−11 46 37.98	13.053	+375	+ 3	0	71	20	0.8	− 21	0	−		
374	+30 38 55.33	13.147	+283	199	+ 1	65	23	1.4	− 28	− 1			
376	+37 43 39.65	12.760	+258	+ 78	− 1	68	60	2.2	− 35	− 3	−24	−11	
377	+72 11 23.27	−12.814	− 9	+ 12	0	68	94	5.4	− 6	− 4	+14	+10	
378	−16 22 4.69	12.753	+386	− 43	0	74	47	1.1	− 24	0			
379	+59 18 58.46	12.697	+156	+ 7	0	74	68	2.4	− 9	− 3	−12	− 4	
380	+29 27 0.84	12.560	+284	+ 76	2	76	55	1.6	− 32	− 2	6	− 5	
383	− 9 43 18.67	12.537	+385	− 244	+ 2	69	22	1.0					
384	−14 27 22.03	−12.210	+393	2	+ 1	72	42	1.8	59	8	14	−16	
385	+27 3 3.74	12.274	+300	− 102	+ 1	67	174	8.4	− 26	2	29	−13	
386	+36 57 37.09	11.819	+271	8	0	70	33	1.2	− 23	+ 3	− 2	− 3	
387	−19 21 17.39	11.889	+412	119	0	70	24	1.5	−123	− 13			
388	−15 21 15.53	11.687	+405	− 78	0	66	22	1.0					

First Section.—(Declination, +82° to −21° 50′.)

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 $\Delta\mu$		Ep. and Wt.			B — N $\Delta\alpha$	B — N $\Delta\mu$	B — A $\Delta\alpha$	B — A $\Delta\mu$
					.0001	.0001	.0001	T	p_α	p_δ	.001	.0001	.001	.0001
			h m s	s	s	s	s				s	s	s	s
389	γ Coron. Bor.	3.9	15 38 32.595	+2.5188	+27	−75	0	72	31	1.1	−32	−4	+10	+7
390	α Serpentis	2.8	15 39 20.511	2.9522	+61	+90	0	68	181	7.2	+4	+1	+36	+8
391	β Serpentis	3.7	15 41 34.330	2.7673	+13	+49	0	73	52	1.8	−21	−6	+25	+7
392	μ Serpentis	3.6	15 44 24.040	3.1270	+88	−59	0	73	49	1.5	+7	−1	+49	+14
393	ϵ Serpentis	3.8	15 45 49.835	2.9875	+65	+83	0	75	98	2.6	+5	+1	+35	+10
395	ζ Ursae min.	4.3	15 47 37.360	−2.2339	+2001	+79	−2	66	107	4.0	−13	−1	+78	+28
396	θ Librae	4.4	15 48 7.833	+3.4100	+134	+69	−1	65	25	1.0	–		–	–
398	γ Serpentis	4.0	15 51 50.024	2.7688	+57	+210	+7	71	69	2.3	−7	0	+29	+11
400	ϵ Coron. Bor.	4.2	15 53 26.824	2.4822	+31	−62	+1	75	47	1.0	+11	+4	+27	+11
403	β^1 Scorpii	2.7	15 59 37.260	3.4817	+141	−8	0	68	109	3.4	+21	+3	+33	+10
404	θ Draconis	4.1	16 0 0.888	+1.1188	+136	−399	−1	66	67	2.7	−36	−8	+12	+8
405	ν^2 Scorpii	4.2	16 6 10.935	3.4807	+135	−8	0	70	10	1.4	+42	+8	–	–
407	δ Ophiuchi	3.0	16 9 6.252	3.1401	+82	−33	+1	71	153	1.4	−8	−2	+29	+8
409	ϵ Ophiuchi	3.3	16 13 1.754	3.1704	+82	+52	0	73	65	2.4	−4	−2	+14	+12
411	τ Herculis	3.9	16 16 44.074	1.8011	+51	−12	0	71	70	1.5	−34	−13	+7	+5
412	γ Herculis	3.8	16 17 30.504	+2.6449	+38	−34	0	73	77	2.4	−4	0	+22	+10
414	ψ Ophiuchi	4.6	16 18 15.047	3.5056	+128	−13	0	69	21	0.9	–	–	–	–
415	η Draconis	2.9	16 22 38.137	0.8046	+183	−25	−1	68	92	2.9	−40	−5	+10	+7
418	ϕ Ophiuchi	4.4	16 25 24.852	3.4287	+109	−38	0	71	24	1.1	–	–	–	–
419	λ Ophiuchi	3.8	16 25 52.151	3.0228	+63	−24	0	72	56	1.6	−4	−2	+38	+11
420	β Herculis	2.8	16 25 55.232	+2.5768	+36	−77	0	76	69	1.8	+13	−1	+7	0
421	15 Draconis	5.0	16 28 10.580	−0.1355	+407	−46	−1	75	70	1.4	+6	+2	+26	+12
423	σ Herculis	4.3	16 30 52.731	+1.9323	+42	−11	0	74	50	1.7	−20	−6	−19	0
424	ζ Ophiuchi	2.7	16 31 39.087	3.2996	+86	+8	0	74	88	2.1	−3	+1	+38	+12
425	21 Scorpii	5.1	16 35 47.312	3.4651	+103	−17	0	72	29	1.2	+13	+1	–	–
427	η Herculis	3.7	16 39 28.042	+2.0550	+39	+28	+1	69	68	3.4	+2	−2	+2	+6
434	κ Ophiuchi	3.4	16 52 56.072	2.8376	+43	−199	0	74	142	3.3	+2	0	+39	+10
436	ϵ Herculis	3.9	16 56 27.800	2.2941	+31	−36	0	73	78	2.2	0	0	+13	+7
437	η Ophiuchi	2.6	17 4 38.542	3.4372	+71	+25	−1	70	91	2.9	+21	+8	+47	+15
439	ζ Draconis	3.2	17 8 29.784	0.1661	+190	−21	−1	74	64	2.1	−8	0	+40	+17
440	α^1 Herculis	3.3	17 10 5.245	+2.7340	+34	−8	0	67	156	6.7	−3	0	+29	+8
441	δ Herculis	3.2	17 10 55.434	2.4628	+33	−18	+1	75	45	1.9	+8	+1	+14	+3
442	π Herculis	3.3	17 11 33.827	2.0881	+33	−23	0	75	57	1.6	+11	+2	+15	+8
443	ξ Ophiuchi	4.4	17 15 0.630	3.5928	+74	+172	+2	69	30	1.0	–	–	–	–
452	β Draconis	3.0	17 28 10.376	1.3537	+50	−15	0	68	108	4.8	0	+2	−2	+6
454	α Ophiuchi	2.1	17 30 17.535	+2.7833	+33	+80	+2	68	180	7.3	−2	0	+39	+11
455	ξ Serpentis	3.7	17 31 51.606	3.4330	+46	−31	0	74	31	0.8	+19	+8	+31	+12
457	o Serpentis	4.7	17 35 17.650	3.3699	+40	−49	0	67	23	1.1	+22	+6	–	–
458	ι Herculis	3.9	17 36 38.484	1.6917	+34	−10	0	74	63	1.7	−56	−13	−7	+5
459	58 Ophiuchi	4.8	17 37 26.261	3.5935	+45	−64	0	66	22	0.8	–	–	–	–
460	ω Draconis	4.9	17 37 32.136	−0.3557	+107	+18	−16	73	42	1.3	−37	+1	−6	+8
461	β Ophiuchi	2.9	17 38 31.939	+2.9623	+27	−28	−1	73	110	3.3	−8	−2	+39	+9
463	μ Herculis	3.5	17 42 32.660	2.3461	+38	−243	+6	73	115	3.0	−17	−4	+21	+6
465	ψ^1 Draconis	4.5	17 43 42.962	−1.0761	+195	+31	+18	73	75	2.4	+20	+11	+58	+22
466	ξ Draconis	3.9	17 51 47.975	+1.0366	+34	+122	−2	71	56	2.2	−66	−9	+7	+11
467	θ Herculis	4.0	17 52 49.398	+2.0562	+25	+1	0	75	43	1.1	−12	−5	+12	+8
468	ν Ophiuchi	3.7	17 53 31.270	3.3014	+25	−8	+1	76	55	1.6	+5	−1	+47	+12
469	35 Draconis	5.0	17 53 55.547	−2.6894	+79	+141	−31	72	42	1.2	+6	+20	+55	+33
470	γ Draconis	2.4	17 54 17.028	+1.3917	+31	−10	+1	65	124	6.1	−20	−3	+3	+8
474	72 Ophiuchi	3.7	18 2 36.515	2.8433	+17	−42	−1	77	92	1.9	+1	+3	+24	+6
475	o Herculis	3.8	18 3 38.486	+2.3392	+21	−1	0	75	45	1.1	−5	+1	+12	+7
478	μ Sagittarii	4.0	18 7 46.992	3.5876	+7	+3	0	67	106	3.0	+27	+7	+50	+16
480	36 Draconis	5.0	18 13 19.223	0.3452	−7	+530	0	66	25	1.8	−52	−5	−10	+4
482	η Serpentis	3.4	18 16 8.123	3.1030	+17	−374	+4	75	103	2.7	+21	+5	+48	+13
485	χ Draconis	3.7	18 22 51.628	−1.0779	−85	+1165	+30	70	73	3.2	−50	−6	+41	+12

First Section — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V.	μ′	100 Δμ′	Ep. T	p_1	p_2	B − N Δδ	B − N Δμ′	B − A Δδ	B − A Δμ′
	° ′ ″	″	.001	.001	.001				.01	.001	.01	.001
389	+26 36 44.00	−11.572	+304	+ 30	− 1	69	38	1.5	− 13	+ 2	−16	− 9
390	+ 6 14 24.14	11.508	+358	+ 38	+ 1	67	174	7.9	− 39	− 5	−27	−13
391	+15 44 4.48	11.442	+338	− 57	+ 1	74	51	1.9	− 20	− 1	−13	− 8
392	− 3 7 27.57	11.208	+382	− 27	− 1	78	38	0.9	− 19	+ 1	+10	− 4
393	+ 4 46 42.72	11.020	+369	+ 57	+ 1	75	85	2.5	− 73	−13	−18	−13
395	+78 6 7.92	−10.949	−267	− 3	+ 1	67	137	6.3	+ 2	+ 1	+ 6	+ 6
396	−16 26 9.43	10.788	+423	+ 121	+ 1	63	25	1.4	–	–	–	
398	+15 59 15.99	11.933	+349	−1297	+ 3	72	55	2.2	− 17	− 8	−23	−13
400	+27 10 2.11	10.584	+312	− 68	− 1	76	56	1.2	− 25	− 1	−17	− 9
403	−19 31 54.72	10.081	+443	− 29	0	67	97	3.6	− 44	− 1	+47	− 8
404	+58 49 55.97	− 9.684	+141	+ 338	− 5	65	71	4.2	− 22	− 1	+ 6	+ 5
405	−19 12 3.32	9.584	+450	− 32	0	69	33	1.5	+ 16	+ 8	–	–
407	− 3 26 13.41	9.179	+409	− 153	0	70	138	4.8	− 60	− 9	− 9	−10
409	− 4 26 56.03	8.988	+417	+ 33	+ 1	75	59	2.0	− 32	− 4	+13	− 6
411	+46 33 4.93	8.699	+240	+ 31	0	73	81	2.9	− 9	+ 2	+22	+ 3
412	+19 23 15.98	− 8.630	+351	+ 39	0	72	67	2.3	− 11	+ 2	−22	−11
414	−19 48 12.65	8.674	+465	− 63	0	65	20	1.0		–	–	–
415	+61 44 25.81	8.204	+110	+ 59	0	66	101	4.7	+ 3	+ 1	+ 8	+ 4
418	−16 23 41.19	8.079	+161	− 38	0	69	23	1.1		–	–	–
419	+ 2 12 9.27	8.092	+407	− 88	0	72	51	1.7	− 49	− 9	− 9	− 7
420	+21 42 26.30	− 8.024	+317	− 24	−1	77	76	2.2	− 12	+ 1	−30	−15
421	+68 59 1.20	7.785	− 16	+ 34	−1	74	65	1.9	+ 3	− 2	+13	+ 7
423	+12 38 35.25	7.565	+261	+ 35	0	63	59	2.0	+ 6	+10	+15	− 3
424	−10 21 53.06	7.521	+119	+ 17	0	76	73	1.9	− 51	− 5	− 8	−11
425	−17 32 55.17	7.208	+171	− 7	0	69	28	1.5	− 17	− 3	–	–
427	+39 6 11.01	− 6.995	+285	− 95	0	68	72	3.7	− 23	− 2	−10	−18
431	+ 9 31 19.04	5.797	+396	− 11	− 3	71	132	3.5	− 28	3	28	−13
436	+31 4 21.46	5.166	+321	+ 21	− 1	71	70	2.0	− 26	3	−33	−12
437	−15 36 1.36	4.709	+189	+ 86	0	73	73	2.6	− 38	5	+16	−13
439	+65 50 15.89	4.417	+ 25	+ 20	0	73	66	2.5	+ 1	+ 3	+ 8	+ 1
440	+11 30 14.87	− 4.301	+391	+ 27	0	65	157	7.1	− 16	2	23	12
441	+21 57 24.95	4.123	+352	− 163	0	74	17	1.8	− 26	5	19	−11
442	+36 55 17.93	4.207	+299	− 2	0	77	54	1.3	28	1	26	12
443	−21 0 20.02	4.117	+518	− 207	+ 2	66	23	1.0				
452	+52 22 31.00	2.769	+196	+ 6	0	68	118	5.9	− 20	3	+10	0
454	+12 37 57.16	− 2.826	+405	− 235	+ 1	67	180	7.8	11	0	5	8
455	−15 20 8.51	2.521	+497	66	0	74	25	0.9	59	6	+33	− 7
457	−12 49 18.95	2.168	+189	− 55	− 1	72	21	0.9	106	17		–
458	+16 3 33.67	2.012	+246	6	0	75	71	3.2	11	6	+20	+ 3
459	−21 38 1.78	2.025	+521	− 55	1	62	17	0.9				
460	+68 18 15.17	− 1.635	50	+ 327	0	74	52	1.9	+ 11	+ 8	+25	+ 11
461	+ 1 36 31.88	1.724	+130	+ 151	0	76	93	3.0	50	7	19	11
463	+27 16 11.36	2.276	+338	− 751	3	72	101	2.8	23	2	21	11
465	+72 11 52.61	1.691	−156	268	0	71	71	2.5	5	0	+ 7	+ 7
466	+56 53 17.71	0.612	+153	+ 75	+ 2	67	48	2.5	20	1	+16	+ 3
467	+37 15 18.88	− 0.624	+300	+ 1	0	74	11	1.6	20	0	19	10
468	− 9 15 41.07	0.685	+181	118	0	79	15	1.3	11	+ 1	+13	6
469	+76 58 31.06	0.292	−390	+ 239	+ 3	77	10	1.8	12	3	+11	+ 6
470	+51 30 1.71	0.526	+203	26	0	65	133	7.1	23	2	1	3
474	+ 9 32 58.07	+ 0.310	+111	+ 92	1	77	73	1.9	39	5	3	5
475	+28 11 51.71	+ 0.320	+341	2	0	75	47	1.4	27	0	13	8
478	−21 5 6.33	0.676	+522	5	0	69	71	2.4	19	3	+11	10
480	+61 21 48.08	1.193	+ 57	+ 28	+ 8	64	35	1.9	+ 7	+ 1	+34	+ 8
482	− 2 55 29.86	0.711	+115	699	5	73	91	3.0	54	8	16	13
485	+72 11 22.27	1.627	−111	− 369	+17	74	78	3.5	+ 23	+ 5	+ 11	+ 9

FIRST SECTION (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B — N		B — A	
					.0001	.0001	.0001	t	p_α	p	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
487	α Lyrae	0.1	18 33 33.146	+2.0309	+ 10	+ 174	3	67	159	7.2	−16	− 1	+17	+ 9
489	110 Herculis	4.3	18 41 21.171	2.5803	+ 17	18	+ 3	73	17	1.6	+11	+ 2	+ 7	+ 4
490	β Lyrae	Var.	18 46 23.277	2.2144	+ 11	+ 3	0	68	134	4.8	+ 7	− 1	+11	+ 7
492	50 Draconis	5.4	18 49 36.178	−1.9122	− 552	− 35	− 8	70	32	0.8	+15	− 4		
493	o Draconis	4.7	18 49 13.560	+0.8877	− 16	+ 105	0	70	45	1.6	−67	−11	+ 2	+ 4
494	θ Serpentis	4.3	18 51 14.906	+2.9826	− 5	+ 31	0	71	47	1.9	+22	+ 3	+17	+13
495	ε Aquilae	4.2	18 55 5.022	2.7218	+ 6	− 11	+ 1	72	78	2.2	− 5	− 2	+27	+ 7
496	γ Lyrae	3.3	18 55 12.171	2.2437	+ 13	− 1	0	75	64	2.0	+11	+ 4	+15	+ 9
498	o Sagittarii	3.9	18 58 41.465	3.5970	− 55	+ 52	0	64	25	1.2	–	–		–
501	ζ Aquilae	3.0	19 0 48.836	2.7570	+ 1	− 6	+ 1	70	140	3.9	+ 5	+ 2	+32	+10
502	λ Aquilae	3.5	19 0 56.535	+3.1841	− 21	− 18	+ 1	77	66	2.1	+ 7	+ 2	+39	+ 9
504	π Sagittarii	3.0	19 3 19.038	3.5699	− 59	− 4	0	70	52	1.8	+ 3	+ 1	+36	+11
505	43 Sagittarii	5.0	19 11 17.094	3.5126	− 62	− 8	0	75	48	1.7	+29	+ 7	–	–
506	δ Draconis	3.2	19 12 31.998	0.0256	− 235	+ 173	− 2	66	95	4.5	−11	− 2	+12	+19
507	κ Cygni	4.0	19 14 47.526	1.3881	− 30	+ 70	− 2	71	72	2.5	− 7	− 2	+ 1	+ 5
508	ρ Sagittarii	4.0	19 15 52.423	+3.4822	− 63	− 17	0	67	29	1.4	–	–	–	–
509	τ Draconis	4.6	19 17 28.712	−1.1276	− 588	− 318	−11	72	76	3.0	−65	− 8	+36	+15
510	δ Aquilae	3.4	19 20 27.394	+3.0253	− 19	+ 169	0	70	151	4.5	+ 1	+ 1	+40	+12
512	β Cygni	3.1	19 26 41.306	2.4187	+ 10	− 2	0	72	72	2.7	0	0	+19	+ 8
513	ι Cygni	3.9	19 27 11.087	1.5134	− 25	+ 20	− 2	71	57	2.4	−20	− 3	− 4	+ 5
515	κ Aquilae	4.9	19 31 30.737	+3.2293	− 45	+ 3	0	73	42	1.3	− 6	− 2		–
516	θ Cygni	4.7	19 33 45.576	1.6086	− 23	− 30	− 4	71	43	2.3	−33	− 7	+ 5	+ 7
517	55 Sagittarii	5.0	19 36 17.980	3.4344	− 76	+ 42	0	64	25	1.2	0	+ 2	–	–
518	56 Sagittarii	5.1	19 40 31.786	3.5033	− 91	− 95	+ 1	70	26	1.3	+26	+ 4	–	–
519	γ Aquilae	2.8	19 41 30.336	2.8523	− 11	+ 9	0	67	164	7.5	+ 3	+ 2	+31	+10
520	δ Cygni	3.0	19 41 50.975	+1.8755	+ 1	+ 50	0	72	59	1.6	−30	− 5	+12	+11
521	α Aquilae	0.9	19 45 54.256	2.9274	− 19	+ 361	− 2	66	166	6.9	− 5	0	+25	+ 8
522	ε Draconis	4.0	19 48 30.849	−0.1819	− 441	+ 160	+ 1	70	72	1.8	−42	− 9	+37	+16
523	β Aquilae	3.9	19 50 24.073	+2.9469	− 15	+ 23	+ 3	67	167	7.6	− 5	− 1	+27	+ 8
525	θ Aquilae	3.4	20 6 8.744	3.0967	− 43	+ 21	0	75	106	3.0	+ 9	+ 1	+53	+13
526	31 Cygni	3.9	20 10 28.961	+1.8889	+ 4	+ 2	0	74	56	1.2	−39	−12	+ 2	+ 6
527	α^1 Capricorni	4.6	20 12 6.360	3.3283	− 85	+ 10	0	67	87	4.1	+ 5	+ 2	+37	+11
528	κ Cephei	4.4	20 12 15.751	−1.9397	−1676	+ 28	0	73	74	2.4	− 4	+ 4	+45	+16
529	α^2 Capricorni	3.8	20 12 30.424	+3.3318	− 86	+ 40	0	66	137	5.3	+ 5	+ 1	+30	+10
530	β Capricorni	3.2	20 15 23.620	3.3742	− 96	+ 24	0	70	68	2.4	−22	− 6	+40	+13
532	γ Cygni	2.3	20 18 38.348	+2.1522	+ 19	+ 2	0	72	103	4.7	− 7	− 2	+16	+10
533	π Capricorni	5.2	20 21 35.898	3.4385	− 116	+ 8	0	73	37	1.5	+14	+ 4	–	–
534	ρ Capricorni	4.9	20 23 9.466	3.4266	− 115	− 10	0	72	80	1.8	+ 7	+ 3	+37	+13
535	θ Cephei	4.3	20 27 51.285	1.0139	− 152	+ 64	+ 1	69	72	3.0	−15	− 1	+32	+13
536	ε Delphini	4.0	20 28 26.141	2.8665	− 12	+ 6	0	76	104	2.7	− 3	− 1	+30	+ 9
538	β Delphini	3.7	20 32 51.592	+2.8131	− 4	+ 74	0	72	46	1.9	−26	− 8	+32	+11
539	ι Capricorni	5.1	20 34 21.495	3.4200	− 122	− 17	0	71	24	1.0	+11	+ 1	+34	+11
540	α Delphini	3.9	20 34 59.603	2.7866	− 1	+ 45	0	70	75	2.4	− 9	− 2	+28	+10
542	α Cygni	1.3	20 38 1.344	2.0440	+ 22	0	0	66	132	6.8	−17	− 4	−10	+ 5
544	γ^2 Delphini	4.1	20 42 1.151	2.7832	+ 2	− 23	+ 1	69	34	1.1	+17	+ 1	+25	+ 8
545	ε Cygni	2.6	20 42 9.889	+2.4266	+ 28	+ 289	− 1	74	68	2.8	−16	− 5	+12	+ 8
546	ε Aquarii	3.9	20 42 15.806	3.2507	− 84	+ 19	0	75	83	2.0	− 1	+ 2	+49	+18
547	η Cephei	3.6	20 43 15.283	1.2266	− 145	+ 133	−16	66	79	3.4	−10	− 1	− 7	+ 6
549	μ Aquarii	4.8	20 47 15.657	3.2390	− 83	+ 25	0	75	60	1.9	+ 2	0	–	–
551	32 Vulpeculae	5.2	20 50 17.862	2.5555	+ 27	− 6	0	73	77	1.5	−22	− 4	+ 8	+ 4
552	ν Cygni	4.0	20 53 26.669	+2.2346	+ 38	+ 4	0	73	63	2.0	−20	− 4	−12	+ 3
553	η Capricorni	5.0	20 58 42.890	3.4199	− 142	− 30	0	66	22	1.3	−11	− 5	–	–
554	θ Capricorni	4.2	21 0 19.618	3.3779	− 127	+ 57	0	73	53	1.6	+12	+ 6	–	–
555	ξ Cygni	3.9	21 1 17.584	2.1803	+ 42	+ 6	0	72	57	1.6	−13	− 4	− 3	+ 3
556	ν Aquarii	4.5	21 4 8.873	3.2720	− 98	+ 62	0	74	41	1.4	+27	+ 6	+29	+ 9

FIRST SECTION (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ′ .001	100 Δμ′ .001	Ep. T	p	p_δ	B − N Δδ .01	B − N Δμ .001	B − A Δδ .01	B − A Δμ .001	
	° ′ ″	″	″	″	″								
487	+38 41 25.44	+3.203	+294	+279	+3	61	168	7.8	−27	−1	7	−9	
489	+20 27 1.13	3.254	+368	344	0	76	17	1.8	−23	0	18	−13	
490	+33 14 16.96	4.023	+315	−7	0	65	138	5.7	−27	−2	28	−13	
492	+75 18 58.33	4.382	−274	+77	−1	71	25	0.8	+88	+26		−	
493	+59 15 57.66	4.339	+126	+23	+2	61	57	3.5	−15	0	+21	+5	
494	+4 4 23.76	+4.472	+423	+26	0	72	46	1.9	−35	1	−15	−12	
495	+14 55 56.34	4.695	+383	−77	−1	72	62	2.1	−10	+1	−15	−11	
496	+32 33 7.78	4.775	+315	−7	0	71	59	2.0	−30	−2	−33	−16	
498	−21 53 17.03	5.011	+506	−67	+1	65	19	0.8	−	−	−	−	
501	+13 42 52.56	5.155	+385	−102	0	69	143	5.1	−32	−3	−12	−11	
502	−5 1 57.55	+5.178	+415	−90	0	77	67	1.8	−55	−7	−4	−11	
504	−21 10 57.67	5.470	+498	−40	0	72	40	1.7	−27	−4	+17	−14	
505	−19 7 51.62	6.158	+484	−19	0	74	49	1.8	−32	−2	−	−	a
506	+67 29 8.27	6.328	+3	+89	+2	67	114	6.1	−2	+1	+30	+9	
507	+53 11 1.69	6.514	+190	+117	+1	71	77	3.9	−23	−4	+16	+1	
508	−18 2 8.12	+6.535	+477	+19	0	70	25	0.8		−	−	−	
509	+73 10 11.64	6.759	−162	+110	−4	75	68	2.1	−6	0	+19	+9	
510	+2 54 54.69	6.971	+414	+77	+2	70	143	5.2	−35	−4	−17	−11	
512	+27 44 58.03	7.394	+324	−9	0	74	75	2.3	−22	+1	−14	−11	
513	+51 30 59.54	7.566	+202	+122	0	71	71	3.9	−36	−6	+5	−1	
515	−7 14 59.66	+7.792	+131	−2	0	71	12	1.2	−38	−3		−	
516	+49 59 21.57	8.221	+212	+217	0	67	68	4.2	−36	−4	+20	+3	
517	−16 21 30.48	8.201	+454	−17	+1	65	21	0.7	−30	−2	−	−	
518	−20 0 6.13	8.417	+157	−97	−1	68	22	1.0	−67	−9	−	−	
519	+10 22 9.76	8.587	+372	−4	0	65	167	7.9	−19	−1	−15	−12	
520	+44 53 11.40	+8.656	+244	+37	+1	68	75	3.5	−22	−7	+22	−1	
521	+8 36 14.56	9.317	+383	+380	+5	65	172	8.1	−12	+2	−9	−11	
522	+70 0 47.67	9.172	−25	+31	+2	70	66	3.1	+9	+4	+35	+14	
523	+6 9 24.15	8.805	+377	−483	0	66	166	7.8	−37	2	−22	−12	
525	−1 7 5.73	10.488	+380	+3	0	76	86	2.7	−36	3	0	−10	
526	+16 26 16.25	+10.808	+228	+1	0	73	64	2.2	−33	−4	+4	1	o¹ or o²
527	−12 49 2.55	10.932	+402	+6	0	66	65	3.5	24	+1	+22	−9	
528	+77 21 37.15	10.964	−242	+26	0	73	77	2.5	−4	+1	+6	+8	
529	−12 51 17.78	10.961	+403	+5	0	64	135	5.4	−28	−2	+23	−11	
530	−15 5 50.36	11.167	+401	+1	0	70	48	2.1	−35	6	+8	−15	
532	+39 56 10.96	+11.398	+253	−3	0	71	107	4.8	−32	−3	−21	−11	
533	−18 32 22.98	11.600	+404	−13	0	75	36	1.4	−71	10	−		
534	−18 8 39.73	11.702	+400	−22	0	71	63	1.8	−18	−2	+9	−17	
535	+62 39 28.31	12.041	+114	−17	+1	71	71	3.8	−9	+1	+9	0	
536	+10 57 47.51	12.069	+328	−26	0	77	87	2.2	−23	1	−4	−10	
538	+14 14 19.48	+12.364	+318	−37	+1	73	46	1.7	−28	−2	3	−8	
539	−18 29 27.27	12.483	+384	−21	0	72	27	1.3	−81	−14	+37	−9	
540	+15 33 32.65	12.539	+312	−8	0	69	66	2.7	−92	−25	10	12	
542	+44 55 22.17	12.752	+225	−1	0	65	168	8.6	14	+2	+11	2	
544	+15 15 19.30	12.817	+303	−204	0	69	39	1.4	−43	8	−18	−12	
545	+33 35 43.67	+13.352	+267	+322	+3	71	75	3.2	−37	4	20	14	
546	−9 51 43.15	13.003	+355	−34	0	75	69	2.4	−36	4	+17	14	
547	+61 27 0.99	13.923	+131	+820	+1	66	78	5.0	−17	+1	+26	+7	
549	−9 21 31.37	13.331	+346	−35	0	72	61	2.4	13	+4			
551	+27 40 37.52	13.561	+269	2	0	73	70	1.5	36	6	13	9	
552	+40 16 54.75	+13.740	+231	−24	0	73	63	2.0	−40	6	16	12	
553	−20 15 2.02	14.052	+318	−44	0	64	21	1.3	+8	+3			
554	−17 37 19.39	14.129	+342	67	0	72	42	1.6	16	0			
555	+13 31 43.35	14.253	+218	−2	0	73	56	2.2	−53	10	+28	2	
556	−11 46 36.29	14.147	+326	−12	+1	74	50	2.2	60	6	+4	12	

First Section (Declination, +82° to −21° 50′).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B − N		B − A	
					.0001	.0001	.0001	T	p_α	p_μ	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
557	ζ Cygni	3.5	21 8 10.792	+2.5515	+ 10	− 2	0	71	130	3.4	− 3	0	+10	+ 8
558	τ Cygni	3.9	21 10 47.911	2.3926	+ 16	+ 131	− 2	68	17	2.0	−29	− 7	− 7	+ 5
559	α Equulei	4.1	21 10 49.523	3.0001	− 27	+ 38	0	77	66	1.8	+ 7	+ 5	+29	+ 9
560	α Cephei	2.6	21 16 11.590	1.4353	− 69	+ 217	+ 2	66	131	5.4	−31	− 7	+15	+12
561	ι Capricorni	4.3	21 16 10.786	3.3460	− 129	+ 22	0	69	11	1.3	− 1	0	–	–
562	1 Pegasi	4.2	21 17 27.700	+2.7736	+ 19	+ 71	0	76	39	1.1	− 7	− 2	+82	+ 7
565	β Aquarii	3.3	21 26 17.711	3.1608	− 71	+ 10	0	70	148	1.6	− 5	− 2	+40	+11
566	β Cephei	3.4	21 27 22.310	0.7909	− 350	+ 22	0	76	123	5.6	−21	− 4	+38	+16
567	ε Capricorni	4.7	21 31 28.959	3.3652	− 148	+ 6	0	71	20	1.1	–	–	–	–
568	ξ Aquarii	4.8	21 32 25.757	3.1971	− 82	+ 77	0	74	63	1.8	+ 3	+ 2	–	–
569	γ Capricorni	3.8	21 34 33.100	+3.3294	− 131	+ 131	0	68	53	2.1	+ 1	+ 2	+33	+11
571	ε Pegasi	2.5	21 39 16.471	2.9161	− 5	+ 18	0	71	150	4.1	+ 3	+ 2	+29	+ 9
572	κ Pegasi	4.2	21 40 6.981	2.7145	+ 47	+ 21	0	77	37	0.8	−19	− 4	+15	+ 7
573	δ Capricorni	3.0	21 41 31.349	3.3163	− 125	+ 179	+ 1	68	64	2.4	+ 5	+ 3	+48	+14
574	μ Capricorni	5.2	21 47 50.713	3.2751	− 112	+ 211	0	72	47	2.0	+26	+ 7	–	–
576	α Aquarii	3.2	22 0 38.887	+3.0826	− 41	+ 9	0	67	168	6.6	− 9	0	+32	+10
577	ι Aquarii	4.3	22 1 2.237	3.2443	− 111	+ 23	0	71	52	1.5	+10	+ 1	+30	+12
579	ι Pegasi	3.9	22 2 21.313	2.7903	+ 63	+ 220	+1	73	55	1.7	−16	− 2	+16	+ 8
580	θ Pegasi	3.7	22 5 9.336	3.0266	− 11	+ 184	0	71	53	1.9	−14	− 3	+26	+ 8
581	ζ Cephei	3.6	22 7 23.014	2.0751	+ 115	+ 9	0	72	74	2.8	−37	− 9	− 9	+ 3
582	θ Aquarii	4.3	22 11 33.445	+3.1684	− 75	+ 74	0	71	108	3.1	+ 1	+ 1	+29	+11
585	γ Aquarii	3.9	22 16 29.491	3.0997	− 42	+ 81	0	73	108	3.4	− 5	0	+35	+ 9
586	β Lacertae	4.6	22 19 37.577	2.3522	+ 157	− 17	+ 1	69	40	2.1	−31	−10	+ 3	+ 4
587	σ Aquarii	4.8	22 25 21.362	3.1781	− 87	0	0	70	61	2.2	− 8	0	–	–
588	δ² Cephei	Var.	22 25 27.381	2.2191	+ 169	+ 13	0	61	55	2.7	−20	− 2	+ 1	+ 6
589	α Lacertae	3.8	22 27 10.220	+2.4643	+ 170	+ 145	+ 1	73	58	1.6	−12	−12	− 5	+ 5
590	η Aquarii	4.1	22 30 13.092	3.0840	− 30	+ 60	0	73	132	3.2	+ 6	+ 4	+36	+11
591	κ Aquarii	5.1	22 32 34.671	3.1081	− 49	− 52	0	68	22	1.2	−19	− 4	–	–
593	ζ Pegasi	3.6	22 36 28.467	2.9910	+ 24	+ 53	0	70	126	3.5	− 7	− 1	+28	+11
595	η Pegasi	3.1	22 38 18.805	2.8072	+ 110	+ 8	0	71	64	2.1	−17	− 3	+ 4	+ 7
596	λ Pegasi	4.2	22 41 12.817	+2.8860	+ 84	+ 11	0	71	56	2.0	+ 3	+ 3	+15	+ 8
598	τ² Aquarii	4.3	22 44 17.893	3.1803	− 98	− 11	0	71	48	1.5	− 8	− 3	+36	+11
599	μ Pegasi	3.7	22 45 10.551	2.8915	+ 92	+ 107	0	73	71	2.3	−20	− 4	+14	+ 6
600	ι Cephei	3.7	22 46 7.133	2.1215	+ 227	− 112	− 1	70	82	3.5	−24	− 2	+ 7	+ 8
601	λ Aquarii	3.9	22 47 23.882	3.1320	− 62	+ 3	0	74	105	3.0	+ 2	+ 1	+38	+11
602	δ Aquarii	3.5	22 49 20.633	+3.1880	− 109	− 33	0	67	42	2.2	+ 5	+ 1	+37	+13
604	ο Andromedae	3.6	22 57 19.112	2.7517	+ 190	+ 20	0	74	57	1.7	+ 3	0	− 2	+ 6
605	β Pegasi	2.7	22 58 55.530	2.9033	+ 120	+ 115	0	74	56	2.1	− 3	− 1	+18	+10
606	α Pegasi	2.6	22 59 16.740	2.9855	+ 58	+ 40	0	67	165	6.6	− 1	0	+23	+ 9
607	φ Aquarii	4.4	23 9 8.619	3.1081	− 43	+ 18	0	71	47	2.2	− 4	+ 3	–	–
608	ψ¹ Aquarii	4.5	23 10 39.175	+3.1156	− 61	+ 248	0	70	32	1.4	−22	− 2	–	–
610	γ Piscium	3.8	23 11 58.871	3.1093	+ 7	+ 502	0	72	120	3.3	− 2	+ 1	+33	+11
612	ψ³ Aquarii	5.2	23 13 45.625	3.1235	− 61	+ 32	0	67	24	1.2	+ 4	+ 5	–	–
613	98 Aquarii	4.2	23 17 13.188	3.1560	− 121	− 87	0	68	19	0.8	+42	+12	–	–
614	κ Piscium	5.0	23 21 18.377	3.0753	+ 1	+ 57	0	75	99	2.4	− 4	+ 1	+33	+11
615	θ Piscium	4.4	23 22 53.707	+3.0415	+ 28	− 88	0	72	34	1.5	0	− 1	–	–
617	λ Andromedae	4.0	23 32 40.014	2.9232	+ 282	+ 150	+ 2	71	61	2.7	−37	− 7	− 1	+ 7
618	ι Piscium	4.3	23 34 18.388	3.0811	+ 32	+ 218	+ 1	71	133	3.7	− 6	+ 2	+28	+ 9
619	γ Cephei	3.4	23 35 11.139	2.4266	+ 753	− 178	−10	65	119	5.7	−45	− 7	+43	+20
620	20 Piscium	5.6	23 42 48.113	3.0844	− 8	+ 63	0	67	22	1.1	–	–	–	–
623	27 Piscium	5.0	23 53 33.217	+3.0712	− 6	− 38	0	66	25	1.3	− 1	− 4	–	–
624	ω Piscium	4.0	23 54 10.548	3.0787	+ 49	+ 101	0	74	144	3.4	− 3	− 1	+34	+12
625	29 Piscium	5.1	23 56 41.954	3.0743	− 2	+ 8	0	67	20	1.1	–	–	–	–
626	30 Piscium	4.7	23 56 49.886	3.0772	− 18	+ 27	0	72	33	1.1	−14	− 2	–	–
627	2 Ceti	4.6	23 58 37.052	3.0762	− 79	+ 13	0	74	51	1.6	+ 1	− 2	–	–

FIRST SECTION — (Declination, +82° to −21° 50′).

No.	Decl. 1900	Ann. V.	Sec. V	μ′	100 Δμ′	T	p	p_s	B − N Δδ	B − N Δμ	B − A Δδ	B − A Δμ	
	° ′ ″		.001	.001	.001				.01	.001	.01	.001	
557	+29 48 59.59	+11.613	+247	− 59	0	69	122	3.5	− 19	+ 2	−16	−10	
558	+37 37 5.88	15.251	+230	+ 427	+ 4	70	51	2.0	47	− 7	−27	−13	
559	+ 4 50 3.27	14.742	+288	− 87	0	77	75	3.0	28	1	+ 4	− 8	
560	+62 9 42.48	15.190	+133	+ 49	+ 2	64	140	7.5	3	0	+ 8	+ 3	
561	−17 15 37.83	15.174	+313	+ 6	0	65	34	1.6	− 14	+ 2			
562	+19 22 35.24	+15.272	+258	+ 59	+ 1	75	51	1.8	− 38	− 5	−23	11	
565	− 6 0 10.41	15.698	+280	− 7	0	67	146	5.1	− 9	+ 4	0	−12	
566	+70 7 18.04	15.768	+ 65	+ 5	0	63	114	6.5	− 2	− 1	+26	+ 8	
567	−19 54 51.19	15.981	+289	− 2	0	67	20	1.1		−	−	−	
568	− 8 18 10.08	16.009	+274	− 24	+ 1	73	60	2.5	− 24	0	−	−	
569	−17 6 50.75	+16.123	+282	− 20	+ 1	69	52	2.4	− 31	3	+23	−11	
571	+ 9 24 58.88	16.384	+241	− 1	0	70	143	4.4	− 22	0	− 6	−10	
572	+25 11 6.68	16.429	+220	− 2	0	75	45	1.5	− 38	−11	− 4	−10	
573	−16 31 52.17	16.202	+269	− 295	+ 1	67	56	2.5	− 12	+ 2	+36	−10	
574	−14 1 21.44	16.814	+255	+ 9	+ 2	71	53	1.9	+ 5	+ 8	−	−	
576	− 0 48 20.80	+17.383	+216	− 6	0	65	167	7.2	− 29	− 4	+ 3	− 8	
577	−14 21 17.82	17.346	+227	− 60	0	69	48	2.0	− 24	+ 2	+20	−12	
579	+24 51 23.37	17.481	+191	+ 18	+ 2	73	54	1.9	− 33	− 2	− 4	− 7	
580	+ 5 42 20.72	17.615	+206	+ 33	+ 1	75	53	1.9	− 32	− 3	+16	− 5	
581	+57 42 29.53	17.682	+135	+ 7	0	71	83	4.3	− 28	− 3	+15	+ 3	
582	− 8 16 52.77	+17.825	+203	− 19	0	71	106	3.6	− 31	− 1	+18	− 6	
585	− 1 53 28.90	18.046	+190	+ 9	+ 1	73	97	3.7	− 52	− 6	+16	− 5	
586	+51 13 40.47	17.965	+138	− 190	0	73	54	3.1	− 27	− 1	+20	+ 2	F 3
587	−11 11 23.30	18.333	+178	− 29	0	69	56	2.4	− 46	− 3	−		
588	+57 51 11.67	18.368	+122	+ 3	0	60	51	3.0	− 15	+ 2	+35	+ 7	
589	+19 46 5.67	+18.438	+135	+ 13	+ 1	72	71	3.3	− 12	0	+12	− 2	F 7
590	− 0 37 58.99	18.474	+164	− 54	0	73	119	3.3	− 38	− 1	+ 1	− 6	
591	− 4 44 38.12	18.491	+161	− 115	0	63	24	1.1	− 21	− 2	−	−	
593	+10 18 33.15	18.718	+118	− 12	0	68	126	4.4	− 13	+ 2	−10	−10	
595	+29 11 52.99	18.752	+135	− 35	0	75	63	1.8	− 13	+ 2	+16	− 2	
596	+23 2 21.31	+18.875	+133	− 11	0	75	66	2.4	− 29	− 5	−19	−14	
598	−14 7 13.82	18.927	+143	− 36	0	69	38	1.6	− 58	3	+15	13	
599	+21 4 24.26	18.943	+128	45	+ 1	73	62	2.0	− 30	− 3	−22	−12	
600	+65 10 27.88	18.893	+ 89	− 121	− 1	69	97	4.8	+ 13	+ 5	+34	+ 9	
601	− 8 6 12.53	19.085	+134	+ 36	0	73	96	3.4	20	0	+10	9	
602	−16 21 9.77	+19.081	+133	− 21	0	69	38	1.9	− 18	+ 5	+25	−10	
604	+41 47 18.19	19.288	+100	− 13	0	73	54	1.6	40	− 4	+ 3	− 5	
605	+27 32 21.72	19.472	+101	+ 133	+ 1	69	67	2.9	− 30	2	− 26	−14	
606	+11 10 1.47	19.311	+105	− 45	0	64	161	6.9	− 36	6	− 21	−14	
607	− 6 35 17.26	19.365	+ 91	− 190	0	69	48	2.3	+ 7	+ 4			
608	− 9 37 57.58	+19.570	+ 91	− 14	+ 1	65	34	2.0	63	− 8			F 91
610	+ 2 44 8.83	19.627	+ 87	+ 49	+ 2	73	111	3.5	30	2	+ 6	6	
612	−10 9 27.05	19.612	+ 83	+ 2	0	63	29	1.6	+ 7	+ 3			F 95
613	−20 38 17.31	19.614	+ 76	− 93	0	72	15	0.5	+ 33	3			14
614	+ 0 42 29.14	19.679	+ 66	90	0	74	89	2.8	− 33	+ 2	+ 2	6	
615	+ 5 49 16.58	+19.742	+ 63	− 43	0	70	44	2.0	− 43	− 2			
617	+15 54 58.78	19.483	+ 42	122	0	70	67	3.7	14	1	+46	+ 3	
618	+ 5 5 3.09	19.487	+ 41	139	0	69	128	4.0	30	3	+ 2	7	
619	+77 1 27.30	20.088	+ 29	+ 158	0	66	156	7.6	0	+ 1	+29	+10	
620	− 3 19 3.30	19.996	+ 25	+ 6	0	66	26	1.4					
623	− 4 6 38.80	+19.972	+ 4	67	0	64	32	1.5	− 19	1			
624	+ 6 18 34.71	19.931	+ 3	109	0	74	141	4.6	− 35	2	9	9	
625	− 3 35 3.29	20.032	2	13	0	65	26	1.5					
626	− 6 34 11.15	20.011	2	34	0	68	32	1.8	12	+ 3			
627	−17 53 33.79	20.038	6	9	0	72	34	1.4	1	+ 5			

Second Section (Declination, 21° 50′ to 82°)

No.	Name and Magnitude		R. A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B − N		B − A	
					.0001	.0001	.0001	T	p_α	p	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		m	h m s	s	s	s	s				s	s	s	s
4	ε Phoenicis	3.8	0 1 20.246	+3.0569	288	+ 110	− 1	72	7	0.2	+ 60	+ 11	+ 65	+ 9
8	ζ Tucanae	4.3	0 14 51.687	3.1544	661	+2718	−55	72	8	0.3	−213	−32	+ 5	+20
9	ι Sculptoris	5.1	0 16 29.816	3.0247	− 135	+ 38	0	81	17	0.3	–	–	–	–
10	β Hydri	2.9	0 20 30.041	3.2218	−1481	+7025	−321	66	39	0.8	− 21	− 2	+ 65	+34
14	α Phoenicis	2.4	0 21 20.554	2.9750	− 228	+ 179	− 2	67	7	0.4	− 6	− 9	+ 75	+ 6
13	β¹ Tucanae	4.5	0 26 57.738	+2.7701	− 113	+ 125	− 3	68	6	0.2	+ 8	+ 8	–	–
27	α Sculptoris	4.3	0 53 47.341	2.8949	− 99	+ 13	0	79	27	0.9	+ 89	+30	+116	+40
31	β Phoenicis	3.4	1 1 37.337	2.6841	− 179	− 11	0	61	9	0.1	+107	+16	+ 79	+15
35	ζ Phoenicis	4.1	1 4 10.979	2.5304	− 219	+ 12	0	68	7	0.3	–	–	–	–
40	γ Phoenicis	3.3	1 24 1.414	2.6100	− 125	− 21	− 1	61	9	0.4	+ 14	+ 1	+ 82	+25
43	δ Phoenicis	3.9	1 27 5.331	+2.5034	− 133	+ 134	− 1	61	8	0.4	–	–	+ 85	+16
45	α Eridani	0.5	1 33 59.159	2.2395	− 129	+ 116	− 2	60	23	0.7	+ 40	+12	+ 82	+21
50	ε Sculptoris	5.4	1 40 57.761	2.8114	− 38	+ 115	− 1	79	18	0.5	+152	+63	+ 70	+25
56	χ Eridani	3.6	1 52 3.983	2.3382	− 97	+ 725	− 6	71	7	0.3	–	–	+128	+54
58	α Hydri	3.0	1 55 37.070	1.8896	− 33	+ 350	− 5	64	11	0.4	+220	+73	− 2	+ 7
61	φ Eridani	3.7	2 12 56.221	+2.1444	− 45	+ 87	− 1	67	7	0.3	+ 71	+25	+ 72	+18
66	κ Fornacis	5.4	2 17 58.022	2.7452	− 7	+ 141	− 1	83	17	0.2	+ 38	+ 3	+ 71	+16
67	δ Hydri	4.2	2 19 58.098	1.0541	+ 290	− 94	+ 3	65	7	0.2	+ 41	+ 4	+ 84	+27
72	ε Hydri	4.2	2 38 2.982	0.9089	+ 334	+ 170	− 3	65	7	0.3	–	–	+ 52	+25
77	β Fornacis	4.2	2 44 54.362	2.5114	− 4	+ 72	+ 1	85	16	0.4	+ 52	− 8	+ 66	+21
80	θ¹ Eridani	3.1	2 54 28.206	+2.2741	− 2	− 51	+ 1	67	9	0.3	− 54	−26	+134	+24
83	Br. 134	4.1	2 57 58.982	2.6143	+ 17	− 107	0	85	22	0.7	− 2	− 3	+ 59	+17
87	θ Hydri	5.4	3 2 2.845	0.0927	+ 715	+ 66	0	72	15	0.3	+145	+32	+100	+37
89	α Fornacis	4.0	3 7 49.396	2.5475	+ 18	+ 251	+ 3	75	24	0.8	+ 33	+11	+ 66	+17
91	Br. 469	4.0	3 15 4.113	2.6675	+ 26	+ 37	0	80	20	0.7	–	–	–	–
97	Br. 495	4.3	3 29 22.265	+2.6492	+ 30	+ 36	0	75	15	1.1	+ 81	+13	+ 57	+17
98	L 1161	4.5	3 33 30.417	2.1528	+ 23	− 1	0	70	7	0.3	–	–	+ 99	+25
102	L 1198	4.1	3 39 7.686	2.2237	+ 23	− 69	0	75	7	0.2	–	–	–	–
104	Br. 530	4.3	3 42 32.733	2.5801	+ 25	− 115	− 3	79	19	0.7	+ 6	− 1	+ 58	+15
106	L 1248	4.2	3 45 42.721	2.2447	+ 25	− 36	0	81	8	0.2	− 39	0	+ 25	+ 15
108	γ Hydri	3.2	3 48 47.092	−0.9783	+1070	+ 125	+ 6	67	31	0.6	+ 51	+30	+108	+48
109	L 1275	5.3	3 49 50.332	+2.2851	+ 26	+ 25	0	85	9	0.2	+ 92	+18	–	–
113	δ Reticuli	4.3	3 57 9.639	0.9390	+ 195	− 9	0	66	7	0.3	+ 44	+11	+ 59	+21
120	α Horologii	3.8	4 10 41.285	1.9867	+ 35	+ 39	− 2	68	6	0.3	− 5	− 2	+ 95	+31
121	α Reticuli	3.4	4 13 8.132	0.7623	+ 214	+ 53	+ 1	64	8	0.4	+ 12	+ 5	+ 58	+19
122	γ Doradus	4.3	4 13 24.362	+1.5680	+ 79	+ 103	+ 2	67	6	0.3	–	–	+ 87	+29
125	L 1441	3.9	4 20 16.846	2.2522	+ 33	+ 50	0	83	17	0.5	+ 2	− 2	+ 43	+11
126	η Reticuli	5.2	4 20 18.401	0.6381	+ 236	+ 123	+ 4	69	7	0.3	–	–	+ 49	+17
129	α Doradus	3.5	4 31 50.134	1.2931	+ 97	+ 66	0	64	8	0.4	+ 24	− 1	+ 61	+26
131	α Caeli	4.5	4 37 20.406	1.9310	+ 40	− 132	− 1	63	8	0.3	+ 86	+16	+ 68	+13
132	β Caeli	5.2	4 38 31.323	+2.1195	+ 39	+ 29	+ 2	77	7	0.3	–	–	–	–
141	ε Leporis	3.1	5 1 13.666	2.5383	+ 32	+ 16	0	73	54	1.3	+ 4	+ 4	+ 14	− 4
151	ε Columbae	3.9	5 27 39.784	2.1297	+ 28	+ 27	− 1	80	10	0.4	–	–	+ 46	+ 7
157	α Columbae	2.7	5 36 1.688	2.1720	+ 27	+ 5	0	68	48	1.5	+ 8	− 1	+ 65	+14
159	β Columbae	3.2	5 47 26.081	2.1137	+ 33	+ 41	+ 4	78	17	0.8	+ 61	+13	+ 67	+18
163	γ Columbae	4.5	5 53 59.505	+2.1266	+ 24	0	0	80	15	0.6	+ 4	+ 4	–	–
164	L 2137	6.1	6 1 35.771	1.7264	+ 30	− 78	+ 3	69	5	0.2	+ 24	+10	+ 44	+ 4
166	κ Columbae	4.5	6 12 59.671	2.1340	+ 21	− 3	+ 1	70	8	0.3	+ 24	7	+ 23	+10
167	ζ Can. Maj.	3.2	6 16 28.471	2.3030	+ 18	+ 9	0	79	21	0.9	+ 74	+14	+ 48	+11
170	α Carinae	−1.0	6 21 43.911	1.3315	+ 9	+ 19	0	60	27	0.7	− 13	− 3	+ 47	+ 7
172	Br. 972	4.6	6 30 51.913	+2.5142	+ 15	+ 9	0	78	18	0.7	− 7	−13	+ 23	+ 8
174	ν Puppis	3.2	6 34 42.111	1.8366	+ 12	+ 9	0	60	9	0.4	− 16	+ 1	+ 66	+16
177	κ Can. Maj.	3.9	6 46 6.354	2.2407	+ 15	− 8	0	76	16	0.6	–	–	+ 29	+ 7
178	α Pictoris	3.3	6 47 9.946	0.6186	− 50	− 104	+ 7	65	9	0.3	− 24	+ 2	+ 43	+22
180	ε Can. Maj.	1.7	6 54 41.753	2.3578	+ 13	+ 4	0	69	72	1.9	+ 11	+ 6	+ 62	+14

SECOND SECTION (Declination, −21° 50′ to −82°)

No.	Decl. 1900	Ann. V. and	Sec. V.	μ' and	100 $\Delta\mu'$	Ep. and Wt. T	p	P	B − N $\Delta\delta$	$\Delta\mu$	B − A $\Delta\delta$	$\Delta\mu$	
	° ′ ″	″	.001 ″	.001 ″	.001 ″				.01 ″	.001 ″	.01 ″	.001 ″	
4	−46 17 57.17	+19.856	− 17	− 187	0	73	9	0.3	− 2	+ 6	+31	+18	
8	−65 27 44.92	21.167	− 38	+1162	− 3	71	11	0.3	−103	10	+66	+15	
9	−29 32 4.23	19.919	− 11	− 76	0	81	12	0.2					
10	−77 49 2.71	20.285	− 50	+ 318	+ 2	68	10	0.8	+ 15	0	+59	+ 7	
11	−42 50 56.80	19.559	− 19	− 101	0	63	11	0.6	− 20	+ 2	+ 6	+ 2	
13	−63 30 32.88	+19.852	− 56	− 56	0	69	9	0.3	− 4	− 1			
27	−29 53 52.63	19.493	−106	− 4	0	78	20	0.9	+ 15	+ 9	− 1	+11	
31	−17 15 15.71	19.313	−112	− 11	0	66	11	0.5	+ 32	+11	−14	− 5	
35	−55 46 49.47	19.284	−111	+ 18	0	72	9	0.3	−				
40	−43 49 50.03	18.499	−144	− 216	0	64	11	0.5	+ 32	+ 9	−29	−11	
43	−19 35 32.48	+18.769	−111	+ 152	− 1	67	8	0.3		−	− 5	− 6	
45	−57 44 40.94	18.358	−139	− 26	− 1	64	21	0.6	+ 37	+14	+35	+ 9	
50	−25 33 8.33	18.081	−183	− 52	− 1	71	18	0.5	− 38	− 1	+30	+14	
56	−52 6 24.10	17.977	−173	+ 279	− 5	75	8	0.2			+18	+19	
58	−62 3 22.76	17.591	−144	+ 41	− 2	65	15	0.5	+ 22	+15	+74	+27	
64	−51 58 30.15	+16.740	−179	− 28	0	67	10	0.4	+ 4	+ 1	+20	+ 6	
66	−24 16 14.35	16.460	−234	− 63	− 1	68	18	0.4	+ 19	+14	− 2	− 7	
67	−69 6 52.03	16.438	− 91	+ 15	+ 1	68	9	0.3	− 16	− 5	+25	+ 4	
72	−68 41 13.19	15.481	− 93	+ 15	− 2	68	8	0.3		−	+12	+13	
77	−32 49 33.60	15.227	−219	+ 119	− 1	80	14	0.5	− 14	− 6	−13	− 3	
80	−40 12 19.02	+14.513	−235	+ 30	+ 1	66	10	0.4	+ 11	+ 6	−20	− 1	
83	−21 0 59.09	14.257	−275	− 12	+ 1	69	19	0.8	− 39	+ 1	− 7	+ 1	τ^3 Eridani
87	−72 17 31.67	14.074	− 17	+ 26	− 1	71	18	0.3	+ 5	+12	+71	+25	
89	−29 22 52.13	14.330	−280	+ 617	− 3	74	21	0.9	+ 6	+10	−13	− 1	
91	−22 7 18.42	13.254	−297	+ 41	0	70	19	0.6			−	−	τ^4 Eridani
97	−21 58 5.34	+12.226	−311	− 21	0	77	14	0.9	+ 19	+18	−21	− 2	τ^5 Eridani
98	−40 36 9.57	11.931	−257	− 28	0	71	8	0.3			50	−11	v Eridani
102	−37 37 45.04	11.476	−269	− 85	+ 1	69	8	0.3			−		h Eridani
104	−23 32 12.20	10.791	−314	521	+ 1	71	17	0.5	−179	−13	−36	− 9	τ^6 Eridani
106	−36 30 10.86	11.010	−277	− 16	0	77	9	0.3	− 10	17	+20	+ 3	v Eridani
108	−74 32 13.81	+10.976	+113	+ 115	− 2	68	31	0.6	8	2	+59	+11	
109	−35 1 10.78	10.763	−286	20	0	75	9	0.3	17	3			f Eridani
113	−61 10 57.18	10.215	−122	23	0	70	8	0.3	128	−20	− 12	1	
120	−42 32 27.11	8.990	−262	213	1	68	8	0.4	+ 72	+17	+24	+10	
121	−62 43 26.75	9.066	−104	+ 54	1	68	11	0.4	+ 6	+10	+ 6	+ 4	
122	−51 11 19.89	+ 9.170	210	+ 179	1	70	8	0.3			+36	+15	
125	−31 11 56.30	8.504	302	+ 51	1	76	13	0.6	+ 21	+12	8	+ 2	δ Eridani
126	−63 37 25.34	8.579	− 90	+ 174	2	71	8	0.3			+49	+26	
129	−55 15 6.00	7.520	−179	− 3	1	67	9	0.4	+ 32	+ 8	+18	+ 5	
131	−12 3 16.82	6.986	265	89	+ 2	68	8	0.3	+ 54	+18	25	7	
132	−37 20 23.08	+ 7.162	−294	+ 184	0	68	6	0.2					
141	−22 30 18.90	5.020	−360	− 65	0	73	37	1.0	+ 17	1	+ 1	+ 3	
151	−35 32 37.81	2.770	309	− 19	0	73	10	0.4			−19	2	
157	−34 7 38.59	2.056	316	− 37	0	69	40	1.2	− 12	+ 1	2	3	
159	−35 48 21.19	1.491	−308	+ 395	1	71	15	0.8	+ 5	+ 7	20	12	
163	−35 17 38.33	+ 0.521	−310	− 5	0	74	13	0.6	− 20	11			
164	−45 2 10.18	+ 0.093	250	+ 233	+ 1	70	7	0.3	+ 71	+ 8	+ 1	2	I G. Pup
166	−35 6 25.69	− 1.059	310	+ 77	0	67	10	0.4	+ 51	+12	+12	+10	
167	−30 1 8.15	1.113	331	3	0	79	20	0.9	+ 52	+20	57	20	
170	−52 38 27.56	− 1.884	193	+ 14	0	63	24	0.7	+ 9	+ 5	+20	+ 7	Argus
172	−22 53 7.15	− 2.671	362	+ 21	0	69	20	0.6	128	14	+12	+ 1	ξ¹ C. Ma
174	−43 6 29.68	3.012	261	18	0	62	11	0.5	22	+ 1	14	2	Argus
177	−32 23 31.35	1.000	−318	+ 6	0	75	13	0.5			39	11	
178	−61 50 1.76	3.834	85	+ 262	+ 1	71	8	0.2	+ 12	+25	38	− 14	
180	−28 50 9.33	4.710	−332	− 1	0	70	50	1.3	23	1	6	+ 2	

SECOND SECTION — (Declination, — 21° 50′ to — 82°).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V		μ and 100 Δμ		Ep. and Wt.			B — N		B — A	
					.0001	.0001	.0001	T	p_α	p_μ	Δα	Δμ	Δα	Δμ
											.001	.0001	.001	.0001
		m	h m s	s	s	s	s				s	s	s	s
181	σ Can. Maj.	3.9	6 57 11.132	+2.3895	+ 12	— 7	0	80	16	0.8	+ 6	— 1	–	–
184	δ Can. Maj.	2.1	7 1 19.527	2.4393	+ 11	— 2	0	74	47	1.3	+ 43	+13	+ 32	+ 7
187	π Puppis	2.7	7 13 36.685	+2.1191	+ 10	— 5	0	71	17	0.9	+ 25	+ 3	+ 64	+11
189	δ Volantis	3.9	7 16 52.910	—0.0170	— 252	— 10	0	65	8	0.2	—350	—11	— 16	+ 1
191	η Can. Maj.	2.4	7 20 8.375	+2.3726	+ 11	— 7	0	74	27	1.3	— 43	—10	+ 20	— 2
194	σ Puppis	3.5	7 26 3.530	+1.9034	+ 8	— 55	+ 2	69	7	0.2	+ 60	+17	–	
197	Br. 1120.	4.4	7 39 47.626	2.4079	+ 12	— 6	0	81	15	0.5	–	–	+ 36	+ 7
198	L 2958	5.1	7 41 41.496	2.1365	+ 12	— 21	0	75	8	0.3	–		+ 66	+20
201	χ Carinae	3.6	7 51 14.197	1.5274	— 30	— 36	0	64	8	0.5	— 3	+ 8	+ 60	+15
203	ζ Puppis	2.3	8 0 4.186	2.1081	+ 13	— 28	0	72	14	0.8	+ 26	+16	+ 30	+ 2
204	ρ Puppis	2.9	8 3 17.418	+2.5545	+ 10	— 65	0	70	83	2.3	+ 9	0	+ 20	0
205	γ Velorum	1.9	8 6 27.093	1.8501	0	0	0	61	12	0.6	— 7	+ 3	+ 67	+20
207	L 3259	4.5	8 14 48.730	2.2445	+ 21	— 96	0	71	9	0.5	–	–	+147	+37
208	ε Carinae	1.7	8 20 27.750	1.2360	— 90	— 31	0	58	12	0.5	+ 20	+ 9	+ 17	+ 7
211	β Pyxidis	3.9	8 36 11.270	2.3475	+ 28	+ 7	0	82	9	0.5	–	–	— 3	+ 6
215	δ Velorum	2.0	8 41 56.569	+1.6579	— 20	+ 23	— 1	65	10	0.5	+149	+58	+ 88	+18
218	L 3639	5.1	8 51 31.676	1.4712	— 53	— 12	0	63	8	0.3	+141	+22		–
222	λ Velorum	2.1	9 4 19.074	2.2049	+ 45	— 21	0	64	11	0.5	+ 21	— 6	+106	+30
225	β Carinae	2.0	9 12 6.251	0.6763	— 357	— 300	— 2	64	21	0.7	+ 16	+10	+ 35	+19
227	ι Carinae	2.2	9 14 24.768	1.6059	— 23	— 40	0	64	20	0.6	+ 49	+15	+ 33	+11
229	θ Pyxidis	5.1	9 17 3.953	+2.6541	+ 35	— 15	0	77	17	0.5	+139	+33	+ 61	+13
230	κ Velorum	2.6	9 19 1.011	1.8563	+ 27	— 18	0	71	8	0.4	+ 31	+15	+ 86	+27
234	ψ Velorum	3.5	9 26 45.692	2.3599	+ 64	— 165	— 1	78	12	0.4	+112	+15	+103	+25
238	υ Carinae	3.0	9 44 36.165	1.5019	— 47	— 21	— 1	65	11	0.4	— 5	+ 4	+ 63	+18
241	φ Velorum	3.7	9 53 21.087	2.1014	+ 94	— 21	0	73	9	0.3	+ 41	+12	+ 58	+15
246	L 4212	4.0	10 10 32.242	+2.5115	+ 118	— 149	— 2	68	7	0.3	+ 62	+ 4	+ 75	+23
248	ω Carinae	3.6	10 11 21.651	1.4316	— 73	— 55	0	72	10	0.3	–	–	— 50	—14
254	L 4319	4.0	10 22 24.645	1.1994	— 222	— 66	— 1	73	10	0.2	–	–	+ 53	+26
255	α Antliae	4.2	10 22 34.568	2.7418	+ 97	— 51	0	77	20	0.6	+ 63	+ 9	+ 90	+25
259	θ Carinae	3.0	10 39 23.280	2.1304	+ 201	— 32	— 1	60	9	0.5	+ 20	+10	+ 43	+18
260	η Carinae	Var.	10 41 10.840	+2.3180	+ 220	+ 4	0	61	18	0.5	+ 35	+ 5	+ 56	+17
261	μ Velorum	2.8	10 42 28.020	2.5692	+ 197	+ 52	0	71	9	0.4	— 10	—14	+ 14	— 3
265	L 4515	4.1	10 49 25.713	2.4219	+ 251	+ 60	+ 1	70	7	0.2	–	–	— 50	—10
272	β Crateris	4.5	11 6 44.348	2.9461	+ 99	— 1	0	76	32	1.5	+ 23	— 1	+ 40	+11
278	π Centauri	4.1	11 16 26.723	2.7219	+ 308	— 37	— 1	67	6	0.3	–	–	+ 47	+14
284	ξ Hydrae	3.6	11 28 4.967	+2.9140	+ 166	— 155	— 1	78	21	0.7	+ 30	+ 3	+ 37	+15
285	λ Centauri	3.3	11 31 10.039	2.7452	+ 151	— 51	— 1	71	8	0.4	+ 99	+18	+ 89	+29
294	δ Centauri	2.8	12 3 10.461	3.0904	+ 382	— 41	0	68	10	0.5	+ 51	+ 9	+ 48	+10
295	α Corvi	4.2	12 3 15.253	3.0869	+ 156	+ 60	+ 1	70	18	1.2	–	–	–	–
296	ε Corvi	3.2	12 4 58.846	3.0791	+ 143	— 47	0	71	65	1.2	+ 4	+ 4	+ 44	+14
297	ρ Centauri	4.2	12 6 25.422	+3.1153	+ 410	— 47	— 1	72	6	0.3	–	–	–	–
299	δ Crucis	3.1	12 9 49.966	3.1590	+ 532	— 58	— 1	68	9	0.4	—184	—79	+ 17	+ 8
302	β Chamael.	4.3	12 12 28.462	3.4221	+1865	— 160	— 8	66	28	0.6	+ 39	+27	— 35	+11
306	ε Crucis	3.5	12 15 57.695	3.2090	+ 585	— 234	— 4	71	5	0.3	–	–	+ 48	+13
309	γ Crucis	1.6	12 25 36.988	3.3001	+ 550	+ 21	+ 1	68	9	0.5	+ 58	+49	+ 17	— 2
311	β Corvi	2.7	12 29 7.981	+3.1437	+ 165	0	0	69	95	2.6	+ 30	+ 7	+ 43	+12
313	γ Centauri	2.4	12 35 59.977	3.2878	+ 417	— 201	— 2	68	10	0.5	— 13	— 6	+ 54	+12
314	β Crucis	1.5	12 41 52.511	3.4722	+ 660	— 66	— 1	60	12	0.6	— 9	— 2	— 2	+ 3
319	δ Muscae	3.6	12 55 23.208	4.0537	+1426	+ 520	+15	74	13	0.3	+ 28	+26	+ 15	+26
325	ι Centauri	3.0	13 14 58.440	3.3583	+ 303	— 282	— 2	75	16	0.9	+ 90	+12	+ 60	+19
329	L 5569	4.0	13 25 14.630	+3.4629	+ 342	— 12	0	70	9	0.3	–	–	+ 82	+24
331	ε Centauri	2.6	13 33 32.950	3.7716	+ 592	— 37	0	67	10	0.5	+ 20	+ 3	+ 36	+ 7
335	ζ Centauri	2.8	13 49 17.963	3.7194	+ 471	— 61	0	69	10	0.5	+ 23	+ 9	+ 77	+23
338	β Centauri	0.8	13 56 45.799	4.1931	+ 848	— 33	0	64	24	0.9	— 29	+ 1	+ 36	+22
339	θ Centauri	2.2	14 0 47.742	3.5155	+ 318	— 431	0	75	18	0.8	+ 22	+ 7	+ 57	+23

SECOND SECTION — (Declination, −21° 50′ to −82°

No.	Decl. 1900	Ann. V. and Sec. V.		μ′ and 100 Δμ′		Ep. and Wt.			B — N Δδ	B — N Δμ′	B — A Δδ	B — A Δμ′	
	° ′ ″	″	″ .001	.001	.001	T	p_δ	p_{μ}	.01	.001	.01	.001	
181	−27 47 29.62	−5.000	−335	−3	0	79	16	0.7	−13	−5	—	—	
184	−26 14 3.91	5.555	−339	−2	0	76	43	1.3	−37	−5	−30	−5	
187	−36 55 4.29	6.328	−290	+1	0	67	16	0.8	+19	+10	+1	−1	[illegible]
189	−67 46 26.76	6.600	+5	0	0	71	9	0.2	+28	+6	+33	+15	
191	−29 6 28.78	6.865	−322	+3	0	74	22	1.1	−27	−4	−21	−6	
194	−43 5 56.19	−7.174	−254	+178	+1	67	10	0.4	−39	−2	—	—	Argus
197	−28 42 56.60	8.466	−315	−10	0	81	14	0.4	—	—	−24	−7	Puppis
198	−37 43 33.01	8.616	−277	−10	0	70	10	0.4		—	−1	−6	ε Puppis
201	−52 12 50.60	9.566	−191	+18	0	68	10	0.5	+22	+12	+16	+4	Argus
203	−39 43 16.97	10.021	−261	+8	0	68	14	0.8	+24	+13	−8	−2	Argus
204	−24 0 57.37	−10.226	−315	+45	+1	71	64	1.8	−36	−7	−37	−5	Argus
205	−47 2 30.54	10.507	−225	+1	0	63	15	0.7	+8	+12	+41	+14	Argus
207	−36 20 57.69	11.035	−266	+89	+1	71	11	0.4	—	—	+22	+7	η Puppis
208	−59 11 15.50	11.519	−142	+13	0	62	13	0.6	−5	+5	+11	+11	Argus
211	−34 57 12.04	12.649	−261	−20	0	73	8	0.4	—	—	+7	+7	
215	−54 20 31.77	−13.109	−179	−94	0	69	13	0.5	−51	+6	+24	+4	Argus
218	−58 50 35.77	13.836	−149	−3	0	66	9	0.3	+20	+16	—	—	b³ Carinae
222	−43 1 43.71	14.435	−217	+5	0	65	11	0.6	+19	+11	−18	−1	Argus
225	−69 18 18.89	14.804	−57	+100	+3	67	23	0.8	+8	+7	+34	+10	Argus
227	−58 51 20.27	15.034	−148	+4	0	67	19	0.6	−16	−2	−10	+8	Argus
229	−25 32 23.19	−15.199	−246	−9	0	74	17	0.6	+109	+23	0	+3	
230	−54 35 0.82	15.301	−168	0	0	70	11	0.5	+38	+19	−30	−11	Argus
234	−40 1 41.14	15.674	−205	+57	+1	76	12	0.4	+28	+18	−58	−23	Argus
238	−61 36 28.87	16.644	−115	+5	0	67	16	0.5	+36	+22	+2	−2	Argus
241	−51 5 30.36	17.066	−153	−3	0	71	12	0.4	+11	+18	−3	+1	Argus
246	−41 37 35.32	−17.771	−160	+32	+1	67	9	0.3	−38	0	−70	−23	γ Velor.
248	−69 32 28.70	17.838	−87	−2	0	74	14	0.4			+6	−4	Argus
254	−73 31 21.33	18.276	−61	−19	+1	74	14	0.3	—	—	+23	+8	l Carinae
255	−30 33 31.27	18.270	−157	−7	0	75	18	0.7	+20	+16	−56	−13	
259	−63 52 13.63	18.804	−99	+16	0	67	11	0.5	+151	+42	+42	+19	Argus
260	−59 9 31.20	−18.869	−106	+4	0	63	19	0.6	+23	+13	−18	−9	Argus
261	−48 53 30.48	18.972	−117	−61	0	73	11	0.4	+13	+20	+8	−3	Argus
265	−58 19 19.23	19.079	−99	+25	0	69	9	0.3			−12	+1	a Carinae
272	−22 16 17.40	19.606	−91	−98	0	76	24	1.0	+13	+8	−18	−3	
278	−53 56 34.97	19.700	−67	14	0	67	9	0.4	—		−16	+1	
284	−31 18 15.71	−19.914	−51	−58	0	76	47	0.7	−19	−4	−19	−15	
285	−62 27 59.21	19.901	−12	−13	0	75	10	0.4	+5	+14	+18	+12	
294	−50 9 55.65	20.061	+15	−16	0	69	13	0.6	+29	+14	+14	+10	
295	−24 10 15.97	20.094	+15	−49	0	70	14	0.9					
296	−22 3 49.07	20.039	+18	+3	0	70	10	0.9	−12	0	−64	−17	
297	−51 48 11.91	−20.060	+22	−21	0	74	8	0.2					
299	−58 11 33.83	20.054	+28	−26	0	69	12	0.5	+40	+13	−35	−13	
302	−78 45 25.04	20.006	+36	+11	0	69	29	0.7	−6	−6	+24	−3	
306	−59 50 54.63	19.909	+11	+89	0	72	7	0.3			+42	+9	
309	−56 33 11.85	20.195	+62	−273	0	69	12	0.5	−62	−13	+10	−2	
311	−22 50 37.48	19.945	+67	−60	0	70	69	2.1	−7	+2	−22	−3	
313	−48 21 38.25	19.818	+83	−18	0	69	12	0.6	+2	+2	+6	+7	
314	−59 8 31.51	19.737	+100	−21	0	65	15	0.7	+9	+9	+4	+6	
319	−71 0 33.96	19.494	+152	−27	+2	74	16	0.4	0	+5	+21	+4	
325	−36 11 5.65	19.080	+161	−96	−1	74	16	0.8	−37	0	+1	+3	
329	−38 53 27.13	18.703	+192	−27	0	69	10	0.4			−9	−9	[illegible]
331	−52 57 28.79	18.428	+226	−28	0	70	11	0.5	−2	+11	+2	+8	
335	−46 17 15.73	17.861	+256	−51	0	70	11	0.5	+18	+13	−9	−3	
338	−59 53 26.04	17.530	+306	−29	0	69	28	0.8	−15	+4	+53	+21	
339	−35 52 41.00	17.853	+262	−527	−3	73	15	0.8	−24	−3	−10	+1	

SECOND SECTION (Declination, −21° 30′ to −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.	μ and 100 $\Delta\mu$	Ep. and Wt.	B − N $\Delta\alpha$	B − N $\Delta\mu$	B − A $\Delta\alpha$	B − A $\Delta\mu$
				.0001	.0001 .0001	T p_α p_μ	.001	.0001	.001	.0001
		M	h m s	s s	s s		s	s	s	s
351	η Centauri	2.5	14 29 9.335	+3.7915 + 390	− 27 0	66 8 0.4	+ 15	+ 5	+ 26	+10
352	α Lupi	2.5	14 35 16.616	3.9677 + 173	− 18 0	68 7 0.4	+ 56	+23	+ 19	+10
360	β Lupi	2.7	14 51 58.778	3.9092 + 393	− 50 0	65 8 0.5	+ 78	+20	+ 46	+ 8
361	κ Centauri	3.4	14 52 39.282	3.8863 + 378	− 7 0	65 6 0.3	−122	−11	+ 79	+28
364	σ Librae	3.6	14 58 12.925	3.5018 + 209	− 56 0	73 32 1.1	− 28	+ 1	+ 35	+15
365	π Lupi	3.8	14 58 18.435	+4.0637 + 451	− 27 0	73 9 0.3	−	−	+ 53	+14
367	κ Lupi	4.2	15 4 58.749	4.1463 + 475	− 117 0	71 7 0.3	+ 19	+ 6	−	
368	ζ Lupi	3.5	15 5 5.933	4.2841 + 548	− 120 0	71 7 0.3	+ 13	+ 6	+100	+22
370	γ Tri. Austr.	3.0	15 9 31.190	5.5348 +1397	− 106 − 1	66 10 0.4	+ 70	+31	+ 34	+21
381	ϵ Tri. Austr.	4.1	15 27 33.854	5.4354 +1125	+ 33 + 3	67 7 0.3	−	−	− 12	+10
382	γ Lupi	3.0	15 28 28.501	+3.9822 + 331	− 16 0	67 10 0.5	+ 11	+ 3	+ 85	+32
394	β Tri. Austr.	3.1	15 46 19.726	5.2148 + 873	− 284 + 7	67 8 0.4	− 4	+ 7	+ 32	+ 8
397	ρ Scorpii	4.0	15 50 42.526	3.6952 + 199	− 12 0	73 12 0.7	−	−	+ 51	+18
399	π Scorpii	3.0	15 52 48.065	3.6210 + 178	− 11 0	76 22 1.1	+ 7	− 1	+ 50	+14
401	η Lupi	3.8	15 53 29.588	3.9627 + 267	− 23 0	71 9 0.4	−	−	−	−
402	δ Scorpii	2.7	15 54 25.147	+3.5403 + 158	− 8 0	71 60 2.0	+ 17	+ 4	+ 26	+ 9
406	δ Tri. Austr.	4.0	16 6 20.020	5.4240 + 783	+ 13 + 1	69 4 0.2	−	−	+ 56	+28
408	L 6764	4.2	16 12 21.328	4.4701 + 373	− 180 0	73 8 0.3	+118	+36	+ 49	+12
410	σ Scorpii	3.0	16 15 6.539	3.6396 + 154	− 8 0	73 11 1.4	+ 8	+ 3	+ 44	+14
413	γ Apodis	3.9	16 18 6.283	9.0568 +3206	− 386 + 4	71 15 0.4	+ 47	+23	+151	+54
416	α Scorpii	1.3	16 23 16.183	+3.6719 + 149	− 6 0	68 93 3.4	+ 4	0	−	−
417	L 6859	4.4	16 24 50.794	3.9122 + 192	− 2 0	82 11 0.4	+ 34	+ 5	+ 94	+41
422	τ Scorpii	2.8	16 29 39.363	3.7279 + 150	− 8 0	72 36 1.4	+ 8	+ 5	+ 46	+17
426	α Tri. Austr.	1.9	16 38 4.380	6.3115 + 889	+ 40 + 3	64 32 0.7	+ 27	+12	+ 18	+18
428	η Arae	3.6	16 41 8.890	5.1591 + 446	+ 42 + 1	73 7 0.3	−	−	+ 65	+30
429	ϵ Scorpii	2.3	16 43 41.120	+3.8783 + 161	− 496 + 1	75 19 1.1	+ 30	+10	+ 48	+ 9
430	μ^1 Scorpii	3.3	16 45 5.720	4.0557 + 177	− 11 0	73 11 0.6	+130	− 7	− 20	+ 3
431	μ^2 Scorpii	3.7	16 45 33.669	4.0547 + 176	− 18 0	78 7 0.3	−	−	− 2	+ 6
432	ζ Arae	3.0	16 50 20.588	4.9485 + 342	− 25 + 1	71 8 0.3	+328	−10	+ 64	+32
433	ϵ Arae	4.2	16 51 36.701	4.7670 + 294	− 9 0	70 8 0.3	+ 31	+ 2	+ 60	+19
438	η Scorpii	3.4	17 4 59.403	+4.2891 + 170	+ 17 + 4	68 10 0.4	+ 33	− 6	+ 34	+ 4
444	θ Ophiuchi	3.3	17 15 52.058	3.6811 + 78	− 1 0	71 70 1.7	+ 19	+ 5	+ 58	+18
445	γ Arae	3.4	17 16 58.542	5.0393 + 225	− 5 0	65 8 0.4	−	−	+ 33	+17
446	β Arae	2.7	17 16 59.153	4.9765 + 217	− 16 + 1	67 8 0.3	− 19	−12	+ 4	+ 2
447	Br. 2198	4.1	17 20 15.755	3.6607 + 72	0 + 1	75 36 1.1	+ 34	+ 9	+ 51	+19
448	υ Scorpii	2.8	17 23 57.872	+4.0747 + 94	− 1 0	82 11 0.4	+102	+23	+ 69	+20
449	α Arae	2.9	17 24 6.648	4.6309 + 116	− 34 + 1	62 10 0.5	+ 18	+ 2	+ 51	+16
450	Br. 2209	4.8	17 25 18.851	3.6577 + 64	+ 3 0	72 25 1.3	−	−	+ 70	+18
451	λ Scorpii	1.8	17 26 49.052	4.0697 + 87	− 3 0	76 16 0.8	+ 16	0	+ 56	+19
453	θ Scorpii	2.0	17 30 7.967	4.3062 + 96	+ 10 0	67 7 0.3	+ 62	+19	+ 41	+19
456	κ Scorpii	2.6	17 35 34.177	+4.1471 + 72	− 5 0	73 12 0.6	− 33	−10	+ 61	+20
462	ι^1 Scorpii	3.1	17 40 35.426	4.1936 + 62	+ 4 0	67 10 0.5	− 23	− 2	+ 45	+11
464	L 7149	3.2	17 43 3.053	4.0819 + 51	+ 48 0	77 7 0.3	−	−	+ 23	+21
472	θ Arae	3.8	17 58 50.846	4.6704 + 17	− 2 + 1	69 7 0.3	+ 66	+ 8	−	−
473	γ Sagittarii	3.0	17 59 23.039	3.8530 + 20	− 42 + 2	79 38 0.9	+ 38	+13	+ 60	+13
476	ϵ Telescopii	4.5	18 3 48.418	+4.4529 + 2	− 18 0	73 8 0.4	+248	+49	+ 77	+19
479	η Sagittarii	3.1	18 10 51.676	4.0600 − 6	− 107 + 2	81 14 0.4	− 4	+ 2	+ 83	+25
481	δ Sagittarii	2.8	18 14 35.547	3.8415 − 8	+ 31 0	76 26 1.0	+ 15	+ 8	+ 46	+14
483	ϵ Sagittarii	1.9	18 17 32.080	3.9823 − 18	− 35 + 1	73 19 1.1	+ 12	+ 6	+ 35	+ 6
484	λ Sagittarii	2.9	18 21 47.970	3.7027 − 12	− 36 + 2	74 50 1.9	+ 6	− 2	+ 47	+12
486	ζ Pavonis	4.0	18 31 21.117	+7.0287 − 431	− 37 +10	71 14 0.4	+ 71	+21	+ 53	+25
488	ϕ Sagittarii	3.3	18 39 24.563	3.7499 − 44	+ 30 0	75 32 1.3	+ 32	+ 5	−	−
491	σ Sagittarii	2.1	18 49 3.903	3.7219 − 55	+ 7 + 1	71 51 1.8	+ 36	+10	+ 56	+17
497	ζ Sagittarii	2.7	18 56 15.013	3.8202 − 78	− 13 0	76 26 1.0	+ 43	+11	+ 71	+20
500	τ Sagittarii	3.5	19 0 41.858	3.7485 − 72	− 44 + 2	78 20 1.0	− 3	+ 2	+ 70	+19

SECOND SECTION. (Declination, −21° 50′ to −82°.)

No.	Decl. 1900	Ann. V. and Sec. V.		μ' and 100 $\Delta\mu'$		Ep. and Wt.			B − N $\Delta\delta$	B − N $\Delta\mu$	B − A $\Delta\delta$	B − A $\Delta\mu$	
	° ′ ″	″	″ .001	″ .001	″ .001	T	p_δ	p_μ	″ .01	″ .001	.01	.001	
351	−11 43 6.99	−15.979	+ 312	− 30	0	68	10	0.5	− 25	+ 2	10	+ 6	
352	−46 57 32.18	15.643	+ 369	− 21	0	67	9	0.5	+ 9	+11	+21	+ 9	
360	−42 43 52.12	14.714	+ 395	− 51	0	67	10	0.5	+ 18	+11	+ 6	+10	
361	−41 42 10.60	14.656	+ 394	− 34	0	67	8	0.4	− 57	− 8	7	−12	
364	−24 53 20.61	14.341	+ 361	− 56	− 1	73	28	1.1	− 55	− 8	10	2	[illegible]
365	−46 39 35.55	−14.314	+ 422	− 35	0	72	12	0.1	−	−	+ 1	+ 3	
367	−48 21 27.18	13.924	+ 441	− 60	− 1	71	9	0.2	− 9	+ 2	−		
368	−51 43 7.07	13.920	+ 456	− 64	− 1	72	9	0.1	− 45	+ 2	+18	+ 9	
370	−68 18 36.08	13.591	+ 599	− 20	− 1	70	11	0.1	+ 54	+22	+60	+22	
381	−65 58 50.09	12.140	+ 630	− 68	0	70	8	0.3	−	−	+64	+20	
382	−40 49 50.06	−12.342	+ 463	− 33	0	66	12	0.5	+ 55	+16	+10	+ 8	
394	−63 7 18.73	11.130	+ 640	−389	− 3	69	11	0.1	+ 50	+18	+11	+17	
39[illegible]	−28 55 19.66	10.750	+ 460	− 31	0	73	13	0.6	−	−	−26	− 3	
399	−25 49 31.59	10.600	+ 453	− 36	0	74	20	1.2	− 24	+12	+17	+ 3	
401	−38 6 39.01	10.546	+ 496	− 34	0	72	10	0.4	−	−		−	
402	−22 20 14.05	−10.480	+ 445	− 37	0	73	53	2.0	− 34	− 2	− 6	+ 2	
406	−63 25 18.07	9.556	+ 700	− 16	0	72	6	0.3	−	−	+13	+ 2	
408	−49 51 36.62	9.126	+ 583	− 52	− 2	71	10	0.4	+ 17	+11	+27	+12	γ² Normae
410	−25 21 10.53	8.891	+ 480	− 33	0	72	34	1.4	− 26	+ 6	−25	− 6	
413	−78 40 20.94	8.700	+1.196	− 78	− 5	74	18	0.3	+ 25	+ 4	+35	+ 1	
416	−26 12 36.85	− 8.247	+ 492	− 35	0	70	70	2.4	− 16	− 7			
417	−34 29 11.58	8.108	+ 525	− 22	0	76	11	0.1	+ 10	+ 6	+ 7	+ 6	N Scorpii
422	−28 0 31.18	7.736	+ 505	− 37	0	75	24	1.0	− 28	− 2	−26	− 7	
426	−68 50 38.13	7.013	+ 866	− 28	+ 1	68	31	0.6	+ 50	+21	+77	+20	
428	−58 51 46.16	6.809	+ 712	− 47	+ 1	73	10	0.3	−	−	−12	− 4	
429	−34 6 12.39	− 6.811	+ 529	−258	− 7	72	17	0.9	+ 23	+ 1	+11	+ 7	
430	−37 52 32.82	6.466	+ 563	− 30	0	69	12	0.6	− 28	− 6	+12	+ 2	
431	−37 50 49.39	6.429	+ 563	− 32	0	69	8	0.4			− 4	+ 3	
432	−55 19 56.09	6.046	+ 690	− 16	0	71	11	0.1	− 4	+ 2	+15	− 1	
433	−53 0 21.28	5.898	+ 667	− 1	0	70	10	0.4	+ 17	+12	19	− 8	[illegible]
438	−43 6 26.42	− 5.062	+ 610	−296	0	67	14	0.5	+ 39	+10	3	6	
441	−21 53 59.27	3.866	+ 528	− 30	0	72	18	1.1	20	+ 6	5	− 3	
445	−56 17 0.20	3.751	+ 724	− 10	0	67	11	0.1			+11	+ 1	
446	−55 26 7.03	3.775	+ 714	− 35	0	70	11	0.4	− 26	− 7	− 1	12	
447	−24 5 0.43	3.589	+ 527	−130	0	73	36	1.1	12	+ 7	0	6	44 Ophiu.
448	−37 12 57.67	− 3.181	+ 588	− 12	0	76	9	0.1	− 10	− 7	+ 5	− 4	
449	−49 47 18.78	3.216	+ 668	− 89	0	66	12	0.6	17	− 6	−20	−10	
450	−23 53 7.46	3.058	+ 529	− 36	0	71	19	0.9		−	−36	− 6	51 Ophiu.
451	−37 1 51.23	2.926	+ 588	− 34	0	73	14	0.8	11	6	− 2	+ 5	
453	−42 56 3.08	2.620	+ 623	15	0	67	9	0.1	25	− 6	+ 2	2	
456	−38 58 12.20	− 2.159	+ 602	− 26	0	71	13	0.6	68	− 11	19	11	
462	−40 5 17.83	1.708	+ 640	− 12	0	72	11	0.1	18	5	56	13	
464	−37 0 10.87	1.158	+ 595	+ 23	+ 1	74	8	0.3			5	+ 1	G Scorpii
472	−50 5 52.88	0.436	+ 681	− 35	0	70	10	0.3	+ 32	+15			
473	−30 25 31.16	− 0.250	+ 561	196	− 1	80	34	0.8	7	+ 2	21	10	
476	−45 58 47.91	+ 0.296	+ 649	37	0	72	10	0.5	+ 16	+ 8	+11	1	
479	−36 47 30.32	0.781	+ 589	169	− 2	71	12	0.5	77	16	16	8	
481	−29 52 11.43	1.237	+ 559	− 39	0	75	21	0.8	18	5	20	7	
483	−31 25 51.70	1.105	+ 578	−127	1	74	18	1.1	34	− 5	+24	+ 11	
484	−25 28 37.41	1.710	+ 536	−191	0	72	37	1.6	1	+ 5	36	11	
486	−74 30 19.23	+ 2.573	+1011	−161	1	75	17	0.2	+ 11	+ 4	+56	+ 8	
488	−27 5 37.04	3.126	+ 538	− 1	0	73	28	1.2	26	+ 2			
491	−26 25 15.74	1.191	+ 528	− 68	0	73	47	1.6	19	+ 7	+ 2	2	
497	−30 1 23.07	1.868	+ 538	3	0	76	22	0.8	+ 15	+16	31	12	
500	−27 49 0.40	1.984	+ 524	263	1	76	18	0.8	30	10	41	13	

Second Section — (Declination, −21° 50′ to −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B — N		B — A	
					.0001	.0001	.0001	T	p_α	p_δ	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
503	α Coron. Austr.	4.2	19 2 40.222	+4.0874	− 122	+ 75	+ 1	77	8	0.3	+ 92	+24	+ 81	+28
514	Br. 2478	4.5	19 30 37.384	3.6555	404	+ 53	0	71	56	1.2	+ 39	+ 9	+ 81	+26
524	Br. 2549	4.6	19 56 30.641	3.6956	− 148	+ 28	0	76	41	1.5	+ 30	+ 5	+ 72	+25
531	α Pavonis	2.0	20 17 44.337	4.7743	− 595	+ 10	+ 2	62	19	0.5	+ 58	+10	+ 50	+16
537	α Indi	3.2	20 39 32.117	4.2372	− 102	+ 41	− 1	65	8	0.4	+107	+14	+ 18	+11
541	β Pavonis	3.5	20 35 57.031	+5.4611	−1164	− 75	0	65	11	0.4	+ 13	+ 5	+ 19	+11
543	ψ Capricorni	4.2	20 40 10.569	3.5591	− 167	− 41	+ 1	77	20	1.0	− 1	0	+ 39	+15
548	β Indi	3.7	20 46 59.780	4.7220	− 734	+ 12	0	71	7	0.3	− 20	− 5	+ 53	+21
563	γ Pavonis	4.2	21 18 10.679	5.0169	−1240	+ 130	−23	65	10	0.1	− 11	−25	+ 36	+14
564	ζ Capricorni	4.1	21 20 57.554	3.4324	− 166	.0	0	74	33	1.3	− 5	− 4	+ 67	+20
575	γ Gruis	3.2	21 47 52.555	+3.6472	− 810	+ 91	0	76	14	0.6	+ 75	+14	+ 95	+31
578	α Gruis	1.9	22 1 55.985	3.8018	− 155	+ 123	0	64	21	0.6	+ 52	+13	+ 53	+12
583	α Tucanae	2.9	22 11 39.181	4.1486	− 846	− 107	+ 2	61	11	0.5	+ 71	+11	+ 27	+14
592	β Octantis	4.4	22 35 50.958	6.4162	−6264	− 295	+19	67	20	0.4	+106	+ 9	+ 75	+28
594	β Gruis	2.1	22 36 41.854	3.6015	− 435	+ 123	− 4	67	8	0.3	− 6	−10	+ 28	− 1
597	ε Gruis	3.7	22 42 30.950	+3.6466	− 516	+ 101	− 1	68	8	0.3	+ 20	+ 8	+ 87	+22
603	α Pisc. Austr.	1.3	22 52 7.579	3.3240	− 211	+ 251	− 1	68	69	1.9	+ 9	− 1	−	−
609	γ Tucanae	4.0	23 11 35.675	3.5292	− 636	− 55	0	66	9	0.4	+ 25	+ 2	+ 62	+26
616	β Sculptoris	4.6	23 27 36.649	3.2291	− 258	+ 77	0	79	11	0.4	+ 41	+ 6	+ 84	+26
621	δ Sculptoris	4.6	23 43 43.107	3.1322	− 160	+ 80	0	73	45	0.8	+ 56	+21	+108	+36

Third Section — (Declinations north of +82° and south of −82°).

No.	Name and Magnitude		R.A. 1900	Ann. V. and Sec. V.		μ and 100 Δμ		Ep. and Wt.			B — N		B — A	
						.0001	.0001	T	p_α	p_δ	Δα .001	Δμ .0001	Δα .001	Δμ .0001
		M	h m s	s	s	s	s				s	s	s	s
28	43 H Cephei	4.5	0 55 1.569	+ 7.3964	+ 1.4817	+ 769	+ 97	74	62	3.3	+ 98	+ 55	− 9	− 13
39	α Ursae min.	2.1	1 22 33.276	+ 25.2219	+20.130	+1377	+589	68	138	7.7	+ 86	+ 10	−239	− 3
117	Gr. 750	6.1	4 5 5.719	+ 17.3154	+ 1.7966	+ 145	+ 37	75	61	1.5	+ 72	+ 21	+ 62	− 7
153	Gr. 944	6.4	5 29 51.630	+ 18.6966	+ 0.5114	+ 184	+ 4	73	30	1.1	+290	+ 56	−	−
179	51 H Cephei	5.2	6 53 44.292	+ 29.6571	− 2.6345	− 469	− 79	72	118	3.7	+302	+ 91	+ 16	+ 24
185	25 H Camelop.	5.3	7 10 3.203	+ 12.8946	− 0.5208	+ 31	− 16	71	50	2.3	−470	−101	−	−
251	30 H Camelop.	5.3	10 18 54.857	+ 7.7197	− 0.8984	− 450	+ 37	78	48	1.0	− 57	+ 19	+ 81	+ 39
303	6 B Ursae min.	6.3	12 14 23.174	+ 0.2482	+ 0.8188	− 737	+208	74	45	3.0	+ 69	+ 14	−	−
315	32² H Camelop.	5.2	12 48 23.221	+ 0.4091	+ 0.2077	− 179	+ 15	69	40	2.9	+ 40	+ 8	−	−
435	ε Ursae min.	4.4	16 56 12.208	− 6.3039	+ 0.3152	+ 72	− 1	66	100	4.2	− 14	+ 15	+ 28	+ 9
477	δ Ursae min.	4.4	18 4 32.798	− 19.4859	− 0.1434	+ 209	− 88	69	142	6.7	+ 78	+ 19	+ 91	+ 36
511	λ Ursae min.	6.6	19 22 29.27	− 67.822	−26.888	−1033	−414	71	122	4.0	−310	+ 3	−829	−203
550	76 Draconis	5.7	20 49 50.617	− 4.0715	− 0.5374	+ 177	+ 2	78	44	1.6	+ 78	+ 46	+119	+ 50
6	σ Octantis	7.2	0 12 29.31	− 0.7813	+ 2.3873	+ 49	− 24	69	24	0.7	−159	− 13	−639	−356
51	L 634	5.6	1 43 7.95	− 3.9460	+ 1.1741	+ 143	− 8	77	20	0.2	+ 98	+ 57	− 82	− 41
200	L 3911	7.8	7 53 1.62	− 14.248	−16.884	− 445	− 8	68	22	0.8	− 61	− 43	−361	−176
224	ζ Octantis	5.5	9 11 14.28	− 7.8696	− 1.6294	−1075	− 69	75	21	0.3	+119	+ 32	−461	−180
328	κ Octantis	5.7	13 24 42.09	+ 8.8359	+ 1.6060	− 753	− 73	75	23	0.3	+ 40	− 2	−180	− 73
343	δ Octantis	4.4	14 10 51.76	+ 9.0873	+ 1.0431	− 514	− 32	68	21	0.5	− 18	− 8	−	−
354	L 5823	6.5	14 38 59.75	+ 24.5623	+ 8.7590	−1814	−181	68	28	0.8	− 49	− 17	− 69	− 27
375	ρ Octantis	5.7	15 20 14.45	+ 13.1258	+ 1.4044	+ 852	+ 16	74	25	0.4	+ 8	+ 12	− 82	− 28
471	χ Octantis	5.2	17 56 4.25	+ 35.718	+ 0.3694	−1151	+494	75	22	0.3	− 92	− 98	−322	−140
499	σ Octantis	5.5	18 59 13.48	+102.133	−38.853	+1108	− 65	70	30	0.9	+273	+ 9	−217	− 55
570	L 6460	6.6	21 37 38.63	+ 68.373	−88.557	+ 56	+814	70	31	0.8	+117	+ 2	−163	−121
584	ν Octantis	5.7	22 12 34.94	+ 12.8332	− 3.1995	− 426	+ 7	69	31	0.8	− 45	− 23	+ 5	− 5
611	τ Octantis	5.6	23 13 9.30	+ 10.9791	− 5.2362	+ 147	− 57	69	34	0.8	−183	− 52	−183	− 78
622	γ¹ Octantis	5.1	23 46 14.49	+ 3.6585	− 0.3151	− 291	+ 22	67	20	0.5	−120	− 40	−	−

Second Section — (Declination, −21° 50′ to −82°).

No.	Decl. 1900	Ann. V.	Sec. V. .001	μ' .001	100 $\Delta\mu'$ .001	Ep. T	Wt. p_δ	Wt. p_μ	B − N $\Delta\delta$ .01	B − N $\Delta\mu'$ .001	B − A $\Delta\delta$ .01	B − A $\Delta\mu'$ .001	
	° ′ ″	″	″	″	″				″	″	″	″	
503	−38 3 36.95	+ 5.302	+572	−112	+ 1	70	8	0.1	+ 21	+ 6	+26	+ 7	
514	−25 6 15.78	7.696	+489	− 26	+ 1	70	39	1.2	− 7	− 1	−40	−13	Sagitt.
524	−27 59 16.59	9.770	+467	+ 12	0	79	31	1.0	− 16	− 1	−29	−15	Sagitt.
531	−57 3 19.72	11.251	+569	− 85	0	64	18	0.6	+ 11	+ 7	−17	− 8	
537	−47 38 24.67	12.305	+484	+ 61	0	66	12	0.5	+ 26	+11	−26	−14	
541	−66 33 44.55	+12.630	+613	+ 18	− 1	68	14	0.4	+ 59	+20	+88	+27	
543	−25 37 49.43	12.740	+391	−158	− 1	75	21	1.0	− 93	−10	−25	− 4	
548	−58 49 52.94	13.325	+508	− 24	0	74	9	0.3	− 44	−15	−32	−15	
563	−65 49 6.91	16.061	+469	+807	+ 1	65	13	0.5	+ 64	+23	+17	+14	
564	−22 50 40.27	15.436	+314	+ 25	0	74	30	1.5	− 10	+ 6	− 2	0	
575	−37 50 6.94	+16.790	+283	− 17	+ 1	71	15	0.7	+ 1	+ 4	+23	+ 8	
578	−47 26 43.45	17.276	+266	−169	+ 1	66	23	0.7	− 4	− 6	+ 7	− 2	
583	−60 15 29.03	17.810	+267	− 38	− 1	66	12	0.5	− 77	− 4	+43	+12	
592	−81 54 20.88	18.713	+326	+ 2	− 2	68	27	0.6	+ 1	0	+65	+ 7	
594	−47 21 27.19	18.721	+179	− 16	+ 1	69	11	0.4	+ 12	+10	+21	0	
597	−51 50 33.87	+18.853	+169	− 59	+ 1	71	11	0.4	− 35	0	+18	+ 7	
603	−30 9 8.29	19.005	+134	−169	+ 1	70	50	1.1	− 7	+ 2	−	−	
609	−58 47 2.12	19.684	+ 99	+ 83	0	68	11	0.4	+ 73	+22	+ 3	− 3	
616	−38 22 16.82	19.861	+ 58	+ 14	0	76	10	0.3	+ 16	+ 8	+57	+25	
621	−28 40 59.91	19.894	+ 24	−102	0	72	28	0.7	+ 68	+31	+ 9	+ 3	

Third Section — (Declinations north of +82° and south of −82°).

No.	Decl. 1900	Ann. V.	Sec. V.	μ' .001	100 $\Delta\mu'$ .001	Ep. T	Wt. p_δ	Wt. p_μ	B − N $\Delta\delta$ .01	B − N $\Delta\mu'$ .001	B − A $\Delta\delta$ .01	B − A $\Delta\mu'$ .001	
	° ′ ″	″	″		″				″	″	″	″	
28	+85 43 14.50	+19.467	−0.268	− 5	− 3	80	65	2.2	− 24	− 1	− 1	+ 1	
39	+88 46 26.49	+18.761	−1.311	+ 1	− 7	68	256	14.2	− 12	− 3	−11	− 3	
117	+85 17 28.72	+ 9.668	−2.219	+ 33	− 2	82	66	1.3	− 34	−10	+11	+ 4	151 H Ceph.
153	+85 8 49.74	+ 2.626	−2.706	+ 1	− 3	74	17	0.6	+ 14	+ 5	−	−	158 H Ceph.
179	+87 12 20.38	− 1.696	−4.197	− 38	+ 7	76	148	4.3	− 15	− 3	+11	+ 7	
185	+82 36 16.00	− 6.077	−1.790	− 14	0	70	36	2.2	− 6	+ 3	−	−	
251	+83 4 2.96	−18.405	−0.470	+ 23	+ 3	79	52	1.0	+ 18	+15	13	0	
303	+88 15 15.19	−19.947	+0.010	+ 60	− 1	76	36	2.1	− 1	+ 2			
315	+83 57 23.45	−19.585	+0.020	+ 17	− 1	70	32	1.8	+ 6	0		−	
435	+82 12 7.67	− 5.511	−0.880	− 3	+ 1	74	130	4.6	− 1	− 2	− 7	0	
477	+86 36 47.61	+ 0.146	−2.837	+ 48	+ 3	74	185	7.8	− 10	+ 1	−14	− 2	
511	+88 59 15.83	+ 7.071	−9.271	+ 11	−14	72	151	4.8	+ 2	+ 2	− 6	+ 1	
550	+82 9 40.11	+13.560	−0.442	+ 27	+ 2	70	52	1.8	+ 10	+ 2	+14	+ 6	
6	−88 55 8.30	+20.024	−0.002	+ 7	0	69	27	0.9	+ 1	+ 1	− 1	+ 2	
51	−85 16 29.30	+18.077	+0.242	+ 16	− 1	78	26	0.2	16	2	−24	− 9	1 G. Oct.
200	−88 34 24.89	− 9.483	+5.691	+ 8	+ 6	69	28	0.8	+ 3	0	−25	−10	λ Oct.
224	−85 15 46.87	−14.816	+0.787	+ 37	+10	77	28	0.3	6	4	− 17	− 9	
328	−85 16 24.78	−18.721	+0.470	− 28	− 4	76	34	0.4	4	5	+ 20	+ 2	
343	−83 12 35.20	−16.882	+0.720	− 15	− 1	67	24	0.6	9	2			
354	−87 44 30.61	−15.478	+2.280	− 62	−17	68	31	1.0	+ 1	0	− 7	− 1	20 G. Oct.
375	−84 7 55.06	−12.798	+1.482	+ 75	+ 9	73	30	0.4	2	− 4	+13	1	
471	−87 39 51.46	− 0.473	+5.490	−129	− 17	74	29	0.4	8	− 5	− 7	1	
499	−89 15 16.46	+ 5.161	+14.14	− 1	+16	69	42	1.1	− 1	0	0	0	
570	−89 19 3.16	+16.270	+5.795	32	0	70	37	0.9	1	2	− 4	1	B Oct.
584	−86 28 33.87	+17.954	+0.835	+ 69	3	71	37	1.0	24	− 6	13	1	
611	−88 1 52.83	+19.615	+0.315	+ 15	1	69	40	1.1	+ 2	0	7	0	
622	−82 34 28.59	+19.998	+0.023	17	0	70	25	0.7	15	1	−		

OBSERVATIONS OF COMET b 1902 (*PERRINE*).

MADE WITH THE 11-INCH EQUATORIAL AT THE SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.,

BY ABBY E. TUCKER.

[Communicated by the Director, MARY E. BYRD.]

1902 Greenwich M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Oct. 8 15 51 56	1	12 , 7	−0 26.57	−4 51.1	19 17 32.91	+41 8 26.3	9.719	0.423	+2.37	+33.5
21 13 58 16	3	8 , 8	+0 29.18	+2 37.4	17 48 59.16	+ 6 10 9.1	9.633	0.758	+2.02	+16.1
25 12 55 16	4	12 , 10	+0 20.61	+1 6.9	17 34 45.73	+ 0 42 9.4	9.614	0.768	+2.05	+13.1
Nov. 2 12 22 18	5	12 , 6	+1 32.72	−2 2.3	17 13 29.64	− 6 42 31.6	9.628	0.781	+2.02	+ 9.2
7 11 8 18	6	8 , 6	−4 4.71	+0 56.2	17 2 5.35	− 9 17 19.7	9.602	0.798	+2.06	+ 7.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	19 47 57.11	+11 12 44.2	Micro. Comp. with *2	4	17 34 23.07	+ 0 40 49.4	Nicolajew. A.G. 1375
2	19 48 31.71	+11 5 46.3	Bonn. A.G. 13479	5	17 11 54.90	− 6 40 38.5	Ottakring. A.G.Zones
3	17 48 27.96	+ 6 7 15.6	Leipzig II. A.G. 8141	6	17 6 8.00	− 9 48 23.7	Paris III, 21734

ELEMENTS AND EPHEMERIS OF COMET a 1903 (*GIACOBINI*).

BY H. R. MORGAN AND ELEANOR A. LAMSON.

[Communicated by Captain COLBY M. CHESTER, U.S.N., Superintendent.]

The following elements were deduced from three normal places derived from observations made at Washington on Jan. 21, 22, 23; 30; and Feb. 5 and 6.

ELEMENTS.

T = 1903 March 15.4644 Gr. M.T.

$$\left.\begin{array}{l} \pi = 136°\ 5'\ 50'' \\ \Omega = \ \ 2\ 21\ \ 6 \\ i = \ 30\ 40\ 20 \end{array}\right\} \text{Ecliptic 1903.0}$$

q = .409201

Residuals (O−C): $\cos\beta\ \Delta\lambda = +1''.6$, $\Delta\beta = -1''.5$

HELIOCENTRIC COORDINATES.

$$x = r[9.999905]\ \sin(225°\ 46'\ \ 7''+v)$$
$$y = r[9.768323]\ \sin(137\ 25\ 31+v)$$
$$z = r[9.908572]\ \sin(134\ 54\ \ 2+v)$$

U.S. Naval Observatory, 1903 *Feb.* 10.

EPHEMERIS.

1903 Gr. M.T.	α	δ	log Δ	Light
	h m s	° ′		
Feb. 20.5	23 43 42	+12 3.8	0.1174	5.9
22.5	23 47 29	12 48.8	0.1038	
24.5	23 51 21	13 33.9	0.0891	8.4
26.5	23 55 17	14 18.4	0.0730	
28.5	23 59 17	15 1.3	0.0554	12.1
Mar. 2.5	0 3 17	15 41.1	0.0361	
4.5	0 7 13	16 16.8	0.0150	18.9
6.5	0 11 2	16 45.1	9.9917	
8.5	0 14 36	17 2.7	9.9663	29.4
10.5	0 17 46	17 5.5	9.9386	
12.5	0 20 25	16 48.2	9.9088	44.5
14.5	0 22 22	16 4.9	9.8771	
16.5	0 23 31	14 50.2	9.8442	62.4
18.5	0 23 49	12 59.3	9.8109	
20.5	0 23 18	10 29.5	9.7784	76.2
22.5	0 22 4	7 20.7	9.7477	
24.5	0 20 17	+ 3 35.4	9.7200	82.1

Brightness on Jan. 19.5 is adopted as the unit.

CONTENTS.

PUBLISHED IN BOSTON, MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, 16 CRAIGIE ST., CAMBRIDGE, MASS. ASSOCIATES, ASAPH HALL AND LEWIS BOSS.
PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, LYNN, MASS. Entered at the Post Office, Boston, Mass., as second-class matter. *Closed Feb. 17.*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 533.

VOL. XXIII. BOSTON, 1903 MARCH 5. NO. 5

ON THE CONVERGENCY OF THE SERIES USED IN THE DETERMINATION OF THE ELEMENTS OF PARABOLIC ORBITS.

By WILLIAM ALBERT HAMILTON.

1. *Introductory.* — The elements of the orbit of a comet are usually determined by means of the data obtained at three separate, complete observations; and it often becomes a question of importance to the astronomer as to the suitability, or perhaps one might say the sufficiency, of such a set of observations to determine with accuracy the required elements. He is confronted on the one hand with a set of formulas somewhat complex in their nature, and which are subject to limitations in their application owing to the properties of the various functions involved in their construction; on the other hand, the data of observation are subject to limitations owing to unavoidable inaccuracies in the construction of the telescope and the multitude of items which fall under the class known as errors. It is thus both a mathematical and a physical problem with which he has to deal, and it becomes important, first that a careful analysis be made of the properties of the formulas and the conditions under which they may be applied, and secondly, that the errors which present themselves in the observations, in spite of the greatest care and skill on the observer's part, may not be allowed to become obscured in the final results of the computation. It has been the purpose of the study of which this paper is a partial result to investigate the formulas for computing cometary orbits from each of these two stand-points. In pursuance of this plan we have investigated among others the nature of the functions usually known as the "ratios of the triangles;" and have found the conditions under which they may be developed into power-series of the time-intervals between the observations. This discussion is given in the first part of this paper. In another part of the investigation, which is not included in the present paper, we have found the effects of the errors of the observations upon the computed elements of the orbit of a comet, using OLBERS's method as a basis of the study. To this is added the results of a computation, by use of the formulas so deduced, of the differentials of error in an actual case.

We proceed to discuss the ratios of the triangles and convergency of the series, using the following notation.

2. *Notation.* — Let t_1, t_2, t_3 denote the first, second, and third times of observation respectively. And if k^2 denote the Gaussian constant, and m the mass of the comet in terms of the mass of the sun taken as unity, then the differential equations of motion of the comet referred to the sun's center as origin of coordinates are

$$\left.\begin{aligned}\frac{d^2x}{dt^2} &= -\frac{k^2(1+m)x}{r^3}\\ \frac{d^2y}{dt^2} &= -\frac{k^2(1+m)y}{r^3}\\ \frac{d^2z}{dt^2} &= -\frac{k^2(1+m)z}{r^3}\end{aligned}\right\} \qquad (1)$$

where r is the heliocentric distance of the comet, and x, y, z are its rectangular cartesian coordinates. In all practical cases m will be infinitesimal in comparison with the mass of the sun, and therefore may be neglected. Furthermore, if we so change the unit of time that the new unit shall be equal to the old when the latter has been multiplied by k, and denote the time when expressed in the new units by $t - t_0$, where t_0 is any particular epoch, we may express these equations of motion very simply thus:

$$\left.\begin{aligned}\frac{d^2x}{dt^2} &= -\frac{x}{r^3}\\ \frac{d^2y}{dt^2} &= -\frac{y}{r^3}\\ \frac{d^2z}{dt^2} &= -\frac{z}{r^3}\end{aligned}\right\} \qquad (2)$$

In these equations the attractions of all the bodies of the solar system are neglected except that of the sun.

3. *Preliminary Notions.* Suppose, now, the coordinates of the comet at the time t_0 to be x_0, y_0, z_0, and its velocities to be

$$\frac{dx_0}{dt}, \frac{dy_0}{dt}, \frac{dz_0}{dt}$$

then, at any other time, the coordinates and velocities are functions of these initial conditions and $t-t_0$; or, as we may write,

$$x = f\left(x_0, y_0, z_0, \frac{dx_0}{dt}, \frac{dy_0}{dt}, \frac{dz_0}{dt}, t-t_0\right)$$

with similar expressions for the other coordinates and the velocities.

Now, it is known from the theory of differential equations* that the coordinates and velocities are expansible into power-series in $t-t_0$ of the form

(3)
$$x = f\left(x_0 \ldots \frac{dx_0}{dt} \ldots 0\right) + \left[\frac{\partial f}{\partial t}\right]_0 (t-t_0) + \left[\frac{\partial^2 f}{\partial t^2}\right]_0 \frac{(t-t_0)^2}{2!} + \ldots$$

which have finite radii of convergency, if r does not vanish for $t-t_0 = 0$.

In the partial derivatives above, $t-t_0$ is to be placed equal to zero after differentiation. Hence from (3) it follows that

$$(4) \qquad \left[\frac{\partial f}{\partial t}\right]_0 = \frac{dx_0}{dt}, \ldots \quad \left[\frac{\partial^n f}{\partial t^n}\right]_0 = \frac{d^n x_0}{dt^n} \ldots$$

From equations (2) we obtain

$$(5) \qquad \left\{ \begin{aligned} \frac{d^2x_0}{dt^2} &= -\frac{x^0}{r_0^3} \\ \frac{d^3x_0}{dt^3} &= \frac{3x_0}{r_0^4}\frac{dr_0}{dt} - \frac{1}{r_0^3}\frac{dx_0}{dt} \\ &\ldots\ldots\ldots\ldots \end{aligned} \right.$$

Equations (5) enable us to find the coefficients for the developments of the type (3), by means of which the coordinates and velocities of the comet at any time t are expressed as power-series of the time-intervals $t-t_0$, the coefficients depending only upon the coordinates and velocities at the initial time t_0. By means of these developments of the coordinates, the so-called ratios of the triangles are built up in the form of series which depend upon particular time-intervals selected from those determined by the three observations. It is of these latter series that we wish to find the conditions of convergency; and it is at once evident that their convergency will depend upon the convergency of the series of the type (3), since the ratios of the triangles are simple functions of the coordinates alone.

4. *Convergency of Series.* — From well known theorems of the theory of functions it follows that any expansions whatever of the ratios of the triangles into power-series for given time-intervals and initial conditions cannot have greater radii of convergency than the values which are determined by the positions of the poles and branch-points of the expressions of those ratios as functions of the time-intervals. First, however, we study the nature of the functions which express x, y, z in terms of t; and from these find the true radii of convergency.

5. *Coordinates as Functions of the Time.* — From the geometrical relations of the orbit of the comet we have the relations

$$\left. \begin{aligned} x &= r[\cos(v+\omega)\cos\Omega - \sin(v+\omega)\sin\Omega\cos i] \\ y &= r[\cos(v+\omega)\sin\Omega + \sin(v+\omega)\cos\Omega\cos i] \\ z &= r[\sin(v+\omega)\sin i] \end{aligned} \right\} \quad (6)$$

where v is the true anomaly, ω is the argument of the latitude of the perihelion, Ω is the longitude of the node, and i is the inclination of the orbit to the ecliptic. The last three quantities are independent of the time; while r and v are expressible in terms of t by means of the relations

$$\left. \begin{aligned} r &= \frac{p}{1+\cos v} \\ \tan\frac{v}{2} + \frac{1}{3}\tan^3\frac{v}{2} &= \frac{2}{p^{\frac{3}{2}}}(t-H) \end{aligned} \right\} \quad (7)$$

where p is the *latus rectum* of the parabolic path of the comet and H is the time of perihelion passage. H and t are thought of as expressed in the units described in section 2 above — a usage which we shall continue throughout this paper.

6. *The Solution of the Cubic.* — By means of equations (7), we are enabled to express x, y and z in terms of the time-intervals $t-H$. In order to do this we introduce the auxiliaries

$$\left. \begin{aligned} \tau &= \frac{3(t-H)}{p^{\frac{3}{2}}} \\ q &= \tan\frac{v}{2} \end{aligned} \right\} \quad (8)$$

Then the second equation of (7) becomes

$$q^3 + 3q - 2\tau = 0 \quad (9)$$

This is the so-called normal form of the cubic in the quantity q. Its solutions by CARDAN's formula are

$$\left. \begin{aligned} q_1 &= \gamma_1 + \gamma_2 \\ q_2 &= \epsilon\gamma_1 + \epsilon^2\gamma_2 \\ q_3 &= \epsilon^2\gamma_1 + \epsilon\gamma_2 \end{aligned} \right\} \quad (10)$$

where $\gamma_1 = (\tau + \sqrt{1+\tau^2})^{\frac{1}{3}}$, $\gamma_2 = (\tau - \sqrt{1+\tau^2})^{\frac{1}{3}}$, and ϵ, ϵ^2 are cube roots of unity (see BURNSIDE & PANTON's *Theory of Equations*, p. 108).

* JORDAN's *Cours d'Analyse*, Vol. III, p. 99.

7. *Branch-Points.* We want to express q as a power-series in τ, and must therefore find the branch-points and poles of the function. At once we have the branch points $\tau = i$ and $\tau = -i$, where $i = \sqrt{-1}$. Also $\tau = \infty$ is a branch-point, as is easily seen by putting $\tau = \frac{1}{\tau'}$, and letting τ' approach zero. This is the same as putting $\tau = \infty$, and we easily find that all three solutions have the same value at this point. If now we consider a RIEMANN-surface of three sheets whose branch-points are at $\tau = i$, $\tau = -i$ and $\tau = \infty$, then by the theory of functions of a complex variable we know that when the proper cross-cuts are introduced the quantity q is a uniform function of position on this surface.

8. *Connection of the Sheets.* In order to get a clear idea of the surface it is necessary to find what sheets pass into each other at the two branch-points which are in the finite part of the plane. To do this, we need to follow only the purely imaginary values of τ: for the two branch-points in question are on the axis of pure imaginaries. Indeed, we may also consider the branch-point $\tau = \infty$ to be on this same axis.

In order to simplify matters, and at the same time to render the reasoning clearer, we make the transformation,

$$\tau = i \cos\theta \tag{11}$$

where θ is real or complex. Then q_1 and q_2 become

$$\begin{cases} q_1 = [i(\cos\theta - i\sin\theta)]^{\frac{1}{3}} = -ie^{-\frac{\theta i}{3}} \\ q_1 = [i(\cos\theta + i\sin\theta)]^{\frac{1}{3}} = -ie^{-\frac{\theta i}{3}} \end{cases}$$

And, since we may write $\epsilon = e^{\frac{2\pi i}{3}}$, $\epsilon^2 = e^{-\frac{2\pi i}{3}}$, we obtain from (10),

$$(12)\quad \begin{cases} q_1 = -i(e^{\frac{\theta i}{3}} + e^{-\frac{\theta i}{3}}) = -2i\cos\frac{\theta}{3} \\ q_2 = -i(e^{\frac{i}{3}(\theta-2\pi)} + e^{-\frac{i}{3}(\theta-2\pi)}) = -2i\cos\left(\frac{\theta-2\pi}{3}\right) \\ q_3 = -i(e^{\frac{i}{3}(\theta+2\pi)} + e^{-\frac{i}{3}(\theta+2\pi)}) = -2i\cos\left(\frac{\theta+2\pi}{3}\right) \end{cases}$$

Now, from (11), if θ takes real values, τ is purely imaginary, and takes values between $\tau = i$ and $\tau = -i$; while if θ is a pure imaginary, τ takes pure imaginary values with moduli greater than unity. Only when θ is complex does τ take real or complex values. Hence, for our purpose, we need consider only imaginary values of θ, or real values of θ, in order to find the connection of the sheets.

We must notice, also, that τ is a periodic function of θ; hence, when we wish τ to trace the line between the two branch-points but once, we take the primitive period and consider this alone. Now, in order that τ may take only pure imaginary values while passing from $\tau = i$ to $\tau = -i$, θ must take the real values between 0 and π, and therefore $\frac{\theta}{3}$ will take the real values between 0 and $\frac{\pi}{3}$. We get the following correspondence for θ, τ, q_1, q_2, q_3.

θ	τ	q_1	q_2	q_3
0	i	$-2i$	i	i
$\frac{\pi}{2}$	0	$-i\sqrt{3}$	0	$i\sqrt{3}$
π	$-i$	$-i$	$-i$	$2i$

Denote the branch-points $\tau = i$ and $\tau = -i$ by A and A' respectively. Then, from the table above we find, according to the period selected, that the two values q_2 and q_3 become equal when τ approaches A; but when τ arrives at A' along the path selected this does not repeat itself; but instead we have $q_1 = q_2$. Hence sheets q_2 and q_3 are connected at $\tau = i$; while q_2 and q_1 are connected at $\tau = -i$. It follows that if we start at $\tau = O$ in the τ-surface and make a complete circuit once around A, then O and $i\sqrt{3}$ will change places. If we draw branch-cuts from A to infinity and from A' to negative infinity, the continuation of the sheets when crossing these cuts will be

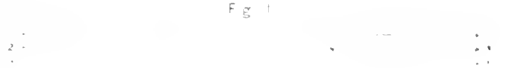
$$\text{Along } A \text{ to } \infty \ldots\ldots \frac{1.2.3}{1.3.2}$$

$$\text{Along } A' \text{ to } -\infty \ldots\ldots \frac{1.2.3}{2.1.3}$$

All three sheets are connected at $\tau = \infty$. A section along the axis of pure imaginaries will appear as in Fig. 1.

It will be of importance for what follows to notice that in the θ-plane the portion which is bounded by the axis of pure imaginaries and the line $\theta = \pi$ is a conform representation of the whole τ-plane — each sheet being represented once in the fundamental region.

Fig. 1



9. *Poles of q in the Sheets.* It is well known that where z is a complex variable, the function e^z can become infinite only for infinite values of z. Hence it follows from (12), that q cannot become infinite except for infinite values of θ. Moreover, owing to the periodicity of the function e^θ which makes $e^{\theta+2\pi i} = e^\theta$, the above infinite value of θ must be either purely imaginary, or perhaps complex with the imaginary part of the complex expression infinitely great. But, from (11), such a value of θ gives τ infinite. Consequently, it follows that q cannot become infinite

except for infinite values of τ. Hence, there are no poles of q in the finite parts of the sheets of the RIEMANN-surface.

10. *Zeroes in the Sheets.* By use of (12) we are also enabled to find at once the zeroes of q in the τ-surface. The general condition for the vanishing of q is given by either of the equations

$$\begin{cases} \cos\dfrac{\theta}{3} = 0 \\ \cos\left(\dfrac{\theta \pm 2\pi}{3}\right) = 0 \end{cases}$$

These are virtually the same, since we may get the one from the other by putting $\theta = \theta' \pm 2\pi$. It is then only necessary to find the value of θ for which $\cos\dfrac{\theta}{3} = 0$. Now, by methods well known in the theory of trigonometry (see CHRYSTAL'S *Algebra*, Vol. II, Chap. 29), it is readily proved that the only values of θ (real or complex) which satisfy this condition are

$$\theta = \frac{2n+1}{2}\pi$$

where n is a positive or negative integer or zero. It follows that only one zero of q is to be found in each fundamental region of the θ-plane for each of the sheets of the RIEMANN-surface. Thus, for the first sheet, it is that value which corresponds to the value of θ which gives, by use of (11), $\tau = 0$. We have already seen in the table following (12) that only one branch of q vanishes at this point: and that the particular one which vanishes thus is dependent entirely upon the sheet of the RIEMANN-surface in which τ is found.

11. *Résumé.* We note here the following summary of results as to critical points upon the RIEMANN-surface upon which q is a function of position:

(13)

$$\begin{cases} \text{Zeroes at} \begin{cases} \tau = 0 \text{ in the } \tau\text{-surface} \\ \theta = \dfrac{\pi}{2} \text{ in the fundamental region of } \theta\text{-plane} \end{cases} \\ \text{Poles — None in the finite part of the plane} \\ \text{Branch-points} \begin{cases} \tau = i \\ \tau = -i \\ \tau = \infty \end{cases} \end{cases}$$

12. *Rational Functions of q and τ.* At this place we state the following theorem which will be useful for later work.

Every rational function of q and τ is a uniform function of position on the same RIEMANN*-surface as that which describes q as a function of τ, and its branch-points are at the same places.* (See FORSYTHE, *Theory of Functions*, page 369). It is to be remembered, however, that this theorem does not apply to the *zeroes* and *poles* of such a rational function of q and τ. These may be located differently, as described in (13).

13. *Coordinates x and y as functions of τ.* We may write the first two equations of (6) as follows:

$$\left.\begin{aligned} x &= [c\cos v - s\sin v]\left(1+\tan^2\frac{v}{2}\right) \\ y &= [c_1\cos v - s_1\sin v]\left(1+\tan^2\frac{v}{2}\right) \end{aligned}\right\} \quad (14)$$

where $p = r(1+\cos v)$; and

$$\left.\begin{aligned} \frac{2c}{p} &= \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i \\ \frac{2s}{p} &= \sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i \\ \frac{2c_1}{p} &= \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \\ \frac{2s_1}{p} &= \sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i \end{aligned}\right\} \quad (15)$$

Using the relations

$$\cos v = \frac{1-\tan^2\frac{v}{2}}{1+\tan^2\frac{v}{2}} \quad \text{and} \quad \sin v = \frac{2\tan\frac{v}{2}}{1+\tan^2\frac{v}{2}}$$

we may write (14), where we put $\tan\dfrac{v}{2} = q$, in the form

$$\left.\begin{aligned} x &= c - 2s\,q - c\,q^2 \\ y &= c_1 - 2s_1 q - c_1 q^2 \end{aligned}\right\} \quad (16)$$

Now, in the equations (16) c, c_1, s and s_1 are constants, independent of τ; and, by the theorem given in the last article, x and y, considered as functions of τ, are functions of position on the same RIEMANN-surface which defines q as a function of τ, and the branch-points of x and y are $q = i$, $q = -i$ and $q = \infty$. Moreover, since, c, c_1, s and s_1 are constants and never infinite, x and y cannot become infinite except where q becomes infinite, viz.: at $\tau = \infty$. Hence we have

THEOREM: *x and y have poles in the* RIEMANN*-sheets only at $\tau = \infty$: and they have branch-points at $\tau = i$, $\tau = -i$ and $\tau = \infty$.*

It follows from the above that x and y are holomorphic functions of τ in the sheets of the RIEMANN-surface, except at the points $\tau = i$, $\tau = -i$ and $\tau = \infty$. Therefore, they may each be expanded into power-series with argument $\tau - \tau_0$ in the vicinity of any point $\tau = \tau_0$. These series will be convergent inside of a circle with center τ_0 and radius reaching from τ_0 to the nearest of the points $\tau = i$ or $\tau = -i$.

14. *Radius of Convergency.* If in (3) we replace t and t_0 by their corresponding values in τ by relation (8), we

have just such an expansion as described in the last article. If we should at the same time take $\tau_0 = 0$, the expansion in x becomes of the form

$$x = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 + \ . \ . \ .$$

where a_0, a_1, &c., are constants. This series will be convergent inside a circle whose center is $\tau_0 = 0$, and with radius unity reaching up to the branch-points $\tau = i$ and $\tau = -i$. Hence, the true radius of convergency in this case would be $|\tau| = 1$; or from (8)

$$(17) \qquad t - H = \frac{p^3}{3}$$

Suppose in (17) we give to p any value, say 1, which would correspond to a perihelion distance of 0.5; then if we make $k = \frac{1}{60}$, which is its approximate value, the case under supposition would give, as the limit for the time-interval for which the expansion of x into power-series would be convergent, the value 20 days. The same period would hold for the corresponding expansion of y.

If τ_0 were any finite point not equal to zero, say some point on the real axis of the τ-plane, then the radius of the true circle of convergency would be larger than that given above. In this case the radius would be $R_\tau = \sqrt{1+\tau^2}$, which holds for both the x and y series. The radius of convergency of the corresponding series in t is at once deducible from the series in τ through the relations (8). The relation is always

$$(18) \qquad R_t = \frac{p^2}{3} R_\tau$$

where the subscripts denote the argument of the series.

Since in an absolutely convergent series we are at liberty to change the order of the terms at will, we may express (3) and the corresponding equation in y by use of coefficients of the kind given in (5), as follows:

$$(19) \qquad \begin{cases} x = Ax_0 + B\dfrac{dx_0}{dt} \\[2mm] y = Ay_0 + B\dfrac{dy_0}{dt} \end{cases}$$

where A and B for their first few terms are

(20)

$$A = 1 - \tfrac{1}{2}\frac{t^2}{r_0^{\,3}} + \tfrac{1}{2}\frac{t^3}{r_0^{\,4}}\frac{dr_0}{dt} + \frac{t^4}{24}\left[\frac{1}{r_0^{\,6}} - \frac{12}{r_0^{\,5}}\left(\frac{dr_0}{dt}\right)^2 + \frac{3}{r_0^{\,4}}\frac{d^2r_0}{dt^2}\right] + \ . \ . \ .$$

$$B = t - \tfrac{1}{6}\frac{t^3}{r_0^{\,3}} + \tfrac{1}{4}\frac{t^4}{r_0^{\,4}}\frac{dr_0}{dt} + \tfrac{3}{10}\frac{t^4}{r_0^{\,5}}\left(\frac{dr_0}{dt}\right)^2 + \ . \ . \ .$$

In these series we have taken $t_0 = 0$. They may be written as series in $t - t_c$ by use of the Weierstrassian theory of the continuation of power-series.

Since a power-series serves in every way to define the behavior of the function from which it is derived as long as we remain within its circle of convergence, we can deal with the series (19) as with quantities which obey all the laws of ordinary algebra (association, commutation, &c.), such as ordinary polynomials or rational quantities, and the resulting series will be convergent. (See CHRYSTAL'S *Algebra*, Vol. II, pp. 139–143.)

15. *Ratios of the Triangles.* We denote the triangles between the positions of two radii vectores of the comet's orbit by the expression $[r_i, r_j]$ where i and j denote the orders of any two of the three observations; also, in general, we denote the three coordinates of the first, second, and third observations by the subscripts 1, 2, 3 respectively. Now the ratios of the triangles $[r_i, r_j]$ are equal to the ratios of their projections on any plane, which may be expressed thus:

$$(21) \qquad \begin{cases} \dfrac{[r_2, r_3]}{[r_1, r_2]} = \dfrac{x_2y_3 - y_2x_3}{x_2y_1 - y_2x_1} \\[3mm] \dfrac{[r_1, r_2]}{[r_1, r_3]} = \dfrac{x_2y_1 - y_2x_1}{x_1y_3 - y_1x_3} \end{cases}$$

Let now $x_2, y_2, z_2, \dfrac{dx_2}{dt}, \dfrac{dy_2}{dt}, \dfrac{dz_2}{dt}$ be taken as the zero-values of the coordinates and velocities in the expansions (19) and (20). Then we get

$$(22) \qquad \begin{cases} x_1 = A_1x_2 + B_1\dfrac{dx_2}{dt} \\[2mm] y_1 = A_1y_2 + B_1\dfrac{dy_2}{dt} \\[2mm] x_3 = A_3x_2 + B_3\dfrac{dx_2}{dt} \\[2mm] y_3 = A_3y_2 + B_3\dfrac{dy_2}{dt} \end{cases}$$

where A_1, B_1, A_3, B_3 are defined by

(23)

$$A_1 = 1 - \tfrac{1}{2}\frac{(t_1 - t_2)^2}{r_2^{\,3}} + \ . \ . \ . \qquad B_1 = (t_1 - t_2) - \tfrac{1}{6}\frac{(t_1 - t_2)^3}{r_2^{\,3}} + \ . \ .$$

$$A_3 = 1 - \tfrac{1}{2}\frac{(t_3 - t_2)}{r_2^{\,3}} + \ . \ . \ . \qquad B_3 = (t_3 - t_2) - \tfrac{1}{6}\frac{(t_1 - t_2)^3}{r_2^{\,3}} + \ . \ .$$

Now, the series (22) are all convergent within the same circle. It follows that, since $x_2, y_2, \dfrac{dx_2}{dt}, \dfrac{dy_2}{dt}$ are not in general equal to zero, the series (23) are also convergent in this same circle (see CHRYSTAL'S *Algebra*, Vol. II, p. 178, 5). It follows also, since for such series the law of distribution holds (*Ibid.*, pp. 142–143), that the products A_1B_3 and A_3B_1 are also convergent series. Hence, from the law of addition, we have that $A_1B_3 - A_3B_1$ is also convergent.* (*Ibid.*, p. 141).

* Prof. HARZER, in Kiel Publications, Vol. XI, Sect. 2, has shown from a direct consideration of the series by processes of successive simplifications and diminishing of the realm of convergency, that the time intervals may be taken small enough so that the series converge. The same result can be inferred from JORDAN'S *Cours d'Analyse*, Vol. III, p. 99. These results are of little practical value because the radius found differs so widely from the true value. By the method of this paper we have the *upper limit of the time-intervals for which the series are convergent and that for any case that may arise whatever.*

We are now at liberty to substitute the values of x_1, y_1, x_3, y_3, as given by (22) in the ratios on the right of (21). We get after the substitution indicated, and by cancelling the factor $x_2\frac{dy_2}{dt} - y_2\frac{dx_2}{dt}$, from the two members of the ratio.*

$$(24)\quad \begin{cases} \dfrac{[r_2\,,\,r_3]}{[r_1\,,\,r_2]} = -\dfrac{B_3}{B_1} = \dfrac{(t_3-t_2)}{(t_2-t_1)}\left[1-\frac16\dfrac{(t_3-t_2)^2-(t_1-t_2)^2}{r_2^3}+\frac14\dfrac{(t_3-t_2)^3+(t_2-t_1)^3}{r_2^4}\dfrac{dr_2}{dt}+\ldots\right] \\ \dfrac{[r_1\,,\,r_2]}{[r_1\,,\,r_3]} = \dfrac{B_1}{A_1B_3-A_3B_1} = \dfrac{(t_2-t_1)}{(t_3-t_1)}\left[1+\frac16\dfrac{(t_3-t_1)^2-(t_2-t_1)^2}{r_2^3}-\frac14\dfrac{(t_3-t_2)[(t_1-t_2)(t_3-t_1)-(t_2-t_1)^2]}{r_2^4}\dfrac{dr_2}{dt}+\ldots\right]\end{cases}$$

where A_1, B_1, A_3, B_3, have the meaning given by (23).

Now, B_1 and B_3 are series with arguments t_1-t_2 and t_3-t_2 respectively. They hold: *i.e.*, are convergent, as long as t_1-t_2 and t_3-t_2 obey the relations

$$(25)\quad \begin{cases} |t_1-t_2| < \dfrac{p^{\frac32}}{3}\sqrt{\dfrac{9}{p^3}(t_2-H)^2+1} \\ |t_3-t_2| < \dfrac{p^{\frac32}}{3}\sqrt{\dfrac{9}{p^3}(t_3-H)^2+1}\end{cases}$$

Also the series $A_1B_3-B_1A_3$, which has two arguments, viz.: t_1-t_2 and t_3-t_2, is convergent so long as (25) hold.

16. *The Zeroes of B_1 and $A_1B_3-B_1A_3$.* If B_1 should vanish, or if $A_1B_3-B_1A_3$ should vanish, then the fractions on the right of (24) evidently would no longer be legitimate. As to this question we state two theorems which are easily proved; but the proofs of which are too long to present here. They are as follows:

Theorem I. The expressions $x_1y_3-y_1x_3$ and $x_1y_2-y_1x_2$ can vanish only for real values of v_3-v_1.

Theorem II. In all cases where the times of the observations are distinct, and where the differences of the longitudes of the comet in its orbit are not equal to an odd multiple of π, the expression $(r_i\,,\,r_j)$ cannot vanish, and the expressions $\dfrac{B_3}{B_1}$, $\dfrac{B_1}{A_1B_3-B_1A_3}$ are legitimate fractions which may be expressed as series, each of which is convergent for all

$$|t_3-t_2| < \frac{p^{\frac32}}{3}\sqrt{1+\tau_2^2} \quad\text{and}\quad |t_1-t_2| < \frac{p^{\frac32}}{3}\sqrt{1+\tau_2^2}$$

The first terms of these series are written out in the right members of (24).

Computation by Use of the Series. The fractions on the right of (24) have both numerators and denominators in the form of series. Their radius of convergency is

$$R_t = \frac{p^{\frac32}}{3}\sqrt{1+\tau_2^2} \quad : \quad \text{where } \tau_2 = \frac{3}{p^{\frac32}}\,(t_2-H)$$

If we make $k = \frac{1}{60}$, the last line of the following table will give the corresponding maximum intervals of time in days for different values of p for which the series are convergent when t_2 is taken equal to H:

TABLE I.

p	4.0	3.0	2.5	2.0	1.5	1.25	1.0	.8	.6	.4	.25	.2	.1	.08	.05	.02
q	2.0	1.5	1.25	1.0	.75	.63	.5	.4	.3	.2	.125	.1	.05	.04	.025	.01
da	160.0	103.4	79.8	56.0	36.2	27.4	20.0	14.1	9.0	5.0	2.5	1.8	.6	.44	.22	.06

It is evident that for any particular value of p the time-intervals should be well within the limit of values for which the series are convergent. This is especially true if we would have the most rapid convergence — a thing most desirable from the standpoint of the computer. In fact, as is well known, it is imperative to have this convergence so rapid that at most but one or two terms will give sufficiently approximate values of the ratios. The reason for this is at once evident when we consider that the series are transcendental in character. Thus the quantities $\frac{dr_2}{dt}$ and r_2 which enter into the terms higher than the first are essentially unknown from the start, and cannot even be guessed at with any degree of certainty until an approximate value of p has been obtained. It cannot be too strongly insisted upon, therefore, that, in order to get the closest determination of the ratios of the triangles, the greatest care must be taken to secure a set of time-intervals which, by their coordination with the parameter of the orbit in hand, will make the series rapidly convergent. It is true that this is more or less a question of trial to start with; yet when a value of p has been once computed by means of any set of time-intervals, it will be seen at once whether the value so obtained is one for which the series are sufficiently convergent for the time-intervals employed. If this is not the case, then new time-intervals should be taken and the computation made over again.

Beloit College, Beloit, Wis.

*Prof. HARZER, in *l.c.*, Sect. 1, has carried the expansion to the 10th degree in the time-intervals.

OBSERVATIONS OF HELIOMETER COMPARISON-STARS,

MADE WITH THE 6-INCH TRANSIT CIRCLE OF THE U. S. NAVAL OBSERVATORY.

By Professor M. UPDEGRAFF and Computer J. C. HAMMOND.

[Communicated by Captain C. M. Chester, U.S.N., Superintendent.]

The following are the results of observations of the stars comprising the last five lists of heliometer comparison-stars proposed by Sir David Gill, H.M. Astronomer, Cape of Good Hope, in his circular of April 2, 1901. They are a continuation of the lists published in *A.J.*, No. 528.

The same notation has been employed as in the publication of the previous lists. The instrument was reversed during each series except that for *Uranus*. The mean epochs of the observations are in the order of the lists, 1902.57, 1902.62, 1902.62, 1902.65 and 1902.80 respectively.

On the nights that Mr. Hammond observed, the microscopes on Circle A were read, and all the recording was done by Mr. H. R. Morgan.

The lists here given and those in *A.J.* No. 528 are not corrected for magnitude equation.

Fundamental Stars, 1902.0.

Star	Mag.	R.A.	Decl.
		h m s	° ′ ″
σ *Scorpii*	3.1	16 15 13.810	−25 21 28.05
ρ *Ophiuchi*	4.7	16 19 42.392	−23 13 14.79
τ *Scorpii*	2.9	16 29 46.810	−28 0 46.36
μ *Sagittarii*	4.0	18 7 51.139	−21 5 4.77
δ *Sagittarii*	2.8	18 14 43.213	−29 52 11.76
λ *Sagittarii*	2.9	18 21 55.370	−25 28 33.98
30 *Sagittarii*	6.1	18 41 57.006	−22 16 27.65
π *Sagittarii*	3.0	19 3 56.171	−21 10 16.41
ψ *Sagittarii*	4.9	19 9 31.920	−25 25 32.83
d Sagittarii	5.0	19 11 54.089	−19 7 38.97
h_2 *Sagittarii*	4.6	19 30 44.654	−25 6 0.31
f Sagittarii	5.1	19 40 38.766	−19 59 48.60
62 *Sagittarii*	4.6	19 56 38.001	−27 58 56.88
4 *Capricorni*	6.0	20 12 15.977	−22 6 46.01
β *Capricorni*	3.2	20 15 30.391	−15 5 27.66
π *Capricorni*	5.2	20 21 42.760	−18 31 59.01
υ *Capricorni*	5.4	20 34 28.324	−18 29 1.46
θ *Capricorni*	4.2	21 0 26.360	−17 37 20.97
ι *Capricorni*	4.3	21 16 47.479	−17 15 7.34
γ *Capricorni*	3.8	21 34 39.757	−17 6 18.18
κ *Capricorni*	4.8	21 37 11.221	−19 18 47.07
δ *Capricorni*	3.0	21 41 37.976	−16 54 19.65
λ *Aquarii*	3.9	22 47 30.144	− 8 6 4.16
β *Piscium*	4.6	22 58 53.397	+ 3 17 32.54
φ *Aquarii*	4.4	23 9 14.839	− 6 34 58.61
λ *Piscium*	4.6	23 37 2.753	+ 1 14 26.40
25 *Piscium*	6.3	23 48 3.590	+ 1 32 44.72
27 *Piscium*	5.0	23 53 39.361	− 4 5 58.66
30 *Piscium*	4.7	23 56 56.055	− 6 33 31.31

Stars for *Uranus*, 1901, 1902, 1903 and 1904.
Observers, Updegraff and Hammond.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″	
a	5.7	16 36 7.95	−19 44 13.6	5
b	8.0	16 37 12.49	−21 9 23.3	3
c	6.5	16 39 14.92	23 0 6.4	5
d	8.7	16 43 14.15	−20 16 59.6	2
e	7.0	16 43 44.24	21 40 49.8	4
f	8.5	16 44 4.58	−23 16 44.3	2
g	7.3	16 48 47.44	21 43 40.3	5
h	5.6	16 50 53.31	22 59 41.7	5
k	7.0	16 51 39.20	21 18 46.2	5
l	7.5	16 57 26.42	23 0 39.7	5
m	6.6	17 0 20.64	−21 25 44.8	5
n	8.7	17 2 38.13	23 5 53.0	1
o	7.7	17 6 28.34	−22 48 21.2	1
p	6.8	17 6 47.60	21 29 44.9	1
q	7.0	17 12 7.94	−23 57 54.1	1
r	8.8	17 12 59.96	22 36 12.5	1
s	4.5	17 15 7.77	−21 0 28.9	5
t	8.0	17 17 17.27	−22 51 54.6	1
u	6.5	17 18 50.29	21 21 0.8	5
v	4.5	17 20 23.07	21 5 8.2	4
w	8.5	17 22 46.69	−22 29 59.6	5
x	4.9	17 25 26.13	23 53 43.9	5
y	6.5	17 29 21.73	−21 58 11.4	1
z	8.0	17 29 33.76	−24 33 10.4	1
α	7.9	17 31 49.57	−23 19 43.2	1
β	6.3	17 32 51.45	21 51 17.4	1
γ	7.8	17 34 51.15	23 47 1.0	5
δ	5.0	17 37 33.42	21 38 9.0	1
π	8.8	17 38 53.04	21 38 27.2	1
ε	8.3	17 39 8.51	22 50 46.5	1
ρ	7.4	17 43 57.85	−21 40 30.7	1
μ	8.7	17 44 15.31	21 54 6.4	1
ζ′	7.0	17 45 40.89	−22 53 27.3	1
σ	6.2	17 48 52.08	24 52 3.8	1
λ	7.0	17 50 27.35	21 56 22.8	1
ν	8.0	17 50 31.80	23 22 27.9	1
ξ	4.6	17 53 48.52	−23 48 26.3	1
η	8.1	17 53 57.50	−25 4 46.0	3
ζ	6.0	17 55 58.15	22 46 40.7	5
τ	6.5	17 59 9.89	−21 24 44.4	5
υ	6.4	18 1 18.68	21 27 15.1	1
θ	8.3	18 1 48.32	−23 6 58.8	3
φ	5.3	18 5 41.51	23 43 16.4	5

Probable error of a single observation in α, 0s.016; in δ, 0″.30 for Circle A, 0″.33 for Circle B.

STARS FOR *Saturn* AND *Jupiter*, 1901.

OBSERVERS, UPDEGRAFF AND HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	5.9	18 27 51.27	21 6 20.0	7	*m*	8.0	18 45 51.61	21 16 10.5	5
b	8.0	18 29 26.55	−22 10 3.9	6	*n*	5.0	18 48 15.18	−22 51 56.1	7
c	6.3	18 32 2.36	21 28 14.3	6	*q*	3.5	18 51 53.03	−21 11 8.6	6
d	5.8	18 32 33.12	−23 35 19.5	6	*r*	6.5	18 55 13.30	−22 50 1.0	6
e	6.2	18 35 53.01	23 55 29.7	7	*s*	3.9	18 58 18.66	−21 53 7.0	6
f	7.3	18 37 25.72	−22 30 23.0	6	*t*	7.1	19 2 19.22	−23 20 10.0	6
g	5.7	18 38 18.11	−25 6 34.1	6	*u*	8.6	19 3 37.85	−22 32 1.9	5
h	5.6	18 40 26.05	−22 29 12.0	6	*v*	3.1	19 3 56.17	−21 10 16.5	6
k	8.1	18 41 58.91	−23 21 19.2	6	*w*	7.3	19 8 16.76	−22 13 38.0	6
l	6.1	18 44 57.05	−22 16 28.0	6					

NOTE: *p* too faint to observe.

Probable error of a single observation in *α*, 0s.016; in *δ*, 0″.23 for Circle A, 0″.27 for Circle B.

STARS FOR *Saturn*, 1902.

OBSERVER, UPDEGRAFF.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	5.1	19 40 38.77	−19 59 48.4	4	*d*	8.2	19 46 8.67	−19 56 43.9	7
b	7.2	19 40 42.13	−21 45 39.9	6	*e*	6.7	19 53 45.76	−22 28 37.2	7
c	8.2	19 44 25.61	−23 1 35.7	7	*f*	7.0	19 54 48.28	−20 7 29.8	6

Probable error of a single observation in *α*, 0s.024; in *δ*, 0″.38 for Circle A, 0″.35 for Circle B.

STARS FOR *Jupiter*, 1902, AND *Saturn*, 1903.

OBSERVERS, UPDEGRAFF AND HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	5.6	20 24 16.88	−18 54 27.6	6	*n*	7.0	20 46 39.90	−20 0 39.5	6
z	8.3	20 27 41.93	−21 13 19.3	7	*m*	6.9	20 47 57.17	−19 29 0.9	7
y	7.8	20 28 49.62	−19 43 57.0	6	*l*	6.0	20 49 15.67	−18 17 40.3	6
x	8.5	20 29 49.78	−18 7 26.5	7	*a*	6.2	20 54 2.10	−19 24 55.0	7
w	7.2	20 30 46.26	−20 55 26.7	6	*b*	6.5	20 55 20.88	−17 54 47.2	6
v	8.7	20 33 55.27	−20 0 58.2	7	*c*	8.0	20 58 29.93	−18 29 56.9	7
u	5.3	20 34 28.30	−18 29 1.4	6	*d*	4.3	21 0 26.36	−17 37 20.4	6
t	8.8	20 36 25.33	−21 37 26.3	7	*e*	6.9	21 1 56.72	−19 28 49.0	7
s	7.3	20 38 18.65	−19 41 14.1	6	*f*	7.7	21 4 11.60	−16 5 59.2	4
r	8.0	20 41 2.64	−17 31 8.0	6	*g*	8.3	21 4 53.67	−17 21 21.5	4
q	8.0	20 41 48.18	−18 58 15.2	7	*h*	8.0	21 5 1.20	−18 43 44.9	4
o	8.0	20 43 35.29	−20 59 16.2	6	*k*	6.4	21 9 37.70	−17 45 2.3	6
p	6.7	20 43 47.08	−18 23 51.2	7					

Probable error of a single observation in *α*, 0s.020; in *δ*, 0″.39 for Circle A, 0″.32 for Circle B.

STARS FOR *Jupiter*, 1903.

OBSERVER, HAMMOND.

Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.	Star	Mag.	R.A. 1902.0	Decl. 1902.0	Obs.
		h m s	° ′ ″				h m s	° ′ ″	
a	4.2	23 9 14.86	−6 34 38.8	12	*g*	7.0	23 21 30.36	−7 8 45.6	5
b	5.7	23 14 19.16	−5 39 35.5	6	*h*	8.2	23 21 35.73	−5 46 18.0	6
c	6.5	23 15 11.04	−4 27 9.4	4	*i*	6.3	23 24 28.13	−5 4 0.2	6
d	6.5	23 15 37.79	−6 26 34.9	4	*k*	6.8	23 25 57.96	−6 49 40.4	6
e	8.0	23 16 10.58	−7 33 36.1	4	*l*	7.2	23 28 25.83	−4 56 32.1	6
f	8.9	23 19 44.81	−5 35 3.8	7					

Probable error of a single observation in *α*, 0s.014; in *δ*, 0″.25 for Circle A, 0″.18 for Circle B.

OBSERVATIONS OF COMET *b* 1902 (*PERRINE*).*

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER MCORMICK OBSERVATORY.

By J. P. McCALLIE.

1902 Charl. M.T.	✳	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 10 12h 7m 1s	1	5 , 10	−1m 8s.74	+4′ 30″.1	3h 3m 53s.12	+39° 18′ 55″.1	n0.867	0.221	+4s.43	−2″.1
27 11 39 24	2	8 , 6	−1 34.68	+2 36.8	0 46 57.21	+55 15 52.0	0.836	n0.215	+6.01	+18.5
28 16 13 0	3	10 , 8	+1 19.01	+4 13.9	0 22 12.45	+56 11 0.5	1.052	9.736	+5.92	+21.8
29 15 11 59	4	10 , 8	−1 33.84	−3 38.0	23 57 21.79	+56 16 12.9	1.058	9.643	+5.80	+24.3
Oct. 1 14 39 10	5	10 , 8	+1 22.01	+4 18.1	23 0 14.05	+56 52 11.8	0.987	n9.904	+5.11	+30.2
7 13 2 31	6	8 , 8	−1 18.53	−3 59.1	20 6 0.70	+11 16 13.7	0.986	0.587	+2.55	+34.6
8 12 52 36	7	10 , 8	+1 13.29	+2 38.1	19 45 45.69	+40 18 11.5	0.961	0.640	+2.33	+33.2
11 10 10 18	8	d11 , 12	−0 0.38	+1 22.8	18 29 54.36	+21 12 0.2	0.862	0.653	+2.02	+23.9
17 8 50 30	9	8 , 8	+1 10.29	−1 13.0	18 8 54.27	+13 45 8.2	0.815	0.660	+1.99	+20.0
21 7 54 15	10	10 , 8	+0 37.83	+6 0.8	17 19 7.82	+ 6 13 32.1	0.785	0.698	+2.03	+16.1
22 7 55 18	11	d 8 , 8	−0 28.84	−1 40.1	17 45 3.1	+ 4 39.9	0.793	0.709	+2.04	+15.4
24 7 31 3	12	8 , 8	−0 54.20	−6 50.7	17 38 1.89	+ 1 56 31.8	0.782	0.722	+2.05	+14.0
28 7 6 33	13	d14 , 12	−0 16.88	−2 15.6	17 26 5.95	− 2 29 15.1	0.786	0.743	+2.04	+11.4
29 6 30 39	14	6 , 5	−2 55.86	+3 35.5	17 23 30.1	− 3 24.8	0.751	0.750	+2.05	+11.1
31 6 11 3	15	4 , 4	−3 1.40	−8 6.0	17 18 26.64	− 5 7 54.0	0.742	0.759	+2.01	+10.4
Nov. 1 6 9 42	16	8 , 8	+1 13.08	−7 2.3	17 15 59.89	− 5 55 27.5	0.750	0.762	+2.02	+ 9.6

Mean Places of Comparison-Stars for the beginning of the year.

✳	α	δ	Authority	✳	α	δ	Authority
1	3h 4m 57s.70	+39° 11′ 22″.9	Lund, A.G. 1631	9	18h 7m 11s.99	+13° 46′ 1″.2	Leipzig I, A.G. 6476
2	0 48 25.88	+55 12 56.7	Camb., (U.S.) A.G. 100	10	17 18 27.98	+ 6 7 15.6	Leipzig II, A.G. 8141
3	0 21 17.52	+56 5 54.8	Hels.-Gotha, A.G. 322	11	17 45 29.9	+ 4 41.3	DM. +4°3523
4	23 58 49.83	+56 19 56.6	Hels.-Gotha, A.G. 14633	12	17 38 54.02	+ 2 3 7.7	Albany, A.G. 5893
5	22 58 46.93	+56 47 23.2	Hels.-Gotha, A.G. 13683	13	17 26 20.78	− 2 27 11.2	Paris III, 22273
6	20 7 16.68	+11 19 38.0	Bonn, A.G. 13855	14	17 26 23.9	− 3 28.6	DM. −3°4120
7	19 44 0.07	+40 15 3.0	Bonn, A.G. 13394	15	17 21 25.97	− 4 59 59.0	Paris III, 22146
8	18 29 52.74	+21 7 13.8	Berlin B, A.G. 6540	16	17 14 14.76	− 5 48 35.4	Paris III, 21919

NOTES. Those comparisons marked *d* were taken directly by micrometrical measurements. — Corrections for refraction have been made. Oct. 7. Seeing 2. Images fuzzy. — Oct. 17. Seeing 1. Comet very faint. Hazy and full moon. — Oct. 21. Tail extends across 5-inch finder. — Oct. 28. Seeing 3. Slightly tremulous.

*From *Supplement to Nos. 531–532.*

OBSERVATIONS OF COMET *b* 1902 (*PERRINE*).*

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY.

[Communicated by Captain COLBY M. CHESTER, U.S.N., Superintendent.]

These observations give a correction of +5s in α and −1′.2 in δ to the ephemeris of ELIS STRÖMGREN in A. N. 3821.

1903 Washington M.T.	✳	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Feb. 5 11h 0m 8s	1	23 , 5	+1m 37s.18	5′ 24″.2	8h 3m 49s.73	33° 23′ 32″.7	n7.890	0.922	+2s.26	17″.7
" 5 11 16 17	1	20 , 4	+1 32.90	4 32.7	8 3 45.45	33 22 44.2	8.177	0.922	+2.26	17.7

Mean Place of Comparison-Star for the beginning of the year.

✳	α	δ	Authority
1	8h 2m 10s.29	−33° 17′ 50″.8	C.G.C. 10728

Δα determined by transits. First observation by FREDERICK, second by DINWIDDIE.

*From *Supplement to Nos. 531–532.*

THE MISSING *DURCHMUSTERUNG* STAR +44°3585.

By ZACCHEUS DANIEL.

On 1900 September 14, while examining the region near *α Cygni* with the 10-inch telescope I found the star DM. +44 3585, 9ᵐ.4, missing from the place given on the chart. The region was carefully examined on 1900 Oct. 18, Nov. 14, Dec. 19; 1901 July 20; 1902 Aug. 8, Sept. 10; and 1903 Feb. 5, but no star brighter than the eleventh or twelfth magnitude was ever seen within 10′ of the place. In addition to the dates given above, I have looked for the star with the same result, probably at least fifty times without recording the observation.

The *Bonner Sternverzeichniss* contains the following position of the missing star:

$$\alpha = 20^h\ 42^m\ 55^s.3 \quad ; \quad \delta = +44^\circ\ 29'.3\ (1855).$$

This is $1^m\ 52^s$ following and 9′ north of 7156 *RR Cygni*. There are two stars of about the twelfth magnitude near this position and one of them may be the star in question, but that can be determined only by exact measurements, which I have not the facilities for making at present. However, no variation in the brightness of either star has been noticed.

Bucknell University, Lewisburg, Penn., 1903 *February* 7.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1901 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
					Eurynome.					
	h m s		m s	′ ″	h m s	° ′ ″			s	″
Aug. 20 10 44 52	1	10 , 5	+0 16.46	+ 6 33.5	22 9 14.30	−3 59 17.8	*n*9.240	0.773	+4.10	+25.9
22 11 16 50	2	11 , 6*t*	+4 19.23	−10 56.7	22 7 26.71	−4 11 26.1	*n*8.541	0.777	+4.12	+26.0
Sept. 3 10 15 5	3	12 , 12	−0 43.27	+ 6 55.6	21 57 2.60	−5 30 3.5	*n*8.636	0.787	+4.17	+26.5
4 10 28 39	4	12 , 12	+0 6.12	− 1 41.5	21 56 13.53	−5 36 53.9	*n*8.824	0.788	+4.17	+26.6
7 9 20 7	5	12 , 12	−0 19.37	− 0 45.4	21 53 51.72	−5 57 20.0	*n*9.235	0.787	+4.17	+26.7
9 10 21 38	6	28 , 7*t*	+1 20.26	+ 6 7.5	21 52 18.33	−6 11 22.9	*n*8.527	0.793	+4.17	+26.6
					Parthenope.					
Oct. 16 9 43 25	7	31 , 7*t*	−1 30.11	− 3 31.3	2 41 21.77	+7 27 27.8	*n*9.545	0.688	+4.51	+18.1

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	22 8 53.74	−4 6 17.2	U.S.N. Obs'y Tran. Cir. Pos.	5	21 54 6.92	−5 57 1.3	½(Mun. I 29974+Rümk. 9700 +W.B. XXI, 1209)
2	22 3 3.36	−4 0 55.4	U.S.N. Obs'y Tran. Cir. Pos.				
3	21 57 11.70	−5 37 25.6	½ Mu. I 30092+Mu. II 12203	6	21 50 53.90	−6 17 57.0	½(Mun. I 29848 +Schj. 8922)
4	21 56 3.24	−5 35 39.0	U.S.N. Obs'y Tran. Cir. Pos.	7	2 42 47.37	+7 30 41.0	Leipzig II, A.G. 1033

U.S. Naval Observatory, 1903 *February* 17.

CONTENTS.

PUBLISHED AT 15 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, LYNN, MASS. *Closed March 4*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 534.

VOL. XXIII. **BOSTON, 1903 MARCH 25.** **NO. 6**

METHOD OF FORMING THE RIGHT-ASCENSIONS OF THE CATALOGUE OF 627 PRINCIPAL STANDARD STARS (*A.J.* 532–533).

By LEWIS BOSS.

In developing a new system of right-ascensions of the principal stars we must examine the testimony of observation from three practically distinct points of view. First, we must ascertain the relation of the stars to the moving equinox; secondly, we must find the errors of observation depending for their argument on the right-ascension of the star observed which, combined with the correction for equinox, we designate as $\Delta\alpha_\alpha$; and thirdly, we must evaluate the errors of observation having declination as the argument, $\Delta\alpha_\delta$, which are due to the failure of the several transits to describe an accurate hour-circle on the sky.

In a subsequent chapter will be exhibited the observed values of $\Delta\alpha_m$, the personal equation dependent on magnitude of the star observed, for each catalogue of observation. From the deductions drawn therefrom it will appear that, in a further approximation toward a better system, upon the basis of the catalogue under consideration, it will be feasible and necessary to take into account this fourth factor in the problem; but in establishing the present system this element was not considered.

The rigorous treatment of our problem implies the solution of equations containing a multitude of unknown quantities; but practically it cannot be handled in this form. The solution must be reached through a series of approximations taking into consideration the principal elements separately and in succession. In the present attempt the neglect of terms in $\Delta\alpha_m$ has doubtless influenced the results for $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$ in a sensible degree, yet this effect must always have been minute. With the present catalogue as a basis the convergence toward the true result ought to be both rapid and satisfactory.

Corrections Having the Form, $\Delta\alpha_\alpha$.

It will be more convenient to consider first the motion of the equinox, together with periodic terms having the argument right-ascension. The combination of these terms we shall call $\Delta\alpha_\alpha$.

For this purpose I have taken as my starting point the right-ascensions of 97 stars contained in the list of "Time Stars" for which NEWCOMB gives five-year ephemerides in his work on *Standard Clock and Zodiacal Stars* (pp. 300–314). In general the choice was confined to stars situated between +30° and −20° of declination; but *α Scorpii*, *α Lyrae*, and *α Pisc. Austr.*, were included. Of the stars between the chosen limits of declination, *α Can. Maj.* and *α Can. min.* were omitted on account of periodicity in proper motion; 11 *Orionis*, *ν Orionis*, *b Ophiuchi*, 1 *Aquilae*, *τ Aquilae*, 1 *Pegasi*, *π Aquarii*, and *θ Piscium* were omitted as insufficiently observed for the purpose required. The choice of this preliminary system, N_1, presents manifest advantages for testing questions relating to $\Delta\alpha_\alpha$, since both points involved have been investigated by Professor NEWCOMB, and with a degree of success which leaves little opportunity for improvement, even with the aid of the additional accumulation of observations which has appeared since these investigations were made.

Position and Motion of the Equinox.

Let us first examine the question as to what correction may be required for the preliminary right-ascensions, N_1, on account of error in the adopted equinox. This equinox was established in NEWCOMB's work on the *Equatorial Fundamental Stars*, and includes the testimony of observations down to about 1870.

Subsequently, in the course of his planetary investigations, Professor NEWCOMB had occasion to investigate anew the position and motion of the equinox. From observations of the sun he found as the correction to the right-ascensions of N_1 (*Astronomical Constants*, pp. 88 and 96),

$$+0''.07 \quad 0''.31\,T$$

wherein T is the interval in centuries from 1850. He also derives similar corrections through the observations of *Mercury*, *Venus*, and *Mars*, and combining these with the result from solar observations, in a manner not readily presented in brief, obtains as the correction for N_1,

$$0''.48 + 0''.30\,T$$

Later, (*A.J.*, XV, p. 188) Professor NEWCOMB gives as revised values of the secular terms, $-0''.37\,T$ and $+0''.40\,T$ as the results, respectively, for observations of the sun alone and of these in combination with those of the planets.

The question whether the result from observations of the planets should have any weight is one which cannot be decided off-hand; but it may fairly be doubted whether the relative weight for the planetary observations should be so great as that which Professor NEWCOMB has assigned. For the further purposes of this investigation I prefer to employ a result from solar observations alone.

In looking over Professor NEWCOMB's tables from which his $\alpha + dl''$ and dl'' are computed (pp. 22 and 30, *Astronomical Constants*) it will be seen that considerable weight has been attributed to the older observations to which I have attached no weight in forming the present system of right-ascensions. In discussions of this kind for the determination of fundamental quantities it seems to me that there are few meridian observations of a date earlier than 1820 which deserve any consideration whatever. In the discussion of secular terms the apparent advantage of employing these older observations is very tempting; but it can be shown that these supposed advantages are really illusory, since the unknown systematic errors of these older observations are liable to be so great in comparison with those of the more precise modern observations as to overbalance the advantage due to the greater interval. A strong illustration of this fact will be found in the comparison of the declinations of AUWERS's reduction of BRADLEY's observations with those of the present Standard Catalogue.

In the present instance, however, Professor NEWCOMB's employment of the older observations has not materially influenced the result. I have taken from his tables the observed values of $e\,(=\alpha + dl'')$ and dl'' which result from observations of mean date, 1820 and later, except that in the case of the Paris results I have not employed the observations made previous to the mean for 1837. Adopting NEWCOMB's weights, I have solved the equations for e and dl'' with the following result for the correction of the right-ascension of N_1:

$$+0''.04 \quad -0''.28\,T$$

in which T is the interval in centuries from 1850. This is almost precisely the same as that already cited which NEWCOMB obtains from all the observations of the sun.

The latest observations included in Professor NEWCOMB's results are those of Greenwich for 1892. In *A.J.*, XXI, 111–2, Professor NEWCOMB discusses the Washington observations of the sun, 1894–1899, and finds for the correction of the right-ascension of N_1:

$$+0^s.009$$

He also shows that a combination of the equinox determinations at Greenwich, 1893–1899, results in a similar correction to N_1 of

$$+0^s.005$$

From NEWCOMB's discussion of the solar observations to 1892 already cited (see *Astronomical Constants*) the correction of N_1 for 1896 would be

$$-0^s.015$$

As previously shown this is confirmed by the discussion which excludes observations earlier than 1820. But, since the Washington observations would give a correction of $-0^s.053$, in the mean to Professor NEWCOMB's ephemeris right-ascensions of the sun, he finally expresses the opinion that, at the date in question, the system, N_1, appears to require a correction of $+0^s.02$ or $+0^s.03$.

Without prolonging the examination further I think it may safely be stated that, so far as the existing determinations of the position and motion of the equinox from observations of the sun throw light on the question, it is uncertain whether the position of the equinox upon which N_1 is founded requires a positive or a negative correction at any epoch. Because I am actually in doubt upon this point I have not ventured upon any correction whatever. I have adopted the equinox of N_1, preferring to await a new influx of determinations before attempting to correct it. It is earnestly to be hoped that these will soon be offered not from Greenwich only, but also from many other observatories situated in latitudes more favorable for the purpose.

Examination of N_1 as to the Periodic Part of $\Delta\alpha$.

The system, N_1, may be regarded as having been established after an exhaustive and trustworthy discussion of systematic errors depending on the right-ascension. This may be regarded as the special feature of NEWCOMB's work on the equatorial fundamental stars. He called the attention of observers to the special means whereby such errors could be eliminated from their observations; and his precepts have been followed with effect at the Greenwich Observatory and elsewhere. At the same time very little has been contributed during the last thirty years which can be utilized as an independent test of N_1 as to periodic errors in $\Delta\alpha_\alpha$. I think the following short table comprises all of these contributions. The numbers are in the sense of corrections to N_1, and result from final comparison with my catalogue of 627 standard stars, which virtually represents it. The comparison includes all stars between $+30°$ and $-22°$ of declination.

Observatory	Corrections to N_1	
Pulkowa, 1865	$-0^s.0022 \sin\alpha$	$+0^s.0033 \cos\alpha$
Greenwich, 1882	$+0.0038$	-0.0026
Pulkowa, 1884	-0.0028	$+0.0056$
Greenwich, 1894	-0.0001	-0.0001

As will be seen the corrections are practically insignificant. If we combine the three later determinations, giving Greenwich 1882 half weight, because during that period the process of eliminating an initial periodic error in the right-ascensions was in progress, we have as the correction to N_1:

$$1888, \qquad -0^s.0004 \sin\alpha \ +0^s.0017 \cos\alpha$$

A very slight alteration in the method of comparison, either as to the limits of declination employed, or as to the representative of N_1, might alter these numbers either way by more than their entire amount.

It might also be remarked that in Volume II (p. (viii)) of the Strassburg Annals there is a quasi determination of the periodic correction, $\Delta\alpha_\alpha$, required for the *Fundamental-Katalog* of Dr. AUWERS. From this I find,

$$+0^s.0135 \sin\alpha \ +0^s.0076 \cos\alpha$$

as the corresponding correction of N_1. But under the circumstances set forth in the discussion cited this cannot be regarded as entitled to very great weight.

For the foregoing reasons I have considered the Catalogue of 627 Standard Stars to require no present revision as to errors of the form $\Delta\alpha_\alpha$.

It seems rather surprising that so little attention by recent observers has been given to the elimination of this form of error in their observations. Whenever, for any reason, observations of the stars are made in the daytime, it is comparatively easy to arrange the night observations in such a way as to render this elimination possible. There has been no period in the history of astronomy when such strenuous attention has been given, as now, to the perfection of clocks and of their installation. Elaborate meridian marks have been established. Thus the means exist for the most rigorous determination of the periodic part of the error, $\Delta\alpha_\alpha$, in the adopted right-ascensions of the observing lists: and yet they are seldom employed for this purpose. In order to appreciate the situation, one has only to cite the observations of BESSEL, STRUVE, and other observers near their time, who used ordinary clocks exposed to the extreme variations of temperature in the observing rooms, and who were yet able to produce systems of fundamental right-ascension almost wholly free from error of this form.

Table I exhibits the final results for determination of $\Delta\alpha_\alpha$, derived from a comparison of each catalogue of observation with the right-ascensions of the Catalogue of 627 Standard Stars between the limits of $+30°$ and $-22°$ of declination, — the constant part, at the same time, being determined from the limits, $+37°.5$ to $-22°$.

TABLE I.

LIST OF FINAL DETERMINATIONS OF $\Delta\alpha_\alpha$ FOR THE PRINCIPAL CATALOGUES.

Greenw. (A-B),	1755	$-0^s.079$	$-0^s.010 \sin\alpha$	$+0^s.001 \cos\alpha$
Königsb. (North),	1820	−0.034	0.000	0.000
Königsb. (Zod.),	1823	+0.025	+0.014	−0.009
Dorpat,	1824	−0.020	+0.011	+0.001
Cape,	1830	+0.013	+0.032	+0.014
Abo,	1829	+0.015	+0.003	−0.001
Greenwich,	1830	−0.062	−0.009	+0.001
St. Helena,	1832	−0.046	0.000	−0.019
Cape,	1833	+0.021	−0.003	−0.014
Cambridge,	1834	−0.024	−0.004	+0.005
Madras (D),	1835	−0.052	+0.017	−0.010
Cape,	1837	−0.004	−0.011	+0.002
Greenwich,	1838	+0.082	−0.028	+0.012
Greenwich,	1844	+0.023	−0.018	+0.017
Radcliffe,	1845	−0.023	−0.024	+0.016
Paris,	1845	+0.025	−0.008	+0.002
Pulkowa,	1845	+0.023	0.000	−0.004
Santiago,	1851	+0.004	−0.008	+0.008
Greenwich,	1851	0.000	0.015	+0.011
Washington,	1856	+0.015	−0.024	+0.017
Greenwich,	1857	+0.019	−0.009	+0.016
Radcliffe,	1857	+0.032	−0.017	+0.002
Cape,	1859	+0.024	−0.012	+0.010
Paris,	1860	+0.038	−0.018	+0.012
Melbourne,	1862	+0.047	−0.030	+0.015
Greenwich,	1864	+0.032	−0.009	+0.009
Cape,	1865	−0.012	0.000	−0.001
Brussels,	1865	$+0^s.050$	$-0^s.037 \sin\alpha$	$+0^s.018 \cos\alpha$
Harvard,	1865	−0.028	−0.017	−0.004
Pulkowa,	1865	−0.005	+0.002	−0.003
Melbourne,	1867	+0.035	−0.033	+0.015
Washington,	1871	−0.001	−0.003	−0.002
Greenwich,	1872	+0.029	−0.007	+0.010
Madras,	1875	+0.017	+0.004	−0.005
Harvard,	1875	−0.006	+0.001	−0.007
Pulkowa,	1876	+0.002	+0.005	−0.005
Paris,	1876	+0.010	−0.024	+0.018
Cape,	1876	+0.037	−0.006	+0.009
Cordoba,	1877	0.000	−0.032	+0.022
Melbourne,	1877	+0.034	−0.010	+0.024
Greenwich,	1882	+0.035	0.004	+0.003
Cape,	1883	+0.019	−0.011	+0.004
Pulkowa,	1884	+0.020	+0.003	0.006
Radcliffe,	1885	+0.017	0.002	+0.008
Strassburg,	1886	+0.016	0.002	0.001
Cape,	1889	+0.022	0.005	+0.002
Madison,	1890	+0.008	−0.002	0.000
Berlin,	1890	+0.020	−0.004	+0.004
Lisbon,	1890	+0.016	−0.001	0.002
Greenwich,	1894	+0.045	0.000	0.000
Mt. Hamilton,	1895	+0.028	−0.004	0.004
Berlin,	1895	+0.020	−0.002	+0.002
Albany,	1898	0.000	0.000	0.000

In all cases I have taken the formula, $a \sin\alpha + b \cos\alpha$, as the expression to be adopted for the periodic part of $\Delta\alpha_\alpha$. This expression is naturally suggested by the association of the observations with the annual cycle of the seasons, and with the diurnal range of temperature. It is doubtless true that in particular instances the real corrections may deviate quite sensibly from this form, but the result of my experience is that this formula usually represents the observed residuals for $\Delta\alpha_\alpha$ in a satisfactory manner. But, even were this formula more deficient than it actually is in the representation of the errors of observation, a strong reason for its adoption would still exist in the requirements of investigations based upon a study of observed right-ascensions. The formulas for determination of precession and solar motion, for instance, contain terms of the form, $a \sin\alpha + b \cos\alpha$; so that in preparing observations for such an investigation it would be more desirable to remove all vestiges of an error of this form than it would be to have even a more exact removal of the errors of observation on the whole associated with a less perfect removal of the periodic term. In fact, curve-drawing in the representation of residual phenomena should be looked upon as an unsatisfactory process to be avoided whenever practicable, and when circumstances warrant the use of a formula.

The original values of $\Delta\alpha_\alpha$, which I obtained from comparison with NEWCOMB'S *Standard and Zodiacal Stars* in the manner already described, do not differ very materially from those of Table I in the case of any important catalogue. Following are the values of $\Delta\alpha_\alpha$, employed throughout the computations for the catalogue, for which the differences from the corresponding values from Table I are much larger than usual.

Dorpat,	1824	$-0^s.020$	$+0^s.010 \sin\alpha$	$+0^s.008 \cos\alpha$
Cape,	1837	-0.004	-0.015	-0.002
Greenwich,	1838	$+0.076$	-0.033	$+0.007$
Greenwich,	1857	$+0.019$	-0.013	$+0.017$
Melbourne,	1862	$+0.047$	-0.024	$+0.014$
Harvard,	1865	-0.024	-0.010	-0.008
Melbourne,	1867	$+0.035$	-0.034	$+0.010$
Cordoba,	1877	$+0.002$	-0.028	$+0.018$
Radcliffe,	1885	$+0.017$	-0.008	$+0.008$

Investigation of Errors having the Form, $\Delta\alpha_\delta$.

Having ascertained that the system, N_1, in the light of all the observations, does not require a decided correction that is discoverable either as to the adopted equinox or as to the periodic part of $\Delta\alpha_\alpha$, we proceed next to construct a system which shall be as far as possible, in like manner and degree, free from errors depending on the declination. In the attempt to do this it will be advisable to recur to the observations themselves, and to build up from them an independent system. If now we could find a list of as many as 200 stars, well distributed in declination, of which the right-ascension of each star had been determined in each valuable authority with approximately the same weight for each star, our task would be comparatively simple. We could first settle upon the standard weight in the fundamental sense to which each catalogue of observation is entitled, and by use of these weights, after correction of each catalogue for $\Delta\alpha_\alpha$, compute the right-ascension of each star from a combination of all the authorities entitled to weight in the fundamental sense. We should at once have our fundamental catalogue, from comparison with which the systematic errors of each of the principal catalogues of observation could be determined with practical finality. A slight subsequent improvement of the right-ascensions of each individual star might then be obtained through the employment of the systematic corrections already ascertained, in a new solution for each star in which the weights would be assigned according to the value of each catalogue in the purely differential sense; so that many valuable series of observations would then be introduced which make no pretensions to value as independent determinations.

In practice the case is far different. I have been unable to find more than 46 stars for which the ideal conditions are even approximately fulfilled; and this number is too few for anything better than a rough preliminary determination of the systematic corrections, $\Delta\alpha_\delta$. Perhaps the necessary method of procedure will be more easily understood through a brief exposition of the successive steps actually adopted in the present investigation.

In arriving at my published right-ascensions three successive approximations were employed. It will not be necessary to exhibit these in detail, since the only object in doing this would be to afford the means of judging what degree of confidence attaches to the final result, — an object which is sought otherwise through a special discussion further on. But some account of the preliminary steps may have an interest.

First, 45 stars (and *Polaris*) were selected (see Table II) on the ground that they are the stars of which the positions are most effectively determined in the greatest number of important catalogues. All the catalogues were then corrected for the values of $\Delta\alpha_\alpha$ determined in the manner already described. The positions may now be supposed to be affected only by systematic errors of the form, $\Delta\alpha_\delta$.

The next process is to obtain a series of values of α and μ for these 45 stars which shall be as nearly as possible homogeneous. At this stage it is not important that the systematic errors should be zero; though it is very desirable that the casual errors of computed mean α and μ should be as small as possible, and that, whatever the systematic errors of these computed values of right-ascension may be, they should be approximately the same within any one comparatively narrow zone of declination. Therefore in selecting the catalogues to be employed in this preliminary

step, precision in the differential sense is of more importance than fundamental character. Hence the average weight assigned to each catalogue for service in this preliminary operation was virtually based upon its precision in the differential sense. This first essay is not for the purpose of computing a system of right-ascensions free from systematic error, but to provide a system which can be corrected easily and with accuracy through the means which actually exist.

The 45 values of α and μ obtained in this way must next be freed from systematic error of the form, $\Delta\alpha_\delta$. Each catalogue which is entitled to any weight for the determination of $\Delta\alpha_\delta$ furnishes a correction of each of the 45 computed right-ascensions that are common to it. Within any comparatively narrow zone we may suppose that these corrections will be approximately the same, except for the casual errors of observation; so that, if we combine four stars in a single zone, and our catalogue of observation contains only two of these stars, the mean of the two corrections will be systematically the same as would have been the mean of all four had the catalogue contained all of them, — the only difference being that the casual error may be larger, and therefore less likely to be compensated by those of other catalogues in the same zone. This process is entirely analogous to that which is followed in the reduction of broken transits.

Thus for each of the nine zones (see Table II) into which the 45 right-ascensions were divided, a "zone-correction" was formed from the corrections of the preliminary right-ascension given by each catalogue supposed to have weight in the determination of $\Delta\alpha_\delta$. As a matter of fact there were few catalogues which did not contain nearly all the stars in each of the zones. The deficiencies were greatest north of $+30°$ of declination.

Taking now one of the zones as a representative of the mean of the stars in that zone we find that we have a series of corrections to that representative given by each of the selected catalogues, arranged in chronological order. The list of selected catalogues, together with the weights assigned to each, is exhibited in the first three columns of Table III; but Lisbon 1890 must be excepted, since it was not published until after the final computations for right-ascension had been completed. The mean date of each catalogue as given in the second column was estimated for the purposes of this computation, and the adopted weights appear in the third column. In assigning these weights various restrictions will suggest themselves to the investigator. In the first place the casual errors of observation may be so great, or the number of observations may be relatively so small, as to vitiate the value that a catalogue might otherwise have for this purpose. Cape 1830 (Fallows) falls in this category. Cambridge 1834 (Airy) is very near the boundary of rejected catalogues for this reason.

The contributions of an observatory during any given period must be judged as a whole, — obviously so, when the same transit is concerned in the production of distinct series of right-ascensions in comparatively close succession. Thus, each of the contributions from the Greenwich Transit Circle has received a weight less than that to which it would have been entitled had it stood as the sole representative of the observations of that instrument. On the other hand, when the extreme interval in a long series of productions, like those from the Greenwich Transit Circle, is so great, the total weight assigned may be much greater than that which could possibly be due to a single representative.

Another question is presented in the criterion of independence relative to terms in $\Delta\alpha$ that should be exacted. The character of the meridian, or hour-circle, described by the transit, as fixed by the definitive reduction of the observations, depends essentially upon three elements:

1. *Upon the character of the determinations of collimation.* Frequent reversal of the transit eliminates this question in a material degree. The Lisbon observations, where the transit was reversed in the observation of each star, illustrates this in a remarkable manner. For the non-reversible transit a systematic error in this respect amounting to Δc introduces a systematic error in the final result equal to

$$\Delta c\left(\tan\frac{P}{2} - \tan\frac{P'}{2}\right)$$

in which P is the polar distance of the star observed, and P' is the effective mean polar distance of the clock stars.

2. *Upon the character of the pivots.* For the most part we must rely upon our experience and judgement as to the quality of work produced by the respective makers of instruments. In a few instances the pivots have been investigated in a manner which inspires confidence in the results. On the other hand, my impression is that the pivots now produced by the best makers are so perfect that the errors originating through irregularities in them are not very important in relation to those from other sources. The error in right-ascension introduced by a relative ellipticity of pivots has the form, $e \sin(2\delta + \zeta)\sec\delta$, in which ζ is an auxiliary angle to be determined. This error is most to be feared in non-reversible transits; but even in the case of a reversible transit, where either of the axes of the relative ellipse happens to be nearly parallel to the line of collimation, this form of error is not eliminated. In many instances where the systematic correction is large and reaches a maximum at about 45° of declination, while it is quite small near the pole, I suspect that the instrumental error may be due to relative ellipticity of pivots. The right-ascensions of Dorpat 21, Greenwich 50, Harvard 65, Brussels 65 and Albany 98 may be open to suspicion of this kind.

3. *Upon the determination of the polar deviation, n.* To secure independence in this respect the value of n should be ascertained through successive transits of close circumpolar stars. If this criterion were to be insisted on it would considerably restrict the choice of catalogues to be employed in determination of $\Delta\alpha_\delta$. This restricted choice I have indicated through the weights assigned in the fourth column of Table III. But we may be permitted to look upon the right-ascensions of the close circumpolar stars, especially of such stars as α and δ *Ursae minoris*, as something approximating the nature of astronomical constants. Admission into the problem of determination of $\Delta\alpha_\delta$ of certain catalogues of right-ascensions which rest on predicted right-ascensions of the close circumpolar stars would mean on the one hand that we are virtually assigning a larger weight than nominally appears to those catalogues upon which the predicted positions depend; and on the other hand that we regard the errors, $\Delta\alpha_\delta$, to be feared in the region intermediate between the pole and the equator as likely to be notably larger than those which we find in investigations of the polar zone. This is really the case, and seems to justify the decision to assign some weight in the determination of $\Delta\alpha_\delta$ to certain series of right-ascension in the reduction of which predicted places of the close circumpolars have been employed. But these catalogues were not used in determination of $\Delta\alpha_\delta$ for the polar zone. (See Table III.)

Added to these three conspicuous elements to be considered in determining $\Delta\alpha_\delta$ are others of a nature less certain and more obscure. These in some instances may have exerted a sensible influence upon the errors of the catalogues. There are systematic errors arising from faulty illumination of the transit threads. There is also the error at the zenith which may exist relative to the apparent direction of transit. This, in the case of ROMBERG at Pulkowa, may have been quite sensible, and there are slight traces of it in other instances. In general, however, the evidence is that this form of error is little to be feared. Associated with this are errors in the line of collimation which may arise from looseness, or weakness, in the structure of the transit, — especially in the fastening of the objective and ocular. I suspect that the right-ascensions of the Washington Transit Circle (1865–1890) may have suffered somewhat from this cause. The respective zenith-points of the principal observatories in the northern hemisphere range over more than 20° of declination; and, therefore, the results for $\Delta\alpha_\delta$ from the mass of observations afford an efficient test for any one of them in respect to these two sources of errors. Personal equations dependent on declination of the star observed may also exist, but their effects would probably be merged to a considerable extent in the determination of the polar deviation of the transit.

Resuming now the zone-corrections for the amendment of the preliminary right-ascensions of 45 stars we may form equations in each zone from the corrections given by the respective catalogues arranged in chronological order, and weighted in the manner already described. Let $\Delta\alpha_0$ be the common correction to the preliminary right-ascensions for 1875 of a given zone, and $\Delta\mu_0$, the common correction for the proper motions. We then form equations of condition in the usual manner, and obtain from the resulting normal equations adopted values of these quantities. Then, correcting each preliminary right-ascension in a given zone by the values of $\Delta\alpha_0$ and $\Delta\mu_0$ for that zone we arrive at right-ascensions for each star which, provisionally, may be regarded as free from systematic error of the form, $\Delta\alpha_\delta$, so far as this result can be obtained from the testimony under consideration.

As already pointed out, a slight inconsistency attaches to these solutions in the fact that, for a zone here and there and for a few catalogues, the normal weight was reduced. It was necessary to take this course in some instances on account of the casual errors which might arise from the unusual weakness of the catalogues in question for certain of the zones, — especially for those north of +30°. Practically, however, the results of this inconsistency are not important; and in many instances they are remedied in a later approximation.

Furthermore, it should be explained that in the case of ROMBERG's Pulkowa catalogue for 1875, of BATTERMANN's Berlin catalogue for 1895, and of Greenwich 1890, the catalogue right-ascensions were modified in this part of the work. ROMBERG's catalogue right-ascensions were first corrected in a way to remove the systematic corrections which he applied (*Int.*, p. 8) in order to reduce them to systematic conformity with Pulkowa, 1865. In this way we may suppose that we have obtained the right-ascensions as they would have been given by the meridian circle itself; and I have assumed that these would be valuable as an independent factor in the determination of $\Delta\alpha_\delta$. A similar course was followed with the Berlin catalogue for 1895 by the use of systematic corrections printed in the introduction (p. 11) which exhibit the deviation of the instrumental meridian from that of the *A.G.C.* In the case of Greenwich 1890 I have applied the correction for systematic error of collimation in the form recommended in the introduction (p. 5) of the catalogue.

But in the tables of systematic correction, $\Delta\alpha_\delta$, hereafter to be given, the numbers are applicable to the catalogue positions.

The results obtained for the 45 stars through the combination of operations already described are exhibited in Table II. The spaces indicate the division into zones. The corrections, $\Delta\alpha$ and $\Delta\mu$, when added to the right-ascensions of the 45 stars, produce those of the final cata-

logue after all the approximations. It will be seen in a general way that this first approximation represents in the systematic sense the final result with a very considerable degree of accuracy. In no case is the correction for an individual star above $0^s.01$ sec δ, and usually it is much less. Systematically the results of this first approximation are practically the same as those of the final catalogue. This, of course, could not have been foreseen at the time; but if the case had been otherwise the same final result would have been reached.

TABLE II.

Right-Ascensions, B′, of 46 Primary Standard Stars Determined in the First Approximation, Defined through the Corrections to them Given by the Right-Ascensions, B, of the Final Catalogue for the Epoch 1875.

	B − B′				B − B′		
	δ	$\Delta\alpha$	$\Delta\mu$		δ	$\Delta\alpha$	$\Delta\mu$
		.001	.0001			.001	.0001
α Urs. min.	$+89^\circ$	-25^s	-13^s	*α Androm.*	$+28^\circ$	$+2^s$	-1^s
				β Tauri	28	0	0
ζ Urs. min.	+78	+ 2	− 1	*β Geminor.*	28	−3	0
γ Cephei	76	+14	+ 9	*α Cor. Bor.*	27	−1	−1
β Urs. min.	75	+ 6	+ 6	*α Arietis*	23	+1	+1
β Cephei	70	+12	+ 6				
				α Bootis	+20	−3	−1
α Urs. Maj.	+62	− 5	− 4	*α Tauri*	16	+1	0
α Cephei	62	−21	− 8	*β Leonis*	15	−2	−1
α Cassiop.	56	− 2	− 2	*α Herculis*	15	0	−1
γ Urs. Maj.	54	+ 7	− 3	*α Pegasi*	15	+1	+1
				γ Pegasi	11	−2	0
θ Urs. Maj.	+52	− 2	− 1	*α Leonis*	13	−2	0
β Draconis	52	+12	+ 5	*α Ophiuchi*	13	−2	−2
γ Draconis	52	− 6	− 4				
η Urs. Maj.	50	+ 2	− 2	*γ Aquilae*	+10	0	0
				α Aquilae	9	−1	0
				α Orionis	7	+4	0
α Persei	+49	− 1	+ 1	*α Serpentis*	7	0	0
ι Urs. Maj.	49	− 7	− 1	*β Aquilae*	6	−2	0
α Aurigae	46	− 7	− 2	*α Ceti*	+4	1	−1
α Cygni	45	+ 3	+ 3	*α Aquarii*	−1	−2	0
				β Orionis	−8	+1	+1
α Can. Ven.	+39	+ 1	+ 2	*α Hydrae*	8	0	0
α Lyrae	39	− 1	− 2	*α Virginis*	10	0	0
β Androm.	35	− 1	+ 1	*α Capricorni*	13	−1	+1
β Lyrae	33	+ 1	− 1	*α Librae*	15	−1	−1

With this preliminary Standard Catalogue of 45 Stars between +79° and −15° of declination the first preliminary corrections of the principal catalogues were determined, embracing those enumerated in Table III, together with others valuable in the differential point of view. In drawing the curves of correction minor peculiarities were not regarded at this stage; but the most painstaking care was exercised to secure practical equality of positive and negative deviations from the general trend of the curves of correction on the part of individual residuals. This is really one of the most critical operations in the entire process, and one upon which the actual gain of precision in successive approximations very much depends; since a fairly good approximation to a perfect standard catalogue already obtained may subsequently be lost through an unlucky combination of errors in the adopted curves of correction.

The corrections were adopted at this stage for stars between +80° and −20° of declination. These preliminary systematic corrections were now employed in determining standard right-ascensions for 113 additional stars for which the weight of determination in view of the collected material seemed to be the greatest. The adopted weights in the star-solutions were now based upon the supposed value of the various catalogues in the differential sense. We now have a standard catalogue of 158 stars to which may be added 70 others within the limits, −20° to −40°, which were taken from the best determined stars of my paper on 179 *Southern Standard Stars* (*A.J.* 450), modified by systematic corrections exhibited in my paper published in the *Astronomical Journal*, No. 499, since I considered those to have reached a grade of approximation in the progress toward a true normal system comparable with that now attained for the 158 northern standard stars. Preliminary to plotting the revised zone-corrections for curve-drawing all of them were multiplied by cos δ, a process which is absolutely necessary to the attainment of real precision in the higher declinations. In drawing these curves more attention was given to subsidiary inflections in the trend of corrections. In regard to minor deviations of the curves from a bold general sweep they are abundantly justified by experience. There is logical reason for such deviations in the probable minute deformations of the pivots of transit instruments combined with other sources of error. In many instances where, on account of the comparative poverty of material in this approximation, I have resisted the inclination to follow an indicated deflection from a free sweeping curve, on a later approximation, with much more material, I have been compelled to recognize it. In general, throughout, I have not pushed the maxima and minima of these curves to the extremes indicated by the observations, but at such points I have usually left an interval from the zone-correction (mean for 5°) equal to its probable error, and in some cases to as much as three or four times the probable error. The weights of the group-correction being known it is possible with care to arrange the residual errors left by the curve so that to some extent they shall follow the law of distribution indicated by the theory of probable error.

TABLE III.

Observed Corrections, — $\Delta\alpha_c \cos\delta$, to the System of Right-Ascensions of 627 Standard Stars, Given by Catalogues of Observation.

Northern Hemisphere.

Decl. of Zone				+80°			+60°			+45°			+30°			+15°			0		
Catal. and	Date 1800+	Weights		**	p	C	**	p	C	**	p	C	**	p	C	**	p	C	**	p	C
						s			s			s			s			s			s
Dorpat	15	2	2	23	2	−.003	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Konigsb.	23	3	3	8	3	+.002	12	3	+.001	18	3	+.016	5	3	−.010	10	3	.000	5	3	+.008
Dorpat	21	3	3	19	3	−.006	29	3	−.018	31	3	−.022	30	3	−.015	21	3	−.001	18	3	+.005
Abo	29	3	–	12	3	−.016	20	3	−.018	32	3	−.011	29	3	+.001	26	3	+.005	33	3	+.001
Greenw.	30	1	1	30	1	−.007	37	1	−.035	49	1	−.018	50	1	−.014	72	1	−.012	67	1	+.007
Cape	33	2	2	.	.	.	.	.	.	4	0	−.066	11	1	−.037	16	2	−.001	18	2	+.016
Camb.	31	1	1	6	0.5	−.011	14	0.5	−.002	8	0.5	−.002	20	1	.000	47	1	−.009	44	1	+.004
Cape	37	2	1	.	.	.	.	.	.	3	0	−.034	16	1	−.016	43	2	+.015	34	2	+.007
Greenw.	38	1	1	18	1	+.031	19	1	+.032	16	1	+.004	26	1	+.007	48	1	+.009	51	1	−.007
Greenw.	11	2	2	22	2	+.015	26	2	+.028	16	2	+.007	32	2	+.011	58	2	−.002	53	2	−.005
Radcl.	45	1	–	38	1	+.009	38	1	−.015	50	1	−.016	19	1	−.023	50	1	−.018	50	1	+.024
Paris	45	3	–	35	0	−.009	32	3	−.014	47	3	−.003	56	3	+.006	71	3	−.005	75	3	.000
Pulkowa	45	8	8	29	8	−.005	36	8	−.001	51	8	+.007	55	8	+.005	61	8	+.001	50	8	−.005
Greenw.	51	1	–	24	1	+.021	19	1	+.014	35	1	−.002	52	1	+.011	70	1	+.001	66	1	−.004
Wash.	56	2	–	33	0	−.001	30	2	+.016	41	2	−.002	47	2	−.008	73	2	−.001	71	2	+.002
Greenw.	57	2	2	26	2	+.007	21	2	+.025	33	2	+.026	45	2	+.022	68	2	+.007	67	2	−.012
Radcl.	57	1	–	28	1	+.009	26	1	+.029	38	1	−.007	49	1	+.004	69	1	−.010	68	1	−.011
Cape	59	1	1	.	.	.	.	.	.	5	0	−.033	25	1	−.007	11	1	+.007	43	1	−.002
Paris	60	3	–	16	0	−.001	21	3	−.007	36	3	−.011	56	3	−.006	72	3	−.003	71	3	+.001
Melb.	62	1	1	.	.	.	.	.	.	3	0	−.006	22	1	+.019	38	1	+.002	29	1	.000
Greenw.	64	2	2	33	2	−.002	20	2	−.003	34	2	+.011	38	2	+.018	67	2	+.001	63	2	−.010
Bruss.	65	2	–	36	0	+.009	31	2	+.050	47	2	+.012	63	2	+.007	70	2	+.009	75	2	−.001
Harv.	65	1	–	31	0	+.005	34	1	−.023	24	1	−.016	33	1	−.015	41	1	−.006	38	1	+.008
Pulkowa	65	10	10	30	10	+.008	37	10	+.009	50	10	+.015	56	10	+.010	59	10	−.003	18	10	−.008
Melb.	67	2	2	.	.	.	.	.	.	3	0	−.024	18	1	−.006	30	2	−.007	28	2	+.007
Wash.	71	4	4	37	4	+.003	31	4	+.010	54	4	+.015	58	1	−.009	74	4	−.008	61	4	+.009
Greenw.	72	2	1	37	2	−.012	36	2	−.002	46	2	.000	60	2	+.013	73	2	+.003	66	2	−.009
Harv.	75	2	–	37	0	+.003	60	2	−.004	50	2	−.002	56	2	−.005	64	2	−.001	49	2	.000
Pulkowa	76	4	–	27	0	+.032	36	4	+.023	53	4	−.004	62	4	−.008	71	4	+.002	64	4	+.003
Paris	76	2	–	.	.	.	8	0	+.024	35	2	−.015	56	2	−.007	64	2	−.005	68	2	+.003
Cape	76	1	–	.	.	.	.	.	.	.	.	.	24	1	−.003	54	1	+.006	64	1	−.002
Cordoba	77	3	2	.	.	.	.	.	.	.	.	.	14	2	+.006	32	3	−.003	49	3	+.003
Melb.	77	1	–	.	.	.	.	.	.	9	0	−.006	27	1	−.005	49	1	−.004	56	1	+.014
Greenw.	82	2	2	37	2	−.010	35	2	−.005	17	2	+.001	62	2	+.008	71	2	.000	65	2	−.005
Cape	83	2	2	.	.	.	.	.	.	27	0	+.013	51	2	+.011	65	2	.000	64	2	+.005
Pulkowa	84	10	10	27	10	+.002	35	10	+.002	50	10	+.001	55	10	+.002	58	10	+.005	46	10	−.003
Strassb.	86	8	8	12	8	−.001	19	8	−.016	36	8	−.009	22	8	+.003	50	8	+.006	58	8	−.008
Cape	89	2	2	.	.	.	.	.	.	18	0	−.021	57	2	−.006	74	2	−.001	68	2	+.006
Madison	90	3	–	17	0	−.006	34	3	−.014	46	3	−.004	57	3	−.008	62	3	−.003	45	3	+.004
Lisbon	90	2	–	.	.	.	9	2	+.001	44	2	+.007	48	2	+.003	57	2	−.003	13	2	−.005
Greenw.	91	3	3	37	3	.000	31	3	−.001	53	3	+.002	63	3	+.003	75	3	.000	76	3	−.001
Mt. Hamil.	95	3	–	33	0	−.015	.	.	.	.	.	.	15	3	.000	28	3	−.007	50	3	+.004
Berlin	95	2	–	.	.	.	33	2	−.002	41	2	+.011	49	2	.000	58	2	+.004	38	2	+.002
Albany	98	3	1	11	3	−.011	3	1	−.013	14	3	−.027	17	3	−.031	29	3	−.004	21	3	+.006

TABLE III.

OBSERVED CORRECTIONS, $-\Delta\alpha_s \cos\delta$, TO THE SYSTEM OF RIGHT-ASCENSIONS OF 627 STANDARD STARS GIVEN BY CATALOGUES OF OBSERVATION.

SOUTHERN HEMISPHERE.

Decl. of Zone		−15°			−30°			−45°				−60°				−80°			
Catal. and Date	1800+	**	p	C	**	p	C	**	p	C'	C	**	p	C'	C	**	p	C'	C
				s			s			s	s			s	s			s	s
Konigsb.	**23**	**7**	**3**	**+.011**															
Dorpat	**21**	**15**	**2**	**+.013**															
Abo	29	33	2	−.011															
Greenw.	30	93	1	+.023	30	0	+.028												
Cape	**33**	**24**	**2**	**+.011**	**16**	**2**	**−.006**	**24**	**2**	**+.005**	**+.006**	**25**	**2**	**+.005**	**+.005**	**16**	**2**	**.000**	**−.001**
Camb.	31	61	1	.000	8	0	+.011												
Cape	37	55	2	−.007	63	2	−.001	63	2	−.001	−.009	56	2	−.003	−.013	33	2	.000	−.012
Greenw.	38	57	1	−.015	21	0.5	+.005												
Greenw.	**44**	**57**	**2**	**−.002**	**20**	**1**	**+.011**												
Radcl.	45	61	1	+.014	14	0	+.010												
Paris	45	92	3	.000	37	2	+.002												
Pulkowa	**45**	**61**	**6**	**−.001**															
Greenw.	51	88	1	−.006	24	0.5	−.005												
Wash.	56	88	2	+.001	61	2	−.009	7	0	−.065	−.065								
Greenw.	**57**	**67**	**2**	**−.012**	34	1	−.004												
Radcl.	57	76	1	+.017	29	0	+.035												
Cape	59	40	1	.000	57	1	−.012	57	1	−.025	−.030	60	1	−.027	−.031	38	1	.000	−.002
Paris	60	85	3	+.007	30	2	+.021												
Melb.	62	37	1	−.009	33	1	+.005	16	1	+.015	+.012	25	1	+.022	+.018	33	1	−.004	−.009
Greenw.	**64**	**80**	**2**	**−.006**	**21**	**1**	**+.001**												
Bruss.	65	90	2	−.015	37	1	−.014												
Harv.	65	39	1	+.011	27	1	+.036												
Pulkowa	**65**	**39**	**6**	**+.002**															
Melb.	**67**	**24**	**2**	**+.009**	**35**	**2**	**+.006**	**65**	**2**	**−.006**	**+.002**	**62**	**2**	**+.010**	**+.018**	**12**	**2**	**+.003**	**+.012**
Wash.	**71**	**75**	**1**	**+.008**	**51**	**1**	**+.009**	**13**	**1**	**+.003**	**+.003**								
Greenw.	72	81	2	.000	28	1	−.002												
Harv.	75	66	2	+.004	31	2	−.009												
Pulkowa	76	51	4	.000															
Paris	76	84	2	+.007	21	1	+.002												
Cape	76	78	1	−.013	65	1	−.037	69	1	−.012	−.026	65	1	+.023	+.016	13	1	.000	+.003
Cordoba	**77**	**96**	**3**	**−.011**	**66**	**3**	**−.005**	**71**	**3**	**+.023**	**+.018**	**65**	**3**	**+.017**	**+.011**	**13**	**3**	**+.002**	**+.002**
Melb.	77	79	1	−.012	39	1	−.039	4	0	. . .	−.062	11	1	+.003	−.003	29	1	+.005	+.009
Greenw.	**82**	**85**	**2**	**.001**	**36**	**1**	**−.002**												
Cape	**83**	**89**	**2**	**−.011**	**53**	**2**	**−.030**	**51**	**2**	**−.001**	**−.018**	**13**	**2**	**+.001**	**.008**	**39**	**2**	**+.002**	**+.001**
Pulkowa	**84**	**37**	**6**	**−.003**															
Strassb.	**86**	**84**	**8**	**−.002**	**32**	**5**	**+.006**												
Cape	**89**	**84**	**2**	**+.001**	**65**	**2**	**−.024**	**78**	**2**	**.017**	**.027**	**60**	**2**	**.011**	**−.020**	**37**	**2**	**.000**	**.007**
Madison	90	15	3	+.011															
Lisbon	90	53	2	+.007	20	2	+.006												
Greenw.	**91**	**97**	**3**	**.005**	**37**	**2**	**.001**												
Mt. Hamil.	95	28	3	+.010	30	3	+.007	5	1	−.003	−.003								
Berlin	95	21	2	−.006															
Albany	98	21	3	+.014	66	3	+.030	13	0	+.019	+.019								

Provided with this second set of systematic corrections, $\Delta\alpha_\delta$, the catalogue of standard stars was now expanded to include 401 stars distributed over the entire sky; and the process already described was repeated, except that new positions were not computed for the 158 northern standard stars, nor for the 70 included in the zone $-20°$ to $-40°$. The curves of correction ascertained in this approximation were the ones finally adopted in computing the right-ascensions which appear in the catalogue. They do not differ in any very material way from the curves of correction derived from the final catalogue-places themselves, which are the corrections to be published at the end of this series of papers.

In relation to the two zones embraced within a radius of 8° from either pole it is proper to say that an amount of labor was expended upon this part of the work which was possibly not fully justified by its importance. The final positions of NEWCOMB, or of AUWERS, were as far as possible utilized as the basis of position to be corrected. An ephemeris of each star at five-year intervals and extending over the entire range of observation was computed by means of the trigonometrical formulas, assumed proper-motions having been taken strictly into account. Annual and secular variations were then computed for each epoch, and also the third derivative, where necessary, by means of differences in the secular variations. Then the work of expansion was tested by mechanical integration with the result that in no case was more than a trifling modification found necessary. The observations were then compared with the ephemeris, and the usual mode of procedure, ending with the formation of zone-equations, was followed, the residuals as in other instances relative to high declinations having been first multiplied by $\cos\delta$. In case of the northern zone the first approximation was reached by means of a special treatment of the four stars, *α Urs. min.*, 51 H *Cephei*, δ and λ *Urs. min.* In the case of *α Urs. min.*, which played the principal role, I employed all the strictly independent determinations known to me, whether contained in the principal catalogues or not. In deciding upon the curves of correction for the polar zones their trend in the adjoining zones was taken into account.

In extending the work from $-40°$ to the southern pole a process was followed which is set forth in connection with the test-computation (Table III), except that the zones tre ted were 5° instead of 15° in width.

Final Test of the Right-Ascensions.

In order to test the entire work and to perfect the curves of correction, I have carefully compared the Catalogue of 627 Standard Stars (including also about 100 standard right-ascensions which remain unpublished) with each of the catalogues of observation employed in its construction, and with some others for which it is of special interest to know the systematic corrections. An abridged outline of the results for all the catalogues to which weight was assigned in the systematic sense is exhibited in Table III. This part of the work, as well as all other essential operations connected with the formation of the catalogue, was performed in duplicate.

Throughout the work the values of $\Delta\alpha_\alpha$ for the principal catalogues were retained as they were first computed in the manner already described; but in this final revision of the systematic corrections a new determination of each individual value of $\Delta\alpha_\delta$ was obtained by means of comparison with the Catalogue of 627 Standard Stars.

The observed values of $\Delta\alpha_\delta$ were computed for zones 5° wide in such a manner that the mean of each zone should fall very nearly at some multiple of 5°. Throughout this and all similar computations in this work the differential weights were rigorously employed, and the resulting weights were always attached to the combinations that were made in means. For the purposes of the present illustration, and in order to bring the entire matter into a form convenient for general inspection, the results for these zones of 5° were condensed into zones 15° in width, except for the polar zones of which the radii are 22°.5.

The table exhibits in the first column the name of the observatory. In the second column is found the estimated mean epoch of observation; in the third, the weights which have been assigned to the respective catalogues in the fundamental sense in this investigation as to $\Delta\alpha_\delta$; in the fourth, the weights which might have been assigned under more strict rules of selection commented upon in the foregoing. The line of numbers for each catalogue in this latter class to which has been assigned a weight of two or more is printed in full-faced type. For the respective zones indicated in the headings of the succeeding columns are given the values of $\Delta\alpha_\delta \cos\delta$ derived from comparison of the observed right-ascensions of the various catalogues with the computed right-ascensions of the standard catalogue in the sense, observed *minus* computed. The number of stars on which each comparison rests, and the specially assigned fundamental weights (in general accordance with those of the third column) are also given. As for the weights in the differential sense, these are omitted for economy of space. The corresponding probable error of the values of $\Delta\alpha_\delta \cos\delta$, to which any weight is assigned, in no case exceeds $\pm 0^s.01$, and is sometimes nominally very slightly greater than $\pm 0^s.001$.

For the generality of catalogues of a date later than 1840 the nominal differential probable error in the principal zones will average about $\pm 0^s.0025$, so that much the larger part of the discrepancies is due to systematic differences.

The residuals for Pulkowa 1875, Greenwich 1894, and Berlin 1895 have been modified to represent the respec-

tive instrumental meridians in the manner previously explained.

Thus the quantities C or C'' (in the sense Obs.—Comp.) are corrections applicable to the published right-ascensions of this standard catalogue given by the respective catalogues of observation. They are discordant; but by means of the weights attached, or by means of those in the fourth column, or through any other system of weights, we may determine in each of the zones any general correction of the standard catalogue which may still be required. This determination I have effected for each system of weights; and the results are exhibited in Table IV. It will be noticed that in the zones for $-15°$, $-60°$, and $-80°$ there are two sets of values of $\Delta\alpha_\delta \cos\delta$, C'' and C. The first, C'', is that which is employed with the attached weights, and C is employed with the weights of the fourth column. The latter, C, is the original value of $\Delta\alpha_\delta \cos\delta$ resulting directly from the comparison. C'' was formed in the following manner. After solution of the equations formed from the values of C (with weights attached) in zones $-15°$ and $-30°$, the values of the observed quantities in those columns were corrected for the concluded values of $\Delta\alpha_0$ and $\Delta\mu_0$ obtained from the solutions. These were trifling in amount as will be seen from the results in Table IV. Then for the Southern catalogues their errors of the form $\Delta\alpha_\delta$ at the declination, $-22°$, are supposed to be known from the means of the corrected values taken from zones $-15°$ and $-30°$. The close circumpolar zone was supposed to give another value of these at $-87°$ in which some confidence may be felt, since the discordances in the various independent determinations are exceedingly small as will be seen from the following list of them.

Systematic Polar Deviations $\Delta\alpha_\delta \cos\delta$ (O—C) at $-87°$.

		O—C			O—C
Cape	33	−0.001	Cape	76	+0.006
Cape	37	−0.013	Cordoba	77	+0.001
Cape	59	0.001	Melbourne	77	+0.007
Melbourne	62	−0.006	Cape	85	+0.001
Melbourne	67	+0.009	Cape	95	0.006

The further computations of the far southern zones are conducted under the supposition that the corrections due to the catalogues of observation, supposed to be known at $-22°$ and $-87°$ from superior weight of material, vary uniformly between these points, and that the residuals, C, after removing these are observed corrections to the system of the standard catalogue.

The principle adopted in this course is empirical, and is based upon experience with the values of $\Delta\alpha_\delta$ for the catalogues of the northern hemisphere. If the correction, C, is comparatively large at a distance of 20° or 30° from the zero usually occurring near $+10°$ of declination, and especially if in that region it is found to be increasing toward the pole it is not likely to change sign at a declination smaller than $+75°$, or $+80°$. Under such conditions the use of a preliminary interpolated correction is much more likely to decrease the peculiar error of any one catalogue of observation than it is to increase it. On the other hand if the original value of the correction at $-22°$ is small the others are not materially affected by this process.

Putting the matter in another way, our procedure gives us in the values of C'' approximately those which would result from the respective series of observation if they should be recomputed with assumed right-ascensions of the close polar stars taken from the present catalogue, and of the clock-stars also taken from the same source, but confined between the limits of $-7°.5$ and $-37°.5$ of declination.

TABLE IV.

Solution of Zone-Equations to Determine Observed Systematic Corrections of the Right-Ascensions of the Standard Stars.

	Solution A				Solution B			
Zone	$\Delta\alpha\cos\delta$ 1875	$100\,\Delta\mu\cos\delta$	$\Delta\alpha$ 1900	$100\,\Delta\mu$	$\Delta\alpha\cos\delta$ 1875	$100\,\Delta\mu\cos\delta$	$\Delta\alpha$ 1900	$100\,\Delta\mu$
	s	s	s	s	s	s	s	s
+80	+0.0004	+0.001	+0.0035	+0.006	+0.0010	0.000	+0.0058	0.000
60	+0.0011	+0.001	+0.0026	+0.002	0.0001	−0.007	−0.0038	−0.014
45	+0.0004	0.004	0.0008	0.006	+0.0033	0.008	+0.0020	−0.011
30	0.0000	+0.002	+0.0005	+0.002	+0.0032	+0.011	+0.0037	+0.012
+15	0.0000	+0.002	+0.0004	+0.002	+0.0010	+0.006	+0.0025	+0.006
0	−0.0007	0.005	−0.0019	0.005	0.0026	0.009	0.0049	0.009
−15	+0.0006	0.003	−0.0001	−0.003	+0.0013	−0.011	0.0022	0.011
30	+0.0004	+0.008	+0.0029	+0.009	0.0010	−0.002	0.0016	0.002
45	−0.0005	−0.007	−0.0017	0.010	0.0069	0.028	0.0195	−0.040
60	+0.0050	+0.005	+0.0126	+0.010	−0.0018	0.013	0.0102	0.026
−80	+0.0012	+0.003	+0.0109	+0.017	−0.0003	+0.004	+0.0040	+0.023

This would be advantageous only under the supposition that the system can be established from existing observations with very much greater weight for zones, $-15°$, and $-30°$ than for zones further south. As a matter of fact the computed weight for zone, $-15°$, is six times that for $-45°$. Under all the circumstances, therefore, I think the result from solutions employing the values of C'' for the southernmost zones is greatly to be preferred.

The results for all the solutions are presented in Table IV, which exhibits under the designation, "Solution A," those in which the attached weights (corresponding to those in the third column) were employed; and also under the designation, "Solution B," the results which are obtained when the weights are taken from the fourth column of Table III under a more rigid criterion as to the independence of catalogues.

Upon careful consideration of this question with the present opportunity for reconsideration after the computations had been laid aside for many months, it seems to me that the results from Solution A are to be preferred not only as to the far southern stars, but throughout. From the north pole down to $-15°$ the differences in the results from the two solutions are not, however, of serious importance.

Probable Error of the System as to $\Delta\alpha_\delta$.

Regarding the discrepancies, C' and C'' in Table III, as due entirely to systematic differences the nominal probable errors of the system as to $\Delta\alpha_\delta$ can be computed for the several zones. These have been derived for Solution A. They are shown in the following table in equatorial seconds in order to correspond to Table III, and to facilitate the comparison between different zones.

Zone	Mean Ep.	(Probable Errors) × (cos δ) for α, ep.	$100\,\mu$	α, 1900	α, 1755
		s	s	s	s
+80	1863	±.0014	±.005	±.0024	±.006
60	1865	.0021	.009	.0038	.010
45	1866	.0016	.007	.0028	.008
30	1867	.0012	.005	.0021	.006
+15	1867	.0006	.003	.0010	.003
0	1867	.0007	.003	.0013	.004
−15	1867	.0009	.004	.0016	.005
30	1872	.0020	.011	.0036	–
45	1867	.0033	.016	.0061	–
−60	1866	.0035	.017	.0071	–

The probable errors under "α, ep." are the minima for right-ascension and correspond to the epochs in the preceding column. Those in the last column correspond equally for epochs not far from 1975. Zone $-80°$ is omitted, since from the nature of the computation the computed probable errors for that zone would be much too small. The probable errors for zones near the equator are necessarily small, since they are very near the artificial zeros of $\Delta\alpha_\delta$.

These probable errors might be regarded as fair indications of the uncertainty of the system as to $\Delta\alpha_\delta$ could we be assured that the weights employed in Solution A (Table III) are sufficiently homogeneous. We have the means for a rough test of this point in the residuals for the zones $+30°$ $+15°$, and $+60°$. From these I find as the probable error of the unit of weight, $\pm 0^s.013$ computed from 67 residuals having weights 1 and 2; while from 45 residuals having weight greater than 2 I find the probable error of the unit to be $\pm 0^s.016$. This agreement is, perhaps, as good as could have been anticipated; and, at any rate, it is good enough to inspire some confidence in the reality of the probable errors of the table within a reasonable limit of uncertainty.

The entire probable error of the system would depend upon a proper combination of these probable errors for $\Delta\alpha_\delta$ with the probable errors appertaining to the position and motion of the adopted equinox. It would be very difficult to estimate what common error may possibly subsist in all the observations of the sun for right-ascension; but in reference to the centennial motion of the equinox I think, perhaps, that $\pm 0^s.02$ may not be regarded as an underestimate of its probable error.

In basing opinions upon probable errors like these it should always be borne in mind that the unfavorable chance may be taken. Mathematically there is one chance in five that the true error will turn out to be twice, and one chance in 23 that will turn out to be thrice, the correctly computed probable error. Therefore, aside from an error which may be common to all the right-ascensions, and due to error in adopted equinox, and assuming that our computations of probable error as to $\Delta\alpha_\delta$ are fairly well founded, we may consider that there is small chance that the systematic error as to $\Delta\alpha_\delta$ for 1900 in our right-ascensions is numerically larger than $0^s.01$ sec δ for any point in the northern hemisphere, or larger than $0^s.02$ sec δ for any point in the southern hemisphere: and that generally it will be much less.

Systematic Corrections of the Form, $\Delta'\alpha_\alpha \tan\delta$.

Hitherto in our discussion the correction $\Delta\alpha_\alpha$, principally due to periodic error in the adopted places of clock-stars, has been regarded as equally appertaining to stars at all distances from the equator, as if it were the sole correction of the kind which is needed. But it is evident that we may have an additional periodic error of this kind due to imperfection in the determination of polar deviation of the transit. In general this error could be represented by the expression, $\Delta'\alpha_\alpha \tan\delta$. For some of the catalogues included in the list of Table III such a term appears to be sensible: and the same may be true of others. For example: we have made a thorough comparison of PIAZZI's Catalogue with the present standard. $\Delta\alpha_\alpha$ was determined precisely

in the same manner as for other catalogues, after the application of a preliminary correction for $\Delta\alpha_\delta$. We have

$$\Delta\alpha_\alpha = +0^s.118 + 0^s.002 \sin\alpha - 0^s.018 \cos\alpha$$

The individual residuals of the comparison in question, at all declinations, were then freed from the effect of both $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$. The error dependent on $\tan\delta$ was then supposed to have the periodic form which has been assigned to $\Delta\alpha_\alpha$ throughout this work. The solution of the equations formed in the three zones resulted as follows:

Zone	Weight	$\Delta'\alpha_\alpha$	
$-12°$ to $-22°$	0.3	$-0^s.114 \sin\alpha$	$+0^s.046 \cos\alpha$
$+30$ $+59$	1.0	-0.068	$+0.043$
$+60$ to $+89$	1.0	-0.098	$+0.054$
Adopted:		$-0.087 \sin\alpha$	$+0.048 \cos\alpha$

The last expression, $+0^s.099 \sin(\alpha + 151°) \tan\delta$, has been adopted as a supplementary correction of Piazzi's right-ascensions. The use of it greatly improves the accordance of Piazzi with other authorities.

Very likely a periodic term of the form adopted may really vary quite materially from the true form of the correction in some hours of right-ascension; but the danger of attempting to draw a curve under the circumstances seems to me to hold out greater possibilities of error than those which attach to the adoption of the formula.

The method of computation adopted in the foregoing illustration concerning Piazzi's right-ascensions will serve to illustrate the procedure with other catalogues. It seemed to be desirable to have some evidence whether such terms have existence in any case. For this purpose the residuals for all catalogues, corrected for $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$, were divided into zones, as follows:

$+79°$ to $+70$	-22 to -50
$+69$ to $+60$	-51 to -65
$+59$ to $+30$	-65 to -79

The residuals in each zone were collected in weighted means covering in each instance not more than two hours of right-ascension. Then in each zone the means were multiplied by mean $\cot\delta$ for that zone; afterward the results for all zones were combined in one general mean. When it was clearly seen that the mean residuals in the several zones indicated no decided trace of a periodic law, either in common or separately, no further discussion of them was attempted. The following list embraces all catalogues for which it was considered to be advisable to evaluate a formula of correction. Those marked with an asterisk are the only ones which appear to be very sensible in relation to the probable error of determination.

I am scarcely prepared to recommend any of these for adoption except, perhaps, those for Radcliffe 57, Melb. 62, and Brussels 65. Dr. Auwers finds a very decided periodic variation for Melb. 62 of the form, $\Delta\alpha_\alpha \tan\delta$, applicable south of declination $-50°$. This limitation seems to be fully warranted.

Corrections of the Form, $\Delta\alpha_\alpha \tan\delta$.

Dorpat,	21*	$(-0^s.014 \sin\alpha$	$-0^s.002 \cos\alpha) \tan\delta$
Greenwich,	30	$+0.005$	$+0.007$
Cambridge,	30	-0.010	$+0.010$
Madras,	35	-0.016	$+0.005$
Greenwich,	41	-0.001	-0.011
Washington,	56	$+0.009$	-0.011
Radcliffe,	57*	-0.007	$+0.024$
Paris,	60	-0.007	$+0.005$
Melbourne,	62*	$+0.022$	$+0.023$
Greenwich,	64	$+0.006$	$+0.004$
Brussels,	65*	$+0.035$	-0.003
Greenwich,	72	$+0.007$	-0.006
Pulkowa,	75	-0.015	$+0.005$
Harvard,	75	-0.003	$+0.005$
Strassburg,	85*	-0.008	-0.004

The systematic corrections to the standard catalogue of right-ascensions indicated in Solutions A (Table IV), are so small that they can safely be neglected at present. For their improvement it is desirable that the several observatories of the southern hemisphere in possession of suitable instruments should devote at least one or two years to the observation of the principal and secondary standard stars. So far as possible all other stars down to the seventh magnitude, which are situated between $-30°$ and the southern pole, should also receive two or three observations each on the part of two or three different observatories. For clock-stars one should employ the principal standard stars situated between $+30°$ and $-22°$ of declination, and should determine the polar deviations of the transits, where practicable, from successive transits of close polar stars.

Several observatories of the United States and of Southern Europe should likewise make special observations in the southern sky upon stars down to the seventh magnitude, at least, working from $-10°$ or $-15°$ of south declination to a zenith-distance of $76°$ or $77°$. It would facilitate future investigations of systematic error if the observed right-ascensions of the clock-stars were also to be included in the respective catalogues for those nights when eight or more have been observed.

Especially, the relation of instrumental meridian to the meridian of the standard catalogue from which the clock-stars are taken should be determined through preliminary computations; so that, either by correcting the standard positions, or by correcting the instrumental results, systematically consistent clock-corrections may be determined from stars in widely separated parallels of declination. The Pulkowa Catalogues for 1855 and 1875 furnish good examples of this method, which is indispensable to the attainment of the best results. The neglect of this precaution has proved a serious blemish in the reduction of several valuable series of observations.

THE BENJAMIN APTHORP GOULD FUND.

Applications for grants of money in aid of astronomical investigation may be made by letter to any of the Directors undersigned stating the amount desired, the nature of the proposed investigation, and the manner in which the money is to be expended. The following information is given for the guidance of applicants.

The BENJAMIN APTHORP GOULD FUND was established in 1897 by Miss ALICE BACHE GOULD, to advance the science of astronomy, and to honor the memory of her father by ensuring that his power to accomplish scientific work shall not end with his death. The principal is $20,000, vested in the National Academy of Sciences as Trustee. The income is to be administered by the undersigned and their successors to assist the prosecution of researches in astronomy.

In recognition of the fact that during Dr. GOULD's lifetime his patriotic feeling and ambition to promote the progress of his chosen science were closely associated, it is preferred that the Fund should be used primarily for the benefit of investigators in his own country or of his own nationality. But it is further recognized that sometimes the best possible service to American science is the maintenance of close communion between the scientific men of Europe and of America, and that therefore, even while acting in the spirit of the above restriction, it may occasionally be best to apply the money to the aid of a foreign investigator working abroad.

In all cases work in the astronomy of precision will be preferred to work in astrophysics, both because of Dr. GOULD's especial predilection and because of the present existence of generous endowments for astrophysics.

Finally, the BENJAMIN APTHORP GOULD FUND is intended for the advancement and not for the diffusion of scientific knowledge, and is to be used to defray the actual expenses of investigation, rather than for the personal support of the investigator during the time of his researches, without absolutely excluding the latter use under exceptional circumstances.

In addition to the above call for applications the Directors, desiring to stimulate the participation of American astronomers in the attempt to bring up the arrears of cometary research, renew the offer to them of the sum of $500 for computation of the "definitive" orbits of comets (see list in *A.J.* 493, p. 104); this sum to be distributed at the average rate of $100 for each computation, — the amount to vary according to the relative difficulty of the computation, and to be determined by the Directors of the GOULD FUND.

LEWIS BOSS, SETH C. CHANDLER, ASAPH HALL.

1903 *March*.

OBSERVATIONS OF THE DECLINATION OF *VESTA*,

MADE WITH THE 5-INCH VERTICAL CIRCLE, AT THE U.S. NAVAL OBSERVATORY,

BY GEORGE A. HILL.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

Date		Wash. M.T.	Obs'd Decl.	O — C
		h m s	° ′ ″	″
1902 July	2	13 4 45	−22 14 17.7	+2.5
	5	12 50 14	22 34 37.1	+2.0
	8	12 35 36	22 54 56.6	+3.5
	11	12 20 52	23 15 8.8	+2.8
	13	12 11 2	23 28 21.3	+4.0
	14	12 6 6	23 35 0.8	+1.5
	16	11 56 15	23 47 55.9	+3.7
	22	11 26 50	−24 25 0.5	+1.9
1902 Aug.	3	10 29 27	−25 27 57.5	+0.7
	7	10 11 6	25 45 4.7	−1.8
	22	8 56 38	−26 31 16.8	−1.0

These places are corrected for parallax, and the comparison in the last column has been made with the ephemeris of *Vesta*, as published in the *British Nautical Almanac for* 1902.

OBSERVATIONS OF THE RIGHT-ASCENSION OF *VESTA*,

MADE WITH THE 5.3-INCH TRANSIT INSTRUMENT AT THE U.S. NAVAL OBSERVATORY,

BY EVERETT I. YOWELL.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

The clock correction was obtained from a group of four clock stars, level and collimation by reversal over the mercury, both before and after the series; azimuth from λ *Ursae minoris* or 51 H *Cephei* s.p. The residuals O—C have been obtained by direct comparison with the ephemeris in the *British Nautical Almanac for* 1902.

Date		Clamp	Observed R.A.	O — C
			h m s	s
1902 July	2	E	19 46 6.15	+4.31
	11	E	37 35.04	+4.38
	16	W	32 36.96	+4.35
	22	E	26 45.81	+4.40
	31	W	19 18 49.99	+4.27
1902 Aug.	7	E	19 13 53.57	+4.12
	18	W	9 6.41	+3.81
	21	E	8 29.91	+3.69
	29	W	8 22.68	+3.54
Sept.	4	E	9 12.42	+3.37
	13	W	13 52.02	+3.15
	16	E	15 47.36	+3.07
	27	W	24 52.16	+2.76
	29	E	26 50.64	+2.78
Oct.	3	W	31 2.76	+2.69
	9	E	37 57.87	+2.53
	12	E	53 41.00	+2.36
	25	E	19 59 23.62	+2.26

OBSERVATIONS OF COMET *d* 1902 (*GIACOBINI*).

MADE WITH 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA.

BY J. P. McCALLIE.

1902 Charl. M.T.	✱	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
Dec. 5 13 10 2	1	8 , 6	+2 32.80	−1 4.5	7 16 49.71	−1 32 54.3	n9.149	9.746	+1.33	12.7
" 7 13 29 52	2	8 , 7	+1 13.69	0 21.8	7 16 10.34	−1 14 18.4	n8.810	9.744	+1.36	13.1

Mean Places of Comparison-Stars for 1902.0.

✱	α	δ	Authority
1	7 11 12.61	−1 28 37.2	Paris III, 9000
2	7 11 52.29	−1 11 10.5	1 München, 2471

Approximate corrections for refraction have been applied.

In my observations of Comet *b* 1902, in *A.J.* 533, p. 57, the values of log $p\Delta$ in R.A. were given in arc instead of in time, so that log 15 should be subtracted from them throughout.

Charlottesville, Va.

OBSERVATIONS OF MINOR PLANETS,

MADE AT THE VASSAR COLLEGE OBSERVATORY,

BY MARY W. WHITNEY AND CAROLINE E. FURNESS.

1901–2 Greenw. M.T.	✱	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				(354) *Eleanora.*						
Nov. 8 16 17 3	1	12 , 6	+0 15.26	+ 2 6.6	5 41 16.90	− 1 37 7.9	n9.511	0.776	+1.28	− 1.1
20 16 8 30	2	10 , 4	−1 11.50	− 1 8.6	5 34 57.27	2 22 39.3	n9.414	0.783	+1.54	0.0
21 15 29 21	2	8 , 4	−1 51.05	3 15.8	5 31 17.73	− 2 25 16.7	n9.492	0.782	+1.55	− 0.2
Dec. 5 16 2 52	3	4 , 4*	−0 16.94	+ 1 39.6	5 23 4.39	− 2 40 39.9	n9.173	0.788	+1.79	1.3
				(82) *Alkmene.*						
Dec. 18 15 25 56	1	12 , 6	−0 55.68	− 4 10.0	7 5 49.65	+27 19 8.3	n9.519	0.468	+5.81	−16.7
Jan. 11 15 29 33	5	10 , 8*	0 22.20	− 6 1.9	6 12 37.19	+28 6 18.5	n8.998	0.324	+2.50	− 9.5
13 14 36 16	6	10 , 8	−0 41.05	10 0.3	6 10 13.72	+28 7 52.3	n9.280	0.358	+2.50	9.3
				(356) *Liguria.*						
Jan. 25 14 28 13	7	8 , 8*	0 37.85	− 1 10.7	7 9 39.58	+35 21 7.3	n9.263	0.066	+2.74	− 9.7
28 14 0 58	8	6 , 4	+1 44.96	+ 2 58.2	7 7 13.59	+35 11 49.5	n9.325	0.105	+2.71	9.1
30 13 31 18	9	8 , 8*	−0 13.60	+ 5 31.6	7 5 15.52	+35 2 54.9	n9.404	0.158	+2.69	9.0
				(17) *Thetis.*						
Jan. 27 11 15 23	10	8 , 8*	+0 28.12	+ 0 24.5	7 55 36.04	+20 5 17.5	9.402	0.551	+2.44	−13.4
30 11 52 52	11	8 , 8*	+0 3.27	5 33.7	7 52 37.19	+20 19 25.7	9.195	0.522	+2.44	−13.4
Feb. 3 11 1 25	13	7 , 9*	0 27.86	+ 1 12.2	7 48 52.25	+20 37 22.4	9.326	0.531	+2.45	13.2
4 13 19 55	14	7 , 4	+2 44.89	+ 1 16.9	7 47 58.46	+20 41 37.5	9.361	0.536	+2.46	13.1
5 12 30 28	14	12 , 6	+1 51.46	+ 5 19.8	7 47 8.04	+20 45 40.4	n9.544	0.591	+2.47	13.1
				(347) *Pariana.*						
Feb. 4 15 0 18	15	8 , 8*	+0 22.69	− 5 33.6	9 10 57.15	+32 9 58.6	9.142	0.291	+2.56	15.7
5 14 17 35	15	8 , 8*	0 35.45	+ 2 45.2	9 9 59.02	+32 17 57.5	9.467	0.306	+2.57	15.6
10 15 37 8	17	7 , 4	+1 9.90	5 29.5	9 5 5.04	+32 56 5.6	9.150	0.161	+2.62	14.8
11 12 46 56	17	8 , 8*	+0 19.71	+ 0 27.6	9 4 14.85	+33 2 2.8	9.645	0.169	+2.62	14.7
14 14 21 19	18	10 , 8	+1 23.76	+ 3 49.4	9 1 21.69	+33 21 5.3	n9.418	0.238	+2.64	14.2
15 12 41 50	18	10 , 8	+0 33.23	+ 9 4.8	9 0 31.16	+33 26 21.0	9.628	0.534	+2.64	14.2
				(68) *Leto.*						
Mar. 10 16 5 14	19	8 , 9*	+0 16.79	− 9 30.3	11 12 13.39	+15 18 51.5	9.004	0.588	+2.46	17.0
11 16 9 32	20	4	. . . . .	+ 1 12.6	. . . . .	+16 3 50.6	. . .	0.580	. . .	16.8
11 16 17 10	20	8 *	0 29.38	. . . . .	11 8 49.16	. . . . .	8.601	. . .	+2.47	. . .
18 15 9 20	21	10 , 8	+0 57.89	− 7 1.7	11 5 33.27	+16 16 39.4	9.131	0.585	+2.50	16.7

1902 Greenw. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(103) *Hera.*						
Mar. 18 16 27 33	22	8 , 8*	+0 11.05	− 2 53.3	12 28 50.98	+ 3 24 0.3	*n*9.145	0.738	+2.52	−15.9[1]
25 16 1 20	23	10 , 5*	+0 0.61	− 3 34.1	12 23 19.64	+ 4 10 30.8	*n*9.105	0.730	+2.55	−16.0[1]
26 15 11 23	23	12 , 6	−0 17.00	+ 2 54.3	12 22 32.03	+ 4 16 59.2	*n*9.175	0.730	+2.55	−16.0[1]
				(393) *Lampetia.*						
Apr. 24 15 5 40	24	5 , 6*	−0 4.50	+ 0 11.5	11 39 27.18	− 7 32 8.4	8.882	0.823	+2.52	−18.7[1]
May 8 15 51 4	25	5 , 5†	+0 37.64	− 5 14.2	11 35 26.39	− 5 33 52.4	9.116	0.802	+2.37	−18.2[1]
9 14 27 42	25	8 , 8	+0 31.34	+ 1 11.0	11 35 20.08	− 5 26 54.2	9.091	0.808	+2.36	−18.2[1]

* Δα measured directly.
† Observation made with square bar micrometer.

[1] Mary W. Whitney, Observer.
[2] Caroline E. Furness, Observer.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	5 40 27.36	− 1 39 15.6	Nicolajew A.G. 1464	14	7 45 11.11	+20 40 33.7	Berlin B, A.G. 3144
2	5 36 4.23	− 2 21 30.7	Schjellerup 1879	15	9 10 31.90	+32 15 27.9	Leiden A.G. 3809
3	5 23 16.54	− 2 42 18.2	Strassburg A.G. Zones	16	9 2 5.65	+32 56 7.3	Leiden A.G. 3763
4	7 6 39.52	+27 23 35.0	Camb. Eng. A.G. 3815	17	9 3 52.52	+33 1 19.9	Micr. Comp. with *16
5	6 42 56.89	+28 12 29.9	Camb. Eng. A.G. 3509	18	8 59 55.29	+33 17 30.4	Leiden A.G. 3750
6	6 41 22.27	+28 18 1.9	Camb. Eng. A.G. 3492	19	11 11 54.14	+15 58 38.8	Bonn VI
7	7 10 14.69	+35 28 27.7	Lund. A.G. 3767	20	11 9 16.07	+16 2 54.8	Berlin A. A.G. 4390
8	7 5 25.92	+35 9 0.4	Lund. A.G. 3736	21	11 4 32.88	+16 23 57.5	Berlin A, A.G. 4369
9	7 5 56.13	+34 57 29.3	Lund. A.G. 3740	22	12 28 37.41	+ 3 27 9.5	Albany A.G. 4512
10	7 55 5.48	+20 5 6.4	Berlin B, A.G. 3212	23	12 23 16.48	+ 4 14 20.9	Albany A.G. 4493
11	7 52 31.48	+20 25 12.5	Berlin B, A.G. 3194	24	11 39 29.16	− 7 32 31.2	Munich 1 7151
12	7 48 26.38	+20 26 10.7	½[Bonn VI + Yar. 3280]	25	11 34 46.38	− 5 28 20.0	Munich I 7045
13	7 49 17.66	+20 33 23.4	Micr. Comp. with *12				

SEARCHING EPHEMERIS FOR APPEARANCE IN 1903 OF COMET 1896 V.

[Abridged from M. Ebell's communication in *A.N.* 3848.]

Berlin M.T.	Assumed Per. Pass. June 6.5			Assumed Per. Pass. June 22.5			Assumed Per. Pass. July 8.5		
	α	δ	Br.	α	δ	Br.	α	δ	Br.
1903 Apr. 3.5	21 24.5	− 5 50	0.86	20 55.0	− 7 14	0.83	20 25.3	−8 32	0.79
11.5				21 17.6	− 5 20	0.95			
19.5				21 40.5	− 3 17	1.09			
27.5				22 3.7	− 1 7	1.23			
May 5.5	23 0.3	+ 2 44	1.29	22 27.1	+ 1 8	1.39	21 51.7	−0 32	1.50
13.5				22 50.8	+ 3 24	1.55			
21.5				23 14.7	+ 5 41	1.72			
29.5				23 38.7	+ 7 54	1.88			
June 6.5	0 37.6	+11 5	1.77	0 2.8	+10 1	2.03	23 23.2	+8 43	2.46

Unit of brightness assumed as for 1897 January 4, when last seen. At discovery in 1896 it was 11M–12M (Br. = 2.93).

CONTENTS.

Published at 16 Craigie St., Cambridge (Boston Postal District), Mass., semi-monthly, by S. C. Chandler. Address, Cambridge, Mass.
Associate Editors, Asaph Hall and Lewis Boss Price, $5.00 the Volume Press of Thos. P. Nichols, Lynn, Mass. *Closed March 24*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 535.

VOL. XXIII. BOSTON, 1903 APRIL 7. NO. 7

NEW FORMULAS FOR FINDING THE MEAN ERROR OF AN OBSERVATION AND SOME LIKELY ERRORS OF THE MOST PROBABLE VALUES OF THE UNKNOWN QUANTITIES IN INDIRECT OBSERVATIONS.

By J. MIDZUHARA.

When a limited number of observation-equations containing independent unknown quantities is given, it is usual, in finding the mean error of the single observation, to adopt the well known formula of GAUSS, which is said to be best approximative; but whether it is truly approximative or not may be questioned, for, it was not the result of the comparison with the true one. I have lately spent much time on the discussion of this point, and finally conceived a formula sensibly different from that of GAUSS: and as the consequence of this discussion I have also found formulas giving some likely errors (I have adopted this name to distinguish from the word "probable error" in the usual meaning) of the most probable values of the unknown quantities. The details of the results of this discussion are as follows.

1. *Notation.* It is to be noticed that in the present paper I have adopted, for the most part, the notation described in the Spherical and Practical Astronomy, Vol. II, by CHAUVENET, except some few parts which will be especially explained as occasion requires.

2. *On a Substitution of Mean Value for the True One.* The square of the true error of $[au]$ is $[au]^2$, and the square of its mean error $[aa]\epsilon^2$; the system of such relations will be expressed, for shortness, by saying that "the mean value of

$\{[au]^2 \pm [bu]^2 \pm \ldots\}$ is equal to $[aa]\epsilon^2 \pm [bb]\epsilon^2 \pm \ldots$."

The expression which we have now to determine is

$$\text{(A)} \qquad [au]\,\Delta x + [bu]\,\Delta y + [cu]\,\Delta z + \ldots$$

and, remembering that (for example)

$$\begin{aligned} \text{mean value of } [au][bu] &= \text{mean value of } \frac{[(a+b)u]^2 - [au]^2 - [bu]^2}{2} \\ &= [ab]\epsilon^2 \end{aligned}$$

it may be easily seen that the mean value of every term of the expression (A) is ϵ^2, which has been substituted for the true by GAUSS; but since the ratios of the different functions of the errors to the corresponding mean values are not necessarily constant (see Art. 3 and last part of Art. 5), it is possible that, even if any function of the errors be identical with that mean value, yet another function of the errors can not be considered so; that is to say, there must be, sometimes, a better value than the mean to be substituted for the true one. On this account I shall now discuss by what expressions the functions

$$[au]^2,\ [bu]^2,\ [cu]^2 \ldots\ldots$$
$$[au][bu],\ [au][cu],\ [bu][cu] \ldots\ldots$$

of which the expression (A) is composed, must be substituted. It is sufficient to consider the case of three unknown quantities.

3. *Discussion of the Functions* $[au]^2$, $[bu]^2$, $[cu]^2$ and $[au][bu]$, $[au][cu]$, $[bu][cu]$.

Let us suppose that

$$[aa] = [bb] = [cc]$$

and

$$[au]^2 > [bu]^2 > [cu]^2$$

then we may put

$$\begin{aligned} [au]^2 &= (A+B)[aa]\epsilon^2 = (A'+B')[aa]\epsilon^2 && (1) \\ [bu]^2 &= (A-B)[aa]\epsilon^2 = (A''+B'')[aa]\epsilon^2 && (2) \\ [cu]^2 &= (A'-B')[aa]\epsilon^2 = (A''-B'')[aa]\epsilon^2 && (3) \end{aligned}$$

and therefore

$$\begin{aligned} [au][bu] &= \pm\sqrt{A^2-B^2}\,[aa]\epsilon^2 && (4) \\ [au][cu] &= \pm\sqrt{A'^2-B'^2}\,[aa]\epsilon^2 && (5) \\ [bu][cu] &= \pm\sqrt{A''^2-B''^2}\,[aa]\epsilon^2 && (6) \end{aligned}$$

where $A, A', \ldots$ $B, B', \ldots$ are indeterminate positive numbers. Now, if we put

$$(7)\qquad [au]\,[bu] = A\,.\,f[ab]\,\epsilon^2\,^{*}$$

$$(8)\qquad [au]\,[cu] = A'.\,f[ac]\,\epsilon^2$$

$$(9)\qquad [bu]\,[cu] = A''.\,f[bc]\,\epsilon^2$$

we get

(10)

$$[cu]^2 = \frac{A\left\{1+\sqrt{1-\left(\frac{f[ab]}{[aa]}\right)^2}\right\}\left\{1-\sqrt{1-\left(\frac{f[ac]}{[aa]}\right)^2}\right\}[aa]\,\epsilon^2}{1+\sqrt{1-\left(\frac{f[ac]}{[aa]}\right)^2}}$$

(11)

$$= \frac{A\left\{1-\sqrt{1-\left(\frac{f[bc]}{[aa]}\right)^2}\right\}\left\{1-\sqrt{1-\left(\frac{f[ab]}{[aa]}\right)^2}\right\}[aa]\,\epsilon^2}{1+\sqrt{1-\left(\frac{f[bc]}{[aa]}\right)^2}}$$

which, if we have

$$(12)\qquad [ab] = [ac] = [bc].$$

give $\quad f[ab] = f[ac] = f[bc] = \pm[aa]$;

and since, for instance, we may change the values of $[ab]$ and $[ac]$, without changing the value of $[bc]$, in the equations (12), and moreover the latter may be any value between zero and $[aa]$, for all possible values of $[ab]$, $[ac]$ and $[bc]$ we must always have

$$f[bc] = \pm[aa],$$

and, by the same reasoning,

$$f[ab] = f[ac] = \pm[aa].$$

Therefore, from (1), (2), (7) and (10), we must also have

$$(13)\qquad \left\{\begin{aligned}[au]^2 &= [bu]^2 = [cu]^2 = A\,[aa]\,\epsilon^2\\ &= A \times \text{mean value of } [au]^2 = \text{\&c.}\end{aligned}\right.$$

and

(14)

$$\left\{\begin{aligned}[au][bu] &= \pm[au][cu] = \pm[bu][cu] = \pm A[aa]\,\epsilon^2\\ &= \pm\frac{A\,[aa]}{[ab]} \times \text{mean value of } [au][bu] = \text{\&c.}\end{aligned}\right.$$

* It is easily seen that we can not adopt the following assumption:

$$(7)'\qquad [au]\,[bu] = A\,[ab]\,\varepsilon^2$$

$$(8)'\qquad [au]\,[cu] = A'\,[ac]\,\varepsilon^2$$

$$(9)'\qquad [bu]\,[cu] = A''\,[bc]\,\varepsilon^2$$

as that of GAUSS. For let us consider the following particular case:

$$\begin{array}{rrrl} a & b & c & \\ +1 & +1 & +1 & [ab] = \tfrac{1}{2}\,[aa] \\ +1 & +1 & +1 & [ac] = \tfrac{1}{2}\,[aa] \\ +1 & +1 & -1 & [bc] = 0 \\ -1 & +1 & -1 & \end{array}$$

$$\begin{array}{llllll} \text{then since} & & [bc] = 0 & & & \\ & \text{from} & (9)' & \text{we} & \text{have} & [cu] = 0 \\ \text{therefore} & \text{''} & (8)' & \text{''} & \text{''} & A' = 0 \\ \text{''} & \text{''} & (3) & \text{''} & \text{''} & B' = 0 \\ \text{''} & \text{''} & (1) & \text{''} & \text{''} & [au] = 0 \\ \text{''} & \text{''} & (7)' & \text{''} & \text{''} & A = 0 \\ \text{''} & \text{''} & (1) & \text{''} & \text{''} & B = 0 \\ \text{''} & \text{''} & (2) & \text{''} & \text{''} & [bu] = 0 \end{array}$$

Thus, in order that the hypotheses (7)′, (8)′, (9)′ shall be true, we must have $[au] = [bu] = [cu] = 0$ which can not be accepted.

which show that the ratios of $[au]^2$, &c., to their mean values are not the same as those of $[au]\,[bu]$, &c., to the corresponding mean values. The ambiguous signs of the expression of $[au]\,[bu]$ (for instance) may be determined by the following consideration. Let us transform the expression

$$\frac{[bu]}{[au]}$$

by substituting mean values for true ones, as follows:

$$\frac{[bu]}{[au]} = \frac{[bu]\,[au]}{[au]^2} = \frac{[ab]\,\epsilon^2}{[aa]\,\epsilon^2} = \frac{[ab]}{[aa]} \qquad (15)$$

$$\frac{[bu]}{[au]} = \frac{[bu]^2}{[au]\,[bu]} = \frac{[bb]\,\epsilon^2}{[ab]\,\epsilon^2} = \frac{[aa]}{[ab]} \qquad (16)$$

then comparing both the results it is evident that the result (15) generally gives a less value, and the result (16) a larger one, showing again that the ratios of the different functions of the errors to the corresponding mean values are not necessarily constant; but each of them determines the sign of

$$\frac{[bu]}{[au]}$$

to be identical with that of $[ab]$, and the geometrical mean of them is equal to unity; therefore we have probably

$$\frac{[bu]}{[au]} = \frac{[ab]}{[ab]_0} \qquad (17)$$

or

$$[au]\,[bu] = \frac{A\,[ab]\,[aa]\,\epsilon^2}{[ab]_0} \qquad (18)$$

where $[ab]_0$ denotes the numerical value of $[ab]$. Now, since the above results come from the supposition that

$$[aa] = [bb] = [cc]$$

to apply them for the practical purpose we must first transform the observation-equations so that the transformed equations have the relation

$$[aa] = [bb] = [cc]$$

as may be easily effected in the following manner.

4. *Transformation of the Observation-Equations.* If the series of the coefficients $a, b, c, \ldots\ldots$ of the unknown quantities $x, y, z, \ldots\ldots$ in the observation-equations be multiplied by constant quantities L_1, L_2, L_3, ... such that

$$[aa]\,L_1^2 = [bb]\,L_2^2 = [cc]\,L_3^2 = \ldots\ldots$$

and, at the same time, the unknown quantities be divided by the same constants $L_1, L_2, L_3, \ldots\ldots$ respectively, then it is evident that the new values of $[vv]$, $[uu]$, and of $[au]\,\Delta x$, $[bu]\,\Delta y$, $[cu]\,\Delta z$, ..., are the same as those

of the original equations, and the new values of $[aa]$, $[bb]$, $[cc]$, ... now become $[aa]L_1^2$, $[bb]L_2^2$, $[cc]L_3^2$, ..., all of which are equal. In the following article it is to be understood that we have always treated of such transformed equations, though their notations are not distinguished between the new and the old equations.

Now, before we consider the expression

$$[an].\Delta x + [bn].\Delta y + [cn].\Delta z + \ldots .$$

in the general case, it is more convenient to first treat the case of two unknown quantities: for it is very simple and the results of its discussion may be extended to the general case almost by similar considerations.

5. *Determination of the Mean Error of an Observation and Some Likely Errors of the Most Probable Values of the Unknown Quantities when the Observation-Equations contain Two Unknown Quantities x and y.* By solving

$$(19)\qquad \begin{aligned} [aa].\Delta x + [ab].\Delta y &= [an] \\ [ab].\Delta x + [bb].\Delta y &= [bn] \end{aligned}$$

we have

$$(20)\qquad \Delta x = \frac{[an][bb] - [bn][ab]}{[aa][bb] - [ab]^2}$$

and therefore

$$(20')\qquad [an].\Delta x = \frac{[an]^2\left\{[aa] - \dfrac{[bn]}{[an]}[ab]\right\}}{[aa]^2 - [ab]^2}$$

which, being transformed by the equations (17) and (13), becomes

$$(21)\qquad [an].\Delta x = \frac{[an]^2\{[aa] - [ab]_0\}}{[aa]^2 - [ab]^2}$$

$$(22)\qquad = \frac{A\epsilon^2}{1 + \dfrac{[ab]_0}{[aa]}}$$

By the same reasoning we have the same expression of $[bn].\Delta y$, and therefore we have

$$(23)\qquad \epsilon^2 = \frac{[vv]}{m - \dfrac{2A}{1 + \dfrac{[ab]_0}{[aa]}}}$$

Now from this result it is to be observed that if we put $A = 1$ and $[ab]_0 = 0$ then the equation (23) is identical with that of GAUSS; and as the value of $[ab]_0$ increases, the denominator of ϵ^2 gradually increases; when the value of $[ab]_0$ becomes a maximum (for instance the case in which $a' = b'$, $a'' = b''$, $a''' = b'''$, &c.), it becomes the same form as that in the case of the single unknown quantity, which coincides with the practical condition since in that case $x + y$ may be considered as the single unknown quantity.

I shall now consider some likely values of Δx and Δy. Let us first consider the rigorous expressions of Δx, Δy, $\Delta x + \Delta y$, as follows:

$$\Delta x = \frac{[an][bb] - [bn][ab]}{[aa]^2 - [ab]^2} \qquad (24)$$

$$\Delta y = \frac{[bn][aa] - [an][ab]}{[aa]^2 - [ab]^2} \qquad (24')$$

$$\Delta x + \Delta y = \frac{[an] + [bn]}{[aa] + [ab]} \qquad (24'')$$

Now, since $[aa] = [bb]$, it is quite probable that the first terms of the expressions of Δx and Δy are larger than the second terms of them respectively, and therefore the signs of Δx and Δy must be the same as those of $[an]$ and $[bn]$ respectively; but when the value of $[ab]$ is positive and not small, it being quite probable that $[an]$ and $[bn]$ have the same signs, Δx and Δy must then have the same signs, so that they vary with $\Delta x + \Delta y$: therefore, since it is evident from the above equation (24'') that $\Delta x + \Delta y$ does not increase infinitely as the value of $[ab]$ increases, the values of Δx and Δy must also be so: this evidently proves that

$$\text{mean error of } x = \frac{\sqrt{[aa]}\,\epsilon}{\sqrt{[aa]^2 - [ab]^2}}$$

$$\text{mean error of } y = \frac{\sqrt{[bb]}\,\epsilon}{\sqrt{[aa]^2 - [ab]^2}}$$

are not approximate values of Δx and Δy respectively, for they gradually increase as the value of $[ab]$ increases, and finally become infinity when $[ab] = [aa]$. Indeed, the most likely values of Δx and Δy will be obtained in the following manner; viz., from the equation (21) we have immediately

$$\Delta x = \frac{[an]\{[aa] - [ab]_0\}}{[aa]^2 - [ab]^2} \qquad (25)$$

$$= \frac{[an]}{[aa] + [ab]_0} \qquad (26)$$

where $[an] = \sqrt{A[aa]}\,\epsilon$. By the same reasoning

$$\Delta y = \frac{[bn]}{[aa] + [ab]_0} \qquad (27)$$

Therefore, adding (26) to (27) we have

$$\Delta x + \Delta y = \frac{[an] + [bn]}{[aa] + [ab]_0} \qquad (28)$$

which is identical with the rigorous equation (24''). It is also to be noticed that by comparing the value of Δx in the equation (26) with the mean value of it we have

$$(\Delta x)^2 = \frac{([aa] - [ab]_0)A}{[aa] + [ab]_0} \times \text{mean value of } \Delta x$$

which shows that the ratio of (Δx) to its mean value is a function of $[ab]_0$.

I have, now, to consider the general case.

6. *Determination of the Mean Error of an Observation and Likely Errors of the most Probable Values of the Unknown Quantities when the Observation-Equations contain any number of the Unknown Quantities* x, y, z, ...

Let

D = the determinant formed from all of the coefficients of the unknown quantities in the normal equations.

D_a, D_b, D_c, ... = the minors corresponding to the constituents $[aa]$, $[bb]$, $[cc]$, . . . respectively.

D_{ab}, D_{ac}, ... = the minors corresponding to the constituents $[ab]$, $[ac]$, ...

then since

$$(33)\qquad D.\Delta x = [an]\,D_a + [bn]\,D_{ab} + [cn]\,D_{ac} + \dots$$

we have

$$(34)\qquad [an]\,\Delta x = \frac{[an]^2}{D}\left\{D_a + \frac{[bn]}{[an]}D_{ab} + \frac{[cn]}{[an]}D_{ac} + \dots\right\}$$

but since, by (17)

$$\frac{[bn]}{[an]} = \frac{[ab]}{[ab]_0}$$

and similarly

$$\frac{[cn]}{[an]} = \frac{[ac]}{[ac]_0}$$

we have

$$(35)\qquad [an].\Delta x = \frac{[an]^2}{D}\left\{D_a + \frac{[ab]\,D_{ab}}{[ab]_0} + \frac{[ac]\,D_{ac}}{[ac]_0} + \dots\right\}$$

By the same reasoning

$$(36)\qquad [bn].\Delta y = \frac{[bn]^2}{D}\left\{\frac{[ab]\,D_{ab}}{[ab]_0} + D_b + \frac{[bc]\,D_{bc}}{[bc]_0} + \dots\right\}$$

. . . .

Therefore, if we put

$$(37)\qquad \Sigma D_a = D_a + D_b + D_c + \dots$$

$$(38)\qquad \sum\frac{[ab]\,D_{ab}}{[ab]_0} = \frac{[ab]\,D_{ab}}{[ab]_0} + \frac{[ac]\,D_{ac}}{[ac]_0} + \dots + \frac{[bc]\,D_{bc}}{[bc]_0} + \dots$$

we have

$$(39)\qquad [an]\,\Delta x + [bn]\,\Delta y + [cn]\,\Delta z + \dots = \frac{[an]^2}{D}\left\{\sum D_a + 2\sum\frac{[ab]\,D_{ab}}{[ab]_0}\right\}$$

or

$$(40)\qquad \epsilon^2 = \frac{[vv]}{m - \frac{A\,[aa]}{D}\left\{\sum D_a + 2\sum\frac{[ab]\,D_{ab}}{[ab]_0}\right\}}$$

This equation gives the best value of the mean error of the single observation. But for the practical purpose we must find some approximate formula.

Now, since the values of $[ab]_0$, $[ac]_0$, ... are usually small in comparison with $[aa]$, $[bb]$, ... we may approximately substitute the mean of $[ab]_0$, $[ac]_0$, $[ad]_0$, . . . $[bc]_0$, $[bd]_0$, ... for each of them involved in the equations (35), (36), ... ; then since

$$D = [aa]\,D_a + [ab]\,D_{ab} + [ac]\,D_{ac} + \dots \qquad (41)$$

comparing this with (35) we have, nearly,

$$[an]\,\Delta x = \frac{[an]^2}{D}\left\{D_a + \frac{D - [aa]\,D_a}{\lambda^2}\right\}$$

$$= \frac{[an]^2\left\{D - (\alpha^2 - \lambda^2)\,D_a\right\}}{D\lambda^2} \qquad (42)$$

where

$\alpha^2 = [aa] = [bb] = \dots$

λ^2 = the mean of $[ab]_0$, $[ac]_0$, ... $[bc]_0$, $[bd]_0$, ...

It is to be noticed that though the formula (42) was deduced from the supposition that the values of $[ab]_0$, $[ac]_0$, &c., are small, yet it is well satisfied to the particular case in which the values of $[ab]$, $[ac]$, &c., are equal to each other, even if they are large.

Now since, accurately to the second order of $\frac{\lambda^2}{\alpha^2}$ in the expression of D, we may put

$$D = \begin{vmatrix} \alpha^2 & \lambda^2 & \lambda^2 & \dots \\ \lambda^2 & \alpha^2 & \lambda^2 & \dots \\ \lambda^2 & \lambda^2 & \alpha^2 & \dots \\ \vdots & \vdots & \vdots & \end{vmatrix} \qquad (43)$$

by the application of the following theorem:

$$D = \phi(\lambda^2) - \lambda^2\frac{\partial\phi}{\partial(\lambda^2)}$$

where

$$\phi(\lambda^2) = (\alpha^2 - \lambda^2)^\mu$$

we have

$$D = (\alpha^2 - \lambda^2)^{\mu-1}\left\{\alpha^2 + (\mu-1)\lambda^2\right\}$$

$$D_a = D_b = D_c = \dots = (\alpha^2 - \lambda^2)^{\mu-2}\left\{\alpha^2 + (\mu-2)\lambda^2\right\}$$

Substituting these, (42) becomes:

$$[an]\,\Delta x = \qquad (44)$$

$$\frac{[an]^2\left\{(\alpha^2-\lambda^2)^{\mu-1}[\alpha^2+(\mu-1)\lambda^2] - (\alpha^2-\lambda^2)^{\mu-1}[\alpha^2+(\mu-2)\lambda^2]\right\}}{\lambda^2(\alpha^2-\lambda^2)^{\mu-1}\left\{\alpha^2+(\mu-1)\lambda^2\right\}}$$

$$= \frac{A\,\epsilon^2}{1 + (\mu-1)\frac{\lambda^2}{\alpha^2}} \qquad (45)$$

and therefore

$$\epsilon^2 = \frac{[vv]}{m - \frac{A\mu}{1 + (\mu-1)\frac{\lambda^2}{\alpha^2}}} \qquad (46)$$

which shows that the value of ϵ generally increases with

that of μ, except that, when $\lambda^2 = \alpha^2$, they become independent of each other.

Now, the best values of Δx, Δy, ... must be found from (35), (36), ...; but their practical values may be easily found from the formula (41), and similar ones, as follows:

(47)

$$\Delta x = \frac{[an]}{\alpha^2 + (\mu - 1)\lambda^2}$$

(48)

$$(\Delta x)^2 = \frac{\left(1 - \frac{\lambda^2}{\alpha^2}\right) A}{\left\{1 + (\mu - 1)\frac{\lambda^2}{\alpha^2}\right\}\left\{1 + (\mu - 2)\frac{\lambda^2}{\alpha^2}\right\}} \times \left\{\begin{matrix}\text{mean value} \\ \text{of } (\Delta x)^2\end{matrix}\right.$$

. . . .

therefore

(49)

$$\Delta x\, \Delta y = \frac{[an][bn]}{\{\alpha^2 + (\mu - 1)\lambda^2\}^2}$$

. . . .

I shall now give a few theorems which may be easily proved from the results of the above discussion.

A. All of the expressions $[an]\Delta x$, $[bn]\Delta y$, $[cn]\Delta z$, ... have positive signs.

This may be evidently seen from the form of the equation (45), which is the approximate value of the second member of (34).

B. $\Delta x \Delta y$, $\Delta x \Delta z$, ... have the same signs with $[an][bn]$, $[an][cn]$, ... respectively.

This immediately follows from the theorem *A.* It is also evident from (49).

C. $\Delta x \Delta y$, $\Delta x \Delta z$, ... have the same signs with $[ab]$, $[ac]$, ... respectively.

This immediately follows from the theorem *B* and the following hypothesis:

"$[an][bn]$, $[an][cn]$, ... have the same signs with $[ab]$, $[ac]$, ... respectively."

which is quite probable when the numerical values of $[ab]$, $[ac]$, ... are large.

D. All the terms of the equation $[an] = [aa]\Delta x + [ab]\Delta y + [ac]\Delta z + ...$ have the same signs.

This may be easily proved by multiplying the equation by Δx, and comparing every term of it with the theorem *C.* As a corollary of the present theorem we may say that:

If, whilst the values of $[aa]$ and $[an]$, $[bn]$, ... are put in unchanged, the numerical values of $[ab]$, $[ac]$, ... be increased then those of Δx, Δy, Δz, ... are decreased.

Tokyo Astronomical Observatory, 1902 October.

E. If we have $a = \pm b' = \pm c'' = ... , a' = \pm b = \pm c'' = ...$, [illegible] the value of $[an]\Delta x + [bn]\Delta y + [cn]\Delta z + ...$ becomes the same as that in the case of one unknown quantity.

This is evident from the equation (45).

F. The equations

$$[an] = [aa]\Delta x + [ab]\Delta y + [ac]\Delta z + ...$$
$$[bn] = [ab]\Delta x + [bb]\Delta y + [bc]\Delta z + ...$$

. . . .

are just satisfied by my values of Δx, Δy, Δz, ... but not by the mean values of them.

It is also important to compare the following results:

$$\text{mean value of } (\Delta x)^2 = \frac{D_1}{D}\epsilon^2 \qquad (a)$$

$$\Delta x = \frac{[an]}{\lambda^2}\left\{1 - (\alpha^2 - \lambda^2)\frac{D_1}{D}\right\} \text{ (from (42))} \qquad (b)$$

For clearness of thought, let us take the following example; viz., let there be given observation-equations involving any number of unknown quantities in which the approximate value w_0 of the unknown quantity w is known, and they be solved by supposing that

1. $w = w_0$
2. $w = w_0 + \Delta w$

then from (*a*) we can not say that, even if the value of ϵ^2 in the second solution be less than that in the first, the mean value of $(\Delta x)^2$ in the second solution is less than that in the first (see my theorem in the *A.J.*, No. 521); but from (*b*) we may say that, if the values of λ^2s and of ϵs in both solutions be equal, the value of Δx in the second solution is generally less than that in the first.

We must now determine the value of *A.* But this being, indeed, a very vague problem, so that we can not now find any reliable value of it, we are compelled still to adopt "$A = 1$," which agrees with the theory of the mean errors.

Finally, it is to be remarked that, since we have hitherto supposed that the observation-equations have been transformed to have the relation $[aa] = [bb] = [cc] = ...$, for instance, to apply the formula (46) for the original equations we must understand that

$$\frac{\lambda^2}{\alpha^2} = \text{the mean of } \frac{[ab]^2}{\{[aa][bb]\}}, \frac{[ac]^2}{\{[aa][cc]\}}, ...$$

OBSERVATIONS OF COMET 1900 II,

MADE AT THE U.S. NAVAL OBSERVATORY,

BY GEORGE K. LAWTON.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1900 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
July *27 11 11 31.7	1	25 , 5	−0 56.50	+ 0 1.2	2 17 48.04	+21 13 38.1	n9.617	0.535	+3.02	+ 7.1
27 15 13 46.3	1	25 , 5	−0 55.60	+ 3 36.7	2 17 18.91	+21 17 13.6	9.574	0.497	+3.02	+ 7.1
30 15 1 25.0	2	25 , 5	−2 0.74	+ 3 6.8	2 51 29.75	+33 29 51.7	n9.611	0.317	+3.28	+ 4.1
31 15 21 11.9	3	25 , 5	+3 11.26	+ 5 54.8	2 52 56.13	+36 38 38.1	n9.588	0.147	+3.10	+ 3.7
Aug. 1 15 35 30.5	4	20 , 4	+2 3.32	+ 1 30.8	2 54 27.15	+39 11 50.1	n9.578	9.921	+3.50	+ 2.6
3 15 18 16.8	5	19 , 4	−3 56.47	− 4 16.7	2 57 51.13	+45 49 30.7	n9.577	n9.310	+3.73	+ 0.2
5 15 41 29.6	6	25 , 5	−3 43.28	− 1 59.1	3 1 58.07	+51 36 47.6	n9.635	n9.982	+4.03	− 1.5
6 14 23 22.3	7	25 , 5	−1 25.56	− 7 55.7	3 4 10.88	+51 14 41.5	n9.808	7.598	+4.21	− 2.2
7 15 18 21.2	8	25 , 5	+2 49.70	+ 0 36.3	3 6 51.43	+57 2 26.1	n9.742	n0.138	+4.12	− 2.7
10 14 54 4.7	9	20 , 4	+3 4.90	−12 1.0	3 16 30.58	+64 27 47.0	9.887	n0.283	+5.12	− 1.9
17 11 0 31.0	10	20 , 4	−3 45.09	+ 3 12.1	4 1 58.81	+77 52 44.2	n0.313	0.527	+7.77	−11.0
17 12 23 52.0	10	19 , 4	−3 0.81	+ 9 25.3	4 2 43.09	+77 58 57.4	n0.344	9.993	+7.77	−11.0
28 12 34 40.5	11	17 , 17	−0 50.9	− 2 6.2	11 14 12.5	+84 43 23.3	9.401	0.865	−9.9	−11.4
30 13 17 17.7	12	14 , 8	+3 50.4	− 6 28.2	12 11 16.3	+83 13 52.2	9.484	0.871	−9.0	−10.5
Sept. 1 11 16 28.4	13	4 , 4	−0 37.7	− 4 9.7	12 42 55.1	+82 5 24.0	0.336	0.771	−8.1	− 7.6
2 11 52 51.8	14	19 , 4	−3 17.4	− 2 42.8	12 55 14.6	+81 21 59.4	0.228	0.810	−7.1	− 6.4
*23 11 41 59.6	15	21 , 9	+0 43.18	+ 0 36.7	14 16 42.74	+70 49 37.1	9.914	0.848	−2.73	− 3.1
Oct. 15 11 13 42.7	16	18 , 4	−1 59.27	− 2 49.5	14 50 57.93	+66 16 56.1	9.639	0.904	−2.35	− 4.2
17 12 12 10.0	17	15 , 6	+5 20.44	+ 2 26.8	14 53 54.22	+66 5 45.8	9.442	0.919	−2.28	− 6.0
19 12 7 25.0	18	16 , 7	−1 33.30	+ 4 37.7	14 56 18.17	+65 56 50.6	9.438	0.920	−2.31	− 5.2
20 11 13 7.4	18	†11 , 16	−0 7.60	+ 0 55.8	14 58 13.85	+65 53 8.4	9.574	0.911	−2.33	− 5.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	2 18 11.49	+24 13 29.5	Berlin B, A.G. 852	10	4 5 36.13	+77 49 43.1	Kasan, A.G. 655
2	2 53 27.21	+33 26 43.5	Leiden, A.G. 1122	11	11 15 13.3	+84 45 43.9	Carrington 1679
3	2 49 41.47	+36 32 39.6	Lund, A.G. 1498	12	12 7 34.9	+83 20 30.9	Carrington 1815
4	2 52 20.63	+39 40 17.0	Lund, A.G. 1517	13	12 43 40.9	+82 9 41.3	Carrington 1897
5	3 1 46.87	+45 53 47.2	Bonn, A.G. 2630	14	12 58 39.4	+81 24 48.6	Carrington 1936
6	3 5 37.32	+51 38 48.5	Camb., U.S., A.G. 1113	15	14 16 2.29	+70 49 3.8	Dorpat, A.G. Zones
7	3 5 32.23	+51 22 39.4	Camb., U.S., A.G. 1111	16	14 55 59.55	+66 19 50.1	Newcomb 951
8	3 3 57.31	+57 1 52.5	Hels.&Gotha, A.G. 2813	17	14 48 36.06	+66 3 25.0	Christiania, A.G. 2215
9	3 13 20.56	+64 39 52.9	Hels.&Gotha, A.G. 2920	18	14 58 23.78	+65 52 18.1	Christiania, A.G. 2235

* The observations from July 27 to Sept. 2 were made with the 12-inch equatorial; those from Sept. 23 to Oct. 20 on the 26-inch equatorial. † Micrometer measurement. NOTE: — Oct. 20, "Exceedingly faint for all measures."

2387 —GEMINORUM.

A cable dispatch of March 27, from Dr. KREUTZ, announced the discovery by Prof. TURNER, at Oxford University Observatory, of a new star in the position,

$$\alpha = 6^h\ 37^m\ 48^s \quad , \quad \delta = +30°\ 3'$$

but which might possibly be a variable. Its magnitude on March 16 was 8.0. The cipher words of the dispatch, *trouble valerian*, are construed as meaning that it was not visible on the plates of February 16.

On March 28, telegrams were received from Harvard College Observatory, communicating a dispatch from Prof. HALE, at Yerkes Observatory, giving the position,

$$1903 \text{ Mar. } 27.750, \quad \alpha = 6^h\ 37^m\ 49^s \quad , \quad \delta = +30°\ 2'\ 38''$$

and the magnitude as 8.5; and stating that the spectrum contains bright lines or bands; also a dispatch from Capt. CHESTER, Superintendent of Naval Observatory, stating that the star was photographed there on March 27, and that the magnitude was 8.5, color red.

By Prof. BARNARD's note printed on another page of this number, the accurate position, observed at the Yerkes Observatory, is

$$\alpha = 6^h\ 37^m\ 48^s.96 \quad , \quad \delta = +30°\ 2'\ 38''.3 \quad (1900.0)$$

OBSERVATION OF THE POSITION OF TURNER'S "*NOVA*," (2387 — *GEMINORUM*).

By E. E. BARNARD.

The position of this star was observed with the micrometer of the 10-inch on March 27. No comparison-star was available for direct measurement. The comparison-star used (A.G. Camb., Eng., 3482), was too distant in declination for differences of right-ascension. A star of 12^M was therefore used as an intermediate.

$$\Delta\alpha \ \textit{Nova} \text{ and } 12^M \text{ star}, \ 5' \ 5''.1 = 23^s.49 \ (4)$$
$$\Delta\delta \quad " \qquad " \qquad 2 \ 16.0 \ (3)$$

The 12^M star was s.f.

The small star was then referred to A.G. 3482.

$$\Delta\alpha \ 3482 \text{ and } 12^M \text{ star} \quad 1^m \ 58^s.36 \ (12)$$
$$\Delta\delta \quad " \qquad " \qquad 3 \ 41.6 \quad (2)$$

The 12^M star n.p.

The *Nova* was also referred direct to 3482 in declination.

$$\Delta\delta \quad 5' \ 57''.0 \ (2) \quad \textit{Nova} \text{ north.}$$

These observations give

$$\textit{Nova} - 3482. \quad \Delta\alpha = -2^m \ 21^s.85$$
$$\textit{Nova} - 3482. \quad \Delta\delta = +5 \ 57''.3$$

Yerkes Observatory, Williams Bay, Wis., 1903 *March* 28.

The place of the comparison-star (A.G., Camb., Eng. 3482) is

$$1903.0 \quad \alpha = 6^h \ 40^m \ 22^s.31 \quad \delta = +29° \ 56' \ 30''.7$$

This gives the place of the *Nova*.

1903.0	$6^h \ 38^m \ 0^s.46$	+30° 2′ 28″.0
1900.0	6 37 48.96	+30 2 38.3

At $11^h \ 0^m$, a direct estimate made the *Nova* $8^M.2$. It was estimated to be $\frac{2}{10}$ of a magnitude less than the comparison-star, which is given as $8^M.2$ in DM. This would make the *Nova* $8^M.4$ on the DM. scale. It was of a strong red color.

Careful tests were made for focus with a power of 700 diameters. The results are

Focus for *Nova*.	2.12 inches	(4 obs.)
Focus for 8^M star.	2.10 "	(6 obs.)
Focus, *Nova* − Star =	+0.02 inch.	

The difference is too small to mean anything, though it is in the right direction for a *Nova*.

NOTES ON VARIABLE STARS. — No. 37.

By HENRY M. PARKHURST.

Approximate Maxima. When from scarcity of decisive observations it becomes necessary to substitute the maximum from the elements for the observed maximum, as indicated by E in the column of weights, there may yet be an indication from the light-curve shown by the observations, of an approximate maximum, which may be given by its Julian date in the column headed "Mag.," and thus either confirm the elements or suggest their approximate correction.

RESULTS OF OBSERVATIONS.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W.	Mag.	Factors	Remarks
				1902						
6888	*RW Sagittarii*									No period ascertained
6892	*RX Sagittarii*	Max.	6018	Sept. 15	6		E	6045 :		320 : *A.J.* 476
6900	*W Aquilae*	Max.	6011	Oct. 18	7		E			*A.J.* 393
7118	*X Aquilae*	Min.	6066	Nov. 12	10		E			*A.J.* 547
7155	*RR Aquilae*	Max.	6096	Dec. 12	7		5			Approximate period 359
7162	*RS Aquilae*									Period about 400 : probably
7242	*S Aquilae*	Max.	6049	Oct. 26	92		E			Correction apparently increasing
7244	*RW Aquilae*	Max.	6114	Dec. 30	194	− 1	6			Period 7.87. *A.J.* 490
7261	*R Delphini*	Max.	6113	Dec. 29	48	− 66	6	7.9		Shortening of period confirmed
(7116)	— 19°5892									Third catalogue, supplement
7458	*V Delphini*	Max.	6042	Oct. 19	8		E	6107 :		Probably much later
(7484)	— *Cygni*	—		—						Third catalogue, supplement
(8104)	— *Aquarii*									" " "
(8106)	— *Pegasi*									" " "
7896	*V Pegasi*	Max.	6118	Jan. 3	9		E	6089 .		Probably much earlier
8230	*S Aquarii*	Max.	6058	Nov. 1	56		E	6058		
8290	*R Pegasi*	Max.	6106	Dec. 22	50	− 2	6	10.1		Correction probably too small
8373	*S Pegasi*	Max.	6161	Feb. 15	48	− 19	5			Low in west
8512	*R Aquarii*	Max.	6127	Jan. 12	86	− 16	9	6.14	1.22 0.91 16	
8562	*Z Aquarii*	Max.	4984	Nov. 25			2	6095 .		1899
8622	*W Ceti*	Max.	6122	Jan. 7	7	+ 72	7			Sky very uncertain

Individual Observations.

Including Observations by Arthur C. Perry

6888 *RW Sagittarii.*

(Cont. from 456, Comp. Stars 421)

Julian	Calendar	Mag.
	1899	
4843.6	July 7	10.44
4867.6	31	9.4 P
	1900	
5228.6	July 27	10.0
5246.6	Aug. 14	9.6
5257.6	25	9.7
5280.5	Sept. 17	9.7
5312.5	Oct. 19	9.1
5318.5	25	9.2
	1901	
5609.6	Aug. 12	10.0
5673.5	Oct. 15	9.2
	Range of 7 dates	
	1902	
6082.5	Nov. 28	9.7
6085.5	Dec. 1	9.6

6892 *RX Sagittarii.*

(Cont. from 456, Comp. Stars 421)

Julian	Calendar	Mag.
	1899	
4843.6	July 7	13]
4928.6	Sept. 30	12] P
	1900	
5228.6	July 27 to	
5318.5	Oct. 25	13]
	6 dates	
	1901	
5609	Aug. 12 to	
5661	Oct. 6	13]
	6 dates	
	1902	
6082.5	Nov. 28	10.0
6085.5	Dec. 1	9.8

6900 *W Aquilae.*

(Cont. from 456, Comp. Stars 339)

Julian	Calendar	Mag.
	1901	
5609.6	Aug. 12	10.9
5631.6	Sept. 3	11.0
5634.6	6	11.2
5643.6	15	11.4
5665.6	Oct. 7	13]
	1902	
6089	Dec. 5	9.8 :
6102	18	9]

7118 *X Aquilae.*

(Cont. from 490, Comp. Stars 339)

Julian	Calendar	Mag.
	1902	
6082	Nov. 28	12.1]
6102	Dec. 18	11.8]

7155 *RR Aquilae.*

(Cont. from 490, Comp. Stars 393)

Julian	Calendar	Mag.
	1901	
5609.5	Aug. 12	12]
5632.5	Sept. 4	12]
5643.5	15	12]
5664.5	Oct. 6	9.1
5665.5	7	9.00_2
5668.5	10	8.77
5671.5	13	8.85_2
5677.5	19	8.91_2
	1902	
6082.5	Nov. 28	8.0
6085.5	Dec. 1	8.2
6089.5	5	8.2
6092.5	8	8.2
6101.5	17	8.1
6102.5	18	8.0
6107.5	23	8.1
6111.5	27	8.2
6114.5	30	8.2

7162 *RS Aquilae.*

(Cont. from 490, Comp. Stars 464)

Julian	Calendar	Mag.
	1901	
5607.6	Aug. 10 to	
5664.5	Oct. 6	13]
	4 dates	
	1902	
6082	Nov. 28 to	
6102	Dec. 18	13]
	3 dates	

7242 *S Aquilae.*

(Cont. from 425, Comp. Stars 464)

Julian	Calendar	Mag.
	1902	
6085	Dec. 1	9.3
6102	18	9.4

7244 *RW Aquilae.*

(Cont. from 490, Comp. Stars 464)

Julian	Calendar	Mag.
	1902	
6085.5	Dec. 1	8.7
6090.5	6	8.6
6091.5	7	8.8
6092.5	8	8.6
6093.5	9	8.7
6101.5	17	8.7
6102.5	18	8.9
6103.5	19	8.8
6107.5	23	8.9
6108.5	24	9.1
6111.5	27	8.7
6112.5	28	8.6
6114.5	30	8.6
6115.5	31	8.8

7261 *R Delphini.*

(Continued from 498.)

Julian	Calendar	Mag.
	1901	
5585	July 19	9.7
5601	Aug. 7	11.2
	1902	
6085	Dec. 1	8.9
6089	5	8.6
6092	8	8.8
6101	17	8.4
6102	18	8.7
6107	23	8.8
6111	27	7.9
6115	31	8.1
	1903	
6121	Jan. 6	8.7

(S.DM)
(7416) −19°5892.

Julian	Calendar	Mag.
	1896	
3777	Aug. 5	8.9
3809	Sept. 6	9.0
3860	Oct. 27	8.7
3897	Dec. 3	8.7
	1897	
4111	July 5	8.8
4114	8	8.9
4144	Aug. 7	8.63_2
4166	29	9.08_2
4215	Oct. 17	8.97_2
4256	Nov. 27	8.82_2
	1898	
4587	Oct. 24	8.92

7458 *V Delphini.*

(Cont. from 482, Comp. Stars 311)

Julian	Calendar	Mag.
	1902	
6085	Dec. 1	10.7]
6102	18	10.7]
6107	23	10.0 :
6111	27	11.5

(7184) — *Cygni.*

Julian	Calendar	Mag.
	1896	
3776	Aug. 4 to	
	1897	
3918	Jan. 23	12.5
		±.3
	10 dates	
	1897	
4078	June 2 to	
4283	Dec. 24	12.5
		±.3
	5 dates	

(7184) — *Cygni.* Cont.

Julian	Calendar	Mag.
	1898	
4579	Oct. 16 to	
4639	Dec. 15	12.5
		±.3
	3 dates	

7896 *V Pegasi.*

(Continued from 482.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	9.8
6101	17	9.8]
6111	27	11.3]

(8104) — *Aquarii.*

(Continued from 372.)

Julian	Calendar	Mag.
	1896	
3813	Sept. 10	12.2
3870	Nov. 6	12.5
	1897	
4138	Aug. 1	12.2
4257	Nov. 27	12.3
	1898	
4545	Sept. 12	12.0

(8106) — *Pegasi.*

(Continued from 372.)

Same dates as (8104)
No perceptible change

8230 *S Aquarii.*

(Continued from 487.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	9.2
6111	27	10.5
	1903	
6123	Jan. 8	10.7]

8290 *R Pegasi.*

(Continued from 487.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	10.1
6103	19	10.4
6107	23	9.9
6114	30	10.6

8373 *S Pegasi.*

(Cont. from 487, Comp. Stars 464)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	11.5
6103	19	9.9
6107	23	9.8
6114	30	9.7
	1903	
6123	Jan. 8	9.5
6128	13	9.3

8373 *S Pegasi.* Cont.

Julian	Calendar	Mag.
	1903	
6132	Jan. 17	9.3
6136	21	9.3
6143	28	8.9
6145	30	8.7
6151	Feb. 5	8.5
6158	12	7.93_2
6160	14	7.76_2
6168	22	8.4

8512 *R Aquarii.*

(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	7.9
6092	8	7.6
6101	17	7.4
6102	18	8.1
6107	23	7.7
6114	30	6.8
	1903	
6123	Jan. 8	6.2
6127	12	6.0
6128	13	5.8
6132	17	6.78
6136	21	6.76_2
6141	26	6.8

8562 *Z Aquarii.*

Julian	Calendar	Mag.
	1899	
4984.6	Nov. 25	8.1 P
	1902	
6101	Dec. 17	8.5
6102	18	8.8
6107	23	8.4
6114	30	9.0
	1903	
6123	Jan. 8	9.7
6127	12	9.8
6128	13	9.6
6136	21	10]

8622 *W Ceti.*

(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	8.3
6101	17	9.0
6102	18	7.8
6107	23	7.8
6114	30	7.8
	1903	
6123	Jan. 8	7.8
6128	13	7.4
6132	17	7.91
6136	21	8.49
6141	26	8.7

CONTENTS.

Published at 16 Craigie St., Cambridge (Boston Postal District), Mass., semi-monthly, by S. C. Chandler. Address, Cambridge, Mass.
Associate Editors, Asaph Hall and Lewis Boss. Price, $5.00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. *Closed April 6.*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 536.

VOL. XXIII. BOSTON, 1903 APRIL 23. NO. 8

DETERMINATION OF ABSOLUTE MAGNITUDE-EQUATION FOR THE CATALOGUE OF 627 STANDARD STARS (*A. J.* 531–2).

By LEWIS BOSS.

In the computation for the right-ascensions of the Catalogue of 627 Standard Stars the numerical data for each star occupy a single sheet. Upon each is entered at the outset, in the chronological order of catalogues, the comparison, O–C, of each catalogue of observation with a previously assumed right-ascension and annual motion. This is the permanent basis of the subsequent computations. In succeeding columns are arranged the residuals corrected for systematic errors of the form, $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$, in the successive approximations. The last column on each star-sheet contains the final residuals, C–O, resulting from the definitive computations for the Catalogue of Standard Stars. These residuals are freed from the effect of $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$ according to the values of these quantities adopted in the final computation, — values which are, in all cases, close approximations to those which have been finally computed subsequently to the formation of the definitive right-ascensions. It is evident, therefore, that these residuals are well suited for the accurate discussion of any type of systematic correction which it is desired to investigate independently of $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$.

Throughout the computations for the catalogue it had seemed to me altogether probable that the subject of magnitude-equation is not yet amenable to precise formulation and definite numerical treatment. But when the operations for deducing the finally adopted values of $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$ had been completed, it occurred to me that certain points of interest relative to the magnitude-effect ought not to be neglected. Abundant well tested material for such discussions was already available in a perfectly convenient form; and no relatively great loss of labor would result in case the investigation should prove to be barren of definite conclusions.

Once entered upon, the course of the investigation followed almost without volition on the part of the computer, and without reference to his preconceived notions. Therefore it is quite natural that the results should be presented in the order in which they were obtained. This course is especially appropriate since each conclusion stands as it was first produced and without subsequent revision.

Description of Table I.

The entire investigation derives its material of observation from Table I; therefore, for a clear comprehension of what follows, it will be necessary to understand the construction of this table. All the "star-sheets" were arranged in the order of magnitude in three zones, as follows:

Zone	Limits
I	+60 to +30
II	+29 to −22
III	−23 to −50

The adopted magnitudes are those of the Harvard Photometry.

Then in each zone the separate residuals, C–O, for each catalogue are collected in groups falling as nearly as may be upon an even magnitude. Zone II (the equatorial zone) is readily recognized in the tables through the group of eight stars of mean, $0^m.9$. It is in the second column for observatories having northern latitude and in the first column for those having southern latitude. Each of the mean residuals is a statement of the mean observed correction given by the Standard Catalogue to the catalogue of observation in question, as a result of the mean by weights of all the residuals contained in the given zone for the particular group of magnitudes to which it refers.

Since very closely approximate values of $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$ were employed in the derivation of the final residuals upon the star-sheets, and since the means of Table I involve all hours of right-ascension and a wide range of declination, it is evident that any outstanding errors which could be attributed to defective knowledge of $\Delta\alpha_\alpha$ and $\Delta\alpha_\delta$ are practically of no importance at all, except possibly in a few instances where the number of stars involved in the mean is very small. We may, therefore, assume that Table I exhibits the effect of differences of magnitude only, combined with casual errors of observation. For each zone

is given in the first column the number of stars participating in the respective means, and in the second column the combined weight upon a unit of which the probable error is ±0.01. Where the probable error is supposed to be greater than ±0.015 the weight is suppressed.

To prepare the mean residuals for examination it was decided to take as the zero of reference, 3ᴹ.5, and to establish this through the arithmetical mean of the groups for 3ᴹ.0 and 4ᴹ.0. This procedure is illustrated by the following example in which the residuals for Zone 11 in the case of Cordoba 1875 are treated.

Mag.	**	Wt.	$\Delta'\alpha_m$	$\Delta\alpha_m$
M			s	s
0.9	8	7	+0.006	+0.008
2.0	13	9	+0.003	+0.005
3.0	44	30	−0.001	+0.001
4.0	66	30	−0.003	−0.001
5.0	50	17	−0.004	−0.002

The numbers in the column, $\Delta'\alpha_m$, are the mean residuals as they were directly computed from the separate residuals collected from the star-sheets. The mean for the groups, 3ᴹ.0 and 4ᴹ.0, is −0ˢ.002. Adding to the numbers in column, $\Delta'\alpha_m$, +0ˢ.002, so as to make the sum of the residuals for 3ᴹ.0 and 4ᴹ.0 zero, we derive the adopted residuals in the last column, $\Delta\alpha_m$, which are thus supposed to be referred to 3ᴹ.5 as the zero-origin. In precisely this manner, without exception, all of the residuals for each of the zones were referred to the zero-origin, 3ᴹ.5; and where this was not possible for a certain zone on account of the scanty weight for one or the other of the residuals for 3ᴹ.0 or 4ᴹ.0, that zone has been omitted from the computations.

All the work up to this point was done in a purely mechanical way upon a uniformly prescribed plan, carried out with undeviating rigor, regardless of what might seem to be discordances or anomalies.

The last column in each of the sub-tables which make up Table I is the mean by weights of the individual zones; and the weight belonging to each residual is the sum of the corresponding weights in the separate zones. It is scarcely necessary to remark that much the larger part of the weight in the final residuals is due to the equatorial zone.

TABLE I.

Final Results for Magnitude-Equations, with Observed Values of $\Delta\alpha_m$.

Greenwich 1755 (A.–B.)				$\Delta'\alpha_m = -^s.004$				$\Delta\alpha_m = -^s.011$	
	**	p	$\Delta\alpha_m$		**	p	$\Delta\alpha_m$		$\Delta\alpha$
M			s	M			s	M	s
				0.9	8	2	−.001	0.9	−.001
1.8	11	1	+.037	2.0	14	2	+.002	1.9	+.014
3.1	26	1	.000	3.0	51	5	−.002	3.0	−.002
3.9	38	1	.000	4.0	121	10	+.002	4.0	+.002
4.8	5		−.070	5.0	87	6	−.009	5.0	−.011

Konigsberg 1825				$\Delta'\alpha_m = -^s.004$				$\Delta\alpha_m = -^s.012$	
				0.9	8	5	+.004	0.9	+.001
1.6	9	3	+.027	2.1	7	4	+.029	1.9	+.028
3.1	7	2	−.005	2.9	8	5	+.005	2.9	+.001
3.9	6	1	+.005	4.3	3	1	−.007	4.1	.000
4.7	3	1	+.001					4.7	+.001

Dorpat 1824				$\Delta'\alpha_m = -^s.004$				$\Delta\alpha_m = -^s.011$	
				0.9	8	7	+.012	0.9	+.012
1.9	15	2	−.019	2.1	8	7	+.001	2.0	−.002
3.0	17	1	−.010	2.9	16	8	+.001	2.9	.000
3.9	23	1	+.010	3.9	25	5	−.001	3.9	+.001
4.7	4	–	+.004	5.0	10	1	−.015	5.0	−.012

St. Helena 1830				$\Delta'\alpha_m = ^s.000$				$\Delta\alpha_m = -^s.008$	
0.9	8	2	−.008					0.9	−.008
2.1	10	2	.000	2.1	18	1	−.006	2.1	−.002
3.0	32	5	.000	3.0	39	1	+.001	3.0	.000
4.0	47	5	.000	4.0	50	1	−.001	4.0	.000
5.0	35	3	−.005	4.9	30	1	−.021	5.0	−.008

Abo. 1830				$\Delta'\alpha_m = -^s.004$				$\Delta\alpha_m = -^s.012$	
				0.9	8	9	+.010	0.9	+.010
1.9	12	7	+.002	2.1	8	9	+.004	2.0	+.003
3.0	13	6	+.006	3.0	22	13	+.004	3.0	+.005
4.0	16	6	−.006	4.0	44	14	−.004	4.0	−.005
4.9	4	1	−.006	5.1	26	7	−.006	5.1	−.006

Greenwich 1830				$\Delta'\alpha_m = -^s.003$				$\Delta\alpha_m = -^s.010$	
				0.9	8	2	+.002	0.9	+.002
1.9	15	1	−.002	2.0	14	2	−.004	2.0	−.004
3.1	26	1	−.005	3.0	51	5	+.005	3.0	+.003
3.9	12	1	+.005	4.0	121	8	−.005	4.0	−.004
4.8	4	-	+.024	5.0	73	5	−.011	5.0	−.010

Cambridge 1830				$\Delta'\alpha_m = +^s.002$				$\Delta\alpha_m = -^s.006$	
	**	p	$\Delta\alpha_m$		**	p	$\Delta\alpha_m$		$\Delta\alpha_m$
M			s	M			s	M	s
				0.9	8	2	−.005	0.9	−.005
				2.0	13	2	−.020	2.0	−.020
				3.0	28	3	+.003	3.0	+.003
				4.0	56	5	−.003	4.0	−.003
				5.1	62	5	−.001	5.1	−.001

Cape 1840				$\Delta'\alpha_m = ^s.000$				$\Delta\alpha_m = -^s.008$	
0.9	8	4	−.004					0.9	−.004
2.1	9	4	.000	2.1	17	2	+.007	2.1	+.002
2.9	31	10	+.001	3.0	40	1	+.005	2.9	+.002
4.0	48	10	−.001	4.0	48	4	−.005	4.0	−.002
4.9	10	2	−.021	4.9	28	2	+.001	4.9	−.010
5.8	8	1	+.023					5.8	+.023

Greenwich 1840				$\Delta'\alpha_m = +^s.009$				$\Delta\alpha_m = +^s.001$	
				0.9	8	3	−.027	0.9	−.027
				2.1	11	4	−.006	2.1	−.006
				2.9	35	10	−.005	2.9	−.005
				4.0	59	10	+.005	4.0	+.005
				4.9	55	6	+.014	4.9	+.014
				5.9	6	1	+.009	5.9	+.009

Greenwich 1845				$\Delta'\alpha_m = +^s.007$				$\Delta\alpha_m = -^s.001$	
				0.9	8	8	−.015	0.9	−.015
1.9	11	2	−.016	2.0	12	10	−.007	2.0	−.008
3.1	17	2	+.020	2.9	38	25	−.003	2.9	−.002
3.9	31	1	−.020	4.0	37	25	+.003	4.0	+.002
4.7	2	–	+.069	4.9	61	15	+.013	4.9	+.016
				5.9	8	2	+.022	5.9	+.022

Radcliffe 1845				$\Delta'\alpha_m = -^s.008$				$\Delta\alpha = -^s.016$	
				0.9	8	2	.000	0.9	.000
1.9	14	1	+.022	2.1	10	2	+.029	2.0	+.026
3.1	16	1	−.001	2.9	36	6	+.007	2.9	+.006
3.9	33	2	+.001	4.0	69	8	−.007	4.0	−.006
4.7	4	-	−.017	5.0	53	5	−.017	5.0	−.017

Pulkowa 1845				$\Delta'\alpha_m = +^s.002$				$\Delta\alpha_m = -^s.005$	
				0.9	8	6	−.003	0.9	−.003
1.9	15	11	−.004	2.0	14	16	−.001	2.0	−.002
3.1	27	17	−.006	3.0	51	50	−.001	3.0	−.002
3.9	41	25	+.006	4.0	112	96	+.002	4.0	+.003
4.8	3	1	+.004	5.0	21	8	−.001	5.0	.000

Paris 1845 $\Delta'\alpha_m = +^s.003$ $\Delta\alpha_m = -^s.005$

M	**	p	Δα (s)	M	**	p	Δα (s)	M	Δα (s)
				0.9	8	5	−.006	0.9	−.006
1.9	15	8	−.007	2.0	11	15	−.011	2.0	−.010
3.1	24	14	+.007	3.0	18	43	−.001	3.0	−.001
3.9	35	6	−.007	4.0	119	56	+.004	4.0	+.003
4.8	5	1	−.006	4.9	72	28	+.002	4.9	+.002
				6.1	12	3	−.006	6.1	−.006

Santiago 1850 $\Delta'\alpha_m = -^s.004$ $\Delta\alpha_m = -^s.012$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	3	+.008	0.9	+.008
				2.1	10	3	+.002	2.1	+.002
				2.9	29	9	+.006	2.9	+.006
				4.0	39	6	−.006	4.0	−.006
				5.0	25	2	−.008	5.0	−.008

Greenwich 1850 $\Delta'\alpha_m = +^s.003$ $\Delta\alpha_m = -^s.005$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	7	−.003	0.9	−.003
1.9	13	3	−.001	2.0	14	9	−.010	2.0	−.009
3.0	23	3	+.004	3.0	50	27	−.003	3.0	−.002
3.9	27	3	−.001	4.0	107	34	+.003	4.0	+.002
				4.9	61	14	+.009	4.9	+.009
				6.0	14	1	+.001	6.0	+.001

Cape 1860 $\Delta'\alpha_m = -^s.003$ $\Delta\alpha_m = -^s.011$

M	**	p	Δα	M	**	p	Δα	M	Δα
0.9	8	4	+.016					0.9	+.016
2.1	10	3	+.003	2.1	16	3	+.003	2.1	+.003
2.9	28	9	+.003	3.0	37	1	−.006	2.9	.000
4.0	46	12	−.003	4.0	46	2	+.006	4.0	−.002
5.1	52	10	.000	4.8	24	2	−.011	5.1	−.002

Washington 1860 $\Delta'\alpha_m = +^s.002$ $\Delta\alpha_m = -^s.006$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	6	.000	0.9	.000
1.8	14	3	−.024	2.0	13	8	+.003	2.0	−.004
3.0	24	3	+.019	3.0	18	27	+.002	3.0	+.004
3.9	31	3	−.019	4.0	112	42	.002	4.0	−.003
4.8	1	–	−.048	4.9	70	28	+.007	4.9	+.006
				6.0	14	5	+.009	6.0	+.009

Greenwich 1860 $\Delta'\alpha_m = +^s.001$ $\Delta\alpha_m = -^s.007$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	9	−.001	0.9	−.001
1.9	11	5	+.014	2.0	14	11	+.001	2.0	+.005
3.0	13	1	+.002	3.0	13	10	.000	3.0	.000
3.9	22	5	−.002	4.0	98	63	.000	4.0	.000
4.7	5	–	+.027	4.9	67	34	+.005	4.9	+.005
				6.0	14	8	+.001	6.0	+.001

Radcliffe 1860 $\Delta'\alpha_m = -^s.005$ $\Delta\alpha_m = ^s.013$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	2	.002	0.9	.002
1.9	15	1	+.005	2.0	14	2	+.002	2.0	+.003
3.0	20	1	.011	3.0	17	9	+.005	3.0	+.004
3.9	24	1	+.011	4.0	101	15	.005	4.0	.004
4.8	5		.000	4.9	61	8	.018	4.9	.017
				6.0	12	2	.013	6.0	.013

Melbourne 1860 $\Delta'\alpha_m = -^s.002$ $\Delta\alpha_m = ^s.009$

M	**	p	Δα	M	**	p	Δα	M	Δα
0.9	8	5	−.004					0.9	.004
2.1	10	6	+.002	2.2	10	2	.007	2.1	.000
3.0	33	17	+.003	3.0	19	1	+.002	3.0	+.003
4.0	42	15	.003	4.0	19	2	.002	4.0	.003
5.0	29	9	.007	4.8	8	1	−.007	5.0	.007

Paris 1860 $\Delta'\alpha_m = +^s.004$ $\Delta\alpha_m = ^s.003$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	10	.008	0.9	.008
1.9	15	7	.006	2.0	13	15	.010	2.0	.009
3.1	22	7	.002	3.0	50	45	.002	3.0	.002
3.9	25	6	+.002	4.0	106	70	+.002	4.0	+.002
4.9	4		+.015	4.9	69	29	+.006	4.9	+.006
				6.0	14	5	+.013	6.0	+.013

Greenwich 1864 $\Delta'\alpha_m = ^s.001$ $\Delta\alpha_m = ^s.009$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	14	+.001	0.9	+.001
1.9	11	5	+.002	2.0	13	16	+.003	2.0	+.003
3.1	17	4	+.002	3.0	45	50	.000	3.0	.000
3.9	19	5	.002	4.0	94	75	.000	4.0	.000
4.7	3		−.012	4.9	62	41	−.003	4.9	.003
				6.0	12	7	.000	6.0	.000

Cape 1865 $\Delta'\alpha_m = -^s.006$ $\Delta\alpha_m = -^s.014$

M	**	p	Δα	M	**	p	Δα	M	Δα
0.9	8	4	+.011					0.9	+.011
2.0	13	5	+.010	2.1	13	2	+.011	2.0	+.010
3.0	40	15	+.004	3.0	31	3	+.016	3.0	+.006
4.0	71	25	−.004	4.9	21	1	−.016	4.0	.005
4.9	60	18	−.008	4.9	15	1	+.006	4.9	−.007
6.1	11	3	−.016					6.1	.016

Brussels 1865 $\Delta'\alpha_m = +^s.004$ $\Delta\alpha_m = -^s.004$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	5	−.004	0.9	.004
1.9	14	3	+.006	2.0	13	6	+.003	2.0	+.004
3.1	25	1	−.003	3.0	50	22	−.001	3.0	−.001
3.9	38	1	+.003	4.0	119	34	+.001	4.0	+.004
4.8	6	1	+.030	4.9	70	20	+.012	4.9	+.013
				5.9	13	1	+.010	5.9	+.010

Harvard 1865 $\Delta'\alpha_m = -^s.005$ $\Delta\alpha_m = ^s.013$

M	**	p	Δα	M	**	p	Δα	M	Δα
1.9	9	2	+.024					1.9	+.024
3.0	17	3	−.002	2.9	37	14	.000	2.9	.000
3.8	17	3	+.002	4.0	68	25	.000	4.0	.000
4.8	4	–	+.013	5.0	27	10	−.003	5.0	.002

Pulkowa 1865 $\Delta'\alpha_m = ^s.002$ $\Delta\alpha_m = ^s.010$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	18	+.004	0.9	+.004
1.9	15	18	.000	2.0	14	25	+.009	2.0	+.004
3.1	27	27	.000	3.0	46	71	.000	3.0	.000
3.9	11	12	.000	4.0	105	150	.000	4.0	.000
4.8	3	2	−.010	4.8	9	10	−.008	4.8	.008

Melbourne 1870 $\Delta'\alpha_m = ^s.000$ $\Delta\alpha_m = -^s.007$

M	**	p	Δα	M	**	p	Δα	M	Δα
0.9	8	10	−.008					0.9	.008
2.1	10	12	−.004	2.1	15	4	−.001	2.1	.003
2.9	29	34	−.002	3.0	36	6	+.002	2.9	.002
3.9	28	34	+.002	4.0	34	3	.002	3.9	+.002
4.9	16	15	.006	4.8	21	3	.007	4.9	.006
6.0	1	1	−.007					6.0	.007

Greenwich 1872 $\Delta'\alpha_m = -^s.002$ $\Delta\alpha_m = ^s.009$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	16	+.006	0.9	+.006
1.9	15	11	+.009	2.0	14	20	+.007	2.0	+.007
3.1	26	11	+.010	3.0	49	62	+.001	3.0	+.003
3.9	37	12	.010	4.0	108	98	.001	4.0	.002
4.8	4		.043	4.9	67	51	+.005	4.9	+.004
				6.0	12	9	.006	6.0	.006

Washington 1875 $\Delta'\alpha_m = ^s.001$ $\Delta\alpha_m = ^s.008$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	14	+.003	0.9	+.003
1.9	15	8	.003	2.0	14	19	+.002	2.0	.000
3.1	24	10	+.008	3.0	50	60	+.001	3.0	+.002
3.9	38	14	.008	4.0	108	72	.001	4.0	.002
4.7	4		.019	4.9	55	36	.000	4.9	.000
				6.0	10	6	+.001	6.0	+.001

Pulkowa 1875 $\Delta'\alpha_m = +^s.003$ $\Delta\alpha_m = ^s.005$

M	**	p	Δα	M	**	p	Δα	M	Δα
				0.9	8	7	.005	0.9	.005
1.9	15	6	.015	2.0	14	8	.002	2.0	.008
3.1	27	8	.006	3.0	45	27	.000	3.0	.004
3.9	12	12	+.006	4.0	111	57	.000	4.0	+.001
4.8	6	1	+.035	4.9	36	14	.002	4.9	+.002
				6.1	8	2	+.007	6.1	+.007

Harvard 1875 $\Delta'\alpha_m = +^s.001$ $\Delta\alpha_m = {}^s.007$

M	**	p	$\Delta\alpha_s$	M	**	p	$\Delta\alpha_m$	M	$\Delta\alpha_m$
				0.9	8	14	.000	0.9	.000
1.9	15	11	+.005	2.1	14	14	.002	2.1	+.001
3.1	27	14	−.006	3.0	45	14	+.001	3.0	.001
3.9	40	22	+.006	4.0	98	83	−.001	4.0	+.001
4.7	3	2	+.005	4.9	46	33	.000	4.9	+.005
				6.1	3	2	−.012	6.1	−.012

Cordoba 1875 $\Delta'\alpha_m = -^s.002$ $\Delta\alpha_m = -^s.010$

0.9	8	7	+.008					0.9	+.008
2.0	13	9	+.005	2.1	18	3	+.005	2.0	+.005
3.0	11	30	+.001	3.0	12	5	+.002	3.0	+.001
4.0	66	30	−.001	4.0	53	1	−.002	4.0	−.001
5.0	50	17	−.002	4.8	31	2	+.011	5.0	−.001

Paris 1875 $\Delta'\alpha_m = +^s.003$ $\Delta\alpha_m = -^s.005$

				0.9	8	7	−.010	0.9	−.010
1.9	14	3	−.019	2.0	14	7	−.002	2.0	−.007
3.0	21	5	−.005	3.0	47	37	.000	3.0	−.001
3.9	24	6	+.005	4.0	96	51	.000	4.0	+.001
5.1	2	1	−.006	4.9	67	30	+.006	4.9	+.006
				6.0	13	7	.000	6.0	.000

Cape 1880 $\Delta'\alpha_m = -^s.003$ $\Delta\alpha_m = -^s.011$

0.9	8	6	+.013					0.9	+.013
2.0	14	7	+.005	2.1	18	2	+.016	2.0	+.008
3.0	13	21	.000	3.0	41	1	−.004	3.0	−.001
4.0	86	25	.000	4.0	53	3	+.004	4.0	.000
4.9	54	15	−.008	4.8	29	2	+.022	4.9	−.004
6.1	8	3	−.006					6.1	−.006

Melbourne 1880 $\Delta'\alpha_m = +^s.003$ $\Delta\alpha_m = -^s.005$

0.9	8	5	−.008					0.9	−.008
2.1	12	7	+.001	1.9	9	1	−.009	2.1	−.001
3.0	45	22	−.003	2.9	12	2	−.011	3.0	−.004
4.0	34	29	+.002	4.1	13	–	+.011	4.0	+.002
4.9	16	14	+.005	4.9	11	1	+.004	4.9	+.005
5.9	6	3	+.007					5.9	+.007

Greenwich 1880 $\Delta'\alpha_m = +^s.001$ $\Delta\alpha_m = -^s.007$

				0.9	8	14	−.005	0.9	−.005
1.9	15	7	+.006	2.1	13	17	−.006	2.0	−.003
3.1	24	6	−.005	3.0	50	54	−.001	3.0	−.001
3.9	38	9	+.005	4.0	115	88	+.001	4.0	+.001
4.8	5	1	−.010	4.9	61	48	.000	4.9	−.001
				6.0	14	8	.000	6.0	.000

Pulkowa 1885 $\Delta'\alpha_m = {}^s.000$ $\Delta\alpha_m = -^s.008$

				0.9	8	18	−.002	0.9	−.002
1.9	15	18	+.002	2.0	14	25	+.001	2.0	+.001
3.1	27	26	−.003	3.0	45	72	+.001	3.0	.000
3.9	40	11	+.003	4.0	99	151	−.001	4.0	.000
4.7	2	2	−.015	4.7	6	10	.000	4.7	−.002

Cape 1885 $\Delta'\alpha_m = {}^s.000$ $\Delta\alpha_m = -^s.008$

0.9	8	6	+.010					0.9	+.010
2.1	13	8	−.001	2.1	18	4	−.007	2.1	−.003
3.0	50	35	.000	3.0	40	6	−.001	3.0	.000
4.0	114	76	.000	4.0	34	6	+.001	4.0	.000
4.9	59	30	+.002	4.9	14	3	+.017	4.9	+.003
5.9	6	3	−.007					5.9	−.007

Strassburg 1885 $\Delta'\alpha_m = +^s.001$ $\Delta\alpha_m = -^s.006$

				0.9	8	14	−.004	0.9	−.004
1.9	15	10	−.003	2.0	13	16	−.007	2.0	−.005
3.1	19	8	+.002	3.0	45	47	−.002	3.0	−.001
3.9	19	8	−.002	4.0	85	70	+.002	4.0	+.001
4.7	2	1	+.015	4.9	45	34	+.002	4.9	+.002
				5.7	7	4	−.016	5.7	−.016

Radcliffe 1890 $\Delta'\alpha_m = {}^s.000$ $\Delta\alpha_m = {}^s.008$

M	**	p	$\Delta\alpha_m$	M	**	p	$\Delta\alpha$	M	$\Delta\alpha_m$
				0.9	8	6	+.006	0.9	+.006
1.9	10	4	−.012	2.1	13	9	+.006	2.1	+.004
3.1	19	2	+.013	3.0	17	29	−.001	3.0	.000
3.9	23	2	−.013	4.0	107	17	+.001	4.0	.000
4.9	3	–	+.019	4.9	68	26	+.003	4.9	+.003
				6.0	13	5	+.008	6.0	+.008

Cape 1890 $\Delta'\alpha_m = -^s.005$ $\Delta\alpha_m = -^s.013$

0.9	8	5	+.013					0.9	+.013
2.0	14	9	+.007	2.1	18	4	−.005	2.0	+.003
3.0	51	33	+.003	3.0	42	8	+.001	3.0	+.003
4.0	120	70	−.003	4.0	52	10	−.001	4.0	−.003
4.9	59	26	−.009	4.8	24	4	+.001	4.9	−.008
6.2	6	3	−.017					6.2	−.017

Greenwich 1890 $\Delta'\alpha_m = +^s.002$ $\Delta\alpha_m = -^s.006$

				0.9	8	7	−.014	0.9	−.014
1.9	15	11	.000	2.0	14	21	−.003	2.0	−.002
3.1	27	16	+.002	3.0	51	73	−.002	3.0	−.001
3.9	11	18	−.002	4.0	125	128	+.002	4.0	+.001
4.8	5	1	−.022	4.9	73	70	.000	4.9	.000
				6.0	11	12	+.003	6.0	+.003

Madison 1890 $\Delta'\alpha_m = -^s.005$ $\Delta\alpha_m = -^s.013$

				0.9	8	9	+.014	0.9	+.014
1.9	14	8	+.012	2.1	10	11	−.005	2.1	+.002
3.0	25	13	+.006	3.0	37	17	+.003	3.0	+.004
3.9	39	21	−.006	4.0	80	96	−.003	4.0	−.004
4.6	3	2	+.006	4.7	18	23	−.008	4.7	−.007
				6.0	2	2	−.015	6.0	−.015

Berlin 1890 $\Delta'\alpha_m = +^s.004$ $\Delta\alpha_m = -^s.003$

				0.6	5	4	−.008	0.6	−.008
2.0	13	10	−.010	2.1	10	15	−.003	2.1	−.006
3.1	25	18	−.002	3.0	35	51	−.003	3.0	−.003
3.9	42	26	+.002	4.0	83	111	+.003	4.0	+.003
4.7	3	2	+.001	4.8	14	21	+.005	4.8	+.005
				6.1	3	5	+.012	6.1	+.012

Lisbon 1890 $\Delta'\alpha_m = -^s.001$ $\Delta\alpha_m = -^s.008$

				0.9	8	16	−.004	0.9	−.004
1.7	11	10	+.004	2.0	13	27	+.002	2.0	+.003
3.1	20	19	−.003	3.0	48	100	+.002	3.0	+.001
3.9	33	29	+.003	3.9	96	168	−.002	3.9	−.001
4.7	2	1	+.019	4.8	17	36	−.001	4.8	.000
				6.0	3	5	−.009	6.0	−.009

Berlin 1895 $\Delta'\alpha_m = {}^s.000$ $\Delta\alpha_m = -^s.008$

				0.9	7	8	+.010	0.9	+.010
1.9	14	10	−.001	2.0	10	16	+.004	2.0	+.002
3.0	21	13	+.003	3.0	41	56	.000	3.0	+.001
3.9	40	27	−.003	3.9	68	98	.000	3.9	−.001
4.7	3	2	−.013	5.0	16	24	+.006	5.0	+.005
				6.0	2	4	+.016	6.0	+.016

Mt. Hamilton 1895 $\Delta'\alpha_m = -^s.003$ $\Delta\alpha_m = -^s.011$

1.6	3	1	+.017	1.6	6	6	+.003	1.6	+.006
3.0	4	2	−.011	3.0	21	11	+.002	3.0	+.002
3.9	1	2	+.011	4.0	14	14	−.002	4.0	−.002
5.1	2	1	+.014	4.8	32	32	−.005	4.8	−.004
				6.0	4	4	−.003	6.0	−.003

Albany 1898 $\Delta'\alpha_m = -^s.004$ $\Delta\alpha_m = -^s.012$

				0.9	8	10	+.004	0.9	+.004
1.7	9	4	+.019	2.1	10	11	+.007	2.1	+.010
3.0	6	1	+.008	3.0	27	26	+.003	3.0	+.003
3.9	2	1	−.008	3.9	28	24	−.003	3.9	−.003
				4.9	11	7	−.008	4.9	−.008

Comparison of Eye-and-Ear with Chronographic Transits.

The residuals, $\Delta\alpha_m$, of Table I, so far as they have any meaning, evidently depend upon the difference between the magnitude-equation of each individual catalogue and that of the Standard Catalogue. Therefore, for nearly contemporaneous catalogues at least, we may eliminate the magnitude-equation of the Standard Catalogue in finding the difference of equations between two or more catalogues of observation. This was the first use of Table I that occurred to me.

After 1850 there are still many series of right-ascension determinations which depend upon eye-and-ear transits, which may be compared with the nearly contemporaneous results from chronographic registry. Accordingly I divided the observations into two series: Series A, 1850–1870; Series B, 1875–1890. Washington 1860 and Pulkowa 1865 contain a few observations by the eye-and-ear method, but not in sufficient number to impeach their character as representatives of the method of chronographic registry. The residuals of the equatorial zone (Zone II) alone are used in this differential comparison, in order to avoid any possible suspicion that the final differences may be due to any other form of systematic error than that due to magnitude-equation.

Relative Magnitude Effect: Eye-and-Ear Transits.

Series A.

		0M.9	2M.0	3M.0	4M.0	5M.0	6M.0
		s	s	s	s	s	s
Stgo.	50	+.008	+.002	+.006	—.006	—.008	-
Greenw.	50	.003	—.010	—.003	+.003	+.009	+.001
Cape	60	+.016	+.003	+.003	.003	.000	-
Radel.	60	—.002	+.002	+.005	—.005	—.018	—.013
Paris	60	.008	—.010	—.002	+.002	+.006	+.013
Bruss.	65	—.004	+.003	—.001	+.001	+.012	+.010
Means		+.0012	—.0017	+.0013	—.0013	+.0002	+.0028

Series B.

		0M.9	2M.0	3M.0	4M.0	5M.0	6M.0
Pulk.	75	.005	—.002	.000	.000	.002	+.007
Paris	75	—.010	—.002	.000	.000	+.006	.000
Cape	80	+.013	+.005	.000	.000	.008	—.006
Cape	85	+.010	.001	.000	.000	+.002	.007
Radel.	90	+.006	+.006	—.001	+.001	+.003	+.008
Means		+.0028	+.0012	—.0002	+.0002	+.0002	+.0004

Relative Magnitude Effect: Chronographic Transits.

Series A.

		0M.9	2M.0	3M.0	4M.0	5M.0	6M.0
		s	s	s	s	s	s
Wash.	60	.000	+.003	+.002	.002	+.007	+.009
Greenw.	60	.001	+.001	.000	.000	+.005	+.001
Melb.	60	.004	+.002	+.003	.003	.007	
Greenw.	64	+.001	+.003	.000	.000	.003	.000
Cape	65	+.011	+.010	+.004	.004	—.008	.016
Pulk.	65	+.004	+.009	.000	.000	.008	
Melb.	70	—.008	.004	—.002	+.002	.006	.007
Means		+.0004	+.0031	+.0010	—.0010	.0029	.0026

Series B.

		0M.9	2M.0	3M.0	4M.0	5M.0	6M.0
Wash.	75	+.003	+.002	+.001	—.001	.000	+.001
Harv.	75	.000	.002	+.001	.001	.000	.012
Cord.	75	+.008	+.005	+.001	.001	.002	
Melb.	80	—.008	+.004	.003	+.002	+.005	+.007
Greenw.	80	—.005	.006	.001	+.001	.000	.000
Pulk.	85	—.002	+.001	+.001	.001	.000	
Strassb.	85	—.004	.007	.002	+.002	+.002	.016
Madr.	90	+.014	—.005	+.003	.003	.008	.015
Berlin	90	—.008	.003	—.003	+.003	+.005	+.012
Lisb.	90	—.004	+.002	+.002	.002	.002	
Means		.0006	.0012	.0000	.0001	.0000	.0013

Evidently we may compare the mean [illegible] eye-and-ear observations with those from chronographic [illegible] in the same series without fear that the [illegible] magnitude-equation, or of other forms of systematic [illegible] rection, for the standard catalogue will be of [illegible] importance whatever in the short interval between the mean epochs of the groups within each of the series. [illegible] take the magnitude-equation of the eye-and-ear [illegible] as standard we shall find that the chronographic [illegible] tions require the following corrections:

Eye-and-Ear minus Chronographic Transits.

Summary.

Magn.	Series A	Series B.	Mean
0.9	.0008	—.0034	—.0021
2.0	+.0051	—.0024	+.0014
3.0	.0003	+.0002	.0000
4.0	+.0003	.0003	.0000
5.0	.0031	—.0002	—.0016
6.0	—.0054	—.0037	—.0046

From the last column we may conclude that if the chronographic transits had been corrected by —0s.0008 (M — 3.5) in the mean there would have been close agreement in the results from the two methods of observation. This difference is exceedingly small; and even at the ninth magnitude its effect would be less than 0s.005. We seem to be justified in the opinion that, so far as it relates to stars brighter than the sixth magnitude, the mean difference between the magnitude-equations for eye-and-ear and for chronographic transits is wholly insignificant. This conclusion appears to be based upon an amount of testimony which ought to be sufficient to establish it beyond question; yet the history of the question indicates that there are many instances in which the magnitude-equation for eye-and-ear transits is a small, if not vanishing, quantity; and there seems to me to be a greater proportion of such instances in the employment of the eye-and-ear method than in that of chronographic registry. Nevertheless, if the existence of this state of facts were to be admitted, it does not [illegible] conflict with the conclusion that the mean [illegible] equation for chronographic transits is only [illegible] than for those wherein the eye-and-ear method [illegible]

This conclusion seems to justify two [illegible] importance.

1. The magnitude-equation is essentially [illegible] phenomenon; and its true explanation [illegible] of experimental psychology.

2. When many star-catalogues [illegible] determination of the proper motions of the [illegible] practically independent of the magnitude [illegible] there is no appreciable difference [illegible] equations pertaining to the two methods [illegible] mean magnitude-equation of the [illegible] 1850, when all transits were [illegible] method, must be nearly the same as [illegible] to that date.

On the Law of Magnitude-Equation.

Although the effective range of the star-magnitudes involved is not very great (since the weights outside of the limits $2^M.0$ and $5^M.0$ are comparatively small) it seemed that it might be possible to secure some evidence as to the general law of variation of magnitude-equation for diminishing magnitudes. There seems to be no reason why the progression should be strictly proportional to the magnitude; and there does seem to be a very natural reason why the rate of change in the equation should increase with diminishing steps of brightness, if it is associated in any very close way with the relative difficulty of seeing. Accordingly the means were collected in nearly contemporaneous groups for those catalogues which seemed to have equations smaller than that of the standard, for comparison with those which appeared to have equations decidedly larger than that of the standard. If the individual equations are linear, according to star-magnitude, then the differences of the two groups of catalogues should be likewise linear; and if, in the individual equations there are very sensible terms depending on the second or higher powers of ΔM, then such terms should appear in the mean difference between large and small equations. The material is selected from Zone II only.

For Small minus Large Equations.

Small Equations.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
		s	s	s	s	s	s
Paris	60	−.008	−.010	−.002	+.002	+.006	+.013
Pulk.	75	−.005	−.002	.000	.000	−.002	+.007
Paris	75	−.010	−.002	.000	.000	+.006	.000
Melb.	80	−.008	+.001	−.003	+.002	+.005	+.007
Berlin	90	−.008	−.003	−.003	+.003	+.005	+.012
Means		−.0078	−.0032	−.0016	+.0014	+.0040	+.0078

Large Equations.

		$0^M.9$	$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$
Cape	65	+.011	+.010	+.004	−.004	−.008	−.016
Pulk.	65	+.004	+.009	.000	.000	−.008	–
Cord.	75	+.008	+.005	+.001	−.001	−.002	–
Cape	80	+.013	+.005	.000	.000	−.008	−.006
Cape	90	+.013	+.007	+.003	−.003	−.009	−.017
Madn.	90	+.014	−.005	+.003	−.003	−.008	−.015
Alb.	98	+.004	+.007	+.003	−.003	−.008	–
Means		+.0096	+.0054	+.0020	−.0020	−.0073	−.0135

If the line of means for the "Small Equations" be subtracted from the line of means for "Large Equations" we shall have the observed values of $\Delta'\alpha_m$ necessary to reduce the catalogues of the latter category to harmony with those of the former.

Small minus Large Equations; Values of $\Delta'\alpha_m$.

Magn.	p	Obs'd $\Delta'\alpha_m$	Computed I	Computed II
s		s	s	s
0.9	2	+.0174	+.0187	+.0167
2.0	2	+.0086	+.0108	+.0102
3.0	3	+.0036	+.0036	+.0036
4.0	3	−.0034	−.0036	−.0037
5.0	3	−.0113	−.0108	−.0117
6.0	1	−.0213	−.0180	−.0204

If the residual at magnitude, 2.0, for Madison 90 be rejected, the second of the observed values of $\Delta'\alpha_m$ becomes $+0^s.0101$ instead of $+0^s.0086$.

The weights are estimated for use in the solutions. It is evident at once from an inspection of the observed residuals, $\Delta'\alpha_m$, that these numbers are very nearly proportional to the difference of magnitudes. They testify very strongly both for the reality of the phenomenon of magnitude-equation, and for the stability of its effects upon the work of the observers. The numbers in column I are computed from the formula,

$$-0^s.0072\,(M-3.5)\,;$$

and those in column II from the formula,

$$-0^s.0073\,(M-3.5)-0^s.00034\,(M-3.5)^2$$

Were it not for the very high precision of the observed quantities and their freedom from the imputation of systematic error there would be very slight ground for preferring one of these formulas over the other. However this may be, we seem to be justified in the conclusion that, for the brighter stars, the magnitude-equation is very nearly proportional to star-magnitude, — so nearly so that a rigid investigation of the adopted magnitude-scale itself would be necessary before a more definite conclusion can be maintained.

When the equation is extended to fainter stars we possess some further evidence on this point. Some of the best determinations of magnitude-equation with screens indicate very slight, or no, increase in the rate of the effect with diminishing brightness. In other instances, however, notably in the case of A.G. Cambridge (Eng.) (*M.N.* Vol. LX, Turner), and A.G. Leipsic (*A.N.* 3854) the rate of magnitude-equation appears to increase rapidly with diminishing brightness of the star, so that even a higher power of ΔM than the second is needed to indicate the change of rate.

On the whole we may conclude that for stars brighter than the seventh magnitude the rate of the equation in its relation to magnitude is usually very nearly linear, with a tendency in some cases to increase with diminishing brightness of the star.

The evident trustworthiness and precision of the means upon which the foregoing conclusions were based induced me next to examine the question as to the absolute magnitude-equation of the Standard Catalogue. Contrary to the impressions I had formed from following the published papers on magnitude-equation during more than twenty years, — impressions which had nearly deterred me from making this investigation at all, — I found the individual determinations of magnitude-equation consistent in a degree fully as high as could have been anticipated from a hopeful estimate of their value. That such determinations have hitherto been regarded by many as untrustworthy, if not

entirely illusory, is abundantly evident from positive comments to that effect, from inconsistent conclusions reached in various investigations, from the various devices invented to get rid of the magnitude-equation, from the fact that many observers of skill and experience make no attempt to determine this equation for their own observations, and from the further fact that no one, as yet, has ventured to publish a series of right-ascensions actually corrected for magnitude-equation, although an intention to do this in the future has been expressed by more than one observer.

Every catalogue of observation for which the magnitude-equation is known by observation with screens, or otherwise, furnishes a means of determining the absolute magnitude-equation of the Standard Catalogue. Let us consider these in turn.

Pulkowa 1875. ROMBERG did not determine his magnitude-equation; but as his observations are nearly contemporary with those of the Albany and Berlin (BECKER) A.G. zone-observations for which absolute magnitude-equations were determined, it is possible to utilize the comparison with these zone-observations to determine the absolute equation of ROMBERG's right-ascensions. In order to utilize these comparisons to good advantage it is desirable to know the relations, Berl. − Pulk. and Alb. − Pulk., at the magnitude 1.0. Theoretically, since all of these observations are based upon the same standard catalogue, the *Fundamental-Catalog* of Dr. AUWERS, these differences should be zero. Practically, they may differ sensibly from this value; and the determination of this quantity is the weakest point in the comparison. After most careful attention to all the evidence, direct and indirect, I have adopted the following:

At 1ᴹ.0:	Berlin − Pulkowa	=	0ˢ.000
	Albany − Pulkowa	=	+0.003

Combining with these results the comparison, Berlin − Pulk., contained in the Berlin A.G. Catalogue [*Int.*, p. (10)], and of Albany − Pulk. (*A.J.*, XVIII, p. 118) I find the absolute equation of Pulk. 75 to be:

through Berlin,	0ˢ.0052
through Albany,	0.0037
Pulk. 75, adopted,	0.0045, per magnitude.

Cape 1885. The magnitude-equation of this catalogue is given by Sir DAVID GILL as (*Int.*, p. XLI),

$$0^s.011$$

and this is recommended for adoption. Since the observations were made by the eye-and-ear method this determination of magnitude-equation is probably entitled to much smaller weight than would be assignable to one depending on chronographic registry, because of the great difficulty in securing a number of observations sufficiently great to overcome the casual errors of determination.

Berlin 1890. For KÜSTNER's magnitude-equation we have his statement (*A.N.* 112, p. 118) in connection with the remark of Dr. AUWERS (*A.G.* Berlin AUWERS, p. [illegible]) and also the explanation of KÜSTNER in the Publications of the Bonn Observatory (*Heft* 4, pp. 11 and 12), from which I assume as the equation of Berlin 90,

$$-0^s.0040$$

This depends upon the differential places in a heliometric triangulation.

Cape 90. The details of the determination of magnitude-equation for Cape 90 are given in the introduction to the Catalogue (pp. XL and XLII), and

$$-0^s.0139$$

is there adopted and recommended for application to the right-ascensions of the catalogue.

Berlin 1895. BATTERMANN determined his magnitude-equation in several series. Following is a summary of his determinations taken from the introduction to his catalogue.

1894, usual method,	0ˢ.0100 (M − 1)	− 0ˢ.0008 (M − 1)²
1897, usual method,	0.0103	− 0.0005
By alternate transits,	0.0110	− 0.00116
Reduced aperture,	−0.0070 (M − 1)	

The latter value he seems to consider the most trustworthy, but doubts whether it is applicable to transits with full aperture. In view of all the evidence I adopt as BATTERMANN's equation for bright stars,

$$-0^s.009$$

Mt. Hamilton 1895. In the introduction of his catalogue for 1895 (Pub. Lick Obs., Vol. IV, p. 22) Professor TUCKER states that his equation from screen-observations is

$$0^s.009$$

but he appears to entertain grave doubts as to the reality of the effect. The result is based upon 75 single determinations, and I see no reason why it should not be entitled to confidence.

Albany 1898. The determination of the magnitude-equation for this series of observations rests upon a large number of observations, and has been adopted in the computations for the Albany Catalogue now in progress. This equation is (*A.J.* XXII, p. 99),

$$0^s.0132\,(M-1) - 0^s.00019\,(M-1)^2$$

Bonn 1900. KÜSTNER has given special attention to the determination of his magnitude-equation, and in a paper of great interest (*A.N.* 3778) determines the magnitude-equation of the *Fundamental-Catalog* of Dr. AUWERS to be

$$0^s.005$$

Much evidence bearing on this point is contained in the several introductions of the recent publications of the Bonn Observatory (Nos. 1, 5 and 6). From these sources we get the following determinations of KUSTNER's magnitude-equation, together with the corresponding equation of F.C.

Zone	Kustner	F.C.
	s	s
0 to +18	+0.0070	−0.0043
+18 to +36	−0.0041	−0.0053
+36 to +51	−0.0036	−0.0034

One is struck with the exceptional equation of the observer for zone 0° to +18°. His previous equation at Berlin had been −0ˢ.004, as already stated, and to this value he returned again in the second zone observed at Bonn. But the consistency of the equation for F.C. in the first zone with the other values found in later zones seems to establish the reality of the abnormal change. Since the revised *Fundamental-Catalog* depends essentially on the same modern authorities upon which my Standard Catalogue is based I shall assume that the equation which KUSTNER found for the former is equally applicable to the latter.

We are now ready to combine the various determinations of absolute magnitude-equation to ascertain the absolute equation of the Standard Catalogue. The process is very simple, and needs merely to be indicated. For instance, we find the magnitude-equation of Pulkowa 75 to be $-0^s.0045(M-3.5)$. If we apply this to the right-ascensions of ROMBERG's Catalogue, and then compare the corrected right-ascensions with those of the Standard Catalogue, we shall arrive at the same result that we shall have if we alter the numbers in the last column of the sub-table for Pulk. 75 in Table I by the addition of $+0^s.0045(M-3.5)$. Then reversing the signs of these quantities, so corrected, to make them applicable as corrections to the Standard Catalogue, we shall have the observed corrections given by Pulk. 75 to the Standard Catalogue for the magnitude-effect. Proceeding in this manner we have the following observed values of the magnitude-effect as determined through the respective catalogues of observations.

OBSERVED VALUES OF $\Delta\alpha_m$ FOR DETERMINATION OF THE ABSOLUTE MAGNITUDE-EQUATION OF THE STANDARD CATALOGUE.

Magnitude through	0ᴹ.9	2ᴹ.0	3ᴹ.0	4ᴹ.0	4ᴹ.9	6ᴹ.0	Wt.
	s	s	s	s	s	s	
Pulk. 75	+.017	+.015	+.003	−.003	−.009	−.018	2
Cape 85	+.019	+.018	+.006	−.006	−.018	−.019	1
Berl. 90	+.020	+.012	+.005	−.005	−.010	−.022	3
Cape 90	+.023	+.018	+.004	−.004	−.011	−.021	3
Berl. 95	+.013	+.012	+.004	−.003	−.019	−.038	1
Mt.H.95	–	+.014	+.002	−.002	−.017	−.020	1
Alb. 98	+.028	+.008	+.004	−.002	−.011	–	2
Bonn 97	+.013	+.007	+.002	−.002	−.007	−.012	4

Collecting the means by weight of the vertical columns (assigning weight 2, to the residual at 0ᴹ.9 for Berlin 90 which depends on only five stars) we have in the third column of the following table the mean observed magnitude-equation of the Standard Catalogue.

SUMMARY OF OBSERVED MAGNITUDE-EQUATION FOR THE STANDARD CATALOGUE.

Magn.	p	$\Delta\alpha_m$ (O)	$\Delta\alpha_m$ (C)	C−O
M		s	s	s
0.9	1	+0.0185	+0.0199	+0.0014
2.0	3	+0.0122	+0.0115	−0.0007
3.0	6	+0.0036	+0.0038	+0.0002
4.0	6	−0.0033	−0.0038	−0.0005
4.9	3	−0.0109	−0.0107	+0.0002
6.0	0.5	−0.0193	−0.0192	+0.0001

Employing the relative weights in the second column I find the magnitude-equation of the Standard Catalogue to be

$$-0^s.0077\ (M-3.5)$$

and this is adopted. The fourth column is derived from this equation, and the last column exhibits the discrepancies, C−O. It will be seen that a large weight has been attributed to the determinations of KUSTNER both at Berlin and Bonn. The Berlin determination rests upon the observation of 37 stars, the relative places of which were determined by the heliometric triangulation of GILL. Of these, 31 stars are between the magnitude 7ᴹ.4 and 8ᴹ.5, and only 6 stars are brighter than 7ᴹ.0, of which the brightest is 5ᴹ.7. Yet, on account of the difference of method, together with the high precision and homogeneity of KUSTNER's Berlin observations, it seemed to me that the assigned weight was not relatively too great. It so happens that the result of this determination is very near the mean of all. As to the Bonn determination, it represents the most elaborate series of determinations of magnitude-equations which has been attempted, and were it not for the anomalous result in the first zone (*Heft* 4) there could be no question as to the weight to which it is entitled.

On the whole there seems to be little likelihood that the magnitude-equation of my Standard Catalogue is less than $-0^s.007$, or more than $-0^s.009$. The probable error of the coefficient, $-0^s.0077$, I estimate to be about $\pm 0^s.0005$; though its nominal, or computed, probable error is only $\pm 0^s.00014$, which, in the present instance, has no real meaning.

Provided with the coefficient of magnitude-equation for the Standard Catalogue, determined with a precision apparently very great, the suggestion follows of itself that we should attempt to determine the absolute magnitude-equations of the individual catalogues of observation. In the last column of each sub-table in Table I, under the caption, $\Delta\alpha_m$, are the observed corrections necessary to reduce the individual catalogues into systematic conformity with the Standard Catalogue in so far as star-magnitude is the

argument for systematic correction. By means of conditional equations the systematic difference of magnitude-equation, $\Gamma\alpha_m$, between each catalogue and the Standard Catalogue has been determined, and the result is shown in full-faced type over the middle of each sub-table. These are the equations which, applied to the right-ascensions of the several catalogues of observation, are supposed to bring them into systematic harmony with those of the Standard Catalogue. Adding to these the adopted magnitude-equation of the Standard Catalogue, $-0^s.0077$, we derive the absolute magnitude-equation, $\Delta\alpha_m$, for each catalogue, as shown by the second of the quantities in full-faced type over each of the sub-tables contained in Table I.

For many of the older catalogues the determination of $\Gamma\alpha_m$ is very uncertain, both because of the small range of magnitude involved and also because of the small weight of the observed residuals. At most, the values of $\Delta\alpha_m$ for these are but rude approximations to the true result. For some of the later catalogues the results are entitled to a fair degree of confidence, as appears from the following schedule of those for which it is possible to compare with the magnitude-equations determined by the respective observers.

Comparison of Observed and Computed Equations.

	Comp.	Obs.	C − O
Pulk. 75	−0.0050	−0.0045	0.0005
Cape 85	−0.0078	−0.0110	+0.0032
Cape 90	−0.0125	0.0139	+0.0014
Berlin 90	−0.0033	−0.0040	+0.0007
Berlin 95	−0.0076	0.0090	+0.0014
Mt. H 95	0.0106	0.0090	0.0016
Alb. 98	−0.0115	−0.0130	+0.0015
Bonn 99	. . . .	. . . .	−0.0027

The C − O for Bonn 1899 is inferred through comparison of its determination of the absolute equation of the standard, $-0^s.005$, with the adopted value, $0^s.0077$. The mean difference between the observed and computed values without regard to weight is $0^s.0016$, and, assuming that the error is equally due to the observed and computed values, it follows that the average error of the computed equation is $\pm 0^s.0011$.

Madras 35 (Downing's reduction) and Madras 75, though each is of very low weight, and subject to many anomalous errors, have been investigated for magnitude-equation. The observations extend into three zones. We have,

Madras 35.

Magn.	Zone I			Zone II			Zone III			Means
0.9				8	1	.001				.001
2.0	11	1	.003	11	2	+.004	16	1	.001	+.001
3.0	27	1	.005	51	5	.001	39	1	+.001	.001
4.0	11	1	+.005	123	9	+.001	50	1	.001	+.001
4.9				71	5	.031	30	1	.021	.028
6.0				13	1	.040				.040

From the final means I find for the [illegible] equation, $-0^s.016$; but if a term depending [illegible] power of ΔM be introduced the equation [illegible]

$$0^s.016(M-3.5) \quad 0^s.0014(M-3.5)^2$$

The extension of the present work to stars of [illegible] magnitude will afford a test of the interesting question whether the apparently rapid increase of the [illegible] for fainter stars is merely due to the systematic [illegible] of the catalogue, or to a real deviation of the [illegible]. To some extent this same peculiarity appears in Madras 75, and this raises the question whether this phenomenon may be attributed to a racial peculiarity. The table for Madras 75 follows.

Madras 75.

Magn.	Zone I			Zone II			Zone III			Means
0.9				8	3	.006				.006
2.0	15	1	+.004	14	1	+.006	18	1	−.018	+.001
3.0	27	1	−.019	51	15	−.001	12	2	−.006	.005
4.0	39	1	+.019	123	22	+.004	48	1	+.006	+.005
4.9				71	11	.012	27	1	−.035	−.014
5.9				11	2	.020				.020

The absolute magnitude-equation may be taken as $-0^s.010$; but if a term depending on $(\Delta M)^2$ be introduced it becomes,

$$-0^s.010(M-3.5) \quad -0^s.0024(M-3.5)^2$$

The assumption that the magnitude-equation of the Standard Catalogue does not sensibly vary from one epoch to another rests wholly on the soundness of the conclusion that, in the mean, the equation for eye-and-ear transits is the same as for chronographic registry. As a further check upon this deduction it may be of interest to institute a comparison between the absolute equations as determined in Table I, for the period 1850 to 1890.

Magnitude-Equations Compared.

Eye-and-Ear		$\Delta\alpha$	p	Chronograph		$\Delta\alpha$	p
Stgo.	50	.012	1	Wash.	60	.006	2
Geh.	50	−.005	2	Geh.	60	.007	2
Cape	60	.011	2	Melb.	60	.009	1
Radcl.	60	.013	1	Geh.	64	.009	2
Paris	60	.003	4	Cape	65	.014	1
Bruss.	65	.001	1	Harv.	65	.013	1
Pulk	75	.005	1	Melb.	80	.007	2
Paris	75	.005	3	Geh.	72	.009	2
Cape	80	.011	2	Wash.	75	.008	5
Cape	85	.008	3	Harv.	75	.007	1
Radcl	90	.008	1	[illegible]	75	.010	2
				Melb.	80	.005	1
				Geh.	80	.007	1
				Pulk.	85	.008	2
				Strass.	85	.006	2
				Geh.	90	.006	4
				Madr.	90	.013	1
				Berlin	90	.003	1
				Lisbon	90	.008	1

The relative weights are based partly upon the trustworthiness of the comparisons, and partly upon the number of observers who took part in the various series. The means by weights are presented in the last column of the subjoined statement, and the arithmetical means are in the preceding column.

	Series	Arith. Means	Weighted Means
		s	s
Eye-and-ear	11	−0.0077	−0.0070
Chronographic	19	−0.0082	−0.0078

The result of the direct comparison is confirmed.

Another question arises as to the application of the equations to observations at a great distance from the equator. The testimony from Zones I and III in Table I is entirely insufficient to afford an answer to this question. But since the amount of the equation seems to be practically independent of the magnifying power employed we may infer that the equations apply in full force to 60° of declination at least. It would be very difficult to test this question for higher declinations by actual observation without an expenditure of labor which might be regarded as disproportioned to the end to be attained. Very little harm can result from the assumption that the equation applies to all declinations; since for the higher declinations its actual effect upon the right-ascension is in proportion to cos δ.

It is evident that all our conclusions from this discussion are strictly applicable to observations of the brighter stars only. There is no reasonable doubt, however, that in most instances the inferences may be extended to the observations of stars somewhat fainter than the sixth magnitude,— perhaps to the magnitudes $7^M.0$ or $8^M.0$. As a rule the determination of magnitude-equation by the employment of screens indicates a linear relation of the magnitude-effect down to stars as faint as the eighth, or ninth, magnitude, so that most observers appear to doubt whether any term involving $(\Delta M)^2$ has any real existence. On the other hand there are doubtless exceptions to this rule, as in the cases of the Cambridge and Leipsic A.G. zones already cited. Modifying the result of Professor TURNER's paper on the right-ascensions of the Cambridge A.G. zone (*M.N.*, LX), taking into account the comparison by which he demonstrates the equality of magnitude-equations for Greenwich 80 and Cambridge relative to the brighter stars, and accepting the result of the present discussion which assigns $-0^s.007$ as the magnitude-equation of Greenwich 80, we may tabulate the magnitude-effect of Cambridge A.G. approximately as in the following table.

It is very probable that other equally striking examples of the non-linear effect of the magnitude-equation will be discovered hereafter, so that, whatever the general rule may be, its application cannot be considered as inflexible.

CORRECTION OF CAMBRIDGE A.G. FOR MAGNITUDE-EQUATION.

M	s	M	s
1.0	+0.021	6.0	−0.020
2.0	+0.014	7.0	−0.040
3.0	+0.007	8.0	−0.071
4.0	0.000	9.0	−0.116
5.0	−0.007	10.0	−0.182

Allusion should be made to one source of uncertainty which has not been mentioned in the foregoing discussion. This relates to the fact that many of the transits upon which the comparisons depend were observed in daylight. These, of course, form but a small part of the entire material, and their influence is limited to the fourth magnitude. It may reasonably be doubted whether these follow the same law as the transits obtained at night. To have separated the two classes of observations would have been impracticable in many cases, and a task of forbidding proportions in others. We may suppose, however, that much the same differential effects obtain in the daylight transits as in those of the night, and, therefore, that the commingling of results may not have produced, on the whole, serious anomalies.

In this same connection it should be observed that the determination of equinox-correction is involved in this difference between daylight and night observations; though it is extremely probable that the errors to be feared from this source are not important in comparison with constant errors in observed transits of the sun.

The Catalogue of 627 Standard Stars may readily be freed from the effect of magnitude-equation by the application to the right-ascensions of the correction, $-0^s.0077$ $(M-3.5)$. This will not disturb the position of the adopted equinox to any sensible degree; and in the case of an observer who has thoroughly determined his magnitude-equation, the adoption of this course will doubtless lead to a sensible improvement in the accordance of results for the correction of the clock. This course has been adopted in the reduction of the transits for the Albany catalogue.

The determination of absolute magnitude-equation should be regarded as an indispensable requisite in all observations aiming at precision. There is every reason to believe that this can be accomplished successfully by the use of wire-gauze screens to produce artificial diminution of magnitude, provided the amount of absorption is at least $2^M.5$. The gain in the value of the results more than compensates the comparatively small amount of extra observing and computing involved, even when the determination is, as it should be, much more complete than has usually been the case.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed April 21*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

Nos. 537-538.

VOL. XXIII. BOSTON, 1903 MAY 15. NOS. 9-10

THE TRUE RADII OF CONVERGENCE OF THE EXPRESSIONS FOR THE RATIOS OF THE TRIANGLES WHEN DEVELOPED AS POWER-SERIES IN THE TIME-INTERVALS.

By F. R. MOULTON.

1. Introduction. The computation of the elements of an unknown orbit from three complete observations consists of two distinct parts: (*a*) the determination of the heliocentric coordinates of the body in question at the epochs of the observations; and (*b*) the determination of the elements from the time-intervals and the data given by (*a*). Of these two parts, (*b*) has given most of the theoretical and practical difficulties. In the *Astronomical Journal*, No. 510, the author has given a solution* of (*b*) which from both a practical and a theoretical stand-point leaves little to be desired.† There is one limitation to the application of the method, however, arising from the fact that it depends upon the use of infinite series which converge only for sufficiently small values of the heliocentric angular motions. But their true radii of convergence were exactly determined, and it was remarked‡ that this is not an objection of importance since the same sort of limitation occurs in (*a*).

In this paper it is proposed to find the true radii of convergence for the series employed in (*a*) in all the cases which can arise.§ It will be shown that the series developed in (*b*) always may be used when the time-intervals are such that those occurring in (*a*) are of practical value.

Tables will be inserted to make the theoretical conclusions arrived at of immediate value to the computer without labor on his part. Finally, it will be shown to what extent the question of convergence enters in some of the other principal methods of determining orbits.

2. Expressions for the Ratios of the Triangles. Let the epochs of the three observations be t_1, t_2, and t_3, and let the corresponding heliocentric distances of the body be r_1, r_2, and r_3. The triangles in question are the projections on the planes of reference of those contained between the r_j, taken two at a time, and the chord joining their extremities. There are three of these triangles arising from the three combinations of r_1, r_2, and r_3. The ratios of the projections of the triangles are equal to the ratios of the triangles themselves, the case where the plane of the orbit coincides with one of the fundamental planes being excluded. Therefore the problem may be considered in the plane of the orbit without any loss of generality.

Take the plane of motion as the xy-plane, and suppose the positive end of the x-axis is directed toward the perihelion point. Let the rectangular heliocentric coordinates at the epoch t_j be x_j, y_j, $j = 1, 2, 3$. Then the expression for the ratio of the triangles may be written in the form

$$\frac{x_i y_j - y_i x_j}{x_k y_l - y_k x_l}, \tag{1}$$

where $i \neq j$, $k \neq l$, and $i, j, k, l = 1, 2$, or 3.

It follows from well-known theorems respecting the multiplication and addition of power-series that, if x_j, y_j, etc., are developable into power series with a common realm of convergence, then the numerator and denominator of (1) are separately developable into power-series which converge in this common realm. Therefore the first problem is to find the conditions under which x and y may be expressed as convergent power-series in $t - t_2$. Since the motion is by hypothesis undisturbed these variables are defined by the differential equations

---

* *A General Method of Determining the Elements of Orbits of All Eccentricities from Three Observations.*

† This occasion is taken to make note of the following errors:

On p. 43, column 2, line 5, read "geocentric" instead of "heliocentric."

On p. 45, column 2, at bottom, read "r^2" instead of "r."

On p. 47, column 2, line 12, read

"$\cos^{-1}\left(\frac{-1}{e}\right)$" instead of "$\cos\left(\frac{-1}{e}\right)$"

‡ *loc. cit.* p. 44, column 1.

§ Dr. W. A. Hamilton has given the complete solution of the problem in the case of parabolic orbits in a memoir in *A. J.*, No. 533.

$$(2)\quad \begin{cases} \dfrac{d^2x}{dt^2} = -k^2(1+m)\dfrac{x}{r^3} \\ \dfrac{d^2y}{dt^2} = -k^2(1+m)\dfrac{y}{r^3} \end{cases}$$

Suppose $x = x_0$, $y = y_0$ at $t = t_0$. If the right members of (2) are regular for $x = x_0$, $y = y_0$, it follows that x and y are expansible as power-series in $t-t_0$ which converge so long as the modulus of this argument is sufficiently small.* Consequently, the time-intervals may always be taken small enough in the actual case so that the processes are valid, but the methods of both CAUCHY and HARZER prove only the existence of the limit without giving its true value with any approximation. For example, when the eccentricity of the orbit equals zero,

$$(3)\quad \begin{cases} x = a\cos nt \\ y = a\sin nt \end{cases}$$

where a is the major semi-axis and n is the angular velocity. In this case x and y are expansible as permanently converging power-series in $t-t_0$ for any initial t_0, while the methods of CAUCHY and HARZER give only a small value of $|t-t_0|$. In the case of parabolic orbits HARZER's existence formula† gives results which are 90% in error, the true limit being found by HAMILTON's formula.‡

Before the expression (1) is used in practice the quotient of the two power-series into which they are expanded is taken, and hence the radius of convergence is limited still further by the poles which are defined by the vanishing of the denominator. Therefore two problems are to be solved. (*A*) To find the conditions under which the rectangular coordinates may be expanded as convergent power-series in the time, and (*B*) to find the conditions under which the denominator of (1) may vanish.

3. THE SINGULAR POINTS OF THE RECTANGULAR COORDINATES CONSIDERED AS FUNCTIONS OF THE TIME. Let the following notation be adopted:

- a = major semi-axis,
- e = the eccentricity,
- p = the parameter,
- n = the mean motion,
- T = the time of perihelion passage,
- M = the mean anomaly,
- v = the true anomaly,
- E = the eccentric anomaly in elliptic motion,
- F = the corresponding auxiliary in hyperbolic motion.

* This theorem, which has been the basis for a very large part of the rigorous developments of the theory of differential equations, was given by CAUCHY in *Comptes Rendus, July 4, 1842*, (*Coll. Works, 1st Series, Vol. VII, p. 5, et seq.*). HARZER, in the *Publications of the Kiel Observatory, Vol. XI, p. 24, et seq.*, has arrived at the same results in the special case under present discussion by successive simplifications of the actual series.

† *loc. cit.*, p. 30.

‡ *Astronomical Journal*, No. 533, Eq. (18).

It is necessary to treat elliptic, parabolic, and hyperbolic motion separately, and they will be discussed in this order.

(*a*) *Case of Elliptic Motion.* In elliptic motion the following equations are true:*

$$\left.\begin{aligned} p &= a(1-e^2) \\ n &= \frac{k\sqrt{1+m}}{a^{\frac{3}{2}}} \\ m &= \text{mass of body} = 0 \text{ with sufficient approximation} \\ M &= n(t-T) = E - e\sin E \\ x &= r\cos v = a\cos E - ae \\ y &= r\sin v = a\sqrt{1-e^2}\sin E \end{aligned}\right\}\quad (4)$$

Suppose the value of E at t_0 is E_0, and write $E = E_0 + E_1$. Then the last two equations of (4) give

$$\left.\begin{aligned} x &= a\cos(E_0+E_1) - ae \\ y &= a\sqrt{1-e^2}\sin(E_0+E_1) \end{aligned}\right\}\quad (5)$$

from which it follows that x and y may be expanded into permanently converging power-series in E_1, where E_1 is real or complex, for every finite value of E_0. These series may be written

$$\left.\begin{aligned} x &= \sum_{i=0}^{\infty} a_i E_1^i \\ y &= \sum_{i=0}^{\infty} b_i E_1^i \end{aligned}\right\}\quad (6)$$

where the radius of convergence is infinite.

It follows from the fourth equation of (4) that E is finite for every finite value of M. Consequently x and y may be expanded into converging power-series in E_1, where E_1 is defined by the third equation of (4), for all finite values of M.

Suppose E_1 is expansible as a power-series in $M-M_0$, where M_0 is the value of M at $t = t_0$, for all $|M-M_0| < R$. If this series is substituted in the right members of (6) and the terms are rearranged with respect to powers of $M-M_0$, then power-series will be obtained which, in accordance with a theorem given by WEIERSTRASS,† converge for all $|M-M_0| < R$. It follows, therefore, that to find the conditions under which x and y may be expanded as converging power-series in $t-t_0$, or in $M-M_0$, which is essentially the same because of the relation between M and t, it is only necessary to find the conditions under which $E_1 = E - E_0$ may be expanded as a converging power-series in $M-M_0$.

Consider E as a function M. This function has singular points for certain values of M, and the expansion as a power-series in $M-M_0$ will converge, in accordance with CAUCHY's well-known theorem, for all values of $M-M_0$

* *Introduction to Celestial Mechanics*, pp. 152, 153.

† *Functionen Lehre*, p. 73. The theorem is given for LAURENT series, but this may be thought of as containing the ordinary power-series as a special case.

whose moduli are less than the distance from M_0 to the nearest singular point. The quantity E is defined in terms of M by the equation

$$E - e\sin E = M$$

The problem is to locate the singular points of E considered as a function of M from this transcendental relation. To do this consider the differential equation which relates E and M.

$$\frac{dE}{dM} = \frac{1}{1 - e\cos E} \tag{7}$$

The right member of this differential equation is analytic in E, and, in accordance with CAUCHY's theorem quoted in Section 2 of this paper, in the vicinity of every E_0 for which it is regular $E - E_0$ is developable as a power-series in $M - M_0$ which has a positive radius of convergency. Consequently, at all of those points for which the right member of (7) is regular, E considered as a function of M is regular. Conversely, for every singularity of the right member of (7), E considered as a function of M has a singular point.

The right member of (7) is uniform, and the only singularities are the poles defined by

$$1 - e\cos E = 0 \tag{8}$$

Since E is to be considered as a function of the complex variable M it will in general be complex. Suppose it has the form

$$E = u + \sqrt{-1}\,w$$

where u and w are real. Then (8) becomes

$$\begin{aligned}1 - e\cos(u + \sqrt{-1}\,w) = 1 - e\cos u\cos(\sqrt{-1}\,w)&\\ + e\sin u\sin(\sqrt{-1}\,w) &= 0\end{aligned} \tag{9}$$

Since $\cos(\sqrt{-1}\,w)$ is real while $(\sin\sqrt{-1}\,w)$ is pure imaginary this equation is equivalent to the two equations

$$\left.\begin{aligned}\sin u\sin(\sqrt{-1}\,w) &= 0\\ 1 - e\cos u\cos(\sqrt{-1}\,w) = 1 - e\cos u\cosh w &= 0\end{aligned}\right\} \tag{10}$$

The first of these equations is satisfied by $w = 0$, or $u = \nu\pi$, where ν is zero or any integer. The second equation becomes in these respective cases

$$\left.\begin{aligned}1 - e\cos u &= 0\\ 1 - (-1)^\nu e\cosh w &= 0\end{aligned}\right\} \tag{11}$$

Since $0 \leqq e < 1$ the first of these equations is impossible, and since $\cosh w$ is positive for all values of w the second can be satisfied only if ν is even. Hence the conditions for singular points of the right member of (7) are given by

$$\left.\begin{aligned}u &= 2\nu\pi\ , \quad (\nu = \text{zero or any integer})\\ 1 - e\cosh w &= 0\end{aligned}\right\} \tag{12}$$

As w^2 varies from zero to infinity $\cosh w$ is an increasing monotonic function whose limits are unity and infinity; hence, for every $0 < e < 1$, the second equation of (12) is satisfied by one, and but one, value of w^2. The singular points are therefore given by

$$\left.\begin{aligned}u &= 2\nu\pi\\ w &= \pm\cosh^{-1}\left(\frac{1}{e}\right) = \pm\log\left(\frac{1 + \sqrt{1 - e^2}}{e}\right)\end{aligned}\right\} \tag{13}$$

where the logarithm is taken to the Naperian base.

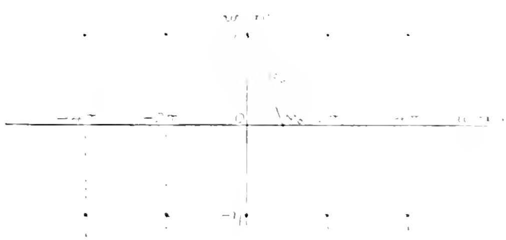
From this point on let u and w represent those values determined by (13). The real problem is to find for what values of M singular points occur, but so far only the values of E for which they occur have been found. KEPLER's equation at once enables us to solve the problem. Let $M = \xi + \sqrt{-1}\,\eta$; then

$$\xi + \sqrt{-1}\,\eta = E - e\sin E = u + \sqrt{-1}\,w - e\sin(u + \sqrt{-1}\,w)$$

Expanding the last term, putting $u = 2\nu\pi$, and equating real and imaginary parts, it is found that

$$\left.\begin{aligned}\xi &= 2\nu\pi\\ \eta &= w - \frac{e\cos u\sin(\sqrt{-1}\,w)}{\sqrt{-1}} = w - e\sinh w\end{aligned}\right\} \tag{14}$$

Since η is an odd function of w, which is two-valued, η is two-valued. The singular points are located at the x in the following figure.



Since E is finite for all finite values of M, these singular points are branch points.

It follows from (13) and (14) that the location of the singular points depends upon e alone, and it is easily seen that as e varies from 0 to 1 the singular points travel from infinity along the lines $2\nu\pi$ toward the real axis. In practice M_0 is always real, and, because of the symmetry of the orbit with respect to the major axis, it is sufficient in discussing the radius of convergence to suppose that $-\pi \leqq M_0 \leqq \pi$. The true radius of convergence of E when expressed as a power-series in $M - M_0$ is (see fig.)

$$R_M = \sqrt{M_0^2 + \eta^2} \tag{15}$$

where η is defined by the second equations of (13) and (14).

It follows also from the chain of arguments given above that R_M is the true radius of convergence of the rectangular coordinates and the expressions for the triangles when expanded as power-series in $M-M_0$.

If t_0 is the value of t corresponding to M_0 the true radius of convergence for all these functions when expanded as power-series in $t-t_0$ is

$$R_t = \frac{R_M}{n} = \frac{a^{\frac{3}{2}} R_M}{k} \tag{16}$$

(*b*) *Case of Parabolic Motion.* Dr. HAMILTON treated this case in his memoir giving not only the location of the singular points and the true radius of convergence, but also a complete discussion of the RIEMANN surface associated with the function. His formula for the true radius of convergence is,* in the notation of this paper,

$$R_t = \frac{p^{\frac{3}{2}}}{3k}\sqrt{1+\frac{9k^2(t_0-T)^2}{p^3}} \tag{17}$$

It can also be found more simply by the methods employed in this paper. The rectangular coordinates are expressed in terms of the true anomaly by the equations

$$\left.\begin{aligned} x &= r\cos v = \frac{p\cos v}{1+\cos v} = \frac{p}{2}\left(1-\tan^2\frac{v}{2}\right) \\ y &= r\sin v = \frac{p\sin v}{1+\cos v} = p\tan\frac{v}{2} \end{aligned}\right\} \tag{18}$$

The true anomaly is related to the time by the well-known equation

$$\tfrac{1}{3}\tan^3\frac{v}{2}+\tan\frac{v}{2} = \frac{2k(t-T)}{p^{\frac{3}{2}}} \tag{19}$$

and the differential equation which relates them is

$$\frac{d\left(\tan\frac{v}{2}\right)}{dt} = \frac{2k}{p^{\frac{3}{2}}\left(\tan^2\frac{v}{2}+1\right)}$$

In accordance with the principles employed in the elliptic case $\tan\frac{v}{2}$ is a regular function of t at all points except at those in which

$$\tan^2\frac{v}{2}+1 = 0$$

The solutions of this equation are

$$\tan\frac{v}{2} = \pm\sqrt{-1} \tag{20}$$

Substituting in (19), the values of t for which the function is singular are found to be

$$(t'-T) = \pm\frac{p^{\frac{3}{2}}\sqrt{-1}}{3k} \tag{21}$$

* *Astronomical Journal*, No. 533, Eq. (18).

Then the true radius of convergence for any real t_0 is

$$R_t = \sqrt{(t'-T)^2+(t_0-T)^2} = \frac{p^{\frac{3}{2}}}{3k}\sqrt{1+\frac{9k^2(t_0-T)^2}{p^3}}$$

agreeing with Dr. HAMILTON's results.

(*c*) *Case of Hyperbolic Motion.* In the case of hyperbolic orbits the coordinates, elements, and auxiliaries are related by the following equations: *

$$\left.\begin{aligned} p &= a(e^2-1) \quad , \quad e>1 \\ M &= \frac{k}{a^{\frac{3}{2}}}(t-T) = -F+e\sinh F \\ r &= a(-1+e\cosh F) \\ \cos v &= \frac{e-\cosh F}{-1+e\cosh F} \\ \sin v &= \frac{\sqrt{e^2-1}\,\sinh F}{-1+e\cosh F} \\ x &= r\cos v = ae-a\cosh F \\ y &= r\sin v = a\sqrt{e^2-1}\,\sinh F \end{aligned}\right\} \tag{22}$$

It follows from the last two equations that x and y are expansible into permanently converging power-series in $F-F_0$ for any finite F_0. From the second equation it follows that F is finite for all finite values of M: therefore x and y are expansible into power-series in $F-F_0$ which converge for all finite values of M. By virtue of the theorem of WEIERSTRASS the conditions under which $F-F_0$ may be expanded as a convergent power-series in $M-M_0$ must be found in order to solve the problem.

The problem is treated precisely as in the elliptic case. The differential equation relating F and M is

$$\frac{dF}{dM} = \frac{1}{-1+e\cosh F} \tag{23}$$

The singular points of F considered as a function of M are located by

$$-1+e\cosh F = -1+e\cosh(u+\sqrt{-1}\,w) = 0 \tag{24}$$

Expanding the right member and equating the real and imaginary parts to zero, it is found that

$$\left.\begin{aligned} \sinh u\sin w &= 0 \\ -1+e\cosh u\cos w &= 0 \end{aligned}\right\} \tag{25}$$

The first of these equations is satisfied by $w = \nu\pi$, or $u = 0$. The second equation becomes in these respective cases

$$\begin{aligned} -1+(-1)^{\nu} e\cosh u &= 0 \\ -1+e\cos w &= 0 \end{aligned}$$

Since e is greater than unity, and $\cosh u$ is greater than unity for all values of u, the first equation cannot be satis-

* *Introd. to Cel. Mec.*, pp. 155 and 156.

fied. The singular points are consequently given by the equations

$$u = 0 \quad , \quad -1 + e\cos w = 0 \tag{26}$$

The solutions of the second equation are

$$\pm w = 2\nu\pi + \cos^{-1}\left(\frac{1}{e}\right) \tag{27}$$

where ν is zero or any integer, and $\cos^{-1}\left(\frac{1}{e}\right)$ is the primitive value of this multiple valued function. Hence, substituting $\pm\sqrt{-1}\,w$ for F, where w is defined by (27), in the second equation of (22), the values of M for which the functions have singular points are found to be

$$M = \xi + \sqrt{-1}\,\eta = \mp\sqrt{-1}\,w + e\sinh(\pm\sqrt{-1}\,w)$$
$$= \mp\sqrt{-1}\,w \pm \sqrt{-1}\,e\sin w$$

Therefore,

$$\xi = 0 \quad , \quad \pm\eta = -w + e\sin w \tag{28}$$

Let $\bar{w} = 2\nu\pi + \cos^{-1}\left(\frac{1}{e}\right)$; then (28) becomes

$$\xi = 0 \quad , \quad \pm\bar{\eta} = -w - 2\nu\pi + e\sin w \tag{29}$$

The singular points are, therefore, all on the imaginary axis, distributed symmetrically on each side of the real axis, and occurring at intervals of 2π out to infinity. The only values of M_0 used in practice are real. Hence the true radius of convergence of the expansions of F, and consequently of x, y, and the triangles, as power-series in $M - M_0$, when the motion is hyperbolic, is given by the equation

$$R_M = \sqrt{M_0^2 + \eta^2} \tag{30}$$

The true radius of convergence for expansions in powers of $t - t_0$ is found from the relation between M and t to be

$$R_t = \frac{a^{3/2}}{k} R_M \tag{31}$$

4. *Numerical Results.* The preceding formulas are comparatively simple, and the numerical results can be obtained in any special case without much labor, nevertheless tables of results with convenient intervals for the arguments are desirable. The true radii of convergence when the argument is $M - M_0$ are capable of being tabulated very simply since they depend upon the two parameters e and M_0. They are given by the formulas (15) and (30). In these equations η depends upon e alone. They are defined by (11) and (29) respectively. For convenience the formulas used will be collected here.

When $0 \overline{\overline{<}} e < 1$, then

$$\left\{\begin{aligned} w &= \log\left(\frac{1+\sqrt{1-e^2}}{e}\right) \\ \bar{\eta} &= \bar{w} - e\sinh w \end{aligned}\right. \quad , \quad \begin{aligned} R_M &= \sqrt{M_0^2 + \bar{\eta}^2} \\ R &= \frac{a^{3/2} R_M}{k} \end{aligned} \tag{32}$$

If a quantity M is introduced into the theory of parabolic motion for the sake of uniformity by the equation

$$M = \frac{2k(t-T)}{p^{3/2}}$$

the corresponding equations become, when $e = 1$,

$$\left.\begin{aligned} R_M &= \sqrt{\pi^2 + M^2} \\ R &= \frac{p^{3/2}}{2k} R_M = \frac{p^{3/2}}{3k}\sqrt{1 + \frac{9k^2(t-T)^2}{p^3}} \end{aligned}\right\} \tag{33}$$

When $e > 1$

$$\left.\begin{aligned} w &= \cos^{-1}\left(\frac{1}{e}\right) \\ \eta &= -w + e\sin w \end{aligned} \quad , \quad \begin{aligned} R_M &= \sqrt{M^2 + \eta^2} \\ R &= \frac{a^{3/2}}{k} R_M \end{aligned}\right\} \tag{34}$$

From these equations Table I has been computed.

TABLE I. $\log \eta$.

e	$\log \bar{\eta}$	e	$\log \eta$
0	∞	.95	8.0294
.1	0.3006	1.05	8.0000
.2	0.1181	1.1	8.4624
.3	9.9637	1.2	8.8865
.4	9.8130	1.3	9.1367
.5	9.6539	2.0	9.8357
.6	9.4749	5.0	0.5479
.7	9.2584	10.0	0.9283
.8	8.9690	100.0	1.9931
.9	8.4942	1000.0	2.9993

From this table and the third, last, and third formulas of (32), (33) and (34) respectively the following table of values of $\log\frac{R_M}{k}$ has been computed.

TABLE II. $\log\frac{R_M}{k}$.

e	$M = 0$	$M = 60$	$M = 120$	$M =$ [illegible]
0	∞	∞	∞	∞
.1	2.0651	2.1178	2.2260	[illegible]
.2	1.8826	1.9895	2.1574	2.2965
.3	1.7281	1.9086	2.1258	2.2794
.4	1.5774	1.8553	2.1055	2.2709
.5	1.4183	1.8213	2.0953	2.2660
.6	1.2394	1.8014	2.0898	2.2635
.7	1.0228	1.7905	2.0871	2.2623
.8	0.7331	1.7862	2.0859	2.2618
.9	0.2586	1.7846	2.0855	2.2616
.95	9.7938	1.7844	2.0851	2.2616
1.0	1.5883	1.8583	2.1064	
e	$M = 0$	$M = 30$	$M = 60$	$M = 90$
1.05	9.7794	1.4835	1.7794	1.9605
1.1	0.2185	1.4840	1.7846	1.9606
1.2	0.6509	1.4880	1.7856	1.9610
1.3	0.9011	1.4978	1.7881	1.9622
2.0	1.6001	1.7000	1.8618	1.9983
5.0	2.3123	2.3171	2.3307	2.3515
10.0	2.6927	2.6935	2.6960	2.7000
100.0	3.7575	3.7575	3.7575	3.7576
1000.0	4.7637	4.7637	4.7637	4.7637

The irregularities in the numbers at $e = 1$ arise from the fact that M bears quite different relations to the true anomalies in the three kinds of conic sections.

From Table II the true radii of convergence in t may be computed from the formulas

$$(35)\quad \begin{cases} R_t = \dfrac{R_M}{k} a^{\frac{3}{2}} & \text{if} \quad e<1 \text{ or } e>1 \\[2ex] R_{t_0} = \dfrac{R_M}{k} \dfrac{p}{2} & \text{if} \quad e=1 \end{cases}$$

These formulas are so very simple that it is not worth the labor from a practical point of view to construct tables with different values of a and p. But it is of interest to know about what the time intervals are for typical cases. The average major axis in the case of asteroid orbits, where the elliptic theory has application, is about 2.65, and the following table has been computed from the first part of Table II and the first equation of (35), using this value of a.

Table III. R_{t_0} in days when $a = 2.65$.

e	For $M_0 = 0$	For $M_0 = 60^\circ$	For $M_0 = 120^\circ$	For $M_0 = 180^\circ$
0	∞ days	∞ days	∞ days	∞ days
.1	501.2	553.0	726.0	933.7
.2	329.3	421.1	620.0	854.0
.3	230.7	349.6	573.7	821.0
.4	163.1	302.2	550.1	805.1
.5	113.0	285.9	537.3	796.0
.6	74.9	273.1	530.5	791.5
.7	45.5	266.5	527.3	789.3
.8	23.4	263.7	525.8	788.4
.9	7.7	262.7	525.3	788.0
.95	2.8	262.6	525.2	788.0

It follows from this table that the expansions of the rectangular coordinates and the triangles in power-series in the time are, in the case of the asteroid orbits where e rarely equals .3, valid for more than 200 days whatever position the body may have in its orbit. The time-interval for $M_0 = 0$ decreases very rapidly as e approaches unity, while the change is much less for other values of M_0. The physical reason for this is that the angular velocity changes most rapidly at the perihelion as the eccentricity varies. Since the major axis is kept constant the limit of the orbit as e approaches unity is a straight line of length $2a$.

Perhaps a more satisfactory idea of the effect of a change in the eccentricity upon the radius of convergence can be obtained by keeping the perihelion distance constant. The physical changes in the vicinity of the perihelion point are differences in curvature in the orbit, and the velocity of the body in its orbit. The curvature decreases continually as e varies from zero to infinity, while the velocity continually increases. The decrease in curvature tends to increase the radius of convergence, but the increase of velocity decreases it much more. The major axis is expressed in terms of q and e by

$$\begin{cases} a = \dfrac{q}{1-e} & \text{for} \quad e<1 \\[1ex] p = 2q & \text{for} \quad e=1 \\[1ex] a = \dfrac{q}{e-1} & \text{for} \quad e>1 \end{cases} \qquad (36)$$

Taking $q = 1$ as an example, which will be fairly representative of many comets' orbits, the following table has been computed, taking $M_0 = 0$, or $t_0 = T$. For a different value of t_0 all of the corresponding radii, which are expressed in days, are larger.

Table IV. $\log R_{t_0}$ and R_{t_0} in days for $t_0 = T$ and $q = 1$.

e	$\log R_{t_0}$	R_{t_0}	e	$\log R_{t_0}$	R_{t_0}
0	∞	∞ days	1.0	1.7388	54.8
.1	2.1338	136.1	1.05	1.7309	53.8
.2	2.0281	106.7	1.1	1.7185	52.3
.3	1.9606	91.3	1.2	1.6994	50.1
.4	1.9101	81.3	1.3	1.6853	48.5
.5	1.8698	74.1	2.0	1.6001	39.9
.6	1.8364	68.6	5.0	1.4093	25.7
.7	1.8070	64.1	10.0	1.2614	18.3
.8	1.7819	60.5	100.0	0.7641	5.8
.9	1.7586	57.4	1000.0	0.2643	1.8
.95	1.7153	55.6			

5. *Zeroes of the Triangles.* The conditions under which the coordinates and triangles may be expanded into converging series in the time have been determined for all classes of conics. But in practice the ratios of the expansions of two triangles, instead of the expansions of single triangles, are used. Hence poles of the functions exist for all those values of the argument for which the denominator vanishes. It should be remarked that the expressions for the triangles contain factors which vanish with the time intervals. In the ratios of the triangles the quotient of these factors is taken separately, and the remaining series in the denominator divided into the corresponding one in the numerator. Thus, t_2 being the origin for the expansions,

$$\frac{x_2 y_3 - y_2 x_3}{x_1 y_2 - y_1 x_2} = \frac{t_3 - t_2}{t_2 - t_1} \times \qquad (37)$$
$$\left\{ 1 - \frac{k^2 (t_3 - t_2)^2 - k^2 (t_2 - t_1)^2}{6 r_2^3} + \frac{k^2 (t_3 - t_2)^2 + k^2 (t_2 - t_1)^2}{4 r_2^4} \frac{d r_2}{dt} + \cdots \right\}$$

Hence the problem is to find the zeroes of $\dfrac{x_i y_j - y_i x_j}{t_j - t_i}$. These singularities do not cancel those which were previously found, for the former were all branch points.

It is most convenient in finding the zeroes of the expression above to find them first in terms of the true anomaly. This is easily anticipated, since the triangle vanishes when

the heliocentric motion in the interval is any multiple of π. Of course, this would be the answer to the problem if it were proved that the triangles could not vanish for any complex values of the argument which have smaller moduli. Instead of using t_2-t_1 in the denominator v_2-v_1 will serve the purpose equally well, for these two quantities vanish simultaneously. The expression in question becomes in polar coordinates

(38)
$$\frac{x_1 y_2 - y_1 x_2}{v_2 - v_1} = r_1 r_2 \frac{(\cos v_1 \sin v_2 - \sin v_1 \cos v_2)}{v_2 - v_1} = r_1 r_2 \frac{\sin(v_2 - v_1)}{v_2 - v_1} = 0$$

Now $r_1 = \dfrac{p}{1 + e\cos v_1}$ which cannot vanish for any finite value of v_1, either real or complex. A similar statement applies to r_2. Hence the condition for poles of the ratios of the triangles becomes

(39)
$$\frac{\sin(v_2 - v_1)}{v_2 - v_1} = 0$$

The variable v, depending on the complex variable t, will in general be complex. Suppose it has the form

$$v = \theta + \sqrt{-1}\, q$$

then (39) becomes

$$\frac{\sin\{(\theta_2-\theta_1) + \sqrt{-1}(q_2 - q_1)\}}{\theta_2-\theta_1 + \sqrt{-1}(q_2-q_1)} = \frac{(\theta_2-\theta_1)\sin(\theta_2-\theta_1)\cosh(q_2-q_1)}{(\theta_2-\theta_1)^2+(q_2-q_1)^2}$$
$$+ \frac{(q_2-q_1)\cos(\theta_2-\theta_1)\sinh(q_2-q_1)}{(\theta_2-\theta_1)^2+(q_2-q_1)^2}$$
$$+ \frac{\sqrt{-1}(\theta_2-\theta_1)\cos(\theta_2-\theta_1)\sinh(q_2-q_1)}{(\theta_2-\theta_1)^2+(q_2-q_1)^2}$$
$$- \frac{\sqrt{-1}(q_2-q_1)\sin(\theta_2-\theta_1)\cosh(q_2-q_1)}{(\theta_2-\theta_1)^2+(q_2-q_1)^2} = 0$$

Equating the real and imaginary parts separately to zero, the conditions for poles are found to be

$$\left.\begin{aligned} \frac{(\theta_2-\theta_1)\sin(\theta_2-\theta_1)\cosh(q_2-q_1)+(q_2-q_1)\cos(\theta_2-\theta_1)\sinh(q_2-q_1)}{(\theta_2-\theta_1)^2+(q_2-q_1)^2} &= 0 \\ \frac{(\theta_2-\theta_1)\cos(\theta_2-\theta_1)\sinh(q_2-q_1)-(q_2-q_1)\sin(\theta_2-\theta_1)\cosh(q_2-q_1)}{(\theta_2-\theta_1)^2+(q_2-q_1)^2} &= 0 \end{aligned}\right\} \quad (40)$$

From these equations it is found that

(41)
$$\left\{\begin{aligned} \cos(\theta_2-\theta_1)\ \sinh(q_2-q_1) &= 0 \\ \sin(\theta_2-\theta_1)\ \cosh(q_2-q_1) &= 0 \end{aligned}\right.$$

after dividing out the factor

$$\frac{(\theta_2-\theta_1)^2+(q_2-q_1)^2}{(\theta_2-\theta_1)^2+(q_2-q_1)^2}$$

The solutions of the first equation are

$$\theta_2-\theta_1 = (2\nu+1)\frac{\pi}{2} \qquad \text{or} \qquad q_2-q_1 = 0$$

The second equation becomes in these respective cases

$$\cosh(q_2-q_1) = 0 \quad , \quad \sin(\theta_2-\theta_1) = 0$$

The first equation cannot be satisfied, and the solutions of the second are

$$\theta_2-\theta_1 = \nu\pi$$

Consequently the solutions of (41) are

(42)
$$\theta_2-\theta_1 = \nu\pi \quad , \quad q_2-q_1 = 0$$

In the first equation ν is any integer except zero, for equations (40) are indeterminate for $\theta_2-\theta_1$ and q_2-q_1 simultaneously zero.*

The problem is to find the relation between t_2 and t_1, or more simply between M_2 and M_1, for which these poles occur. The solution of this problem is simple, since t is expressible in terms of v by finite equations in each of the three classes of conics which must be considered separately.

(a) *Case of Elliptic Motion.* The eccentric anomaly is expressed in terms of the true anomaly by means of the equation

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{v}{2}$$

Therefore

$$\left.\begin{aligned} \tan\frac{E_1}{2} &= \sqrt{\frac{1-e}{1+e}}\tan\tfrac{1}{2}(\theta_1+\sqrt{-1}\,q_1) \\ \tan\frac{E_2}{2} &= \sqrt{\frac{1-e}{1+e}}\tan\tfrac{1}{2}(\theta_1+\nu\pi+\sqrt{-1}\,q_1) \end{aligned}\right\} \quad (43)$$

If ν is odd

$$\tan\frac{E_2}{2} = -\sqrt{\frac{1-e}{1+e}}\cot\tfrac{1}{2}(\theta_1+\sqrt{-1}\,q_1) = -\frac{1-e}{1+e}\cot\frac{E_1}{2}$$

If ν is even

$$\tan\frac{E_2}{2} = \tan\frac{E_1}{2}$$

Then the mean anomalies are computed from the equations

$$\left.\begin{aligned} M_1 &= E_1 - e\sin E_1 \\ M_2 &= E_2 - e\sin E_2 \end{aligned}\right\} \quad (44)$$

and the time interval from

$$t_2 - t_1 = \frac{M_2 - M_1}{n} = \frac{a^{\frac{3}{2}}(M_2 - M_1)}{k} \quad (45)$$

* Dr. HAMILTON gave these results for parabolic orbits, *loc. cit.*, p. 54.

In practice t_i and t_j represent two of t_1, t_2, t_3, and t_2 is taken as the origin of time for the expansions. Since t_2 is real it follows that v_2 is real, and then from second equation of (12) that if one of the epochs t_i, t_j is t_2 the other is real. The limit on the time-interval is that the heliocentric motion of the body during it shall be less than 180°. If $t_j - t_i = t_3 - t_1$ there are two quantities limited by but a single equation, and one of them may be taken arbitrarily. From (12) it follows that if one is real they both are, and the modulus becomes a minimum. Hence in all cases $q_i = q_j = 0$. The time-interval must be such that the heliocentric motion during it shall be less than 180°. This leads to the following interesting result. Suppose an observation has been made at t_1, and that two more are to be made; the radius of convergence of the reciprocal of the triangle does not depend upon the way in which the second observation divides the whole interval, at least except so far as the intervals $t_2 - t_1$ and $t_3 - t_2$ are limited by the branch points previously discussed.

(*b*) *Case of Parabolic Motion.* The formula for parabolic orbits is

(16)
$$t_j - t_i = \frac{p^{\frac{3}{2}}}{2k}\left\{\tan\frac{(v_j+\pi)}{2} - \tan\frac{v_i}{2} + \tfrac{1}{3}\left(\tan^3\frac{(v_j+\pi)}{2} - \tan^3\frac{v_i}{2}\right)\right\}$$

(*c*) *Case of Hyperbolic Motion.* The formulas for hyperbolic orbits are

$$(17)\quad\left\{\begin{aligned} \tan\frac{F_i}{2} &= \sqrt{\frac{e-1}{e+1}}\tan\frac{v_i}{2} \\ \tan\frac{F_j}{2} &= \sqrt{\frac{e-1}{e+1}}\tan\left(\frac{v_j+\pi}{2}\right) = -\sqrt{\frac{e-1}{e+1}}\cot\frac{v_j}{2} \\ M_i &= -F_i + e\sinh F_i \\ M_j &= -F_j + e\sinh F_j \\ t_j - t_i &= \frac{M_j - M_i}{n} = \frac{a^{\frac{3}{2}}(M_j - M_i)}{k} \end{aligned}\right.$$

In case the triangle between r_1 and r_3 occurs in the denominator the expansions of the ratios the triangles are never valid when the whole heliocentric motion in the interval of time $t_3 - t_1$ is as much as 180°. Notwithstanding this fact much space has been used in explaining the solutions of certain equations depending on these expansions in the case where this limit is exceeded.* If the heliocentric motion in the whole interval is less than 180°, the series are always convergent unless the intervals are still further limited by the branch points of the functions. The formulas given in section 3, or the tables of section 4, show when this will occur.

6. *Numerical Results.* For practical use a table giving $\log\left(\frac{M_3 - M_1}{k}\right)$ will be most serviceable since it depends only upon the arguments e and v_1. The time in days can be found by multiplying this quantity by $a^{\frac{3}{2}}$ in the case of elliptic and hyperbolic orbits, and by $\frac{p^{\frac{3}{2}}}{2}$ in the case of parabolic orbits. It will be sufficient to give the results for $v_1 = -90°$, $v_1 = 0$, and $v_1 = +90°$ in elliptic orbits, and for $v_1 = -90°$ in the case of parabolic and hyperbolic orbits.

TABLE V. $\log\left(\frac{M_3 - M_1}{k}\right)$.

e	$v_1 = -90°$	$v_1 = 0$	$v_1 = +90°$	e	$v_1 = -90°$
0	2.2616	2.2616	2.2616	1.0	2.1904
.1	2.2026	"	2.3132	1.05	0.3835
.2	2.1350	"	2.3593	1.1	0.8273
.3	2.0568	"	2.4000	1.2	1.2646
.4	1.9647	"	2.4362	1.3	1.5158
.5	1.8510	"	2.4679	2.0	2.2444
.6	1.7161	"	2.4958	5.0	2.9594
.7	1.5358	"	2.5197	10.0	3.3476
.8	1.2789	"	2.5397	100.0	4.4199
.9	0.8349	"	2.5542	1000.0	5.4266
.95	0.3857	"	2.5595	–	–

While the preceding table enables one to compute very simply the limit with sufficient accuracy in any case that can arise, it fails to give one at a glance the way the limit changes with the eccentricity. For this purpose the following table was computed keeping the perihelion distance equal to unity.

TABLE VI. $t_3 - t_1$ in days for $q = 1$, $v_1 = -90°$.

e	$t_3 - t_1$	e	$t_3 - t_1$
0	182.6	1.0	219.2
.1	186.8	1.05	216.3
.2	190.8	1.1	212.5
.3	194.7	1.2	205.6
.4	198.4	1.3	199.5
.5	202.1	2.0	175.6
.6	205.7	5.0	113.9
.7	209.1	10.0	82.5
.8	212.5	100.0	26.7
.9	215.9	1000.0	8.5
.95	217.4		

These intervals for $v_1 = -90°$ are the shortest for which any of the three triangles can vanish when the perihelion distance is unity. If the interval between the first and last observations is less than the number given in the table no trouble can arise from this singularity.

The real question is whether the radius of convergence of the quotient of two triangles is determined by the poles or the branch points. To settle this question for that part of the orbit where the radius of convergence is least compare Tables IV and VI. It is necessary to make some assumption regarding the way the second observation di-

* OPPOLZER, *Bahnbestimmung*, pp. 79, 93. WATSON, *Theoretical Astronomy*, pp. 186–7.

vides the whole interval. For simplicity suppose it divides it into two equal parts. Then, to get the whole interval as determined by the branch points, it is necessary to multiply the numbers of Table IV by two, while those in VI are to be taken as they stand. It is at once seen that so long as $e < .2$ the true radius is determined by the poles defined by the vanishing of the triangle in the denominator; and so long as $e \geqq .3$ the true radius of convergence is determined by the branch points which were found in section 3. This is true for all orbits as well as for $q = 1$ since the limits in both cases depend upon the linear dimensions of the orbit in the same manner.

If the triangle contained between r_1 and r_3 does not occur in the denominator the numbers in Table VI should be multiplied by two, for the whole interval will be about twice as long as $t_3 - t_2$ or $t_2 - t_1$. In this case the branch points determine the true radius of convergence if $e \geqq .1$. In no case does the convergence of the series persist until the whole heliocentric motion gets near 180° except when $e < .2$.

7. *Comparison with the Series used in the Astronomical Journal*, No. 510. In the paper in *loc. cit.*, solving the second part of the problem of determining the elements of an unknown orbit, certain series were employed which were shown to converge if the heliocentric motion in the intervals $t_2 - t_1$ and $t_3 - t_2$ were not too great. The precise radius of convergence for various values of the eccentricity was given in Table I, p. 48. The question of interest is whether the series used in that paper converge so long as those which express the ratios of the triangles. In the determination of orbits the whole interval of heliocentric motion must be less than 180° as has been shown. Consequently, so long as the radius of convergence is greater than 90° for the series used in *A.J.* No. 510, the second part of the problem is solvable by the method developed there if the first part is by the usual method. It is seen from Table I, *loc. cit.* p. 48, that for the middle observation in any part of the orbit the series are valid over an interval greater than 180° if $e \leqq .1$; and, when $t_2 = T$, the interval is greater than 180° for all finite values of e. For $e = 1$, $t_2 = T$, the limit is precisely 360°.

It was shown in the preceding section that the true radius of convergence is defined by the poles as long as the eccentricity has a value less than .3. It follows from Table I, *A.J.* No. 510, that the series developed there for all $e < .3$ are valid for greater intervals than those giving the ratios of the triangles except when $v_2 > 120°$, in which case the limit is a little less for $e = .2$ and $e = .3$. Therefore no trouble can arise in the asteroid orbits in using the method of solving the second part of the problem of orbits which was developed in the former paper. It also follows from Table I, *loc. cit.*, and the results of this paper that in case of parabolic orbits the proposed series converge for greater intervals than those for the ratios of the triangles so long as the true anomaly at the time of the second observation does not exceed 90°. This is taking into consideration only the poles, and the branch points show that the radius of convergence of the series of the former paper are greater than those in this for $e = 1$ for values of v_2 considerably greater than 90°.

This is all that is desired from a practical point of view, for observations of comets are not usually made when the true anomaly is very great. The conclusion is that whenever the series for the ratios of the triangles are of practical use those developed in *A.J.* No. 510 for finding the elements may also be used. It was shown in the former paper that three terms of the series used in finding the elements give results accurate to the sixth place when $e = 1$, $q = 1$ and $t_3 - t_2 < 13$ days, $t_2 - t_1 < 13$ days. In the expression for the ratio of the triangle between r_1 and r_2 to that between r_2 and r_3 the term of the fourth degree contains $\frac{k^4 (t_3 - t_2)^4}{36\, r_2^6}$, the remainder of it vanishing if $t_3 - t_2 = t_2 - t_1$. If $t_3 - t_2 = 13$ days, $q = 1$, and $v_2 = 0$ the numerical value of this term is .00007, giving an error of seven units in the fifth place if it is omitted.

8. *The Convergence of the Series used in Other Methods.* The preceding investigations exhibit in a conspicuous manner the weakness of the OLBERS and GAUSS methods, which can be used only when the intervals of time between the observations are comparatively short. On the other hand the elements are better determined when the points on the orbit are not very near together. Thus, the computer finds himself limited in both directions, and in most cases he can secure satisfactory agreement between theory and observations only by differential corrections based on errors in an ephemeris. A thorough discussion of what intervals are most advantageous in the various possible cases which can arise in practice is much to be desired, and the results contained in this paper furnish a solid foundation for such an investigation.

Another question of much interest and practical importance is whether some of the other methods, which are short enough to be of practical value, are not free from the limitations to which the method of GAUSS is subject. Most conspicuous of these is that first developed by LAPLACE.*

The same general ideas have been followed out by VILLARCEAU in an exhaustive memoir,† which for some unexplained reason is not referred to by later writers using the same fundamental ideas. The process has been carried out to terms of higher order, but at the price of great complexity, by HARZER.‡ More recently LEUSCHNER has

* *Mécanique Céleste*, Vol. I, Part I, Book II, Chap. IV.

† *Annales de l'Observatoire Impérial de Paris*, Vol. III.

‡ *Astronomische Nachrichten*, No. 3371.

developed it* so as to make its practical application at every point as simple as possible. In a very suggestive preface to TISSERAND's *Leçons*, POINCARÉ has commented on the fundamental ideas in the methods of GAUSS and LAPLACE, and has shown that they are essentially the same in the first approximation.

In all of these expositions developments of the geocentric polar coordinates as power-series in the time are used. The present question relates to their convergence. Let the geocentric rectangular coordinates be ξ, η, ζ and the polar coordinates ρ, α, δ. The rectangular coordinates are related to the rectangular heliocentric coordinates by linear equations with constant coefficients, and they have, therefore, the same radius of convergence. The rectangular and polar coordinates are related by the equations

$$(48)\quad \begin{cases} \xi = \rho\cos\alpha\cos\delta \\ \eta = \rho\sin\alpha\cos\delta \\ \zeta = \rho\sin\delta \end{cases}$$

From these equations it follows that

$$(49)\quad \begin{cases} \rho = \sqrt{\xi^2+\eta^2+\zeta^2} \\ \alpha = \tan^{-1}\left(\dfrac{\eta}{\xi}\right) \\ \delta = \tan^{-1}\dfrac{\zeta}{\sqrt{\xi^2+\eta^2}} \end{cases}$$

The singular points of these equations are defined by

$$(50)\quad \xi^2+\eta^2+\zeta^2 = 0 \quad , \quad \frac{\eta}{\xi} = 1 \quad , \quad \frac{\zeta}{\sqrt{\xi^2+\eta^2}} = 1$$

If these equations were expressed in terms of the elements of the body and the earth, and of the time, and if the resulting equations were solved for the time (using complex values) the singular points of the functions would be found. There would be no serious practical difficulty in the matter, since these coordinates are expressible linearly in terms of the heliocentric coordinates which are expressed in terms of the time for all conics by means of well-known equations. It is not intended to enter into the details of this matter here. It is sufficient to point out that when $e > .3$ the developments of ξ, η, and ζ, and consequently of ρ, α, and δ converge only so long as the expressions for the ratios of the triangles converge. In this case the LAPLACIAN method has no advantage from this point of view over the GAUSSIAN. It is to be noted further that the singularities (50) depend not only upon the eccentricity and parameter of the orbit in question, and upon the position of the body in its orbit, but also upon the elements which define the plane of the orbit, and the coordinates of the earth.

It is evident that these many parameters might occur in such a manner that one of equations (50) would be fulfilled for a time-interval of very small modulus, when the method would fail for even short intervals between the observations. For example, the expansions would soon fail if the body were near the pole. This is only a fault of the method, and not an inherent difficulty, for a change to ecliptic coordinates will avoid it. When the eccentricity is less than .3 the LAPLACIAN expansions may converge longer than the GAUSSIAN, but there is no guarantee of it in general. The conclusion is that the two methods are subject to the same general restrictions, though in special cases each may be better than the other.

There are three problems worthy of solution by the more powerful methods of modern mathematics. (*a*) To find under what conditions the data furnished by three observations are *essentially* insufficient to define the elements of the orbit; (*b*) to find under what conditions the same data are insufficient to define the elements by the LAPLACIAN method; and (*c*), the same problem for the GAUSSIAN method. Perhaps the answer in the three cases is the same. Dr. HAMILTON has answered (*a*) in many, if not all, cases of parabolic orbits in a memoir still unpublished, by a direct discussion of the Jacobian of the coordinates with respect to the elements.

* *Publications of the Lick Observatory*, Vol. VII, Part I.

The University of Chicago, 1903 *March* 28.

OBSERVATION OF TURNER'S "*NOVA*," (2387 — *GEMINORUM*).*

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

BY C. W. FREDERICK.

[Communicated by Captain COLBY M. CHESTER, U.S.N., Superintendent.]

1903 W.M.T.	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	Red. to App. Place	
March 31 8h	29 , 6	+1m 17s.54	−2′ 20″.9	6h 38m 1s.73	+30° 2′ 21″.8	+1s.18	−6″.9

Mean Place of Comparison-Star for the beginning of the year.

α	δ	Authority
6h 36m 43s.01	+30° 4′ 49″.6	½ [Leiden A.G. 2783+Cambridge. Eng., A.G. 3447]

The comparisons in α were made by transits.

The position of the *Nova* reduced to 1903.0 is: 6h 38m 0s.54 +30° 2′ 28″.7.

* *From Supplement to No. 536.*

PHOTOGRAPHIC OBSERVATION OF THE MINOR PLANET (60) *ECHO*.*

OBTAINED WITH THE 6-INCH STAR-CAMERA AT THE U. S. NAVAL OBSERVATORY.

By G. H. PETERS.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

The minor planet (60) *Echo*, which was discovered by FERGUSON at this Observatory, was picked up in 1899 by photography. Before its elements were determined it was considered a new discovery, but subsequently was identified as *Echo*.

This asteroid was photographed at the Naval Observatory on April 17, 1903, on a plate exposed from 12^h to 13^h 15^m W.M.T., and the following correction to the position in the *Berliner Jahrbuch* determined.

Correction $\Delta\alpha$ +1^s.7 $\Delta\delta$ −6″.

* *From Supplement to No. 536.*

PHOTOGRAPHIC OBSERVATIONS OF MINOR PLANETS.

OBTAINED WITH THE 6-INCH STAR CAMERA AT THE U. S. NAVAL OBSERVATORY.

By G. H. PETERS.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent.]

The appended corrections to the *Berliner Jahrbuch* positions were determined from photographic trails of the asteroids given below. In the case of (236) *Honoria* no observations are noted since 1890.

Asteroid	Date	Correction	
(83) *Beatrix*	April 27, 1903	$\Delta\alpha$ −2.3	$\Delta\delta$ +13
(236) *Honoria*	April 28, 1903	−6.6	+28
(335) *Roberta*	April 28, 1903	−2.0	+ 3

OBSERVATIONS OF COMET *a* 1903 (*GIACOBINI*).*

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY.

By C. W. FREDERICK.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1903	Wash. M.T.	✱	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
	h m s			m s	′ ″	h m s	° ′ ″			s	″
Jan. 21	6 39 31	1	9 , 1	+0 58.85	+1 24.0	23 0 6.61	+ 2 47 16.0	9.574	0.724	−0.29	+2.7
22	6 33 43	2	9 , 4	1 53.79	0 49.5	23 1 10.86	+ 3 2 7.8	9.570	0.722	0.28	+2.6
23	6 51 1	3	20 , 8	+3 21.25	−0 59.6	23 2 17.39	+ 3 16 55.0	9.595	0.723	0.30	+2.7
30	6 46 22	4	*d*10 , 10	+0 31.59	+9 4.7	23 10 41.63	+ 5 7 13.6	9.611	0.716	0.27	+2.2
Feb. 5	6 46 29	5	10 , 8	+1 15.73	−4 17.7	23 18 53.17	+ 6 53 3.2	9.627	0.711	0.29	+1.8
6	6 40 51	6	*d* 8 , 10	0 44.94	−3 18.8	23 20 21.02	+ 7 11 40.9	9.624	0.708	0.26	+1.8
9	7 8 26	7	30 , 8	+1 46.03	+4 0.4	23 24 55.10	+ 8 9 52.9	9.649	0.715	0.25	+1.7
12	7 0 14	8	30 , 6	+3 46.97	−3 13.3	23 29 42.19	+ 9 10 23.0	9.650	0.711	0.23	+1.6
17	7 17 23	9	*d*10 , 10	−0 13.45	6 25.0	23 38 17.60	+10 57 26.9	9.663	0.716	0.22	+1.1
19	7 15 19	10	15 , 3	+7 57.27	+2 22.0	23 41 55.34	+11 41 49.5	9.665	0.716	0.24	+1.1
20	6 42 55	11	*d*10 , 10	−0 13.15	1 44.3	23 43 44.40	+12 3 52.9	9.654	0.698	0.20	+0.9
22	6 50 58	12	*d*10 , 10	0 25.19	7 0.2	23 47 33.20	+12 49 23.7	9.661	0.702	0.19	+0.8
23	7 3 13	13	19 , 4	+3 4.13	−1 15.8	23 49 30.53	+13 12 21.5	9.667	0.709	0.19	+0.8
25	7 10 53	14	*d*10 , 10	+0 26.81	2 17.2	23 53 28.22	+13 57 59.9	9.674	0.713	0.18	+0.6

Mean Places of Comparison-Stars for the beginning of the year.

✱	α	δ	Authority	✱	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	22 59 8.05	+2 46 19.3	Albany, A.G. 7960	8	23 26 25.45	+ 9 13 34.7	Leipzig II, A.G. 11676
2	23 3 4.93	+3 2 51.7	Albany, A.G. 7980	9	23 38 31.28	+11 3 50.8	Leipzig I, A.G. 9412
3	22 58 56.14	+3 17 51.9	Albany, A.G. 7957	10	23 33 58.31	+11 39 26.4	Leipzig I, A.G. 9387
4	23 10 10.31	+4 58 6.7	Albany, A.G. 8028	11	23 43 58.05	+12 5 36.3	Leipzig I, A.G. 9451
5	23 17 37.73	+6 57 19.1	Leipzig II, A.G. 11622	12	23 47 58.58	+12 56 23.1	Leipzig I, A.G. 9469
6	23 20 33.22	+7 11 57.9	Leipzig II, A.G. 11641	13	23 46 26.59	+13 13 36.5	Leipzig I, A.G. 9461
7	23 23 9.32	+8 5 50.8	Leipzig II, A.G. 11652	14	23 53 1.59	+14 0 16.5	Leipzig I, A.G. 9505

The first observation by W. W. DINWIDDIE. Comparisons in α were directly determined by micrometer when marked *d*.

* *From Supplement to No. 536.*

OBSERVATIONS OF COMET *d* 1902 (GIACOBINI).*

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA,

By T. McN. SIMPSON, Jr.

1903 Charl. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ (α)	log pΔ (δ)	Red. to App. Pl. (α)	Red. to App. Pl. (δ)
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 18 10 59 8	1	– , 9	. . .	−0 53.5	. . . .	+19 22 53.6	. .	9.516	. .	−11.1
11 11 18	1	6 ,	+2 21.66	. . .	6 36 37.06	. . . .	9.164	. .	+1.82	. .
20 11 28 40	2	, 5	. .	−4 51.9	. . . .	+19 58 16.5	. .	9.523	. .	−11.1
11 47 50	2	11 ,	−0 46.52	. . .	6 36 30.78	. . . .	9.562	. .	+1.83	. .
21 11 18 37	3	– , 12	. . .	−1 36.0	. . . .	+20 16 15.9	. .	9.513	. .	−10.9
23 9 13 57	4	– , 8	. . .	+1 15.2	. . . .	+20 49 23.1	. .	9.420	. .	−10.7
9 15 10	4	*d*11 , –	−0 5.96	. . .	6 36 33.66	. . . .	9.028	. .	+1.79	. .
24 9 41 12	5	8 , 8	−1 26.92	+3 39.0	6 36 38.12	+21 6 44.4	9.228	9.428	+1.78	−10.7
25 11 20 38	6	8 , 8	+3 7.98	+2 34.3	6 36 44.31	+21 24 44.1	9.552	9.513	+1.75	−10.4
26 10 17 17	7	*d*16 , 12	−0 20.32	−0 22.3	6 36 51.64	+21 40 40.3	9.407	9.499	+1.75	−10.3
Mar. 3 9 51 56	8	12 , 8	−1 12.11	+1 59.6	6 37 51.23	+23 0 59.1	9.390	9.411	+1.67	− 9.9
4 11 33 31	9	16 , 10	−0 19.22	−1 18.2	6 38 12.32	+23 17 36.4	9.619	9.532	+1.66	− 9.8

Mean Places of Comparison-Stars for 1903.0.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	6 34 13.58	+19 23 58.2	A.G. Berlin, A. 2297	6	6 33 34.61	+21 22 20.2	A.G. Berlin, B. 2505
2	6 37 15.47	+20 3 49.5	A.G. Berlin, B. 2544	7	6 37 10.21	+21 41 12.9	A.G. Berlin, B. 2543
3	6 36 11.92	+20 18 2.8	A.G. Berlin, B. 2533	8	6 39 4.67	+22 56 9.1	A.G. Berlin, B. 2564
4	6 36 37.83	+20 48 18.9	A.G. Berlin, B, 2535	9	6 38 59.88	+23 19 34.4	A.G. Berlin, B. 2563
5	6 38 3.26	+21 3 16.1	A.G. Berlin, B. 2552				

NOTES: *d* refers to direct micrometrical measurements. March 4 — Comet faint, observations interrupted by clouds.

Charlottesville, Va.

* *From Supplement to No. 536.*

RESULTS OF OBSERVATIONS WITH THE ZENITH TELESCOPE, FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA.

By C. L. DOOLITTLE.

The following series is a continuation of that found in the *Astronomical Journal* of October 16, 1891 (No. 509).

The value of the constant of aberration resulting from this series is 20″.513 ±.009.

$$\varphi = 39^\circ\ 58' +$$

1901	IV ″	No.	I ″	No.
Oct. 1	1.87	8	..	..
3	2.01	9	1.97	8
4	2.06	9	2.00	8
5	1.87	9	1.96	6
6	2.19	9	2.12	9
7	2.03	9	1.87	10
8	1.93	9	1.94	9
10	1.93	9	..	..
15	2.15	9	1.81	8
16	1.75	9	2.02	10
17	2.20	8	1.88	10
18	1.97	9	..	..
19	2.18	8	2.17	10
21	2.04	9	2.15	10
23	1.99	9	2.14	10
24	2.06	9	1.94	10
25	2.06	9	2.01	10
26	1.90	9	2.14	10

1901	IV ″	No.	I ″	No.
Oct. 27	2.07	9	2.10	10
29	1.97	9	1.92	10
30	2.09	9	2.13	5
Nov. 1	..	..	2.06	10
2	2.19	9	1.99	10
3	2.19	9	2.00	10
6	2.00	9	2.07	10
7	1.88	8	..	..
8	..	..	2.09	10
9	1.96	8	2.23	10
10	2.06	9	2.06	10
12	2.10	9	..	..
14	1.95	8	..	..
15	..	..	1.98	10
19	2.10	9	2.14	10
20	1.98	9	1.98	10
21	1.96	9	2.01	10
22	1.90	9	2.19	6

1902	I ″	No.	II ″	No.
Jan. 25	2.18	9	..	..
27	..	..	2.31	10
28	2.39	7	..	..
30	2.24	10	..	..
Feb. 2	2.20	10	2.15	10
3	2.07	10	..	..
4	2.20	10	2.13	9
5	1.81	10	..	..
6	2.11	10	..	..
7	2.29	10	2.36	10
8	2.21	10	2.06	10
9	2.32	4	..	..
10	2.35	10	2.39	10
11	1.88	7	2.06	2
13	2.28	10	2.19	10
14	2.30	10	2.22	2
15	2.47	10	2.24	10
18	2.30	10	2.16	10
19	2.49	10	2.31	10
22	..	..	2.21	10
23	2.24	10	..	..
24	1.94	4	..	..
Mar. 1	1.95	5	..	..
2	2.19	6	..	..
3	2.22	10	..	..

1902	I ″	No.	II ″	No.
Mar. 6	2.21	10	2.27	10
9	2.16	3	..	..
10	2.33	10	2.41	10
11	1.87	10	2.02	8
14	..	..	2.23	5

1902	II ″	No.	III ″	No.
May 5	2.40	10	2.49	9
8	2.08	9	2.14	10
9	2.60	10	2.42	10
11	2.34	10	2.35	10
13	..	..	2.46	10
14	2.44	10	2.32	10
15	2.17	10	2.26	10
17	2.26	6	..	..
19	2.36	9	2.24	10
21	..	..	2.00	10
22	2.23	10	2.19	3
24	2.06	10	..	..
28	2.30	10	2.13	10
29	2.09	10	2.03	10
30	2.13	9	2.22	10
31	1.98	10	2.18	10
June 1	2.03	10	2.07	9

1902		II ″	No.	III ″	No.
June	2	2.00	6	..	..
	4	2.51	7	2.45	9
	5	2.10	10	2.48	9
	6	2.10	9	2.02	7
	7	..	..	2.16	10
	8	2.48	10	2.45	8
	9	1.92	8	2.03	10
	10	2.14	5	..	..
	12	2.17	6	..	..
	17	..	..	2.23	10
	19	..	..	2.21	10
	21	..	..	2.25	5
	22	..	..	2.34	10
	23	..	..	2.36	10
	24	..	..	1.99	8
	27	..	..	2.32	2
July	4	..	..	2.01	2

1902		III ″	No.	IV ″	No.
July	8	2.18	10	2.16	7
	10	2.29	10	2.53	9
	11	2.01	10	2.02	8
	12	2.00	10	2.13	9
	13	2.02	10	1.98	9
	14	2.02	10	2.00	9
	15	1.99	8	2.16	9
	16	2.31	10	2.11	9
	17	2.00	10	2.02	9
	22	2.35	10	2.37	8
	23	2.15	2	..	..
	24	2.00	8	..	..
	27	2.02	10	1.94	9
Aug.	1	2.15	7	..	..
	2	2.16	8	2.00	9
	4	2.19	10	2.04	9
	6	..	..	2.37	9
	7	2.02	7	..	..
	8	2.10	1	2.24	9
	9	2.01	10	2.01	6
	11	2.12	10	1.98	9
	12	2.11	10	2.24	9
	14	2.14	10	1.97	9
	16	2.13	9	2.25	9
	17	2.20	10	2.01	9
	18	1.91	10	2.16	9

1902		IV ″	No.	I ″	No.
Aug.	30	1.89	9	..	..
Sept.	2	2.40	1	..	..
	4	2.19	9	..	..
	5	2.30	9	..	..
	6	2.05	9	..	..
	7	1.93	9	..	..
	10	2.12	9	..	..
	11	1.94	9	..	..
	13	2.18	9	..	..
	14	2.07	9	..	..
	15	2.28	9	..	..
	16	2.05	9	..	..
	17	2.18	3	..	..
Oct.	9	..	..	2.07	8
	12	1.98	9	..	..
	14	2.08	9	1.93	10
	15	2.07	9	1.91	10
	19	1.96	9	2.13	10
	20	2.17	8	1.98	10
	21	2.08	9	2.10	10
	22	1.95	9	1.89	10
Oct.	23	1.69	3	..	..
	25	2.17	9	..	..
	28	1.97	9	2.20	9
	29	..	..	1.88	10
	30	..	..	1.89	10
	31	2.06	9	1.95	10
Nov.	1	1.83	9	2.32	10
	2	1.92	9	1.90	9
	3	1.90	7	2.01	6
	7	2.13	7	2.00	7
	14	1.84	9	..	..
	15	2.06	8	..	..
	19	2.05	9	2.10	6
	20	1.97	9	..	..
	21	..	..	1.88	10
	23	1.85	9	1.83	10
	27	1.99	9	1.94	9
	28	1.88	9	1.91	9
	29	1.81	5	..	..
Dec.	1	1.90	8	2.06	10

Weighted Mean Date	ς	No.
1901–2	° ′ ″	
Oct. 9	39 58 1.974	184
Oct. 26	2.060	194
Nov. 13	2.040	192
Feb. 3	2.188	149
Feb. 16	2.261	145
Mar. 7	2.191	87
May 12	2.334	153
May 28	2.114	134
June 7	39 58 2.224	111

Weighted Mean Date	ς	No.
1902	° ′ ″	
June 21	39 58 2.234	57
July 16	2.116	213
Aug. 11	2.112	198
Sept. 9	2.098	103
Oct. 18	2.012	133
Oct. 28	2.008	130
Nov. 24	39 58 1.938	129

Whole number, 2312

MICROMETER OBSERVATIONS OF THE SATELLITE OF *NEPTUNE* IN 1901–1902 AND 1902–1903,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By E. E. BARNARD.

These measures of the satellite of *Neptune* are continuation of the observations previously printed in *A.J.* 508, etc.

The measures have all been made with a power of 700 diameters. It was thought there might be some gain in using this eyepiece, though a lower power would sometimes have shown the satellite better.

The season of 1902–1903 has been a very bad one, and the measures of the satellite in general have been difficult. An unusual amount of cloudy weather has cut down the number of nights on which observations could be made.

As in previous observations the center of *Neptune* was bisected; as the disc is not large, this can be done with great exactness, and it is believed that observations so made, in the case of this planet, are preferable to measures made from the limb or limbs.

The two sets of distance measures were made with the fixed and movable wires interchanged, so that they are essentially double distances.

Observations of the Satellite of *Neptune*, 1901–1902.

1901 *August*.

[illegible] time d h m s	p	s	Comp.
27 15 57 16	92.11	. .	6
16 3 7	. . .	16.06	4
16 9 40	. . .	15.81	4

September.

3 15 11 15	12.05	. .	6
15 17 59	. . .	12.81	5
15 53 40	. . .	13.00	5
16 15 21 57	302.75	. .	7
15 30 23	. . .	12.89	6
15 36 16	. . .	12.85	6
22 15 35 12	295.16	. .	7
15 40 7	. . .	13.60	5
15 43 34	. . .	13.51	5

1901 *September* — Cont.

d	90 time h m s	°	″	Comp.	
23	15 36 3	251.58	. .	6	
	15 41 21	. . .	15.72	4	
	15 45 19	. . .	15.67	4	
24	15 6 31	190.32	. .	9	
	15 13 30	. . .	10.61	5	
	15 17 28	. . .	10.51	5	

October.

d	90 time h m s	°	″	Comp.	
1	15 14 35	101.70	. .	5	
	15 20 54	. . .	15.47	5	
	15 25 44	. . .	15.50	5	
20	16 35 43	12.11	. .	6	
	16 41 9	. . .	13.50	4	
	16 45 7	. . .	13.09	5	
21	13 37 18	325.18	. .	7	
	13 43 12	. . .	11.52	5	
	13 47 33	. . .	11.32	5	
22	11 7 36	35.14	. .	6	The object observed,
	11 14 39	. . .	9.31	5	not the satellite.
	11 19 17	. . .	9.14	5	
29	13 14 22	218.76	. .	6	
	13 22 28	. .	12.16	4	
	13 28 2	. . .	12.59	4	

November.

d	90 time h m s	°	″	Comp.	
12	16 19 58	68.03	. .	5	
	16 24 19	. . .	16.36	4	
	16 27 36	. . .	16.32	4	
18	11 44 37	65.43	. .	6	
	11 49 5	. . .	15.67	4	
	11 52 9	. . .	15.69	4	
19	10 57 0	11.77	. .	6	
	11 2 29	. . .	11.20	4	
	11 6 8	. . .	10.91	4	
26	15 21 28	275.95	. .	6	
	15 26 43	. . .	16.31	4	
	15 30 25	. . .	16.30	5	

December.

d	90 time h m s	°	″	Comp.	
10	11 27 30	148.65	. .	7	
	11 33 8	. . .	10.90	4	
	11 36 35	. . .	11.31	4	
	11 41 24	148.17	. .	5	
15	9 42 0	229.16	. .	6	
	9 49 6	. . .	13.94	4	
	9 51 1	. . .	14.03	4	
16	11 26 4	138.90	. .	6	
	11 31 18	. . .	12.38	4	
	11 35 36	. . .	12.08	5	

1901 *December* — Cont.

d	90 time h m s	°	″	Comp.	
17	10 9 2	88.45	. .	5	
	10 14 51	. . .	16.90	5	
	10 19 1	. . .	17.15	5	
22	8 11 13	138.63	. .	7	Satellite difficult.
	8 19 19	. . .	12.48	5	Seeing excessively
	8 24 24	. . .	10.86	5	bad.
23	8 40 1	85.59	. .	4	
	9 33 1	84.65	. .	5	Question if the satel-
	9 39 48	. . .	16.86	4	lite. There was a
	9 42 42	. . .	16.96	4	fainter object 2″ s.f.
	9 47 41	84.26	. .	6	this.
29	9 54 12	78.59	. .	6	
	9 59 37	. . .	17.19	5	
	10 4 49	. . .	17.10	5	
30	8 17 29	31.13	. .	7	
	8 25 57	. . .	12.63	4	
	8 29 25	. . .	12.92	5	

1902 *January.*

d	90 time h m s	°	″	Comp.	
2	8 10 43	206.83	. .	8	
	8 17 39	. . .	11.78	4	
	8 21 21	. . .	12.08	4	
5	9 42 50	19.16	. .	6	
	9 47 41	. . .	11.34	4	
	9 51 11	. . .	11.64	4	
10	9 32 44	69.51	. .	6	
	9 38 11	. . .	16.45	4	
	9 42 15	. . .	16.69	4	
12	8 24 5	291.85	. .	7	
	8 26 54	. . .	14.01	4	
	8 32 12	. . .	14.03	4	
13	8 2 32	249.83	. .	6	
	8 7 21	. . .	16.14	4	
	8 10 49	. . .	16.41	4	
18	7 52 7	286.27	. .	6	
	7 55 57	. . .	14.85	4	
	7 58 50	. . .	14.80	4	
24	11 41 50	273.06	. .	5	
	11 46 53	. .	16.41	4	
	11 50 44	. . .	16.33	4	
27	7 44 48	97.15	. .	6	Extremely difficult.
	7 50 46	. . .	16.04	5	
	7 54 12	. . .	16.56	5	
31	7 32 2	233.19	. .	5	
	7 36 42	. . .	14.44	4	
	7 40 30	. . .	14.59	4	

1902 *February.*

	90 time			Comp.	
d	h m s	°	″		
2	7 25 23	93.12	. .	5	Excessively difficult.
	7 32 39	. . .	16.34	4	
	7 39 7	. . .	16.65	4	
7	7 13 16	145.63	. .	6	
	7 19 27	. . .	11.53	5	
	7 23 39	. . .	11.13	5	
8	6 49 19	88.66	. .	6	
	6 54 55	. . .	16.83	4	
	6 58 32	. . .	16.68	4	
15	7 0 6	37.53	. .	6	
	7 5 10	. . .	13.28	4	
	7 8 11	. . .	12.96	4	
17	6 42 2	261.50	. .	7	
	6 49 1	. . .	16.59	5	
	6 53 41	. . .	16.59	5	
24	6 48 50	204.25	. .	7	
	6 53 57	. . .	11.70	4	
	6 57 25	. . .	11.72	4	
25	6 44 52	120.49	. .	7	
	6 49 9	. . .	12.98	4	
	6 51 56	. . .	13.23	4	
	March.				
17	8 32 45	341.65	. .	9	
	8 39 34	. . .	10.14	5	
	8 42 54	. . .	10.31	5	
18	8 19 35	274.38	. .	6	
	8 24 35	. . .	15.59	4	
	8 28 24	. . .	15.74	4	
24	7 42 45	270.59	. .	7	
	7 48 59	. . .	15.84	5	
	7 53 4	. . .	16.17	5	
25	7 54 43	227.00	. .	9	
	8 2 40	. . .	14.28	4	
	8 7 19	. . .	13.69	5	
	April.				
6	7 42 4	213.17	. .	7	Single distances: very difficult, and lost in clouds.
	7 49 1	. . .	12.53	5	
8	7 54 55	78.46	. .	7	
	8 0 1	. . .	16.26	5	
	8 3 26	. . .	16.21	5	
13	7 51 17	119.12	. .	6	
	7 56 28	. . .	13.50	5	
	8 0 6	. . .	13.27	5	
14	7 36 33	73.84	. .	6	
	7 41 45	. . .	16.24	4	
	7 45 32	. . .	16.06	4	
15	7 45 35	19.58	. .	5	
	7 49 56	. . .	11.07	4	
	7 53 6	. . .	10.78	5	

OBSERVATIONS OF THE SATELLITE OF *Neptune*, 1902–1903.

1902 *August.*

	90° time			Comp.	
d	h m s	°	″		
25	16 7 47	183.85	. .	5	Exceedingly faint, clouds.
	16 12 55	. . .	10.21	5	
	16 17 18	. . .	10.25	5	
	September.				
1	15 22 18	101.17	. .	8	Difficult.
	15 27 38	. . .	15.56	4	
	15 31 26	. . .	15.32	4	
8	16 21 51	51.54	. .	8	Excessively difficult
	16 29 56	. . .	14.05	5	
	16 34 2	. . .	14.20	5	
9	15 21 34	338.97	. .	7	
	15 28 39	. . .	10.75	4	
	15 32 47	. . .	10.77	4	
15	15 19 56	329.09	. .	7	
	15 27 44	. . .	11.18	5	Very faint and difficult; clouds.
	15 33 52	. . .	11.95	5	
16	15 12 6	273.21	. .	8	Satellite fairly well seen; observations good.
	15 17 13	. . .	16.09	4	
	15 22 3	. . .	15.79	4	
18	15 38 54	141.42	. .	7	Excessively difficult
	15 47 17	. . .	11.46	4	
	15 51 52	. . .	11.97	4	
29	14 41 55	126.34	. .	6	Sky fogging
	14 46 57	. . .	13.45	4	
	15 51 0	. . .	12.96	5	
	October.				
6	15 38 5	116.72	. .	6	
	15 43 4	. . .	14.20	4	
	15 46 57	. . .	14.51	5	
7	13 15 17	77.34	. .	6	Satellite very faint, seeing excessively bad.
	13 23 42	. . .	16.39	4	
	13 30 16	. . .	16.43	5	
13	15 4 27	69.44	. .	7	Satellite difficult
	15 8 50	. . .	16.25	4	
	15 12 28	. . .	15.94	5	
14	14 46 51	5.37	. .	8	Satellite very faint.
	14 54 7	. . .	11.78	6	
	14 59 2	. . .	11.06	5	
27	17 5 13	273.98	. .	10	Satellite very faint.
	17 12 16	. . .	16.93	4	
	17 16 20	. . .	16.65	4	
	November.				
21	11 2 50	30.54	. .	6	Satellite fairly well seen
	11 8 12	. . .	12.27	4	
	11 11 31	. . .	12.03	4	

1902 *December.*

d	90 time h m s	°	″	Comp.	
4	11 52 9	289.37	. .	7	Satellite well seen.
	11 57 15	. . .	11.99	5	
	15 1 11	. . .	15.02	5	
30	9 19 38	332.34	. .	8	Excessively difficult.
	9 27 33	. . .	11.98	4	
	9 31 32	. . .	11.73	6	

1903 *January.*

d	90 time h m s	°	″	Comp.	
12	9 52 57	261.86	. .	6	Question if the satellite: very faint.
	9 59 28	. . .	16.68	4	
	10 4 15	. . .	16.80	4	
19	9 28 24	202.34	. .	7	Very faint; in clouds.
	9 34 3	. . .	11.77	5	
	9 38 28	. . .	11.11	5	
20	7 49 38	123.08	. .	6	Satellite better seen than at any observations this season.
	7 55 16	. . .	13.22	5	
	7 58 58	. . .	13.16	5	

February.

d	90 time h m s	°	″	Comp.	
2	12 30 25	239.03	. .	6	Clouds stopp'd obsns. Excessively difficult.
9	9 24 58	344.22	. .	8	Satellite very faint.
	9 32 44	. .	10.76	5	
	9 37 37	. . .	10.78	5	

Yerkes Observatory, Williams Bay, Wis., 1903 April 15.

1903 *February* — Cont.

d	90 time h m s	°	″	Comp.	
16	8 45 43	272.99	. .	8	Satellite very difficult.
	8 54 16	. . .	16.13	5	
	8 57 56	. . .	15.79	5	
17	8 35 29	231.00	. .	9	
	8 39 29	. . .	14.09	4	
	8 42 47	. . .	13.70	4	
23	8 42 24	223.58	. .	6	Satellite easy.
	8 48 41	. . .	13.31	5	
	8 52 33	. . .	13.19	5	
24	9 39 17	136.30	. .	8	Satellite well seen.
	9 45 33	. . .	12.00	5	
	9 49 8	. . .	12.00	5	

March.

d	90 time h m s	°	″	Comp.	
2	8 16 41	132.06	. .	10	Satellite faint.
	8 22 36	. . .	12.57	5	
	8 26 13	. . .	12.49	5	
25	8 19 27	175.82	. .	7	Satellite well seen.
	8 26 34	. . .	10.63	5	
	8 31 1	. . .	10.58	5	
30	10 2 0	235.41	. .	5	
	10 6 5	. . .	14.57	4	
	10 9 25	. . .	14.15	5	

The times are all six hours slow of Greenwich.

COMET *b* 1903.

[From RITCHIE's Circular, No. 134, of May 8.]

A cable message from Dr. KREUTZ via Harvard College Observatory, received May 2, announced the discovery of a comet by GRIGGS of Thames, N.Z., on April 17, a position secured by Mr. TEBBUTT of Windsor, N.S.W., accompanying the announcement. The latter position is the following:

April 26.8617 Gr. M.T., R.A. $4^h\ 3^m\ 1^s.6$, Decl. $-16°\ 23'\ 25''$.

The daily motion of the object was given, $1^m\ 26'$ in R.A., and south $27'$ in Declination.

A later message gives the following orbit, computed by Dr. KREUTZ from observations of April 26, 29 and May 1.

ELEMENTS.

$T =$ 1903 March 25.51 Greenw. M.T.

$$\left.\begin{aligned}\pi &= 186°\ 41'\\ \Omega &= 213\ \ 15\\ i &= \ \ 66\ \ 30\end{aligned}\right\}\text{Mean Eq. 1903.0}$$

$q = 0.5135$

EPHEMERIS.

Gr. Midnight 1903	R.A. h m s	Decl. ° ′	Br.
May 9	5 14 44	−21 1	0.60
13	5 36 44	−22 3	
17	5 58 8	−22 54	
21	6 18 52	−23 35	0.38

Brightness at discovery = 1.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed May 14.*

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 539.

VOL. XXIII. BOSTON, 1903 JUNE 1. NO. 11

ON THE PHOTOGRAPHIC EFFICIENCY OF A 13-INCH REFLECTOR OF 20-INCHES FOCUS.

By J. M. SCHAEBERLE.

For certain lines of astronomical work the efficiency of the mirror described in this paper seems to be remarkably great. A brief review of the well-known theoretical and practical principles involved may be desirable before comparing the photographic results with the data given by powerful existing telescopes.

If I denotes the intensity of the image of a large celestial area as formed by a theoretically perfect lens or mirror having the focal-length F and diameter D, then according to the principles of geometrical optics we can write

$$I = a \cdot \frac{D^2}{F^2} \tag{1}$$

For a given celestial object the factor a in the above and following equations will be assumed to have the same value for all telescopes near the limit of photographic vision.

If I' is to be the intensity of the same surface in a second telescope whose known aperture is $D' = nD$, then the required focal length F' is given by

$$F' = nF\sqrt{\frac{I}{I'}} \tag{2}$$

When, however, the luminous area is very small the above formulas become sensibly inaccurate, and finally, when the image is that of a fixed star, they are no longer even approximately true.

Long ago AIRY demonstrated mathematically that according to the undulatory theory of light the diameter of the spurious disk of a star, as formed on the optical axis of a theoretically perfect telescope, varies inversely as the diameter of the image-forming surface. So that if, for instance, the effective diameter of the lens or mirror is doubled the image contracts to one-half its former diameter, resulting in a four-fold increase in the light-intensity.

For fixed stars we therefore have the expression,

$$I = a\frac{D^4}{F^2} \tag{3}$$

In order that the intensity of the same star in a second telescope shall be I' the value of F' must now be

$$F' = n^2F\sqrt{\frac{I}{I'}} \tag{4}$$

Equations (1) and (3) show that the ratio of the intensities of the same star in any two telescopes is n^2 times as great as the surface-intensity ratios in the same telescopes. So far as the contrast (between a star and a surface against which it appears projected) depends upon these ratios it may be said to vary with the area of the aperture; for if we assume the aperture to remain constant and the focal length to change, both star and surface would vary according to the same law, so that the increase in contrast due to a diminution of the focal length results simply from the smaller scale of the image. In the former case the change in contrast may be said to be real, in the latter case only apparent.

In visual work there are a number of serious objections to the plan of securing greater intensity by decreasing the focal length indefinitely to a certain limit. If in addition the image-forming surface is composed of a system of lenses, apparently insurmountable errors, due mainly to chromatic aberration, are introduced. Nearly all of these objections are removed when the parabolic reflector is used in connection with the photographic plate, and only such work undertaken which deals with images near the optical axis. The obstacles to be overcome in the attempt to secure the best results on a *very* small scale seem to be almost wholly of a mechanical nature. If a mirror can be figured with such perfection that the microscopic images on the photographic plate have angular diameters not much greater than the corresponding photographic images formed by a powerful telescope, then, with the aid of a microscope it will be possible to study certain special problems with even greater facility than can be done with any visual telescope.

As an effort towards determining how far the power of

a telescope can be increased by diminishing the scale, and still have the theoretical and practical advantages for certain kinds of work outweigh the disadvantages, an extreme case, or rather a case believed to be extreme at the time, was decided on for trial. It may be of interest to give a brief description of the instrument.

Two parabolic mirrors were figured. One with an aperture of 12 inches, and a focal length of 46 inches is, with a power of 360 diameters, used as a Newtonian guiding telescope for the second or principal mirror. This has a clear aperture of 13 inches, and a focal length of 20 inches. The tubes containing these mirrors are bolted together, and are carried by an equatorial mounting of the old English style.

For focussing, a battery of three objectives of a compound microscope is placed so close to the three points of support of the photographic plate, that the latter, or the ground-glass, can just be slipped into position. A small rectangular prism back of the lenses throws the rays to the eye-piece (at the side of the tube). The magnifying power is somewhat greater than 400 diameters, and the diameter of the visual field of view is about three minutes of arc.

As the "expense" item is a rather serious matter in a private undertaking of this nature, a number of deviations from the usual plans were made, and parts not necessary for securing the highest possible degree of accuracy were not finished.

In long exposures a good driving-clock is almost as essential as a well-figured mirror, or a good focal adjustment of the photographic plate. For this purpose a simple governor was made for an old eight-day clock movement, and a few other parts added, the chief one being a carefully cut ⅞-inch steel screw two feet long, with 24 threads to the inch. This screw revolves once in 14 seconds, and thus gives a horizontal motion to a Babbitt-metal nut. Two thin steel bands (whose ends are fastened to the nut and sector-arc of 43 inches radius respectively) make the connection between the clock and the hour axis. A third band on the sector is connected with a simple arrangement for producing a constant pull of about two pounds (a few ounces will move the telescope) to keep the other two bands taut. The clock has simply to overcome the friction in the nut due to the pull of two pounds. The clock-platform is mounted on three wheels, and perfect slow-motion in R.A. is secured by a slight pull on an endless rope, which turns a screw, and thus moves the clock horizontally along a tangent to the sector-arc. The clock can be made to run two hours without re-winding. These parts are all out of the way under the observing floor.

A small finder attached to the main tube serves its usual purpose well. Inclosing the whole is a cylindrical sheet-iron dome eight feet in diameter. The guiding star is always the brightest one to be found within a degree or two of the object to be photographed (the variations in the differential refraction etc. are practically insensible during the short time required for the exposure); the 12-inch mirror is tilted, and the eye-piece shifted laterally to suit each particular case. For more than two years I have been wholly occupied with these optical, mechanical and experimental efforts to increase the efficiency of the photographic telescope (along certain lines) without increasing its size.

The one great difficulty which limits the power of even the largest telescope long before its capabilities have been exhausted, results from the fact that near the limit of vision we have to deal with a luminous area (caused primarily by reflections in our atmosphere, and in a less degree by reflections in space, nebulous areas and faint stars. Instrumental defects tend to increase the trouble).

With a given illumination of the sky (due to any cause exterior to the instrument) it would seem that as soon as the image of this sky-background begins to show on the photographic plate the faintest stars which can ever be photographed with any telescope, under the same conditions of sky, have already made their impressions on the same plate, and are necessarily of greater intrinsic brightness than this background. To make these limiting impressions visible, the contrasts must be increased; assuming the exposure and development of the plate to be the best possible, this can only be done, it would seem, by increasing the diameter of the aperture.

Before the instrument was finished or even commenced, it seemed reasonable to admit that even if the theoretical requirements could be practically fulfilled, the large existing telescopes, of darker field, might in long exposures be able to reveal stars several magnitudes fainter than could be obtained with the contemplated instrument during the comparatively short time a plate could be exposed to advantage in its bright field.

When, therefore, the remarkable fact was made apparent that negatives exposed for less than five minutes with the 13-inch mirror revealed stars apparently beyond the reach of the 36-inch refractor of the Lick Observatory, and also revealed every star shown on a published photograph which had an exposure of two hours in the Crossley reflector (aperture 3 feet, focal length 17.5 feet), the result, although not wholly unlooked for, really exceeded expectations.

In experiments made for the purpose of finding some way to lessen the drawback of a bright field, photographic plates, both common and orthochromatic, varying in sensitiveness from the most rapid to the slowest, were employed, with the expected result that slow plates give the greatest contrasts, but always at the expense of increased exposure-time.

A serious objection to a very long exposure exists when, as in my case, the image of the guiding star is not made by the surface which forms the photographic image. In long exposures the varying stresses as the instrument revolves

on the hour axis may and often do cause a relative drift of images in the two telescopes sufficiently great to become sensible under high powers, thereby annulling to a certain extent the value of the result. The method of the sliding plate-holder used with so much success by COMMON, the Lick Observatory Astronomers, by RITCHIE of the Yerkes Observatory, and others, could evidently not be advantageously employed in the present case. Mr. ROBERTS, the English astronomer, who has done so much valuable work in celestial photography, uses a novel but costly method of his own. For making his remarkable photographs of the Milky-Way Prof. BARNARD used a small achromatic-guiding telescope strapped to a six-inch portrait-lens of about 30 inches focus, which gave a large field fairly well covered. The notable discoveries of Dr. MAX WOLF are made with similarly mounted, but larger, objectives.

The photographs taken with the 13-inch mirror referred to below are all made on commercial SEED plates, No. 27. A 5 x 7 plate is cut into 18 pieces, so that each plate is $\frac{5}{3}$ x $\frac{7}{4}$ inches; the negative proper is near the center of the plate, and ordinarily about 0.1 or 0.2 inch in diameter, which can be increased to 0.5 inch if so desired.

To determine the approximate magnitude of the faintest stars visible, a number of plates were exposed on certain regions covered by a chart which Prof. TUCKER made with the aid of the 36-inch refractor of the Lick Observatory. This chart shows stars down to the 17th magnitude; it is printed in the Publications of the Astronomical Society of the Pacific, No. 37.

In any given region of the chart following *α Leonis* all but the faintest stars are photographed with an exposure of one minute. In a two-minute exposure practically every star, within 4′ or 5′ of the optical axis, in any given region of the chart, is revealed, and near the center of the negative new ones are usually to be detected. Before the development of a plate, exposed for 2^m, is complete, the background image plainly shows on the negative, so that under the ordinary conditions existing here* no material advantage is to be gained by prolonging the exposure much beyond 15 or 20 minutes.

Under favorable circumstances, then, this instrument certainly photographs stars fainter than the 17th visual magnitude in less than five minutes. For determining the photographic magnitude of these fainter stars no general method, making any claim to accuracy, is known. For comparison with other results, the best available data seem to be Professor KEELER's photographs of the *Ring* nebula in *Lyra*, published in the *Astrophysical Journal*, No. 10, p. 193. I have photographed this nebula at various times with the 13-inch mirror, and give here the results secured under the most favorable conditions.

The nebula is just to be recognized on negatives exposed for 4^s. In 8^s-negatives the central star shows plainly, as does also the 13th magnitude (LASSELL 1) close following the nebula. In 16^s-negatives, the nebula and these two stars are quite conspicuous.

In negatives exposed for 32^s LASSELL's Star 3 is just to be recognized (this star according to KEELER is still invisible on his original negative exposed for two minutes in the 3-foot reflector). LASSELL's Star 2 is plainly seen, as is also the 10″ distant companion to the central star. In 64^s-negatives all the stars to be found on the published plate of KEELER's 10-minute negative can be recognized. In negatives of 128^s exposure quite a number of stars are visible which do not show on KEELER's two-hour-exposure plate; they are doubtless to be found on his original negative. In a one-minute exposure the background of the sky is already faintly to be seen on fully developed negatives. In two minutes, as already stated, it is quite strongly impressed upon the plate. As the exposure time is prolonged the density of all objects increases, and the faintest objects can be recognized with greater certainty. I have prolonged the exposure up to 60 minutes, but nearly every star shown on the resulting negative can be found on plates exposed for less than five minutes.

Perhaps the best illustration of the power of this comparatively small instrument is the fact that it has revealed the true form of the *Ring* nebula in *Lyra*. This is now *plainly* shown to be a *two-branched spiral* which starts at the central star, and in a clock-wise direction leaves it on opposite sides near the minor axis. With a spider-line micrometer attached to a compound microscope magnifying 100 diameters, I have measured the following distances from the central star to points where the thread-like arcs

* My little private experimental observatory is surrounded by residences, a dozen or more within a stone's throw. The combinations of smoke and nearby powerful arc lights (for street illumination) which burn at all hours of the night when the moon is not above the horizon, often produce great variations in the sky background, amounting at times to several stellar magnitudes, in an otherwise clear sky.

The brightness of the field of view may be slightly affected by a series of scratches which (when well advanced with the work after long and patient figuring) were made through an accident to the polisher of my machine. The variation in the radius of curvature is so rapid in this mirror, that the slightest indentation, due either to bubbles or scratches, causes the polisher to change the surface in the immediate neighborhood of every defect. I was, on this account, forced to discontinue the figuring somewhat before it should have been done. As a result, a somewhat imperfect polish and small rings still exist.

The general curvature of the whole surface is very satisfactory, and the definition is much the best of all my past amateur efforts in the way of figuring parabolic surfaces. The 12-inch mirror has a good figure, and no scratches.

cross the major axis, beginning at the preceding end. $-45''$, $-36''$, $-28''$, $-19''$, $-10''$, $+12''$, $+21''$, $+28''$, $+36''$, $+41''$; certain of these arcs are plainly double, the mean position is in such cases given above. The one crossing at $+10''$ starts on the *south* side of the star; the one at $-12''$ is the first crossing of the *north* branch, etc. The distances on the minor axis from the central star to tangents parallel to the major axis, for the several individual arcs clearly to be distinguished, are approximately as follows: beginning on the north $-13''$, $-9''$, $+2''$, $+7''$, $+12''$, $+17''$. Those at $-9''$ and $+2''$ correspond to the innermost north and south tangents respectively, etc.

From the central star outward both branches can be traced continuously through arcs of at least 420°, after which they seem to run into each other in projection, forming heavier rings apparently corresponding to the inner edge of the main nebula; at certain other positions they are again farther apart, giving rise to darker areas, and producing the impression that two or three nearly circular non-concentric heavy rings form the main ring. The scale of the whole photographic image is

$$90'' \times 60'' = 0^{\text{in}}.009 \times 0^{\text{in}}.006$$

There seems to be real nebulosity near the 13^{M} star and in the area inclosed by the two tangents from this star to the extreme outer boundary of the nebula.

From the data already given, it appears that, for exposures of two minutes or less, the smaller instrument photographs stars in about one-fourth of the time required by the larger; or, at a given instant reveals stars 1.6 magnitudes fainter. According to equation (3) it should, theoretically, be only 0.8 magnitude, assuming that plates of equal rapidity were used at both instruments. The difficulties connected with the guiding of the CROSSLEY telescope which KEELER mentions in another paper (*Ap. J.*, Vol. XI, No. 5) will almost wholly account for the difference. Also to be considered is the fact that the best rays, amounting to $\frac{1}{16}$ of the total light, are cut off by the diagonal. With the 13-inch no trouble is experienced in keeping the guiding image at the intersection of two spider lines with a power of 360 diameters on the guiding telescope; the plate holder stops only $\frac{1}{100}$ part of the light, and there is no second reflection. The difficulties of guiding should have less effect on the visibility of a nebula, and this is shown in the foregoing comparison. KEELER's negative just reveals the nebula in a 30-second exposure, but I have not yet succeeded in photographing it in less than 4 seconds, while according to equation (1) it should require but 2 seconds; here the purer sky of Mt. Hamilton is clearly in evidence.

As a result of these observations and comparisons, I am inclined to agree with the views expressed by Mr. ROBERTS, that stars much fainter than the 18th or 19th magnitude cannot be photographed with any instrument working near sea-level or at moderate altitudes.

Variations in the sky-illumination for different directions in space, independent of those caused by our atmosphere, could readily be made with a short-focus instrument of this kind placed in a favorable climate and 10,000 feet or so in altitude. By slightly changing the plate after each exposure fifty or more protected images, each a few minutes of arc in diameter, could be made on the same negative in a single night. The exposure time, the zenith distance (= colatitude?) and the development being the same for each image, direct comparisons would be possible. In work of this kind it would be just as necessary to have accurate following as in photographing a nebula. This is plainly evident from an examination of some experimental negatives where the field is two or three degrees in diameter. The aberrational spread of the star images causes the appearance of a sensibly increasing density of the photographic background with increasing distance from the optical axis.

Near the center of the negative an image may have great density, and yet be so small that it is wholly invisible to the naked eye. With poor guiding, however, this would no longer be true, and the resulting background would not represent the actual brightness of the area photographed. For the observations of variable stars the value of the saving in time where very faint stars are under observation can hardly be overestimated.

These faint stars give disks so small that they are far beyond the reach of naked-eye vision; near the optical axis they do not, under favorable conditions, exceed $2''$ in diameter (= 0.0002 inches) in exposure up to five minutes or more. (I have made instantaneous exposures on ϵ *Lyræ*, using SEED's lantern-slide plates; the resulting negatives show the individual components of each pair, as they are successively brought near to the optical axis by moving the telescope in declination.) At the edge of the field of view having a radius of, say $7'$, the brighter stars are considerably enlarged and elongated to the extent indicated in the table below. For the fainter stars, however, only that portion of the image which is near the vertex of the pattern is sufficiently strong to make a record on the plate, so that at a distance of $4'$ or $5'$ from the optical axis these fainter stars still appear quite small, though sensibly elongated.

For preliminary examinations of the negatives, lenses varying from about one-half inch focus to the shortest focus (one-eighth inch or less) conveniently available, should be at hand.

For approximate measures a simple position filar micrometer attached to a compound microscope with powers varying from 20 to 200 diameters (the higher powers for such objects as ϵ *Lyræ* mentioned above, and also for structural detail in nebulas, etc.) can be most advantageously used.

I find that the most satisfactory illumination is afforded by light reflected (or transmitted) by a rough surface which subtends a large solid angle at the negative.

A very effective method of examining the plates, especially those of short exposure, is to use a *dark* background, illuminating the negative with a strong side-light: by this procedure all the fainter objects become luminous.

blurring factor is therefore 1.08. The analytical expression for this factor (*Astr. Jour.*, No. 455) is

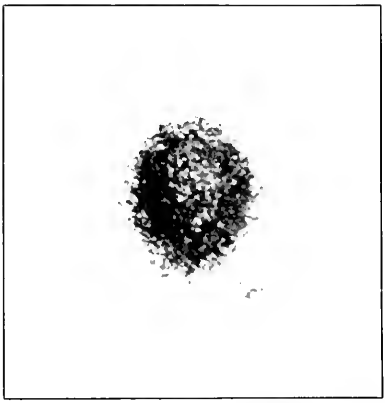
$$\frac{1}{\cos\gamma \cos^2\frac{1}{2}\gamma}$$

which gives, as it should, the same value.

When delicate results are required at considerable dis-

RING NEBULA IN LYRA.

Photographed by J. M. SCHAEBERLE with [illegible] inch Reflector of [illegible] inch Focus, Oct. 30, 1902, exposure [illegible] seconds. Enlarged 150 diameters.



ASTRONOMICAL JOURNAL No. [illegible]

and seventh exposures by the formulas,

$$ax + by + c + x = x_1$$
$$dx + ey + f + y = y_1$$

These in turn were reduced to right-ascension and declination by means of the standard stars published in the bulletins of the astrophotographic congress. Two methods

means of the [illegible] in Circular No. [illegible] of the [illegible]graphic congress. Finally, from these [illegible] equations were formed, and the solar parallax [illegible] least-square solution.

The measurements were made with the [illegible] measuring machine, in duplicate by Mr. Roy Fraser [illegible] myself. The images near the center, which [illegible] were

usually round and well defined. But toward the edge of the plate there was a decided elongation and tail. I have since found that images near the edge can be very much improved by capping down the object-glass to about nine inches, without greatly diminishing the distinctness of the fainter images. A marked constant difference, between Mr. FERNER's measures and my own appears in the distorted images, but does not show in the round stars.

The times and lengths of exposure are found in the following table:

No.	Minneapolis Sid. Time	Exp.	Bar. Ther.	No.	Minneapolis Sid. Time	Exp.	Bar. Ther.
	h m s	m	in.		h m s	m	°
1	22 8 19	2	29.18	8	3 21 49	12	− 2.0
2	22 13 34	2.5	+1°.0 C	9	3 38 34	2.5	–
3	22 21 19	4	–	10	6 36 35	2.5	–
4	0 11 34	2.5	–	11	6 41 31	2.5	–
5	0 19 4	5	–	12	7 25 34	2.5	29.25
6	1 28 34	2.5	–	13	7 43 11	4.5	− 2.7
7	1 38 49	6	–				

The coordinates are measured from the center of the plate, and are corrected for error of run and scale-error. In reducing them to the mean of exposures six and seven, the different stars were weighted according to the roundness of their images.

The coordinates of *Eros* resulting from this reduction are as follows:

No.	x	y	No.	x	y
1	+3.9712	−0.1133	8	+2.2331	−4.0826
2	3.9175	0.1707	9	2.1650	4.2636
3	3.9055	0.2630	10	1.2981	6.7032
4	3.1428	1.9742	11	1.2742	6.7701
5	3.0993	2.0666	12	1.0903	7.3808
6	2.8817	2.5620	13	+1.0119	−7.6224
7	+2.8232	−2.6959			

These places were next reduced to right-ascension and declination. Owing to a mistake in identification, two of the standard stars most favorably situated were not measured. The number of reduction stars were thus reduced to six, and they were not symmetrical with reference to *Eros*.

The stars used were,

Star		α	δ	x	y
		h m s	° ′ ″		
301	+50	1 26 59.882	+50 22 1.06	− 8.4971	−38.6269
331	51	27 58.688	51 19 13.51	+ 1.5515	+18.6538
334	51	28 33.784	51 38 28.98	+ 7.2321	+37.9071
338	51	29 22.197	51 39 7.70	+14.8256	+38.5047
339	51	30 24.334	51 14 15.93	+24.3804	+13.5078
314	+50	1 30 44.595	+50 44 59.15	+27.4953	−15.8812

The reduction-equations resulting from these are,

A, with refraction applied:

$$+0.00256\,x\sec\delta - 0.00125\,y + 0^s.1666 + x\sec\delta + \alpha_0 = \alpha$$
$$+0.11142\,x\sec\delta + 0.00257\,y + 0''.338 \;\; + y + \delta_0 \qquad = \delta$$

B, refraction not applied:

$$+0.00255\,x\sec\delta - 0.00125\,y + 0^s.1663 + x\sec\delta + \alpha_0 = \alpha$$
$$+0.14328\,x\sec\delta + 0.00257\,y + 0''.340 \;\; + y + \delta_0 \qquad = \delta$$

where $x\sec\delta$ is expressed in seconds of time, y in seconds of arc, and

$$\alpha_0 = 1^h\,27^m\,50^s.00 \quad , \quad \delta_0 = 51°\,0'\,37''.0$$

and α and δ are the right-ascension and declination of the star for 1900.0.

By means of these formulas the following right-ascensions and declinations of *Eros* were obtained:

Exp.	α_A	α_B	δ_A	δ_B	Motion α	Motion δ
	h m s	s	° ′ ″	″	s	′ ″
1	1 28 15.334	15.333	+51 0 33.19	33.24	−5.378	−2 36.86
2	28 15.187	15.186	51 0 29.75	29.79	−5.238	−2 32.86
3	28 14.928	14.927	51 0 24.20	24.25	−5.033	−2 26.95
4	28 10.211	10.210	50 58 41.40	41.44	−1.352	−0 39.88
5	28 9.942	9.941	50 58 35.85	35.88	−1.157	−0 34.14
6	28 8.597	8.596	50 58 6.08	6.12	−0.132	−0 3.92
7	28 8.236	8.236	50 57 58.04	58.08	+0.132	+0 3.93
8	28 4.598	4.596	50 56 34.74	34.77	+2.853	+1 25.20
9	28 4.179	4.178	50 56 23.87	23.90	+3.204	+1 35.76
10	27 58.870	58.867	50 53 57.42	57.44	+7.684	+3 52.73
11	27 58.724	58.722	50 53 53.40	53.42	+7.808	+3 56.58
12	27 57.604	57.602	50 53 16.76	16.78	+8.899	+4 30.52
13	1 27 57.426	57.124	+50 53 2.26	2.28	+9.334	+4 44.13

The table also contains the corrections due to the motion of the planet in orbit. From this table the equations of condition were formed. The general equation is represented by

$$1^h\ 28^m\ 8^s.452 + \Delta\alpha - 0^s.3624\,a - a\,m - \alpha = 0$$
$$50^\circ\ 57'\ 59''.00 + \Delta\delta - 8''.80\,a' - a\,m - \delta = 0$$

and the equations of condition are

Exposures			A		B		p
				r		r	
1	$+1\ \Delta\alpha$	$+1.63\ m$	$+0.16 = 0$	$+0.13$	$+0.17 = 0$	$+0.14$	1
2	1	$+1.60$	-0.04	-0.07	-0.03	-0.06	1
3	1	$+1.55$	$+0.10$	$+0.08$	$+0.10$	$+0.08$	1.3
4	1	$+0.13$	-0.01	-0.03	-0.03	-0.02	1
5	1	$+0.36$	$+0.05$	$+0.07$	$+0.06$	$+0.06$	1.3
6	1	0.00	-0.14	-0.11	-0.13	-0.11	1
7	1	-0.10	-0.09	-0.05	-0.07	-0.05	1.3
8	1	-1.01	$+0.30$	$+0.36$	$+0.32$	$+0.37$	0.8
9	1	-1.15	-0.02	$+0.05$	0.00	$+0.05$	1
10	1	-2.08	-0.38	-0.28	-0.36	-0.28	0.7
11	1	-2.09	-0.27	-0.18	-0.25	-0.17	1
12	1	-2.14	-0.39	-0.30	-0.38	-0.30	1
13	1	-2.13	$+0.06$	$+0.16$	$+0.07$	$+0.16$	1.2
1	$1\ \Delta\delta$	-0.26	$+0.33$	$+0.32$	$+0.28$	$+0.31$	1
2	1	-0.24	$+0.03$	$+0.03$	-0.01	$+0.02$	1
3	1	-0.20	$+0.01$	$+0.03$	0.00	$+0.02$	1.3
4	1	$+0.28$	-0.07	-0.09	-0.10	-0.09	1
5	1	$+0.30$	-0.06	-0.08	-0.10	-0.08	1.3
6	1	$+0.32$	-0.30	-0.32	-0.34	-0.33	1
7	1	$+0.32$	-0.13	-0.15	-0.17	0.16	1.3
8	1	$+0.11$	$+0.06$	$+0.05$	$+0.03$	$+0.05$	0.8
9	1	$+0.06$	-0.06	-0.08	-0.10	-0.08	1
10	1	-0.97	$+0.35$	$+0.36$	$+0.34$	$+0.38$	0.7
11	1	-1.00	$+0.22$	$+0.24$	$+0.20$	$+0.25$	1
12	1	-1.32	$+0.12$	$+0.15$	$+0.11$	$+0.17$	1
13	1	-1.15	$-0.11 = 0$	-0.08	$-0.12 = 0$	0.06	1.2

Equations 3, 5, 7, 13 have been given extra weight because two images of the planet were obtained in each of these exposures, and the mean of the measures of these images was used. Less weight was assigned to 8 and 10 because of imperfect images. On account of the long exposure of 8, the weight ought probably to be much less than it is, or the measure rejected altogether.

Normal Equations	A	B
$+14.64\ \Delta\alpha + 0.00\ \Delta\delta - 3.11\ m$	$-0.53 = 0$	$0.57 = 0$
$0.00\ \Delta\alpha + 14.64\ \Delta\delta - 3.95\ m$	$+0.06 = 0$	$0.43 = 0$
$-3.11\ \Delta\alpha - 3.95\ \Delta\delta + 36.32\ m$	$+1.26 = 0$	$+1.21 = 0$

From which,

	A		B	
$m =$	$-0''.034$	$\pm 0''.021$	$-0''.029$	$\pm 0''.021$
$\Delta\alpha =$	$+0.028$	± 0.038	$+0.019$	± 0.038
$\Delta\delta =$	-0.013	± 0.038	-0.021	± 0.038
$r =$		± 0.122		± 0.124
$\Pi =$	8.766	± 0.021	8.771	± 0.021

From the smallness of the absolute terms of the equations of condition, it is evident that the systematic error must be small. Evidently the abnormal size of 10, 11 and 12 explains the variation of the parallax from the accepted value 8".80.

I was not able to investigate the effect of the trail of the asteroid because the latter was too faint to be used as a guiding star in the following telescope.

University of Minnesota, Minneapolis.

ON THE FUNDAMENTAL ELEMENTS OF COMPUTATION IN THEIR RELATION TO SYSTEMATIC STELLAR MOTION.

BY LEWIS BOSS.

The systematic drift of stellar motions, whether it be due to reflected solar motion, or to other causes, is often of the order of the systematic errors of the observations from which the motions are computed. The adopted precessional motion may play an important part. Attention to these points is, therefore, of primary importance.

Let us consider, for example, Sir DAVID GILL's suspected rotation of bright relative to fainter stars (*A.N.* 3800), upon the systematic basis, B, of the right-ascensions published in *A.J.* 531–2. Assuming that his zone-observations are based upon the standard right-ascensions of AUWERS we have (for −10° to −52°) B − Cape 1900 = $+0^s.068$. If GILL used the time stars of the *Berliner Jahrbuch* as for Cape 90, and if $I\delta_s$ for the instrument remained the same, we should have: B − Cape 1900 = $+0^s.065$, nearly as before.

From direct comparison with B (with 31 additional standards in manuscript), I find for this zone:

	Stars	$\Delta\alpha$
B − Taylor	69	$-0^s.132$
B − C 1880	72	$+0.077$

We may therefore put:

$$C\,1900 - \text{Tayl.} = -0^s.200\,;\quad C\,1900 - C\,1880 = +0^s.009$$

Then GILL's comparisons would stand as in the subjoined statement, the numbers in the first three columns having been copied from GILL's article, *A.N.* 3800.

CAPE 1900 — TAYLOR.

	Stars	C—T.	S. Corr.	C—T., corr'd
$5^M.8$	218	$-0^s.188$	$+0^s.200$	$+0^s.012$
7.4	472	−0.315	+0.200	−0.115

CAPE 1900 — CAPE 1880.

	Stars	C—C 80	S. Corr.	C—C 80, corr'd
$6^M.8$	681	$+0^s.011$	$-0^s.009$	$+0^s.002$
7.9	813	−0.014	−0.009	−0.023

We have still several elements of error to consider before we reach the question of rotation.

1. There is the possible error of comparison, including the uncertainty of the fundamental system.

2. There is the possible effect of magnitude equation, which may amount to $-0^s.003\,(M-3.5) - 0^s.0041\,(M-3.5)^2$ in case we assume the equation for C 1900 the same as that for C 1890 (see *A.J.* 536). The corrected numbers for C 1900 − Taylor would then become: $5^M.8$, $+0^s.040$; $7^M.4$, $-0^s.039$. No particular stress can be laid upon this result; but it illustrates actual possibilities.

3. Small modifications may be due to the employment of NEWCOMB's precessions.

From the proper motions computed by AUWERS for the zone +15° to +20°, Professor SEELIGER obtains a test of the supposed rotation (*A.N.* 3865, p. 9). This test shows the rotation to be non-existent; but it leaves an average mean motion for all stars of about $-0^s.10$. This is reduced to zero, if the proper motions are reduced to conformity with NEWCOMB's precessions and the right-ascensions of system, B.

I take this opportunity to refer to the very able and interesting paper of Professor KAPTEYN (*A.N.* 3859), concerning the apex of solar motion. A problem of such extreme difficulty seems to demand in the very first line a thorough investigation of the errors to which the various series of star-positions are liable; and, in the second place, the employment of all the observations that can readily be brought to bear. In no other astronomical investigation do these requirements seem to me more indispensable. An attempt to apply these principles in the discussion of this problem for the brighter stars is in progress at Albany. The recent publication of a catalogue of standard stars (*A.J.* 531–2) is a step in this work. For some time to come our chief anxiety will be to learn what are actually the proper motions. Under these circumstances I wish to defer extended comment on KAPTEYN's paper at present, and will merely refer to certain points.

1. KAPTEYN's criticism (*A.N.*, 3859, pp. 328, 352) upon the systematic corrections employed by me in the discussion of L. STRUVE's Bradley-Pulkowa values of $100\mu_s$ (*A.J.* 501) is well taken. There was an oversight. The numbers, as I computed them, were for the zones −7°.5, +7°.5 and +22°.5, respectively, −0″.86, −1″.14 and −1″.22. How the wrong numbers came to be used is a mystery for which I find no explanation.

2. The determination of a systematic correction of all the proper motions through the discussion of the solar motion itself as KAPTEYN has attempted it (*A.N.* 156, pp. 1–20) seems to me an inadmissible procedure. This delicate element is thereby made to depend upon the mere fortuitous differences in the actual apical positions for the several restricted groups of stars.

3. Of existing determinations of the solar apex those which are based upon proper motions between 0″.1 and 1″.0 seem to be less open to objection than the others. Those based on smaller motions are untrustworthy from lack of thorough and satisfactory treatment of the systematic errors. The very large proper motions are very possibly abnormal.

4. The habit of some computers of setting the limits of proper motion to be employed not according to the total motion, but according to one standard for μ and another μ', is justly criticized by KAPTEYN; but I think he has exaggerated the effect in concrete instances. Taking into consideration the works of BISCHOF, STUMPE, and PORTER (second computation) I cannot think of the declination of the solar apex as much less than +40°.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS PRICE, $5 00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed May 30.*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

Nos. 540-541.

VOL. XXIII. BOSTON, 1903 JUNE 18. NOS. 12-13

METHOD OF FORMING THE SYSTEM OF DECLINATIONS FOR THE CATALOGUE OF 627 STANDARD STARS (*A.J.* 531-2).

By LEWIS BOSS.

The general principle adopted in the formation of declinations for the Catalogue of 627 Standard Stars was the same as for the right-ascensions. First, we endeavor to form a system of declinations of individual stars which may be accepted as sufficiently homogeneous within each zone which is to be treated as a whole for correction. The immediate end desired is that the positions and motions of all the stars within the zone shall require the same corrections, whatever these may be. Thus, the zone-corrections (*n*, Table IV) at various epochs given by the several star-catalogues will be systematically correct relatively to each other, although these catalogues may be very far from having all the stars in common. This zone is now treated as a representative star for which the most probable correction of its computed declination and motion is to be ascertained from the evidence afforded by those star-catalogues which are entitled to weight as independent determinations.

In this work it was assumed that the writer's *Declinations of 500 Stars*, B_1, with its extension southward in *Standard Stars south of* −20° (*A.J.* 448-450), B_2, offered a suitable basis for correction. But in preparing B_1 for this use the declinations of 50 stars were revised by means of recent observations reduced to systematic conformity with B_2. The list of stars forming the basis of operations then consisted of the 210 stars best determined in B_1 with the 179 stars of B_2.

Corrections of the Form $\Delta\delta_\alpha$.

It turned out that the system, B_1, does not appear to require any material correction of the form $\Delta\delta_\alpha$ in order to make it consistent with observations published since that system was formed. It is scarcely necessary to exhibit the original values of $\Delta\delta_\alpha$ which were adopted throughout the computations. As a substitute for these Table I contains the values of $\Delta\delta_\alpha$ which were computed for each catalogue of observation through comparison with the adopted declinations of the Catalogue of 627 Standard Stars, B. In general these values of $\Delta\delta_\alpha$ are in close conformity with those which were computed in the preliminary operations. For one-half of the catalogues the maximum difference between the preliminary and adopted values of $\Delta\delta_\alpha$ does not rise above 0″.02 at any hour of right ascension.

TABLE I. ADOPTED VALUES OF $\Delta\delta_\alpha$.

		sin	cos			sin	cos
Grw.	16	+0.07	−0.09	Bonn	66	+0.37	−0.11
Kon.	21	−0.27	+0.05	Leips.	67	+0.02	0.01
Dpt.	24	+0.15	−0.02	Leid.	67	−0.02	−0.02
Abo.	29	+0.25	−0.12	Mosc.	68	+0.01	−0.11
Grw.	30	+0.16	+0.08	Pulk	69	0.00	+0.04
St. H	32	−0.08	+0.50	Grw.	72	−0.01	0.00
Cape	33	−0.06	−0.16	Madr	75	+0.32	+0.17
Camb.	34	+0.04	−0.19	Harv	75	0.00	0.00
Cape	37	−0.02	+0.01	Pulk.	76	+0.03	+0.06
Grw.	38	−0.05	−0.17	Cord.	76	−0.07	0.00
Radcl.	45	−0.08	+0.05	Paris	76	+0.25	−0.13
Grw.	45	+0.02	0.06	Cape	76*	0.00	0.05
Paris	45	−0.03	−0.07	Melb.	77	−0.06	−0.07
Pulk.	47	−0.03	+0.07	Washn.	77	+0.03	−0.02
Stgo.	51	+0.20	−0.21	Cape	83	0.04	+0.12
Cape	51	−0.18	−0.03	Pulk	84	−0.02	+0.02
Grw.	51	−0.03	−0.07	Radcl.	85	+0.12	+0.07
Pulk.	55	+0.02	+0.03	Str.	86	+0.01	+0.03
Washn.	56	−0.12	0.07	Cape	89	0.01	0.06
Grw.	57	+0.05	+0.01	Ber.	90	0.02	+0.06
Radcl.	57	−0.10	+0.08	Mun.	92	0.01	+0.01
Cape	59	+0.11	+0.03	Grw.	94	+0.03	0.00
Paris	60	+0.14	0.08	Mt H	95	+0.05	+0.05
Grw.	64	0.02	+0.04	W. Obs.	97	+0.23	+0.07
Bruss.	65	+0.08	−0.05	Alb.	98	0.04	0.06
Cape	65	0.00	0.02				

Madr	35	−.07 sin α	−.22 cos α	+.00 sin 2α	+.14 cos 2α
Melb.	62	+.06	.32	+.11	+.23
Grw.	82	+.04	+.08	.07	.02
Madr	90	+.11	−.21	+.10	.00

* For stars north of −20° only. Zone corrections for stars further south

In dealing with the individual star-declinations in this investigation, the work was performed as a continuation of the manuscript computations for B_δ which have been preserved. It became simply necessary to add the results of later observations to the "star-sheets" prepared for B_δ, and then to proceed to the revision by the zone-method as already described in connection with the right-ascensions. In the original work, and consequently in this, some of the older series of declinations were first corrected for terms of the form, $\Delta\delta_\alpha$, required in order to correct for difference from the STRUVE-PETERS values of nutation, precession, etc., adopted in the construction of the several catalogues. For example, corrections on account of nutation, etc., were first applied to the declinations of Königsberg 21 (DÖLLEN'S reduction, *Recueil de Mém., Obs. Cent. de Russie*), Abo 29, and Cape 33, before employing them in the operations for deduction of the normal system. These corrections were:

Königsberg	21,	$-0''.24 \sin\alpha$	$-0''.03 \cos\alpha$
Abo	29,	$+0.24$	-0.04
Cape	33,	-0.01	-0.07

The additional corrections, $\Delta\delta_\alpha$, apparently necessary in order to reduce the respective catalogues to the system, B, are:

Königsberg	21,	$-0''.03 \sin\alpha$	$+0''.08 \cos\alpha$	
Abo	29,	$+0.01$	-0.08	A
Cape	33,	-0.05	-0.09	

The combination of these two sets of corrections makes up the respective values of $\Delta\delta_\alpha$ given in Table I. Similar remarks apply to several other of the earlier catalogues, for which the details are given in B_δ.

In computing $\Delta\delta_\alpha$ the observations were divided into zones: $+80°$ to $+40°$; $+39°$ to $-21°$; and $-22°$ to $-70°$. Usually the values of $\Delta\delta_\alpha$ from the separate years, for a given catalogue, were sufficiently consistent. The following notable differences were found, however:

	Zone	sin	cos
St. Helena 32.	$+39°$ to $-21°$	$+''.06$	$+''.51$
	-22 to -70	$-.23$	$+.50$
Cambridge 34.	$+80$ to $+40$	$-.32$	$-.21$
	$+39$ to -21	$+.17$	$-.19$
Washington 56,	$+80$ to $+40$	$-.27$	$-.10$
	$+39$ to -21	$-.14$	$-.07$
	-22 to -42	$+.17$	$-.02$
Melbourne 68,	$+39$ to -21	$+.05$	$-.02$
	-22 to -70	$-.02$	$-.20$
Madras 75,	$+80$ to $+40$	$+.56$	$+.19$
	$+39$ to -21	$+.32$	$+.10$
	-22 to -60	$+.21$	$+.33$

It would scarcely be advisable, however, to take these differences into account, since they do not, in general, much exceed the limit of uncertainty admissible according to the theory of probable error. It is proposed, however, to put $\Delta\delta_\alpha = 0$ for declinations south of $-20°$ in YARNALL'S Washington Catalogue for 1860.

In the instances where terms in 2α have been introduced, it is for the reason that terms of single period give an unsatisfactory representation of the residuals. With the term of double period taken into account the representation of the observed residuals for Greenwich 82 is very good. A like term ($-0''.04 \sin 2\alpha$) is indicated for Greenwich 94, but the improvement in representation of the residuals is not marked, and the term is not adopted.

EFFECT OF VARIATION OF LATITUDE.

In founding the system as to $\Delta\delta_\alpha$ no special account has been taken of the effect of variation of latitude upon the observed declinations. In the time which could be allotted to the present discussion, it did not seem practicable to investigate the correction required on that account, since to accomplish this in a precise form would have required as much labor as for all the other operations put together. Certain items of testimony on this point are, however, readily available. Some of the older catalogues, like those of Kön. 21, Dpt. 24, Abo 29, and Cape 33, were based upon polar points ascertained from the observation of close circumpolar stars, or of zenithal stars referred to an arbitrary zero. These are technically free from the effect of variation of latitude. Dr. CHANDLER'S discussion of POND'S observations with two mural circles has resulted in declinations which are freed from the effects of latitude-variation by a most exhaustive discussion which seems to leave nothing to be desired (*A.J.* XVI, p. 3). Dr. CHANDLER has also computed the corrections which are required in order to free the Pulkowa vertical circle observations, of mean date 1869, from this effect (*A.J.* 402). Dr. NYRÉN computed and applied this effect for the vertical circle observations of 1885; but Dr. CHANDLER points out that the annual term was neglected. GROSSMAN (*Abh. Kön. Sächs. Ges. d. Wiss.*, Vol. XXVII, p. 206) states that he applied in his reductions corrections for variation of latitude after ALBRECHT'S researches. To these I have added another from the following combination, derived from Table I.

CORRECTIONS OF B HAVING THE FORM $\Delta\delta_\alpha$.

Greenwich	64,	$+0''.02 \sin\alpha$	$-0''.04 \cos\alpha$
"	72,	$+0.01$	0.00
"	82,	-0.04	-0.08
Mean		0.00	-0.04
Washington	77,	-0.03	$+0.02$
W. Long. 38° 31′		-0.02	-0.01
Melb. 68 and 77, W. Long. 215° 1′		$+0.02$	$+0.09$
Mean corr. of B,		$0.00 \sin\alpha$	$+0.04 \cos\alpha$

Reversing the signs of formulas A, and of $\Delta\delta_\alpha$, for Pulkowa 84 and W.-Ott. 97 in Table I, and adding CHANDLER's corrections to Pulkowa 69, we have the following list of corrections to B on account of variation of latitude.

OBSERVED CORRECTIONS OF B, OF THE FORM $\Delta\delta_\alpha$.

Königsberg	21	$+0''.03 \sin\alpha$	$-0''.08 \cos\alpha$
Dorpat	24	-0.15	$+0.02$
Abo	29	-0.01	$+0.08$
Greenw'h (CH.)	29	-0.01	$+0.07$
Cape	33	$+0.05$	$+0.09$
Pulkowa	69	-0.08	0.00
Combination	74	0.00	$+0.04$
Pulkowa	84	$(+0.02)$	(-0.02)
Wien-Ottakring	97	-0.23	-0.07

From various considerations the first and last values appear to be entitled to small weight. Those from Greenwich 29 and Pulkowa 69 are the only ones which depend on a thorough investigation that takes into account the annual term of latitude variation, — the only term which can have introduced a serious uncertainty in B as to terms in $\Delta\delta_\alpha$. If this annual term tends to have a constant value throughout the period of observation, then it is probable that the proper motions of B are virtually free from any sensible inequality of the form $\Delta\delta_\alpha$. This is the particular end desired by the writer in this investigation.

Several circumstances conspire to eliminate a part of the annual term in the observed declinations; so that the full effect of that term may not have appeared in B. Some of the older observations depend upon zenithal, or polar, points derived from observation of stars. Difference of longitude of the observatories tends to diminish the resultant effect. In many observatories the observation of brighter stars is extended over long periods of time; and when observations are made at all hours of the night, as at some of the principal observatories, the resultant effect of the annual term would be somewhat diminished.

When all the elements which conspire to produce errors of the form, $\Delta\delta_\alpha$, are considered, one can scarcely fail to be surprised at the very small discordances of this form that appear in the catalogues of large weight. For a considerable percentage of those catalogues $\Delta\delta_\alpha$ is comparable with the probable error of its determination through comparison with B.

Corrections of the Form, $\Delta\delta_\delta$.

In proceeding to obtain a normal system which may be considered to give the most probable representation of the testimony of observation, we must employ in the first instance only those catalogues which are supposed to give independent declinations. Since the astronomical refraction plays such an important part in the determination of the zenith-distances of stars, a series of observed declinations can scarcely be regarded as independent unless it can be shown that the adopted latitude and refraction constant are consistent with the results of extensive and sufficient observations of circumpolar stars. The strict application of this criterion would reduce the series of independent declinations to a comparatively small number. But the method of comparing catalogues resulting from nearly contemporaneous observations in the two hemispheres appears to offer a legitimate means for increasing the number of virtually independent determinations. This method of deducing conclusions as to the most probable value of the refraction constant for each of the sets of observations compared has been recognized ever since the time when accurate observations were first made in the Southern hemisphere. If we put

ρ'_n = the mean adopted refraction at the pole for the northern observatory,

ρ'_s = the same for the southern observatory,

ρ_n and ρ_s = the respective adopted mean refractions for a star common to the two catalogues — considered positive north of the zenith.

k_n and k_s = 100 times the factors by which the adopted refractions should be multiplied in order to find their corrections,

then each comparison between the observed declinations at a northern and southern observatory, respectively, furnishes an equation of the form,

$$\frac{\rho_n - \rho'_n}{100}\,k_n - \frac{\rho_s - \rho_s'}{100}\,k_s = \delta_s - \delta_n$$

This equation affords a sufficient approximation only when the values of k are relatively small; and it assumes that the true polar point has been already found in the reductions for the catalogue. It is also implied that the error of the adopted instrumental corrections may be neglected. In general, for the better class of catalogues, it may be assumed that the polar point has been well determined, in the sense that the observed declinations above and below the pole, in its vicinity, are the same; and that this has been established by an adequate number of observations. As to the instrumental corrections, it may be said that no determination of them can be regarded as free from sensible error. Errors depending on sin z, however, are partly taken up in k, resulting, perhaps, in a spurious value of that quantity, but in a value which best represents the error of the catalogue for moderate zenith-distances. Errors of the form, cos z, are not apt to be of serious consequence in these comparisons. For instance, in the case of Greenwich this error is zero at declination +13°, while, at the Cape, the length of the arc between +22° and the south pole is independent of the term $a \cos z$. As in all other classes of meridian observation for star catalogues, the error of graduation remains as a very uncertain ele-

ment: and for immunity from the ill effects of this uncertainty we should endeavor to bring to bear upon this problem the results from many different instruments.

Therefore, before forming the first set of zone-equations, several northern and southern catalogues, not otherwise absolutely independent, were compared for the determination of k. Omitting the voluminous details the results for k are exhibited in Table II.

TABLE II. REFRACTION FACTORS FROM CATALOGUE COMPARISONS.

		k	
Greenwich	30	−0.140	+ Redn. to BESSEL's refr.
St. Helena	32	−0.490	+ Redn. to BESSEL's refr.
Cambridge	34	+0.642	
Cape	33	−0.193	
Greenwich	42	+0.068	
Cape	37	−0.231	
Greenwich	57	−0.035	
Cape	59	−0.205	
Greenwich	64	−0.167	
Melbourne	68 (S.)	−0.257	
	(N.)	−0.257	+0.542 ρ
Washington	77	−0.125	
Cordoba	76	−0.412	
Melbourne	77 (S.)	−0.207	
	(N.)	−0.207	+0.512 ρ
Greenwich	82	−0.217	GILL's comparison, I
Cape	83	−0.218	GILL's comparison, I
Greenwich	94	−0.108	
Cape	89	−0.014	

In making the comparison of Greenwich 30 (POND) with St. Helena 32, the zonal means for POND's declinations were first revised to bring them into substantial conformity with the results derived from CHANDLER's reduction of POND's reciprocal observations with two mural circles (*A.J.* XVI, 3). There are 33 stars in common with B. We have:

POND (CH.) — B.

δ	$**$	$-\Delta\delta$
°		″
+65.4	7	+0.06
+40.6	4	+0.22
+25.6	8	+0.09
+11.4	12	+0.20

The range over which these determinations extend is not sufficient for a good determination of k through comparison with a southern catalogue. CHANDLER shows by means of POND's observations of four circumpolar stars culminating at a low altitude that BESSEL's refractions employed in the reductions produce consistent results (*A.J.* XIV, 4).

It seems to me that the extension of Dr. CHANDLER's reduction of POND's declinations to include all of the zenith-distances observed by POND (1825–1835) would be a work surpassing in importance any other of this nature which could be undertaken. Washington 77 was compared both with Cordoba 76 and Melbourne 77, and the results were so adjusted as to produce what was regarded as the best practicable reconciliation of the three catalogues. Some such combination as this was necessary in the case of the Cordoba observations, which were not carried much beyond 60° north of the zenith.

It should be remembered that the corrections, k, are applicable to the refractions employed in the reductions of the respective catalogues: so that for the Melbourne catalogue for 1870 we are to employ as the corrected refraction on both sides of the zenith, BESSEL's $\times$ 0.9937; and for Melb. 80, BESSEL's $\times$ 0.9942. Likewise for Cape 89, where Pulkowa refractions were employed in the reductions, we have as the corrected refraction, approximately, BESSEL's $\times$ 0.9972. The general result indicated in Table II is in favor of refractions virtually equivalent to those of the Pulkowa tables. In the mean, it appears that the Greenwich observations are best satisfied by the refraction, BESSEL's $\times$ 0.9987.

The instrumental corrections required by the Greenwich and Cape transit circles have been investigated very thoroughly. This is especially true as to errors of graduation. Consequently the values of k derived from the mutual comparisons of this series of observations appear to deserve great confidence. So much cannot be said for the other comparisons embraced in Table II: so that, for these, the quantities k, in each individual case ought not to be attributed to refraction alone. However, we may still hope that, in the mean, this process has resulted in the elimination of the larger part of the errors due to the employment of imperfect refractions in the reductions: and that, large as they are, there will also be a tendency toward the elimination of uncorrected instrumental errors. Accordingly the catalogues of Table II, as corrected, together with Kön. 21, Dpt. 24, Abo 29, Pulk. 47, Paris (LAUGIER) 53, Leiden 67, Pulk. 69, Pulk. 84, and Strassb. 86, have been employed in deducing the fundamental system. The circumpolar observations of LAUGIER do not indicate any material correction of the adopted refraction (CAILLET's) so that this series can be regarded as fairly independent. A similar conclusion applies to the declinations of Strassburg 86, where the circumpolar observations are quite well satisfied by the adoption of BESSEL's refractions.

BAUSCHINGER's declinations (Munich 92), though independent, contain few stars outside the circumpolar region, and these are not well distributed for our purpose: so that no use is made of this series in the operations for establishing the system.

GROSSMANN's observations at Wien-Ottakring, 1897, contain a large number of observations of circumpolar stars, and were given weight as independent determinations from

zone $+40°$ northward. A somewhat careful analysis of the declinations leads to the suspicion that anomalies exist in this series of observations which will receive attention further on.

The method of forming the zone-equations from which the observed systematic corrections of B. were ascertained for each zone of $5°$ need not be presented here. In place of this the process is illustrated in Table IV of the present paper, which is intended for an independent test of B. Here the zones are $15°$ instead of $5°$ in width; but the method of computation is the same. Following the columns in which the catalogues with their estimated mean dates of observation are given, the estimated weight of each in the fundamental sense appears under the designation, p'_0. Then, under n, in each zone, is given the mean correction of B indicated by each catalogue of observation, with the weight, p, which represents the precision of n when corrected for supposed systematic error. The unit of p is supposed to have a probable error of $\pm 0''.1$. The residuals, n, corrected for the effect of adopted k (Table II) are entered under the caption n'. These represent the means for correcting the system, B, in order to arrive at an absolute normal system, based upon the best testimony readily available for that purpose. This is accomplished by means of zone-equations of the form,

$$\Delta\delta_0 + T\Delta\mu'_0 = n',$$

T having been reckoned in units of a century from 1875. The weights have been taken from the column headed, p'_0, as already explained, but they have been modified in a few instances when p is very small, and in all cases where the zenith-distance of the stars, upon which the n of a given observatory is based, is greater than $65°$. The weight becomes zero when the zenith-distance is greater than $75°$. For the corrected catalogues enumerated in Table II, however, the modification of weight for zenith-distance was not so marked as for the others. The results for the solution of the zone-equations are given under the heading, "Fundamental Solution," in Table V. It will be seen that this first approximation does not indicate any material correction of the adopted normal system, B. This fact, however, has no special significance, or advantage over the original solution, except to show that B., as prepared for a basis to be corrected, was sufficiently precise and homogeneous for the purpose; and to demonstrate, furthermore, that in the subsequent operations the system as established by the original zone-equations has been preserved with fidelity. With comparatively few numerical errors in the subsequent operations, and especially with want of attention in drawing the curves for $\Delta\delta_0$, this might not have been the case.

Table V exhibits, after the values of $\Delta\delta_0$ and $100\ \Delta\mu'_0$, their respective probable errors, as computed from the equations. Whether these are really valid depends upon the correctness of the relative weights employed. There appears to be no mathematical method of arriving at a decision upon this point. If these probable errors may be regarded as fair approximations to the truth, then it may be said, in general, that the probable systematic error of the declinations for 1900, in the zone, $+25°$ to $-15°$, is about $\pm 0''.07$, and of $100\ \Delta\mu'_0$, $\pm 0''.19$. Farther north the uncertainty is less, and further south it is greater. After the final operations to be described, the nature of the systematic error in B, which may be revealed in the future, should be such that its variation from one zone of $5°$ to an adjacent zone shall be very small. The probable errors attached to the results of fundamental solutions in Table V are largely interdependent from one zone to another; so that, for instance, if a comparatively large correction should be found for zone, $+1°$, then the true corrections for $+15°$ and $-15°$ would probably have the same sign as at $+1°$.

Secondary Revision of the System.

It may be said that the fundamental equations, in a general way, have established the position of the equator among the stars. It now remains to investigate the graduation of the sky intermediate between this equator and the poles with the aid of further evidence of observation available for the purpose.

There are many series of observed declinations which cannot be regarded as offering independent determinations, for which, nevertheless, the instrumental corrections for errors of graduation, flexure, etc., have been carefully investigated, but for which an independent determination of refraction is wanting. For some of these series the zenith-points were determined through assumed declinations of the stars. If this has been done, as at Mt. Hamilton, by means of a restricted zone of stars, or even as at Paris through a less restricted choice that is calculated to yield consistent zenith-points throughout the year, then the testimony of such observations may be of value in smoothing out sinuosities of systematic error in the observed corrections of B and B_1 due to resultant graduation errors affecting the comparatively few catalogues upon the testimony of which the absolute normal system has been founded. Accordingly all the catalogues deemed suitable for the purpose were compared with the system, B_1, which has resulted from the operation just described. It was assumed that the systematic errors of the catalogues could usually be represented by a correction of the form,

$$\Delta\varphi + a \sin z + b \cos z + k\frac{\rho}{100}$$

When $\Delta\varphi$ is omitted the last term was taken as $k\dfrac{\rho - \rho_0}{100}$. $\Delta\varphi$ is not necessarily the true correction of the latitude; it simply represents a constant correction of the declinations. Only in the case of Cape 33 was an attempt made

to determine b. With the ordinary arrangement of observations, as to z.d., it is seldom possible to secure adequate discrimination between a and k. It is, therefore, impossible to attach much importance to either as standing purely for that which it is supposed to represent. The advantage of the formulas in representing the systematic corrections, as a whole, is not seriously impaired. Table III contains the result of the investigation for formulated systematic correction of each of the catalogues employed in the secondary revision of B′.

Table III. Formulated Systematic Corrections.

		$\Delta\delta$ ″	a	k	
Greenwich	15	−0.54	−0.48	. .	
Königsberg	21	. .	+0.05	. .	
Dorpat	24	. .	. .	+0.476	
Abo	29	. .	−0.16	+0.230	
Greenwich	30	−0.07	−0.38	+0.313	+Corr'n. Bradley's to Bessel's refr.
St. Helena	32	−0.55	. .	−0.130	+Corr'n. Young's to Bessel's refr.
Cape	33	−0.49	+0.73	−0.474	+0.58 cos z +Div. Corr.
Cambridge	34	−0.29	−0.04	+0.30	
Cape	37	. .	+1.48	−0.091	(a applies +3° to −72°)
Greenwich	38	. .	−0.04	−0.261	
Greenwich	45	+0.05	. .	+0.031	
Pulkowa	45	. .	+0.32	. .	
Santiago	51	. .	+0.70	−0.150	
Greenwich	51	−0.04	. .	+0.016	
Paris	53	+0.13	−0.56	0.00	
Washington	56	+0.19	−0.61	−0.400	
Greenwich	57	+0.21	−0.17	−0.057	
Cape	59	. .	. .	−0.125	
Paris	60	−0.05	−0.75	−0.567	
Melbourne	62	. .	+0.07	−0.400	
Greenwich	64	. .	. .	−0.084	
Cape	65	. .	. .	−0.068	
Leiden	67	. .	−0.14	. .	
Melbourne	68	. .	. .	−0.346	+Redn. to South refr.
Pulkowa	69	. .	+0.10	−0.162	
Greenwich	72	. .	. .	+0.493	$-0''.60\,(\sin^3 z' - \sin^3 z)$
Harvard	75	. .	. .	−0.202	
Cordoba	76	. .	. .	−0.389	
Paris	76	−0.21	−0.46	−0.376	
Cape	76	. .	. .	−0.109	
Melbourne	77	. .	. .	−0.426	+Redn. to South refr.
Washington	77	−0.10	−0.81	−0.568	
Greenwich	82	. .	−0.06	−0.246	
Cape	83	. .	.	−0.221	
Pulkowa	84	. .	+0.18	+0.055	
Radcliffe	85	−0.12	−0.90	−0.431	
Strassburg	86	. .	. .	+0.005	
Cape	89	. .	. .	−0.030	
Madison	90	. .	. .	−0.190	
Greenwich	94	. .	−0.25	−0.304	
Mt. Hamilton	95	. .	. .	−0.111	
Wien-Ottakr.	97	−0.37	−0.81	−0.250	
Albany	98	. .	. .	. .	

In the majority of instances the formulas of correction for the several catalogues do not appear to require special comment. The application of these corrections to the corresponding values of n (Table IV) results in the values of n'', final corrections to B given by each catalogue of observation. Inspection of the values of n'' for most of the better catalogues indicates that the formulas, in general, represent the discordances from B remarkably well. This must be regarded as very satisfactory for at least two reasons.

1. This could scarcely have been the case with so many catalogues, derived from observations in the two hemispheres, unless the final system, B, is a very fair approximation to a true normal system.

2. The mystery of systematic errors is very largely removed by this showing of the sources from which the greater part of them may have arisen.

In many instances, however, if we compute k from the normal equations, assuming values of a materially different from those contained in Table III, we shall still find that the combined systematic correction down to p.d., 100°, remains substantially unchanged. We should not, therefore, attach too much significance, in those cases, to the relative distribution of the correction between a and k. Some remarks upon individual catalogues may be of service.

Cape 33. This will receive attention later, in connection with the extension of the system southward.

Cape 37. The sine term, a, seems to apply only between +3° and −72° of declination, and to be due to the peculiar treatment of observations by reflection. This catalogue is in need of revision.

Melb. 68 *and* 77. See previous remarks in regard to these catalogues.

Greenwich 72. The formula for flexure adopted in the reductions for this series was $a \sin z \cos^2 z$. If we assume that the formula should have been $a \sin z$, and put $a = -0''.60$ the resulting corrections to the declinations should be,

$$-0''.60\,(\sin^3 z' - \sin^3 z),$$

z' representing the zenith-distance of the pole.

Wien-Ottakring. The formula for this series was derived from a very careful discussion in which the declinations below pole were treated separately from those above. The value of a, as distinguished from k, is entitled to a fair degree of confidence. That the flexure adopted by Grossmann in the reduction of this series can be in error so much as 0″.81 seems scarcely credible. Yet, if the flexure determined by Herz for the same instrument (as quoted by Grossmann, p. 52) had been adopted we should have had as the correction of Grossmann's present declination, $-0''.58\,(\sin z' - \sin z)$, and the discordance from B would have been reduced to a small quantity, so far as this term is concerned. Somewhere between +30° and +45° there seems to be a very large alteration in the systematic

corrections required in order to bring GROSSMANN's declinations into harmony with B. The adopted correction at $+50°$ is $-0''.03$, and at $+25°$, $-1''.01$; at $-45°$, $-0''.19$, and $+30°$, $-0''.85$. Between declinations $+31°\ 47'$ and $+44°\ 52'$, GROSSMANN has only one star, so that it is impossible to analyze the nature of this comparatively abrupt alteration in the difference, B−W.-Ott. The combined testimony of all the recent catalogues is wholly against the hypothesis that there exists a material anomaly in B in this vicinity. If the discordance should be attributed to some defect in the Vienna observations, or in the reduction of them, the most natural source of suspicion would seem to be as to the adopted division correction, though the hypothesis of some looseness in the fastening of the objective, or ocular, may not be wholly excluded.

With the formulated systematic corrections contained in Table III, the zone-equations were revised and solved anew. The results for each zone are contained in the table for $B-B_s$ (*A.J.* 531, p. 21). By way of illustration, and in order, at the same time, to test the system, B, the process is here repeated. The necessary data are exhibited in Table IV. n'' represents the corrections to B which would have been found if the declinations of the individual catalogues had been first corrected for the effect of the formulas in Table III. Following the values of n'' in each column are the weights, p'', which were adopted in the solution of the zone-equations. These are compounded of the weights, p, which are due to the casual error of n, and of a certain maximum weight, p''_0 (Table IV), assigned to each catalogue from an estimate of the probable outstanding errors due to the unavoidable imperfection of the formulas of correction (especially because they do not take account of errors of graduation). The general aim was that the unit of weight should correspond to a probable error of $\pm 0''.1$. This probable error turned out to be $\pm 0''.08$, in the mean, so that the weights, p''_0, were too small on the whole.

The results of the solution of these zone-equations are given under the head of "Secondary Solution" in Table V.

It will be noted that the formulas in Table III are based, not upon B, but upon B_1 and B_2. The results in Table V indicate, therefore, that nothing can be gained by further approximations under the principles adopted. We must either await further observations, or adopt radical improvements of method, before a further gain in systematic accuracy can be anticipated. The questions, what authorities afford independent evidence upon the true system of star-declinations, and what relative weights should be assigned to them, are matters of individual judgment upon which the most competent critics will differ. But this difference of judgment must be somewhat radical before any very material modifications of the present results can be obtained.

TABLE V. RESULTS FROM ZONE-EQUATIONS.

FUNDAMENTAL SOLUTION.

Zone.	Mean Ep.	$\Delta\delta_{1875}$	p.e.	$100\ \Delta\mu'$	p.e.
°		"	"	"	"
+78	1865	−0.017	±0.014	−0.01	±0.06
60	1865	−0.030	.027	−0.04	.12
45	1865	−0.021	.034	−0.06	.15
29	1866	−0.038	.037	−0.05	.18
15	1865	−0.029	.037	−0.19	17
+ 1	1867	−0.025	.059	−0.19	.19
−15	1867	+0.011	046	+0.02	.23
−30	1872	+0.054	±0.071	+0.41	±0.46

SECONDARY SOLUTION.

Zone.	Mean Ep.	$\Delta\delta_{1875}$	p.e.	$100\ \Delta\mu'$	p.e.
°		"	"	"	"
+78	1866	+0.017	±0.013	−0.01	±0.056
60	1864	+0.015	.014	+0.01	.058
45	1866	+0.015	.012	+0.02	0.54
29	1868	+0.007	.010	+0.03	0.45
15	1868	−0.002	.012	−0.02	.054
+ 1	1868	+0.002	.010	−0.05	0.49
−15	1869	−0.003	.008	+0.03	.040
30	1877	+0.005	.024	−0.13	141
45	1872	−0.003	.032	+0.14	193
60	1872	−0.023	.031	+0.06	.189
−81	1873	−0.053	±0.029	+0.07	±0.190

Extension of System to the Southern Hemisphere.

Owing to the scarcity of reliable determinations of declination from observations of the Southern hemisphere during the first half of the nineteenth century, the attempt to secure systematic accuracy in the proper-motions of the far Southern stars is one of very great difficulty. Previous to the first observations made with the Cape Transit circle, there were no measurements of star-declination at observatories in the Southern hemisphere which appear to be entitled to weight as absolutely independent determinations. HENDERSON's work at the Cape is of the first quality, so far as the skill and judgment of the observer is concerned, but his observations cover only one year, and he was handicapped by the remarkable defects of the Jones mural circle. Nevertheless, his observations appeared to offer the only practicable hope for deriving important independent evidence as to Southern declinations previous to 1860. The best plan of procedure seemed to be in accepting the system B as absolute for stars as far southward as −24°, and to employ these with other means in an attempt to determine the instrumental errors of the Jones circle. The solution of the zone observations was carried out definitively, in the first instance, only to −24°. Subsequently, and after first approximate correction of Cape 33, the zone-corrections at −30 and −34 were ascertained with close approximation to the final result. In connection with other corrections an attempt was made to determine the systematic errors of graduation. For this purpose the mean correction for the arc, 32°.5 to 37°.5, was called C_1; that of 57°.5 to 62°.5, C[illegible]; 27°.5 to 32°.5, C[illegible], etc. Circle readings increase toward the north, and north

TABLE IV. MATERIALS FOR ZONE-EQUATIONS. FORMATION OF NORMAL SYSTEM.

Catalogue		+78°					+60°					+45°						+29°				
	p'	p	n	n'	n''	p''	p	n	n'	n''	p''	δ	p	n	n'	n''	p''	p	n	n'	n''	p''
			″	″	″				″					″	″	″					″	
Grw. 16	–	2	+.20	. .	−.13	0.7	4	+.56	. .	−.10	0.8	46	4	+.61	. .	+.02	0.8	5	+.81	. .	+.09	0.8
Kon. 21	2	4	+.04	+.04	+.05	0.8	4	−.20	−.20	−.18	0.8	47	5	+.09	+.09	+.13	0.8	1	+.02	+.02	+.07	0.5
Dpt. 24	3	6	+.13	+.13	+.07	0.5	7	+.34	+.34	+.18	0.6	45	9	+.37	+.37	+.13	0.6	6	+.28	+.28	−.04	0.5
Abo. 29	3	4	+.04	+.04	−.02	1.5	8	+.08	+.08	−.08	1.5	46	12	+.12	+.12	−.11	1.5	8	+.32	+.32	.00	1.5
Grw. 30	1	2	.10	.17	.09	0.4	3	+.21	−.05	−.07	0.5	45	4	+.60	−.03	+.04	0.5	4	+1.10	+.28	+.24	0.5
St. H. 32	0.5	.	.	.	.	.	.	.	.	.	.	46	.	−1.20	+.25	−.02	.	4	−1.15	+.18	−.43	0.2
Cape 33	0.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−.19	−.47	+.14	0.4
Camb. 34	0.5	4	+.01	−.07	−.07	0.3	2	+.36	+.14	+.14	0.3	46	1	+.69	+.56	+.36	0.3	2	+.44	−.01	−.01	0.3
Cape 37	0.5	.	.	.	.	.	.	.	.	.	.	44	1	+.50	−.19	+.21	.	2	+.18	−.26	−.01	0.4
Grw. 38	1	6	+.05	+.05	+.07	1.0	9	+.03	.00	+.14	1.5	45	8	−.04	−.08	+.07	1.5	6	−.06	−.11	+.08	1.0
Grw. 45	1	6	−.04	−.04	+.02	1.0	8	−.46	−.19	−.14	1.5	45	8	+.10	+.06	+.15	1.5	6	−.17	−.22	−.13	1.0
Paris 45	–	7	+.02	. .	+.02	1.0	6	+.47	. . .	+.17	1.0	44	8	−.14	. .	−.14	1.5	7	−.02	. .	−.02	1.0
Pulk. 47	7	12	−.04	−.04	+.02	3.0	11	−.18	−.18	−.02	3.0	45	22	−.27	−.27	−.03	3.0	18	−.36	−.36	−.04	3.0
Stgo. 51	–	.	.	.	.	.	.	.	.	.	.	40	.	+.02	. .	+.70	.	1	−.59	. .	+.27	0.2
Grw. 51	–	5	+.15	. .	+.10	1.0	4	+.18	. .	+.13	1.0	45	6	+.14	. .	+.09	1.0	9	+.02	. .	−.04	1.5
Par.(L)53	1	2	−.32	−.32	. .	.	3	−.34	−.34	. .	.	46	2	−.18	−.18	. .	.	2	+.18	+.18	. .	.
Wash. 56	–	3	−.49	. .	−.10	1.0	2	−.37	. .	−.04	1.0	45	3	−.27	. .	−.04	1.0	1	−.04	. .	+.09	1.5
Grw. 57	3	10	−.11	−.11	+.13	1.5	7	−.30	−.29	−.06	1.5	45	8	−.18	−.16	+.02	1.5	13	−.19	−.17	−.03	1.5
Cape 59	3	.	.	.	.	.	.	.	.	.	.	40	.	−.22	−.81	−.59	.	5	+.17	−.22	−.09	0.8
Paris 60	–	5	+.08	. .	+.26	1.5	6	−.09	. .	−.06	1.5	45	8	−.07	. .	−.15	1.5	12	+.17	. .	−.01	1.5
Melb. 62	–	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−.29	. .	+.13	0.3
Grw. 64	2	11	−.01	+.02	+.01	1.5	6	+.05	+.11	+.08	1.5	44	8	−.09	.00	−.05	1.5	11	−.10	+.02	−.04	1.5
Cape 65	–	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−.01	. .	−.15	0.8
Leid. 67	8	3	−.10	−.10	−.13	1.0	10	+.18	+.18	+.12	1.5	45	15	+.16	+.16	+.06	1.5	9	+.13	+.13	−.01	1.5
Melb. 68	2	.	.	.	.	.	.	.	.	.	.	40	.	−.77	−.22	−.49	.	4	−.13	+.05	−.11	0.6
Pulk. 69	10	19	.00	.00	+.03	3.0	15	−.05	−.05	+.05	3.0	45	30	−.23	−.23	−.08	4.0	26	−.24	−.24	−.03	3.0
Grw. 72	–	12	+.10	. .	−.05	1.0	9	+.40	. .	+.07	1.0	45	10	+.50	. .	+.09	1.0	15	+.51	. .	−.01	1.0
Harv. 75	–	8	−.08	.	−.06	0.6	6	+.06	. .	+.13	0.6	45	9	−.10	. .	.00	0.6	9	−.23	. .	−.10	0.6
Cord. 76	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	5	+.87	+.10	+.09	1.5
Paris 76	–	3	−.02	. .	−.12	1.0	1	+.20	. .	+.04	0.7	44	5	+.31	. .	+.08	1.5	12	+.31	. .	+.03	1.5
Cape 76	–	.	.	.	.	.	.	.	.	.	.	41	.	−.42	. .	−.75	.	2	+.28	. .	+.06	0.7
Melb. 77	1	.	.	.	.	.	.	.	.	.	.	40	.	−1.13	−.41	−1.13	.	5	+.03	+.31	+.16	0.4
Wash. 77	3	23	−.04	−.01	+.10	2.0	11	−.18	−.12	−.12	1.5	45	20	−.05	+.03	−.09	2.0	24	+.31	+.41	+.13	2.0
Grw. 82	3	16	−.07	−.04	−.04	2.0	12	−.20	−.12	−.13	1.5	44	12	−.04	+.08	+.05	1.5	19	−.06	+.09	+.05	2.0
Cape 83	3	.	.	.	.	.	.	.	.	.	.	41	2	+.91	+.21	+.21	.	9	+.15	+.01	+.01	1.5
Pulk. 84	10	26	.00	.00	+.01	3.0	23	−.06	−.06	+.01	3.0	45	11	−.05	−.05	+.06	4.0	36	−.18	−.18	−.03	4.0
Radel. 85	–	7	+.03	. .	−.09	0.8	5	+.51	. .	+.18	0.8	44	4	+.48	. .	−.02	0.8	8	+.74	. .	+.08	0.8
Strass. 86	4	4	.00	.00	.00	2.0	11	+.06	+.06	+.06	3.0	46	15	+.09	+.09	+.09	3.0	9	+.02	+.02	+.02	3.0
Cape 89	3	.	.	.	.	.	.	.	.	.	.	41	3	+.19	+.13	+.10	.	14	−.01	−.04	−.07	2.0
Madn. 90	–	20	+.11	. .	+.15	1.0	11	+.14	. .	+.23	1.0	45	19	+.01	. .	+.13	1.0	21	−.32	. .	−.17	1.0
Grw. 94	3	23	+.10	+.12	+.10	2.0	20	.00	+.04	−.01	2.0	45	26	+.10	+.16	+.07	2.0	31	+.06	+.13	+.02	2.0
Mt. H. 95	–	13	−.05	. .	−.02	1.0	.	.	.	.	.	40	3	−.07	. .	+.01	0.8	3	+.25	. .	+.35	0.8
W-Ott. 97	(2)	8	−.05	−.05	−.13	.	12	−.06	−.06	−.14	.	48	15	+.05	+.05	−.27	.	4	+.98	. .	+.40	.
Alb. 98	–	6	−.07	. .	−.07	2.0	1	−.31	. .	−.31	0.7	45	6	−.05	. .	−.05	2.0	10	+.02	. .	+.02	2.0

Catalogue	−45°					−60°				−81°			
	δ	p	n	n''	p''	p	n	n''	p''	p	n	n''	p''
	°		″	″			″	″			″	″	
St. H. 32	45	2	−.50	+.13	0.2	2	−.14	+.15	0.2	1	−.71	−.46	.
Cape 33	45	3	−.12	−.22	0.4	3	+.25	−.21	0.4	4	+.06	−.01	0.4
Cape 37	44	8	+.38	+.06	0.5	6	+.75	+.04	0.5	5	+.19	−.04	0.5
Stgo. 51	47	1	−.30	+.01	0.2	2	−.23	−.07	0.3	1	+.07	+.08	0.2
Cape 59	45	7	+.06	−.03	0.8	6	+.02	−.05	0.8	8	−.10	−.13	0.8
Melb. 62	43	1	−.19	+.03	0.2	1	+.40	+.50	0.2	4	+.25	+.25	0.3
Cape 65	45	4	−.16	−.21	0.8	6	−.06	−.10	0.8	5	−.23	−.25	0.8
Melb. 68	45	7	+.63	+.39	0.6	6	+.46	+.28	0.6	10	+.15	+.07	0.7

(Continued on opposite page.)

TABLE IV. MATERIALS FOR ZONE-EQUATIONS. — FORMATION OF NORMAL SYSTEM.

Catalogue		p''_0	+15 p	n	n'	n''	p''	±0° p	n	n'	n''	p''	−15 p	n	n'	n''	p''	−30 δ	p	n	n'	n''	p''
Grw.	16	1.0	10	+.72	. .	−.11	1.0	8	+.80	. .	−.11	0.8	9	+.98	. .	.00	0.8	25	1	+1.13	. .	+.12	.
Kön.	21	1.0	2	+.11	+.11	+.17	0.7	1	−.09	−.09	−.02	0.5	1	−.14	−.14	−.07	0.5	.	.	. .	. .	. .	.
Dpt.	24	0.6	5	+.24	+.24	−.19	0.5	4	+.73	+.73	+.12	0.5	2	+.91	+.91	−.16	0.2	.	.	. .	. .	. .	.
Abo.	29	2.0	8	+.44	+.41	+.03	1.5	9	+.57	+.57	+.05	1.5	5	+.70	+.70	−.11	0.5	.	.	. .	. .	. .	.
Grw.	30	0.6	6	+1.50	+.42	+.30	0.5	5	+1.47	−.02	−.15	0.5	6	+2.08	−.12	−.15	0.5	25	1	+2.64	+.31	+.01	.
St. H.	32	0.2	1	−1.22	−.10	−.13	0.2	2	−.89	+.11	−.15	0.2	1	−.60	+.12	−.09	0.2	30	1	+.08	+.79	+.20	0.2
Cape	33	0.5	2	+.06	−.24	+.02	0.4	2	+.12	−.12	+.04	0.4	3	−.16	−.37	+.01	0.4	31	2	−.69	−.86	−.09	0.4
Camb.	34	0.4	3	+.22	−.35	−.35	0.4	3	+.32	−.43	−.43	0.4	3	+1.13	−.03	−.03	0.3	25	.	+2.12	+.21	+.21	.
Cape	37	0.5	3	+.18	−.17	+.01	0.4	3	−.25	−.54	+.13	0.4	6	−.38	−.62	.00	0.5	31	11	−.09	−.30	−.09	0.5
Grw.	38	1.5	8	−.14	−.20	+.01	1.5	7	−.28	−.36	−.03	1.0	7	−.43	−.55	−.03	1.0	25	1	−.61	−.80	.00	.
Grw.	45	1.5	10	+.01	−.05	+.05	1.5	8	−.11	−.19	−.08	1.5	8	+.01	−.11	+.02	1.5	25	1	+.07	−.12	+.05	.
Paris	45	1.5	12	+.03	. .	+.03	1.5	13	+.15	. .	+.15	1.5	13	+.07	. .	+.07	1.5	26	2	−.39	. .	−.39	.
Pulk.	47	4.0	21	−.31	−.31	+.08	3.0	10	−.40	−.40	+.04	3.0	3	−.50	−.50	−.03	1.0	.	.	. .	. .	. .	.
Stgo.	51	0.3	2	−.86	. .	+.02	0.3	1	−.89	. .	−.10	0.2	2	−.98	. .	−.33	0.3	29	1	−.66	. .	−.17	0.2
Grw.	51	1.5	12	−.15	. .	−.21	1.5	11	+.11	. .	+.04	1.5	10	+.09	. .	+.02	1.5	25	1	+.07	. .	−.03	.
Par.(L)	53	−	3	+.18	+.18	. .	.	3	+.42	+.12	. .	.	1	+.27	+.27	. .	.	26	.	−.50	. .	. .	.
Wash.	56	2.0	6	+.06	. .	+.10	1.5	6	+.19	. .	+.18	1.5	6	+.03	. .	+.05	1.5	28	3	−.25	. .	−.04	0.5
Grw.	57	2.0	21	−.27	−.24	−.13	2.0	19	−.03	+.01	+.09	2.0	17	−.12	−.06	+.02	2.0	25	2	−.33	−.23	−.07	.
Cape	59	1.0	11	+.28	−.03	+.09	1.0	10	+.31	+.05	+.15	1.0	10	+.21	−.01	+.08	1.0	30	13	−.03	−.21	−.14	1.0
Paris	60	2.0	16	+.28	. .	+.03	2.0	15	+.21	. .	−.02	2.0	15	+.05	. .	.00	1.5	26	3	−.56	. .	−.05	.
Melb.	62	0.3	3	−.34	. .	+.23	0.3	4	−.85	. .	−.29	0.3	3	−.55	. .	−.08	0.3	30	3	−.40	. .	−.04	0.3
Grw.	64	2.0	19	−.04	+.11	+.03	2.0	14	−.01	+.18	+.09	1.5	15	−.14	+.16	+.01	1.5	25	1	−.63	−.16	−.39	.
Cape	65	1.0	15	+.17	. .	+.07	1.0	14	+.22	. .	+.13	1.0	18	+.11	. .	+.04	1.0	29	8	−.01	. .	−.07	1.0
Leid.	67	1.5	12	+.38	+.38	+.21	1.5	8	+.11	+.11	−.08	1.0	4	−.02	−.02	−.23	1.0	.	.	. .	. .	. .	.
Melb.	68	0.7	7	+.40	+.42	+.29	0.6	7	+.16	+.10	−.02	0.6	5	+.38	+.26	+.16	0.6	30	6	+.41	+.24	+.16	0.6
Pulk.	69	4.0	29	−.35	−.35	−.08	3.0	17	−.34	−.34	+.02	3.0	8	−.48	−.48	−.06	1.0	.	.	. .	. .	. .	.
Grw.	72	1.0	17	+.60	. .	−.11	1.0	14	+1.09	. .	+.09	1.0	13	+1.43	. .	+.07	1.0	25	1	+2.22	. .	+.07	.
Harv.	75	0.7	11	−.27	. .	−.10	0.7	8	−.09	. .	+.13	0.6	8	−.31	. .	−.02	0.6	27	2	−.33	. .	+.09	0.4
Cord.	76	3.0	19	+.60	−.04	−.01	3.0	19	+.47	−.08	−.05	3.0	19	+.43	−.03	−.01	3.0	29	13	+.27	−.13	−.11	2.0
Paris	76	2.0	16	+.25	. .	−.07	2.0	16	+.22	. .	−.08	2.0	15	+.16	. .	−.01	1.5	26	2	−.32	. .	−.14	.
Cape	76	1.0	5	+.16	. .	−.01	0.8	7	+.09	. .	−.05	0.8	8	+.03	. .	−.09	0.8	30	11	+.22	. .	+.12	1.0
Melb.	77	0.4	11	+.43	+.53	+.19	0.4	10	+.27	+.27	.00	0.4	11	+.60	+.52	+.30	0.4	29	6	+.72	+.60	+.41	0.4
Wash.	77	2.0	33	+.26	+.38	−.02	2.0	27	+.24	+.39	−.10	2.0	25	+.31	+.50	.00	2.0	29	13	+.13	+.39	+.13	1.5
Grw.	82	2.0	27	−.28	−.09	−.14	2.0	22	−.11	+.14	+.09	2.0	21	−.12	−.14	−.08	2.0	25	2	−.58	+.04	+.05	.
Cape	83	2.0	18	+.30	−.04	−.04	2.0	22	+.28	.00	.00	2.0	29	+.25	+.02	+.02	2.0	28	11	+.28	+.08	+.08	1.5
Pulk.	84	1.0	11	−.25	−.25	−.08	4.0	25	−.24	−.24	−.06	3.0	13	−.11	−.11	+.03	1.5	.	.	. .	. .	. .	.
Radcl.	85	1.0	11	+.75	. .	−.02	1.0	10	+.74	. .	−.07	1.0	10	+.51	. .	−.20	1.0	25	1	+.50	. .	+.27	.
Strass.	86	1.0	16	+.07	+.07	+.07	3.0	20	+.02	+.02	+.01	3.0	29	+.02	+.02	+.01	1.0	26	5	+.18	+.18	+.46	.
Cape	89	2.0	26	+.03	+.01	−.02	2.0	27	+.01	+.02	.00	2.0	26	+.10	+.09	+.07	2.0	30	18	−.03	−.01	.06	2.0
Madn.	90	1.0	27	−.33	. .	−.15	1.0	20	−.36	. .	−.11	1.0	6	−.29	. .	−.01	1.0	.	.	. .	. .	. .	.
Grw.	94	2.0	39	+.01	+.11	−.03	2.0	38	−.08	+.05	−.08	2.0	35	−.20	−.01	−.06	2.0	25	3	−.52	−.21	−.06	.
Mt. H.	95	1.0	11	+.06	. .	+.17	1.0	19	+.09	. .	+.23	1.0	11	−.20	. .	−.03	1.0	30	10	−.13	. .	−.19	1.0
W-Ott.	97	−	5	+1.05	. .	+.33	.	6	+.79	. .	−.03	.	6	+.53	. .	−.28	.	28	1	+.12	. .	−.11	.
Alb.	98	3.0	19	+.10	. .	+.10	3.0	10	.00	. .	.00	2.0	14	+.07	. .	+.07	2.0	29	21	−.01	. .	.01	3.0

Catalogue		−45 δ	p	n	n''	p''	−60 p	n	n''	p''	−81 p	n	n''	p''
Cord.	76	45	7	+.18	.11	2.0	5	+.07	−.18	2.0	23	+.02	.08	3.0
Cape	76	45	9	+.17	+.09	1.0	10	−.19	−.25	1.0	10	.25	−.28	1.0
Melb.	77	46	1	+.72	+.13	0.3	3	+.47	+.25	0.4	9	+.08	.01	0.4
Washn.	77	40	1	.72	+.03	.	.	. .	. .	.	.	. .	. .	.
Cape	83	46	11	+.16	.04	1.5	9	+.29	+.16	1.5	10	+.07	+.02	2.0
Cape	89	44	18	+.01	.01	2.0	15	−.03	−.05	2.0	13	.00	.00	2.0
Mt. H.	95	40	1	1.18	−.78	.	.	. .	. .	.	.			
Alb.	98	40	1	−.37	.37	.	.	. .	. .	.	. .	.		

z.d.'s are regarded as positive. Then the corrections to declinations are assumed to be:

$$\Delta q + a \sin z + b \cos z + k \frac{p}{100} + C_n = \Delta\delta$$

To reconcile the constant difference between direct and reflected z.d.'s a constant, Δz, is introduced. The northern declinations furnish 16 equations; D — R (see Mem. R.A.S., X, p. 78), 12 equations; and declinations below pole *minus* declinations above pole (Mem. R.A.S., X, p. 73), six equations. The weights are adopted in general accordance with HENDERSON's data wherein 0″.22 is the minimum p.e. of repeated observations on a given object, and the casual error is taken from HENDERSON's table (p. 59), but not smaller than ±0″.56 in any case. Adopted p.e. of unit of weight is ±0″.30.

In the comparison of observations above and below pole (and in two other instances) the grouping did not permit of the direct determination of a single C independent of its next neighbor. In such cases the proportional ratio with which each C was involved was expressed in the equation. Each division correction was represented in at least four of the 34 equations. The normal equations were formed with all rigor, and solved by approximations, — the convergence being rapid and requiring only three repetitions for a precise result. The values of the unknowns resulted as follows:

Δq	$= -0''.49$	C_4	$= -0''.04$
Δz	$= +0.16$	C_5	$= -0.58$
a	$= +0.73$	C_6	$= -0.10$
b	$= +0.58$	C_7	$= -0.19$
k	$= -0.471$	C_8	$= -0.25$
		C_9	$= -0.35$
C_1	$= +0.30$	C_{10}	$= -0.48$
C_2	$= +0.65$	C_{11}	$= +0.59$
C_3	$= +0.20$	C_{12}	$= +0.52$

The leap between C_{10} and C_{11} is especially notable and is fully confirmed by the trend of the individual comparisons which make up the means wherever those graduations occur. The following table exhibits computed and observed values of the discordances upon which the investigation is founded.

This comparison seems to leave nothing to be desired. Only one discrepancy is relatively large (the third in "lower — upper") and this is within admissible limits relatively to its weight. The remarkable feature of this investigation is that, while the declinations north of the Cape zenith are completely reconciled to those of B, the discordance, D — R, which gave HENDERSON so much trouble is accounted for in a manner which seems to be very satisfactory. The observations above and below pole are also satisfactorily accordant. These facts seem to warrant confidence in the determination of the correction for errors of graduation. On the other hand, a small but decided discordance of the latitude obtained in this discussion from that of the modern results for the Cape transit circle subtracts somewhat from the complete satisfaction which might otherwise be felt.

RESULTS OF DISCUSSION OF HENDERSON'S OBSERVATIONS, C — O.

$\Delta\delta$, B — Cape 33.

p	δ	(O)	(C)
	°	″	″
2.2	+45.9	−1.32	−1.26
2.2	38.8	−0.05	−0.08
5.4	30.2	+0.17	+0.14
4.6	26.1	+0.31	+0.48
6.4	21.4	+0.62	+0.63
6.9	15.0	−0.47	−0.36
8.6	10.8	−0.06	−0.20
7.9	5.9	+0.02	−0.06
5.9	+ 1.2	−0.13	+0.01
5.5	− 3.6	−0.19	−0.19
7.5	8.8	−0.42	−0.37
12.2	13.6	+0.15	+0.16
9.7	19.0	+0.36	+0.39
7.0	23.8	+0.79	+0.81
5.0	29.5	+0.38	+0.42
6.5	−33.8	+0.70	+0.61

D — R (Δz).

p	z	(O)	(C)
	°	″	″
0.7	−40.8	+0.89	+0.92
1.5	−34.9	+1.81	+1.29
0.7	−29.6	+1.27	+1.19
6.6	−25.0	+0.90	+1.07
2.2	−15.1	+0.40	+0.21
2.2	+ 5.5	−1.17	−1.35
1.5	+16.0	+0.06	−0.57
1.5	+24.6	−0.46	−0.15
0.7	+32.1	−0.89	−0.30
1.5	+40.4	−1.08	−0.97
1.5	+44.5	−1.48	−1.18
2.2	+50.6	−1.65	−1.63

$\Delta\delta$, Lower — Upper.

p	δ	(O)	(C)
	°	″	″
3.0	−89.0	+0.08	−0.03
3.2	−84.1	+0.49	+0.50
0.9	−78.3	+0.73	−0.44
0.4	−74.7	−0.65	−0.75
1.4	−68.1	−1.38	−0.94
1.2	−63.9	−0.39	−0.05

The latitude of the Jones mural circle given by this discussion is $-33°\,56'\,3''.25 + \Delta q - \Delta z = -33°\,56'\,3''.90$. For the Cape transit circle we have for the seconds of latitude, and their values when the refraction-corrections of Table II are taken into account:

	″	″	″
Cape 59 (p. 10, Int.)	3.55	−0.10	3.65
Cape 83 (p. xlvii, Int.)	3.54	. .	3.54
Cape 89 (p. xxiv, Int.)	3.45	−0.03	3.48

In 1886-1891 GILL also determined the latitude by the TALCOTT method. The declinations of his northern stars were determined with the Pulkowa vertical circle. The southern element of his pairs consisted of close circumpolar stars, observed equally at both culminations. Thus, errors in the assumed declinations of the southern stars were eliminated. The seconds of this result are, 3″.65 (Cape 85, Int. p. xlvii). Assuming that the Pulkowa declinations in question require the systematic correction to B which has been found for Pulkowa 84, the seconds of latitude reduced to B would be 3″.53. Accordingly, we may assume that the mean latitude of the Cape transit circle is 3″.54, and this is numerically smaller than the latitude deduced in this discussion of HENDERSON's observations by 0″.36. For so short a period as one year, the mean latitude applicable to HENDERSON's observations, uncorrected for variation of latitude, may have been sensibly different from the true mean latitude; but this difference can scarcely have been greater than 0″.1.

This discordance in latitude may point to a numerically smaller value of k than was reached in the present discussion; but a more probable explanation is indicated in connection with the term $b \cos z$. If this term had been omitted in the equations we should have had, approximately: $\Delta\varphi = -0''.08$; $a = +0''.78$; $k = -0.50$; with small changes in the correction for errors of graduation. The seconds of latitude would then have been: $3''.49$, in excellent agreement with the modern results; supposing that there is no such thing as secular variation of latitude. The northern declinations would not have been so accordant with B as in the actual solution; but they would not have been altered at any point within 70° of the zenith by so much as $0''.2$; and ordinarily by less than half that amount.

Adopting the result of this discussion of Cape 33, the zone equations, south of −30°, were first solved, omitting St. Helena 32 and Cape 37. Then the corrections for these catalogues and for Melb. 62, were formulated as in Table III. The effect of this process is to assign due weight in the formation of the southern system to these authorities through the medium of their relations to northern stars, and to make the essential weight of Cape 33 in the formation of the system to be about one-half greater than that which is nominally assigned to it in Table IV.

It would seem to be extremely desirable that the zenith-distances observed by McClear for the Cape Catalogue of 1840 should be discussed anew on principles somewhat similar to those which have been adopted in the foregoing discussion of Cape 33. This would afford a much needed check upon the conclusions derived from Henderson's observations. However, we shall soon be in a position to determine whether this treatment of Henderson's observations has resulted in improvement of them. If errors have been introduced thereby, these will be reflected in some degree in the values of proper-motion, and the effects ought to become very sensible in the predicted declinations of B within five, or ten, years from the present time.

OBSERVATIONS OF COMET *d* 1902 (*GIACOBINI*),

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA.

By T. McN. SIMPSON, Jr.

1903 Charl. M.T.	✱	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Apr. 21 9 37 50	1	10 , 4	+0 45.66	+0 50.0	7 20 39.42	+32 5 59.9	9.688	0.464	+0.99	−6.7
27 9 35 5	2	*d*12 , 8	+0 21.08	−6 7.9	7 29 8.57	+32 44 56.5	9.702	0.482	+0.90	−6.5
28 10 27 6	2	8 , 8	+1 52.27	−0 0.7	7 30 39.74	+32 51 3.6	9.734	0.589	+0.88	−6.5
29 9 51 49	3	8 , 6	+2 6.31	−6 18.6	7 32 6.35	+32 56 16.2	9.719	0.525	+0.87	−6.5

Mean Places of Comparison-Stars for the beginning of the year.

✱	α	δ	Authority
	h m s	° ′ ″	
1	7 19 52.77	+32 5 16.6	Leiden, A.G. 3128
2	7 28 46.59	+32 51 10.8	" " 3192
3	7 29 59.14	+33 3 14.1	" " 3199

NOTES.

d refers to direct micrometrical measurement. April 27 — Comet very faint; observation difficult. These observations have been corrected for refraction.

I wish to call attention to a mistake in my observations of this comet published in *A. J.*, Nos. 537–8. Log $p\Delta$ throughout should be increased by 1 in the characteristic. The same correction should be applied to Mr. McCarthy's observations published in *A. J.* No. 534 for the reduction of which I am responsible.

Charlottesville, Va.

OBSERVATIONS OF COMET *a* 1903 (*GIACOBINI*),

MADE WITH THE 11-INCH EQUATORIAL AT THE SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.

By MARY E. BYRD.

1903 Greenw. M.T.	✱	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 20 11 48 19	1	16 , 11	−0 12.97	−1 48.5	23 15 44.86	+12 3 48.7	9.639	0.740	−0.21	+0.9
22 12 6 38	2	12 , 10	0 25.67	6 54.5	23 47 32.72	+12 49 29.1	9.646	0.748	0.19	+0.8
Mar. 2 11 48 24	3	8 , 8	+3 10.04	−1 46.4	0 3 35.29	+15 45 37.1	9.652	0.737	0.19	+0.1
11 48 24	4	8 ,	+3 12.60	. . . .	0 3 35.44	. . . .	9.652	. .	0.19	. .
12 12 10 1	5	10 , 7	1 0.57	6 32.2	0 24 58.47	+17 14 17.2	9.657	0.760	0.12	0.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority
	h m s	° ′ ″	
1	23 43 58.04	+12 5 36.3	Leipzig I, A.G. 9451
2	23 47 58.58	+12 56 23.1	Leipzig I, A.G. 9469
3	0 0 25.41	+15 47 23.7	Berlin A, A.G. 9779
4	0 0 23.03	+15 18 46.9	Berlin A, A.G. 9778
5	0 22 59.16	+17 21 20.2	Berlin A, A.G. 109

OBSERVATIONS OF SUNSPOTS.

MADE AT BOSTON UNIVERSITY OBSERVATORY.

By C. Q. JONES and L. R. TUCKER, Students in Astronomy.

W.M.T. 1902–1903			Groups N	Groups S	Spts. in Gps. N	Spts. in Gps. S	Totals Grps.	Totals Spts.	Def.
	d	h							
Oct.	6	4	1	0	12	0	1	12	G
	6	22	2	0	15	0	2	15	G
	7	4	2	0	17	0	2	17	G
	8	1	2	0	14	0	2	14	G
	9	1	2	0	21	0	2	21	G
	10	0	2	0	4	0	2	4	F
	10	22	1	0	1	0	1	1	P
	13	0	1	0	3	0	1	3	G
	14	3	1	0	1	0	1	1	F
	15	2	1	0	3	0	1	3	G
	15	22	1	0	1	0	1	1	F
	19	23	1	0	3	0	1	3	G
	22	1	0	1	0	14	1	14	F
	23	4	0	1	0	13	1	13	G
	24	1	0	2	0	24	2	24	G
	28	4	0	2	0	8	2	8	F
	30	1	0	1	0	1	1	1	F
	31	0	0	1	0	1	1	1	G
Nov.	14	3	2	1	8	1	3	9	G
	15	0	2	1	6	1	3	7	P
	19	22	1	0	30	0	1	30	G
	20	2	1	0	25	0	1	25	G
	20	23	1	0	25	0	1	25	P
	21	2	1	0	18	0	1	18	F
	22	1	1	0	17	0	1	17	G
Jan.	5	0	0	1	0	2	1	2	F
	5	23	0	1	0	2	1	2	P
	8	1	0	1	0	1	1	1	P
Feb.	5	2	0	1	0	1	1	1	G
	6	1	0	1	0	1	1	1	F
	8	23	0	1	0	6	1	6	G
	10	3	0	1	3	10	2	13	G
	12	1	1	0	2	0	1	2	E

W.M.T. 1903			Groups N	Groups S	Spts. in Gps. N	Spts. in Gps. S	Totals Grps.	Totals Spts.	Def.
	d	h							
Feb.	13	1	1	0	2	0	1	2	P
	19	1	0	1	0	1	1	4	P
	20	0	0	1	0	5	1	5	P
	24	1	1	2	1	4	3	5	F
	24	23	1	2	1	6	3	7	G
	25	1	1	2	1	7	3	8	G
	26	2	1	1	1	9	2	10	G
	27	1	1	1	1	6	2	7	P
Mar.	2	2	1	0	2	0	1	2	G
	4	2	1	0	3	0	1	3	F
	13	2	0	1	0	5	1	5	G
	13	23	0	1	0	4	1	4	G
	25	2	1	0	6	0	1	6	F
	26	3	1	1	4	1	2	5	G
	27	0	1	1	3	6	2	9	G
Apr.	1	1	1	1	5	10	2	15	F
	2	1	1	2	2	6	3	8	F
	3	1	0	2	0	4	2	1	G
	6	1	0	1	0	2	1	2	P
	8	22	0	1	0	5	1	5	G
	9	2	0	1	0	6	1	6	G
	9	22	0	1	0	5	1	5	G
	12	23	0	1	0	3	1	3	P
	20	5	0	1	0	1	1	1	
	22	2	0	1	0	1	1	1	P
	23	0	0	1	0	1	1	1	F
	27	1	3	1	27	4	4	31	G
	28	2	2	1	19	4	3	23	G
	29	1	2	1	28	2	3	30	E
	29	23	2	1	68	9	3	77	G
May	1	0	2	2	32	3	4	35	G
	1	22	2	0	7	0	2	7	P
Totals,			54	50	442	209	104	651	

For explanations see *A.J.* 466.

Observations were made, and no spots seen, as follows: November, 8 days; December, 10 days; January, 9 days; February, 1 day; March, 7 days; May, 4 days. 23 different groups we observed, containing 223 different spots; 13 groups, with 151 spots, were in north latitude, while 10 groups, with 72 spots, were south.

PROJECTION ON *MARS*.

A dispatch *via* Harvard College Observatory, May 27, states that a large projection on *Mars* was found by SLIPHER at Lowell Observatory, Flagstaff, Ariz., May 26^{d} 8^{h} 35^{m} (Mountain Standard Time), in position angle 200°, lasting 35 minutes.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(9) *Metis.*						
Apr. 5 9h 55m 5s	1	27 , 8	−0m 30s.65	+ 2′ 41″.7	12h 48m 45s.85	+ 3° 12′ 6″.1	n9.362	0.713	+2s.29	−12″.8
8 9 11 56	2	17 , 6	−0 47.55	−10 29.3	12 45 55.39	+ 3 24 20.0	n9.444	0.713	+2.30	−12.8
9 11 45 2	2	28 , 6	−1 50.31	− 6 13.0	12 44 52.63	+ 3 28 36.3	8.261	0.705	+2.30	−12.8
10 9 51 28	3	29 , 6	−0 37.86	− 9 16.9	12 44 0.99	+ 3 32 5.1	n9.274	0.708	+2.30	−12.8
17 9 22 13	4	29 , 6	−1 33.19	+ 1 7.5	12 37 48.57	+ 3 53 53.1	n9.268	0.704	+2.29	−12.6
				(230) *Athamantis.*						
Apr. 8 11 21 30	5	30 , 6	−1 38.16	+ 6 7.7	13 13 7.63	−18 11 39.3	n8.988	0.864	+2.64	−11.4
8 11 21 30	6	30 , 6	−3 28.89	+ 2 31.3	13 13 7.60	−18 11 41.2	n8.988	0.864	+2.64	−11.4
10 10 39 4	7	30 , 6	−1 38.97	− 6 53.7	13 11 20.97	−17 55 46.5	n9.213	0.858	+2.64	−11.7
17 10 34 15	8	30 , 6	+1 41.32	+ 0 9.9	13 5 7.06	−16 55 21.1	n9.019	0.858	+2.64	−13.0
18 9 37 7	9	30 , 10	−0 2.77	− 3 41.5	13 4 17.03	−16 46 44.5	n9.317	0.848	+2.64	−13.0
				(60) *Echo.*						
Apr. 18 10 45 11	10	28 , 6	+1 22.18	− 1 26.6	14 3 8.37	−10 27 36.3	n9.266	0.816	+2.62	− 9.1
21 12 12 52	11	29 , 6	+2 25.08	− 7 13.5	14 0 14.85	−10 7 18.1	8.261	0.821	+2.59	− 9.1
27 10 13 2	12	29 , 6	−1 47.52	− 0 13.5	13 54 44.21	− 9 28 10.1	n9.210	0.812	+2.61	− 9.4
29 11 15 39	13	30 , 6	+2 6.09	+ 1 12.6	13 52 53.36	− 9 15 45.3	n8.282	0.815	+2.61	− 9.8
May 2 9 13 30	14	30 , 8	−0 6.69	+ 5 0.1	13 50 35.70	− 9 0 12.7	n9.357	0.804	+2.62	− 9.9
				(83) *Beatrix.*						
Apr. 28 11 19 46	15	29 , 6	−0 55.58	+ 1 5.8	14 16 23.41	−15 17 4.1	n8.830	0.851	+2.75	− 8.1
29 12 10 51	16	30 , 6	+1 28.91	+ 3 18.4	14 15 20.58	−15 14 50.0	8.684	0.851	+2.75	− 8.4
May 2 11 28 8	16	30 , 6	−1 29.23	+ 9 46.9	14 12 22.46	−15 8 21.5	n7.980	0.851	+2.77	− 8.4
6 9 34 36	17	21 , 5	+0 33.94	+ 9 35.6	14 8 33.99	−14 59 42.9	n9.297	0.840	+2.78	− 9.2
11 9 39 3	18	35 , 7	−1 59.47	− 3 51.5	14 1 0.31	−14 49 12.3	n9.154	0.844	+2.78	− 9.2
				(16) *Psyche.*						
Apr. 28 12 53 48	19	29 , 6	−0 48.84	+ 4 6.3	14 41 33.96	−11 32 58.8	8.830	0.829	+2.68	− 6.2
May 4 10 11 39	20	30 , 6	+0 47.58	+ 3 21.2	14 39 52.77	−11 9 2.6	n9.133	0.824	+2.73	− 6.8
4 10 44 57	21	30 , 6	+1 31.24	+ 3 21.5	14 39 52.49	−11 9 2.1	n9.131	0.824	+2.72	− 6.8
7 10 26 10	22	30 , 6	+1 23.08	− 0 19.7	14 37 30.41	−10 57 19.3	n9.159	0.822	+2.74	− 6.9
8 10 26 52	22	30 , 6	+0 35.77	+ 3 33.1	14 36 43.11	−10 53 26.5	n9.126	0.822	+2.75	− 6.9
				(12) *Victoria.*						
May 7 12 14 6	23	30 , 6	+2 18.42	− 1 39.2	15 12 18.48	−19 53 4.3	7.423	0.875	+2.95	− 4.4
8 10 55 13	24	30 , 6	+0 10.53	5 46.2	15 11 26.04	−19 44 18.5	n9.174	0.868	+2.96	− 4.4
9 9 38 57	24	30 , 6	−0 12.72	+ 3 4.2	15 10 32.77	−19 35 28.2	n9.157	0.847	+2.97	− 4.5
11 10 26 12	25	30 , 6	−1 12.79	− 5 8.3	15 8 36.99	−19 16 12.4	n9.254	0.863	+2.97	− 4.4
11 10 51 16	26	30 , 6	+0 45.27	+ 0 55.0	15 8 35.72	−19 16 5.8	n9.114	0.867	+2.98	− 4.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	12h 49m 14s.21	+ 3° 9′ 37″.5	Albany, A.G. 4600	14	13h 50m 39s.77	− 9° 5′ 2″.9	Radcliffe 1890, 3610
2	12 46 40.61	+ 3 35 2.4	" " 4586	15	14 17 16.24	−15 18 1.8	Wash. A.G.Z. 53,113,209
3	12 44 36.55	+ 3 44 34.8	" " 4575	16	14 13 48.92	−15 18 0.0	" " 47,119,209,231
4	12 39 19.47	+ 3 52 58.2	" " 4555	17	14 7 57.27	−15 9 9.3	" A.G.Z. 50, 114
5	13 14 43.15	−18 17 35.6	Wash. A.G.Z. 54,116,204	18	14 5 57.00	14 45 11.6	" " 45, 115
6	13 16 33.85	−18 14 1.1	" " 44, 116	19	14 45 20.12	11 36 58.9	Radcliffe 1890, 3835
7	13 12 57.30	−17 48 41.4	" " 47, 117	20	14 39 2.46	−11 12 17.0	½(Schj. 5220 + Mu. I, 5048)
8	13 3 20.10	−16 55 18.0	" " 44, 116, 204	21	14 38 18.53	−11 12 16.8	Mu. I, 10469
9	13 4 17.16	−16 42 50.0	" " 44, 110	22	14 36 4.59	10 56 52.7	Yarnall, 6156
10	14 1 43.57	−10 26 0.6	½(Mu.I, 9840+Mu.II, 5181)	23	15 9 57.11	19 51 21.0	Cincinnati 1885, 2585
11	13 57 47.18	− 9 59 55.2	Wien, A.G.Z., 56, 250	24	15 11 12.52	−19 38 27.9	" " 2588
12	13 56 29.42	− 9 28 17.2	" " 134, 253	25	15 10 16.84	−19 10 59.7	" " 2587
13	13 50 44.66	− 9 16 48.4	Radcliffe 1890, 3611	26	15 7 47.47	−19 16 56.2	Radcliffe 1890, 3627

NOTES ON VARIABLE STARS. — No. 38.

By HENRY M. PARKHURST.

2689 *Z Puppis*. In the Supplementary Catalogue, *A.N.*, No. 514, No. 2690 should be 2689, the variable discovered by PERRY, and announced in *A.J.*, No. 428. The four observed maxima appear to vary at least 30 days from the average period of 255 days, while both observed minima have occurred within about 60 days from the average times of maxima.

2690 *X Puppis*. The observations of 2690 show 5 maxima, with an average deviation of 11 days from the average period of 155 days. PERRY's observations in April, 1899, *A.J.* 468, nearly correspond with a sixth maximum. The observed minimum of 1903 is consistent with a period of only one-half of this.

Subtangent Process. An illustration of the subtangent method (*A.J.* 400 and 456) occurs in obtaining the maximum for 976, which was lost in the twilight just before it was reached. The 8 observations given below, by smoothing, were reduced to the three following:

$$A \quad 6155.36 \quad 8^{\mathrm{M}}.35;$$
$$B \quad 6166.81 \quad 8.09;$$
$$C \quad 6175.60 \quad 7.92;$$

Making the first tangent from

$$AB \quad 11.45 = -.26: \ 1 \text{ day} = -^{\mathrm{M}}.0227$$

Making the second tangent from

$$BC \quad 8.79 = -.17: \ 1 \text{ day} = -^{\mathrm{M}}.0193$$

The mean of observations A, B, is 6161.08 $8^{\mathrm{M}}.22$;

The mean of observations B, C, is 6171.20 $8^{\mathrm{M}}.00$.

The maximum is assumed to occur at the time $C+t$.

From the first tangent, we obtain for the time $14.52+t$, the vertex, $8.22-(.329+.0227\,t)$.

From the second tangent, we obtain for the time $4.40+t$, the vertex $8.00-(.085+.0193\,t)$.

The subtangents are included within the parentheses. The subtangent being trisected at the vertex, the vertex is at the distance of two-thirds of the subtangent from the respective magnitudes. Multiplying the observed magnitudes by $1\frac{1}{2}$, and then taking the differences, we have the equation,

$$.33 - .244 - .0034\,t = 0$$

whence $t = +25.3$: and hence the maximum is at 6201, 3 days earlier than from the elements, and 8 days after my last possible observation. From the last subtangent, making $t = 0$, the deduced maximum, $7^{\mathrm{M}}.91$ is derived.

RESULTS OF OBSERVATIONS.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W	Mag.	Factors			Remarks
				1902–3							d	
103	*T Andromedae*	Max.	6084	Nov. 30	65	–	E	–	–	–	–	
114	*S Ceti*	Max.	6062	Nov. 8	34	+10	2	–	–	–	–	From light-curve
434	*S Piscium*	Max.	5947	July 16	33	–	E	–	–	–	–	
466	*U Piscium*	Max.	6012	Sept. 19	48	–	E	–	–	–	–	
715	*S Arietis*	Min.	6098	Dec. 11	39	–	E	–	–	–	–	
845	*R Ceti*	Min.	6151	Feb. 5	79	–	E	–	–	–	–	
893	*U Ceti*	Max.	6124	Jan. 9	28	0	4	7.4	–	–	–	[cess
976	*T Arietis*	Max.	6201	Mar. 27	35	– 3	9	7.91	–	–	–	Derived by the subtangent pro-
1113	*U Arietis*	Max.	6123	Jan. 8	9	–43	4	7.7	–	–	–	Elements, *A.J.* 403
1166	*X Ceti*	Max.	6102	Dec. 18	12	–	E	–	–	–	–	Elements, *A.J.* 438
1577	*R Tauri*	Max.	6179	Mar. 5	46	–33	9	7.34	0.65	1.34	19	
1582	*S Tauri*	Max.	6174	Feb. 28	42	–52	9	9.60	2.64	1.69	28	
1717	*V Tauri*	Max.	6136	Jan. 21	64	+30	3	9.7	–	–	–	
1761	*R Orionis*	Max.	6171	Feb. 25	46	+25	3	11.2	–	–	–	Unsatisfactory
1805	*V Orionis*	Min.	5382	Dec. 28	14	–	E	–	–	–	–	1900
"	"	Min.	6180	Mar. 6	17	–	E	–	–	–	–	1903. Reappeared shortly after
1944	*S Orionis*	Max.	6072	Nov. 18	29	–	E	–	–	–	–	Probably later
2013	*U Aurigae*	Max.	5408	Jan. 23	9	– 5	5	–	–	–	–	1901
"	"	Max. A	6210	Apr. 5	11	–17	7	7.70	0.32	0.31	5	
"	"	Max. B	6226	Apr. 21	11	– 1	3	7.8	–	–	–	Subtangent approximation
2080*a*	*Z Tauri*	Max.	6138	Jan. 23	–	–	1	10.5	–	–	–	Probably earlier
2100	*U Orionis*	Max.	6231	Apr. 26	17	–21	9	5.69	0.24	0.19	4	
2266	*V Monocerotis*	Max.	6179	Mar. 5	22	+22	9	7.58	1.33	1.11	9	Obsns. unsatisfactory
2387	— *Geminorum*	–	–	–	–	–	–	–	–	–	–	TURNER's Nova
2404	*X Geminorum*	Min.	–	–	–	–	–	10.3	–	–	–	Slight changes
2445	*W Monocerotis*	Min.	6202	Mar. 28	–	–	7	10.8	–	–	–	An apparent minimum
2475	*X Monocerotis*	Max.	6191	Mar. 17	–	–	5	8.4	–	–	–	Possibly earlier
2689	*Z Puppis*	Max.	6107	Dec. 23	–	–	–	8.2	–	–	–	Period 255
2690	*X Puppis*	Max.	5121	Apr. 11	–	–	7	8.0	–	–	–	1900. Period 155 ?
"	"	Min.	6136	Jan. 21	–	–	8	8.7	–	–	–	
"	"	Min.	6171	Feb. 25	14	–18	7	8.34	–	–	–	

Individual Observations.

Including Observations by Arthur C. Perry.

103 *T Andromedae.*

(Cont from 498. Comp Stars 346.)

Julian	Calendar	Mag.
	1902	
6092	Dec. 8	10]
6103	19	11]
6115	31	12]
	1903	
6133	Jan. 18	12]

111 *S Ceti.*

(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6092	Dec. 8	8.3
6107	23	8.8
	1903	
6133	Jan. 18	9.8

131 *S Piscium.*

(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6111	Dec. 27	12]
6115	31	12]
	1903	
6133	Jan. 18	12]

466 *U Piscium.*

(Continued from 498.)

Julian	Calendar	Mag.
	1899	
4984.6	Nov. 25	11.4r
	1902	
6111	Dec. 27	11.8]
	1903	
6133	Jan. 18	11.8]

715 *S Arietis.*

(Continued from 498.)

Julian	Calendar	Mag.
	1902	
6115	Dec. 31	11]
	1903	
6133	Jan. 18	11]

845 *R Ceti.*

(Continued from 498.)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	12]
6133	18	12]
6189	Mar. 15	12]

893 *U Ceti.*

(Cont from 498. Comp.Stars 346.)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	8.0
6124	9	7.4
6133	18	8.0
6145	30	8.7

976 *T Arietis.*

(Cont from 498. Comp Stars 463.)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	8.9
6133	18	8.3
6145	30	9.0
6168	Feb. 22	8.0
6171	25	7.8
6177	Mar. 3	8.0
6189	15	7.7
6193	19	7.8

1113 *U Arietis.*

(Cont from 498. Comp Stars 314.)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1	8.0
6133	18	7.7
6145	30	8.5

1166 *X Ceti.*

(Cont.from 498. Comp Stars 468.)

Julian	Calendar	Mag.
	1903	
6133	Jan. 18	10.1
6136	21	10.1
6145	30	11.0

1577 *R Tauri.*

(Continued from 498.)

Julian	Calendar	Mag.
	1903	
6116	Jan. 1 to	
6136	21	11]
	3 dates	
6145	Jan. 30	10.3
6168	Feb. 22	8.4
6171	25	7.5
6174	28	7.4
6177	Mar. 3	7.3
6180	6	7.5
6189	15	7.4
6201	27	7.6
6217	Apr. 12	7.8

1582 *S Tauri.*

(Continued from 498.)

Julian	Calendar	Mag.
	1903	
6133	Jan. 18	9.9
6136	21	9.9
6145	30	9.9
6168	Feb. 22	9.6
6171	25	9.7
6174	28	9.5
6177	Mar. 3	9.5
6189	15	10.0
6201	27	10.1

1717 *V Tauri.*

(Cont.from 513. Comp Stars 513.)

Julian	Calendar	Mag.
	1903	
6133	Jan. 18	9.9
6136	21	9.7
6145	30	10.3

1761 *R Orionis.*

(Continued from 498.)

Julian	Calendar	Mag.
	1899	
4984.6	Nov. 25	10.3
	1900	
5347.5	Nov. 23	10.0
5376.5	Dec. 22	10.4
5378.5	24	10.25_2
	1901	
5471.5	Mar. 27	11.5_2
	1903	
6138	Jan. 23	11.9
6168	Feb. 22	11.2
6171	25	11.2
6174	28	11.2
6177	Mar. 3	11.4]
6189	15	11.4]
6201	27	11.4

1805 *U Orionis.*

(Cont. from 468. Comp Stars 319.)

Julian	Calendar	Mag.
	1900	
5376.5	Dec. 22	11]
	1903	
6169	Feb. 23	10.2]
6205	Mar. 31	10.3
6217	Apr. 12	10.2

1911 *S Orionis.*

(Continued from 463.)

Julian	Calendar	Mag.
	1903	
6169.5	Feb. 23	7.4
6172.5	26	8.2
6175.5	Mar. 1	8.51_2
6176.5	2	7.83_2
6177.5	3	8.38_2
6180.5	6	9.08_2
6186.5	12	9.58_2

2013 *U Aurigae.*

(Continued from 468.)

Julian	Calendar	Mag.
	1900	
5376.5	Dec. 22	9.5
5378.5	24	8.94_2
5383.5	29	8.24_2
	1901	
5471.5	Mar. 27	10.46_2
	1903	
6206	Apr. 1	7.55_2
6208	3	8.39_2
6209	4	7.65_2
6210	5	7.76_2
6215	10	8.11
6217	12	7.95_2
6222	17	8.86_2
6223	18	8.13_2
6224	19	6.85_2
6225	20	8.13_2
6226	21	8.16_2
6229	24	8.77

2080a *Z Tauri.*

Julian	Calendar	Mag.
	1903	
6138	Jan. 23	10.5
6168	Feb. 22	11.0
6171	25	11.1
6226	Apr. 21	12

2100 *U Orionis.*

(Continued from 468.)

Julian	Calendar	Mag.
	1903	
6138	Jan. 23	10.4
6168	Feb. 22	10.1
6171	25	10.0
6177	Mar. 3	10.5
6203	29	7.4
6217	Apr. 12	5.7
6222	17	5.7
6223	18	5.36
6224	19	7.99
6225	20	7.61
6226	21	6.48_2
6229	24	5.89_2
6230	25	5.70_2

2100 *U Orionis* Cont.

Julian	Calendar	Mag.
	1903	
6231	Apr. 26	5.82_2
6232	27	5.10
6233	28	5.82
6234	29	6.18
6235	30	6.31
6236	May 1	6.84
6237	2	6.41

2266 *U Monocerotis.*

(Cont from 468. Comp Stars 403.)

Julian	Calendar	Mag.
	1903	
6169.5	Feb. 23	7.6
6172.5	26	8.0
6175.5	Mar. 1	7.1
6176.5	2	7.51_2
6177.5	3	7.92_2
6178.5	4	7.29_2
6180.5	6	7.19
6186.5	12	7.92_2
6201.5	27	7.95_2

2387 *Nova Geminorum.*

Julian	Calendar	Mag.
	1903	
6205	Mar. 31	7.4
6206	Apr. 1	7.51_2
6208	3	7.88_2
6209	4	7.80_2
6210	5	8.51_2
6215	10	8.17_2
6217	12	8.83_2
6222	17	8.07
6223	18	8.71
6224	19	7.62_2
6225	20	8.01_2
6226	21	6.60
6229	24	9.23_2
6230	25	9.39_4
6232	27	9.54_2
6233	28	9.95_2
6235	30	9.13_4
6236	May 1	9.20
6237	2	9.99

2404 *X Geminorum.*

(Continued from 468.)

Julian	Calendar	Mag.
	1903	
6205	Mar. 31	10.3
6206	Apr. 1	10.47
6222	17	10.55
6225	20	10.40
6226	21	9.10
6233	28	10.66

2415 *W M[illegible]*

Julian	Calendar	Mag.
	1903	
6188	Mar. 11	10.4
6189	15	10.02
6192	18	10.08
6200	26	10.94
6205	31	10.91
6206	Apr. 1	10.65_2
6222	17	9.9

2175 *X M[illegible]*

Julian	Calendar	Mag.
	1903	
6189	Mar. 15	8.84
6191	17	8.4
6192	18	8.62_2
6193	19	8.58
6200	26	9.08
6205	31	9.22_2
6206	Apr. 1	9.23_2
6222	17	8.8

2689 *Z Puppis.*

(Cont from 513. Comp Stars 403.)

Julian	Calendar	Mag.
	1899	
4975.7	Nov. 16	12r
	1902	
5832.5	Mar. 23	10.4r
5835.5	26	10.4r
5853.5	Apr. 13	10.2r
5867.5	27	10.2r
5871.5	May 1	9.9r
6106.7	Dec. 22	8.5r
6107	23	8.2
6111	30	8.2
6114.6	30	8.4
	1903	
6121.6	Jan. 6	8.7
6130.6	15	9.1
6132.6	17	9.9
6145.6	30	9.7
6160.6	Feb. 14	10.2
6168.6	22	9.9r
6174.6	28	10.5r
6178.6	Mar. 4	9.9]
6201.6	27	9.9
6208.5	Apr. 3	9.9 r
6222.6	17	9.9
6224.5	19	10.4r

2690 *X Puppis.*

(Cont from 468. Comp Stars 403.)

Julian	Calendar	Mag.
	1901	
5056.6	Feb. 5	8.5r
5081.6	Mar. 2	8.5r
5102.5	23	8.4
5105.5	26	8.4
5123.5	Apr. 13	8.0r
5137.5	27	8.3
5141.5	May 1	8.7
6106.7	Dec. 22	8.2r
6107	23	8.0
6114	30	8.2
6114.6	30	8.0
	1903	
6121	Jan. 6	8.5
6130.6	15	8.4
6132	17	8.7
6145	30	8.7
6160	Feb. 14	8.2
6168	22	8.3
6187	Mar. 13	8.05
6201	27	9.15
6222	Apr. 17	8.5
6224.5	19	8.7r

OBSERVATIONS OF THE COMPANIONS OF *SIRIUS* AND *PROCYON*,

MADE WITH THE 40-INCH REFRACTOR,

By E. E. BARNARD.

Sirius.

Sirius is favorably placed for observation during the worst season for observation here, and as this star is a strong atmospheric test, it has been impossible to get a good series of measures until the latter part of the past winter.

A few scattering isolated measures were obtained with difficulty in the endeavor to get a complete set of two or more nights closely following each other. These are perhaps of little value, but it may be well to print them with the above caution.

			°	″	
1	1899.903 Nov. 25		148.88	4.60	Good
2	1901.885 Nov. 19		133.13	5.34	
3	1902.153 Feb. 25		125.71	5.85	Believed to be good
4	1902.917 Dec. 1		121.90	6.06	
5	1903.054 Jan. 20		123.26	6.18	Angle uncertain

Nos. 1, 3 and 4 were made with the star in the center of field, the others by occulting the bright star by the edge of the field, which makes it specially difficult to determine the angle.

The following measures of the companion were made under fair conditions with the star in the center of the field, and are believed to be very good.

		°	″
1903.147	Feb. 23	119.74	6.28
.150	24	118.53	6.05
.166	Mar. 2	118.95	5.93
1903.154		119.07	6.09

The increasing distance of the companion now makes it an easy object with the great telescope if the seeing is good.

Procyon.

The small companion of *Procyon* is much more difficult than that of *Sirius*, though it is easier now to observe, since its distance has increased. The same remarks apply to this star that are referred to *Sirius*. It requires a very good night to see and measure it.

The following two measures were made with difficulty.

		°	″	
1901.882	Nov. 18	340.53	4.93	Single dist. very diff'lt
.885	19	346.46	5.20	Good
1901.883		343.49	5.06	

One of these angles is bad.

The following measures are good, and were made with the star in the center of the field.

		°	″
1903.147	Feb. 23	350.47	5.17
.150	24	351.11	5.23
.166	Mar. 2	351.50	5.07
1903.154		351.03	5.16

My previous measures, which were printed in *A.J.* 435, 162, 482, are

	°	″
1898.213	326.0	4.83
1899.073	330.6	4.91
1900.055	336.0	5.09

It will be seen that the direct angular motion is about 5° a year. The distance seems not to have materially increased in the past two or three years.

Here are two measures of the old distant companion.

		°	″
1901.882	Nov. 18	345.43	60.76
1902.153	Feb. 25	346.29	61.17

In all the measures a magnifying power of 700 diameters has been used.

Yerkes Observatory, Williams Bay, Wis., 1903 *April* 15.

EPHEMERIS OF FAYE'S COMET.

By F. E. SEAGRAVE.

Greenw. Midnight 1903	α h m s	δ ° ′ ″	log r	log Δ
July 1	4 57 52	+18 40 23	0.223590	0.406606
5	5 9 37	+18 41 47	0.225444	0.405785
9	5 21 17	+18 39 58	0.227522	0.404999
13	5 32 50	+18 34 59	0.229814	0.404236
17	5 44 15	+18 26 56	0.232313	0.403489

This ephemeris is based upon elements by STROMGREN, published in a recent number of the *Astr. Nachrichten*. The comet will be in perihelion on June 3, but will be very unfavorably situated for observations, as it will rise only about fifty minutes before the sun. It should be seen towards the middle of July.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed June 17.*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

Nos. 542-543.

VOL. XXIII. BOSTON, 1903 JULY 21. NOS. 14-15

AN EXAMPLE IN PERIODIC ORBITS, THE SECOND-ORDER PERTURBATIONS OF *JUPITER* AND *SATURN* INDEPENDENT OF THE ECCENTRICITIES AND OF THE MUTUAL INCLINATION.

By JAMES PARK McCALLIE.

INTRODUCTORY.

A periodic solution is a particular integral in the problem of three bodies. It is possible only under certain restricting conditions which do not exist in nature. Yet such solutions are of great beauty and interest, and often possess real value in assisting us to obtain a more general solution of the differential equations of motion; as, for example, the periodic orbit used as an intermediary by Dr. G. W. HILL in his "*Researches in the Lunar Theory.*"[1]

Analytically a solution is said to be periodic when the coordinates, referred to axes rotating with a uniform angular velocity, may be expressed in series periodic with respect to the time, while geometrically a periodic orbit is one in which a body, referred to the same rotating axes, returns periodically to the same position with reference to the other two bodies.

LAGRANGE in a very elegant manner discovered the first periodic solutions in the problem of three bodies, which however are without much practical value, since to obtain them he assumed the mutual distances as always being in a constant ratio to each other. These are the straight line and equilateral triangle solutions.

The next periodic solutions were obtained by G. W. HILL.[1] By neglecting the lunar inclination and the solar parallax and eccentricity he finds a particular integral of the equations for the moon's motion about the earth under the influence of the disturbing force of the sun. The curve corresponding to this particular integral is closed when referred to rotating axes, and is what is known as the *variational* orbit of the moon. This is used by Dr HILL as an intermediary instead of the ellipse or modified ellipse of other lunar theorists.

POINCARÉ has shown that there are an infinite number of such solutions that are really distinct. Since POINCARÉ's wholly analytical treatment, the subject of periodic orbits has attracted many astronomers and mathematicians, and a number of memoirs, both analytical and numerical, have been produced. The whole field of periodic orbits is recognized as a fertile one, though by no means easy of entrance.

Of the memoirs on the subject may be mentioned one by C. V. L. CHARLIER,[1] in which he obtains analytically some of the results found by DARWIN in his extensive numerical work on periodic orbits.[2] In the majority of memoirs one mass is assumed infinitesimal, but periodic orbits exist, whether the mass be infinitesimal or not. It is in the case where none of the masses are infinitesimal that I have selected the following numerical example in periodic orbits. The case is purely an ideal one, but it was in the hope that the results might be of some interest to astronomers that the work was undertaken.

The suggestion of the problem is due to Dr. G. W. HILL, and I desire to express my great indebtedness to him, and also my appreciation to Prof. ORMOND STONE for his encouragement and helpful suggestions, and to Mr T. McN SIMPSON, Jr., for checking some of the numerical work

Example. If two masses, small relatively to a third mass, revolve around the latter in coplanar orbits, having no proper eccentricities, they will have symmetrical conjunctions and oppositions, *i.e.*, their conjunctions and oppositions will be symmetrically placed with regard to their mutually perturbed orbits, which will cut the line of syzygies perpendicularly. Let us take the time of such a symmetrical conjunction as the origin of time, and the longitude of this conjunction as the origin of longitudes. The differential equations of the two bodies will then have particular integrals, or periodic solutions, as is shown by HILL and POINCARÉ. Assume the masses of the two planets to be, for the inner, the mass of *Jupiter*, and for the outer, the mass of *Saturn*, with periods also respec-

[1] *The American Journal of Mathematics*, Vol. I.

[1] *Meddelanden från Lunds* [illegible] No. 18.

[2] *Acta Mathematica*, Vol. 21.

tively equal to those of *Jupiter* and *Saturn*, while the mass of the largest body is the mass of the sun. The problem in hand is to find the expressions for the coordinates of the two small bodies as far as the terms proportional to the squares and products of the masses. These terms have been found before, as for instance in HILL's "*New Theory of Jupiter and Saturn*," but they are there mixed up with terms involving the eccentricities, etc., and it is the present purpose to determine them entirely separate from such influences, and in the light of a periodic solution. It may be of some interest to know just how large these terms of the second order are.

Coordinates. I shall refer to the three bodies in question as the *Sun*, *Jupiter*, and *Saturn*. That the latter two may have the same expression for their perturbative functions it is necessary only to use symmetrical differential-equations as explained in TISSERAND, Vol. I, Chap. IV. *Jupiter* is referred to the center of the *Sun* as origin, while *Saturn* is referred to the center of mass of *Jupiter* and the *Sun*. Allowing the subscripts 0, 1, 2 to refer to the *Sun*, *Jupiter*, and *Saturn* respectively, and denoting the masses severally by m_i ($i = 0, 1, 2$), we have for the heliocentric coordinates of *Jupiter* and *Saturn*

$$\xi_1 = x_1 \quad , \quad \xi_2 = x_2 + \kappa_1 x_1$$
$$\eta_1 = y_1 \quad , \quad \eta_2 = y_2 + \kappa_1 y_1$$

where

$$\kappa_i = \frac{m_i}{\mu_i} \quad , \quad \mu_i = m_0 + m_1 + \ldots + m_i$$

If r_i, v_i ($i = 1, 2$) represent the radii vectores and true longitudes of *Jupiter* and *Saturn* respectively, then

$$x_1 = \mathrm{r}_1 \cos v_1 \ , \ y_1 = \mathrm{r}_1 \sin v_1 \ ; \ x_2 = \mathrm{r}_2 \cos v_2 \ , \ y_2 = \mathrm{r}_2 \sin v_2$$

PERTURBATIVE FUNCTION.

The potential of the system is

$$U = \frac{m_0 m_1}{\Delta_{0,1}} + \frac{m_0 m_2}{\Delta_{0,2}} + \frac{m_1 m_2}{\Delta_{1,2}}$$
$$= \frac{m_0 m_1}{\mathrm{r}_1} + \frac{m_0 m_2}{\mathrm{r}_2} + m_1 m_2 F$$

where

$$m_1 m_2 F = m_0 m_2 \left[\frac{1}{\Delta_{0,2}} - \frac{1}{\mathrm{r}_2}\right] + \frac{m_1 m_2}{\Delta_{1,2}}$$
$$= m_0 m_2 \left[\left\{\mathrm{r}_2^2 + \kappa_1^2 \mathrm{r}_1^2 + 2\kappa_1 \mathrm{r}_1 \mathrm{r}_2 \cos(v_2 - v_1)\right\}^{-\frac{1}{2}} - \frac{1}{\mathrm{r}_2}\right] + \frac{m_1 m_2}{\Delta_{1,2}}$$

If we put

$$\frac{\mu_1}{m_0}\mathrm{r}_2 = r_2 \ , \ \mathrm{r}_1 = r_1 \quad , \quad v_2 - v_1 = \theta \ , \ \frac{\mu_1}{\mu_0} = \mathrm{m}$$

F has the approximate expression

$$F = \frac{\mathrm{m}}{r_2}\left[\left\{1 - 2\frac{r_1}{r_2}\cos\theta + \left(\frac{r_1}{r_2}\right)^2\right\}^{-\frac{1}{2}} - \frac{r_1}{r_2}\cos\theta + \frac{1}{4}\frac{m_1}{m_0}\left(\frac{r_1}{r_2}\right)^2\left\{1 + 3\cos 2\theta\right\}\right]$$
$$= F_0 + F_1$$

in which F_1 is the part having as a factor the small mass m_1. Since the planets have no proper eccentricities, and lie in the same plane, the perturbations will depend on the single argument $v_2 - v_1$, or the elongation. Hence it is sufficient to put in the function F, as a *first approximation*,

$$r_1 = a_1 \ , \ r_2 = a_2 \ , \ v_2 = l_2 = n_2 t, \ v_1 = l_1 = n_1 t \ , \ \theta_0 = (n_2 - n_1)t$$

Then F may be written separately in its two parts,

$$\left.\begin{aligned} F_0 &= \mathrm{m}\left[\tfrac{1}{2}\sum_{-\infty}^{+\infty} A^i \cos i\,\theta_0 - \frac{a_1}{a_2^2}\cos\theta_0 + \tfrac{1}{2}A^0\right] \\ F_1 &= \tfrac{1}{4}\,\mathrm{m}\, m_1 \frac{a_1^{\,2}}{a_2^{\,3}}\left[1 + 3\cos 2\theta_0\right] \end{aligned}\right\} \qquad (1)$$

where

$$\tfrac{1}{2}\sum_{-\infty}^{+\infty}{}_i A^i \cos i\,\theta_0 = \frac{1}{a_2}\left[1 - 2\alpha\cos\theta_0 + \alpha^2\right]^{-\frac{1}{2}}$$

In F_0 the value of A^i for $i = 0$ has been taken from under the sign Σ, and so hereafter.

DIFFERENTIAL EQUATIONS OF MOTION.

1. *For Jupiter.* The equations for *Jupiter* in rectangular coordinates are

$$\mu_0 \kappa_1 \frac{d^2 x_1}{dt^2} = \frac{\partial U}{\partial x_1} \quad , \quad \mu_0 \kappa_1 \frac{d^2 y_1}{dt^2} = \frac{\partial U}{\partial y_1}$$

These equations expressed in the polar coordinates r_1, v_1 after the manner of DEPONTECOULANT's equations in the "*Lunar Theory*,"[1] are

$$\left.\begin{aligned} \tfrac{1}{2}\frac{d^2}{dt^2}(r_1^{\,2}) - \frac{\mu_1}{r_1} + \frac{\mu_1}{a_1} &= \mathrm{m}_2\left[r_1\frac{\partial F}{\partial r_1} + 2\int d'F + 2\mathrm{m} g_1\right] \\ \frac{dv_1}{dt} &= \frac{1}{r_1^{\,2}}\left[h_1 + \mathrm{m}_2 \int \frac{\partial F}{\partial v_1}\,dt\right] \end{aligned}\right\} \qquad (2)$$

In these equations the new expressions introduced have the following significance:

$$d'F = \left\{\frac{\partial F}{\partial r_1}\frac{dr_1}{dt} + \frac{\partial F}{\partial v_1}\frac{dv_1}{dt}\right\} dt \quad , \quad \mathrm{m}_2 = \mathrm{m}\, m_2$$

$$-\frac{\mu_1}{2\mathrm{m}_2 a_1} + \mathrm{m}\, g_1 = \text{constant of integration attached to } \int d'F$$

$$h_1 = \text{constant of integration.}$$

2. *For Saturn.* The equations for *Saturn* formed in the same way are (3)

$$\left.\begin{aligned} \tfrac{1}{2}\frac{d^2}{dt^2}(r_2^{\,2}) - \frac{\mu_2 \mathrm{m}^2}{r_2} + \frac{\mu_2 \mathrm{m}^2}{a_2} &= \frac{\mu_2 \mathrm{m}\, m_1}{m_0}\left[r_2\frac{\partial F}{\partial r_2} + 2\int d''F + 2\mathrm{m} g_2\right] \\ \frac{dv_2}{dt} &= \frac{1}{r_2^{\,2}}\left(h_2 + \frac{\mu_2 \mathrm{m}\, m_1}{m_0}\int \frac{\partial F}{\partial v_2}\,dt\right) \end{aligned}\right\}$$

where the corresponding terms have an exactly similar meaning to those employed in *Jupiter's* equations.

3. *Equations Connecting Constants.* The above equations for *Jupiter*, or *Saturn*, are of the second and first

[1] See BROWN's *Lunar Theory*, pp. 16, 17.

order respectively, and are sufficient to determine three arbitrary constants, besides h_i and a_i ($i = 1$ or 2). But since the orbits have no inclinations or nodes there are only four constants, e_i, π_i, n_i, ϵ_i ($i = 1$ or 2), to be determined for each body, and therefore we must have another equation connecting the constants. Three of the constants are immediately determined by the special conditions of the problem. For since the orbits of the planets have no eccentricity other than that caused by their mutual perturbations, and hence their perihelia are indeterminate, we have

$$e_i \sin \pi_i = 0$$
$$e_i \cos \pi_i = 0$$

By reason of the way in which we have chosen our origins of longitudes and of time, ϵ_1 and ϵ_2 are zero. Hence the only two independent constants are n_1 and n_2. The equations above referred to are, for *Jupiter* and *Saturn*, respectively,[1]

$$(4)\quad\left\{\begin{aligned} &\frac{1}{r_1}\frac{d^2 r_1}{dt^2} - \left(\frac{dv_1}{dt}\right)^2 + \frac{\mu_1}{r_1^3} = \frac{m_2}{r_1}\frac{\partial F}{\partial r_1} \\ &\frac{1}{r_2}\frac{d^2 r_2}{dt^2} - \left(\frac{dv_2}{dt}\right)^2 + \frac{\mu_2 m^2}{r_2^3} = \frac{\mu_2 m m_1}{m_0}\frac{1}{r_2}\frac{\partial F}{\partial r_2} \end{aligned}\right.$$

Units Employed. Let us take m_0, the mass of the *Sun*, as our unit of mass, and let the mean distance of the earth from the *Sun* be the unit of length. Then that k, the *Gaussian Constant*, may also be unity, the unit of time must be 58.13245 mean solar days. Hence we may put

$$\mu_1 = 1 + m_1 = n_1^2 a_1^3 \quad , \quad \mu_2 m^2 = (1 + m_1 + m_2)(1 + m_1)^2 = n_2^2 a_2^3$$
$$\alpha = \frac{a_1}{a_2} = \left[(1 + m_1 + m_2)(1 + m_1)\right]^{-\frac{1}{3}} \left(\frac{n_2}{n_1}\right)^{\frac{2}{3}}$$

The values of m_1, m_2, n_1, n_2 are taken from p. 558 of HILL's "*New Theory of Jupiter and Saturn*," and are

$$m_1 = \frac{1}{1047.879} \quad , \quad n_1 = 109256''.62552$$
$$m_2 = \frac{1}{3501.6} \quad , \quad n_2 = 43996''.21506$$

The above mean motions are for a sidereal year. Taking as our values for the mass and mean motion (in a sidereal year) of the earth,

$$m' = \frac{1}{327000} \quad , \quad n' = 1295977''.41516$$

from the equation $a' = (1 + m')^{\frac{1}{3}} n'^{-\frac{2}{3}}$ we obtain the numerical value of a', which, used as the unit of distance, gives

$$\log a_1 = 0.716237409 \qquad \log \mathrm{r} = \log \frac{n_2}{n_1} = 9.604967534$$
$$\log a_2 = 0.979909852 \qquad \log \alpha = 9.736327557$$

Integration of Equations of Motion. In order to solve equations (3) and (4) it seems best to put

$$r_1^2 = a_1^2(1 + u_1 + \delta u_1) \quad , \quad \frac{dv_1}{dt} = n_1 + z_1 + \delta z_1$$
$$r_2^2 = a_2^2(1 + u_2 + \delta u_2) \quad , \quad \frac{dv_2}{dt} = n_2 + z_2 + \delta z_2$$

[1] BROWN's *Lunar Theory*, pp. 16, 17.

where u_1, z_1, u_2, z_2 represent perturbations of the first order with respect to the masses and δu_1, δz_1, δu_2, δz_2 are of the second order.

1. *First Order Terms for Jupiter.* The radius-vector equation for *Jupiter* becomes, to terms of the first order,

$$\frac{d^2 u_1}{dt^2} + n_1^2 u_1 = 2 n_1^2 a_1 m_2 \left[a_1 \frac{\partial F_0}{\partial a_1} + 2 a_1 \int \frac{\partial F_0}{\partial l_1} dt + 2 n_1 g_1 \right] \qquad (5)$$

This linear differential equation of the second order may be solved by indeterminate coefficients. Since its right member is a cosine function of the elongation, θ, only, we put

$$u_1 = 2 \sum_{-\infty}^{+\infty} \mathrm{a}_i \cos i\theta$$

Substituting this value of u_1 in the above equation, and equating coefficients of the same argument on either side we have

$$\mathrm{a}_0 = \mathrm{m}_2 \left[\tfrac{1}{2} a_1^2 \frac{\partial A^0}{\partial a_1} + 2 a_1 g_1 \right]$$

$$\mathrm{a}_{-1} = \mathrm{a}_1 = \frac{\mathrm{m}_2}{\mathrm{r}(1-\mathrm{r})(2-\mathrm{r})} \left[\frac{1-\mathrm{r}}{2} a_1^2 \frac{\partial A^1}{\partial a_1} + a_1 A^1 - \frac{3-\mathrm{r}}{2} \alpha^2 \right]$$

$$\mathrm{a}_i = \frac{\mathrm{m}_2}{(1-\mathrm{r})\{1 - i^2(1-\mathrm{r})^2\}} \left[\frac{1-\mathrm{r}}{2} a_1^2 \frac{\partial A^i}{\partial a_1} + a_1 A^i \right] \quad i = \pm 2 \ldots$$

To the same order the longitude equation is

$$n_1 + z_1 - \frac{h'_1}{a_1^2} = \left[h'_1 + \delta_1 h_1 + \mathrm{m}_2 \int \frac{\partial F_0}{\partial l_1} dt \right] \frac{1 - u_1}{a_1^2} - \frac{h'_1}{a_1^2}$$

where h_1 has been replaced by $h'_1 + \delta_1 h_1$. In the circular orbit $h_1 = h'_1 = n_1 a_1^2$. Hence $\delta_1 h_1$ is a small constant of the order of the masses. Then

$$z_1 = \frac{\delta_1 h_1}{a_1^2} - n_1 u_1 + \frac{\mathrm{m}_2}{\mathrm{m}} n_1^2 a_1 \int \frac{\partial F_0}{\partial l_1} dt \qquad (6)$$

Putting

$$\delta_1 v_1 = \int z_1 dt = \sum_{-\infty}^{+\infty} \alpha_i \sin i\theta,$$

we find

$$-\alpha_{-1} = \alpha_1 = \frac{\mathrm{m}_2}{2\mathrm{r}(1-\mathrm{r})^2(2-\mathrm{r})} \Big[2(1-\mathrm{r}) a_1^2 \frac{\partial A^1}{\partial a_1}$$
$$+ (\mathrm{r}^2 - 2\mathrm{r} + 4) a_1 A^1 - (\mathrm{r}^2 - 4\mathrm{r} + 6) \alpha^2 \Big]$$

$$\alpha_i = \frac{\mathrm{m}_2}{2(1-\mathrm{r})i\{1 - i^2(1-\mathrm{r})^2\}} \Big[2(1-\mathrm{r}) a_1^2 \frac{\partial A^i}{\partial a_1}$$
$$+ \{3 + i^2(1-\mathrm{r})^2\} a_1 A^i \Big] \quad i = \pm 2 \ldots$$

The constant term of z_1 is $\frac{\delta_1 h_1}{a_1^2} - 2 n_1 \mathrm{a}_0$; but we shall define n_1 as the mean motion of *Jupiter* in disturbed as well as in undisturbed orbit, and it will be obtained directly from observation. Hence

$$\frac{\delta_1 h_1}{a_1^2} - 2 n_1 \mathrm{a}_0 = 0$$

Since the arbitraries e_1 and π_1 of the general solution of the problem are zero in this case, and a_1 is not independent of n_1, all the arbitrary constants have now been fixed, for ϵ_1 is zero by the conditions laid down. Hence g_1 is not independent of the other arbitraries, and we find it by means of the first of equations (4). This equation will also enable us to verify the preceding work, inasmuch as the coefficients of $\cos i\,\theta_0$ on each side of the equation should be identical. To terms of the first order the equation is

$$\frac{d^2u_1}{dt^2} - 4n_1z_1 - 3n_1^2u_1 = 2\,\frac{\mathrm{m}_2}{\mathrm{m}}\,n_1^2a_1^2\,\frac{\partial F_0}{\partial a_1}$$

Substituting in this the above values of u_1 and z_1 we find

$$g_1 = -\tfrac{1}{3}\,a_1\,\frac{\partial A^0}{\partial a_1}\,,\quad\text{or}\quad \mathrm{a}_0 = -\tfrac{1}{6}\,\mathrm{m}_2\,a_1^2\,\frac{\partial A^0}{\partial a_1}\,.$$

2. *First Order Terms for Saturn.* The radius-vector equation for *Saturn* is

$$(7)\quad \frac{d^2u_2}{dt^2} + n_2^2u_2 = 2\,\frac{m_1}{\mathrm{m}}\,n_2^2a_2\left[a_2\,\frac{\partial F_0}{\partial a_2} + 2\int\frac{\partial F_0}{\partial l_2}\,dt + 2\mathrm{m}\,g_2\right]$$

Let
$$u_2 = 2\sum_{-\infty}^{+\infty}\mathrm{b}_i\cos i\,\theta_0$$

In forming $a_2\,\partial F_0/\partial a_2$ we make use of the relation

$$a_2\,\frac{\partial F_0}{\partial a_2} + a_1\,\frac{\partial F_0}{\partial a_1} = -F_0$$

Then

$$\mathrm{b}_0 = \frac{m_1}{2}\left[4a_2g_2 - a_1a_2\,\frac{\partial A^0}{\partial a_1} - a_2A^0\right]$$

$$\mathrm{b}_{-1} = \mathrm{b}_1 = \frac{m_1\nu^2}{(1-\nu)(1-2\nu)}\left[\frac{1-\nu}{2}\,a_1a_2\,\frac{\partial A^1}{\partial a_1} + \frac{1+\nu}{2}\,a_2A^1 - \alpha\right]$$

$$\mathrm{b}_i = \frac{m_1\nu^2}{(1-\nu)\{i^2(1-\nu)^2-\nu^2\}}\left[\frac{1-\nu}{2}\,a_1a_2\,\frac{\partial A^i}{\partial a_1} + \frac{1+\nu}{2}\,a_2A^i\right]$$

The differential equation for longitude of *Saturn* is, to terms of first order,

$$(8)\qquad z_2 = \frac{\delta_1h_2}{a_2^2} - n_2u_2 + \frac{m_1}{\mathrm{m}}\,n_2^2a_2\int\frac{\partial F_0}{\partial l_2}\,dt$$

Putting

$$\delta_1v_2 = \int z_2\,dt = \sum_{-\infty}^{+\infty}\beta_i\sin i\,\theta_0$$

we find

$$-\beta_{-1} = \beta_1 = \frac{m_1\nu^2}{2(1-\nu)^2(1-2\nu)}\Big[2\nu(1-\nu)a_1a_2\,\frac{\partial A^1}{\partial a_1} + (1+2\nu^2)\,a_2A^1 - (1+2\nu)\alpha\Big]$$

$$\beta_i = \frac{m_1\nu^2}{2(1-\nu)^2\,i\{i^2(1-\nu)^2-\nu^2\}}\Big[2\nu(1-\nu)\,a_1a_2\,\frac{\partial A^i}{\partial a_1} + \{2\nu+\nu^2+i^2(1-\nu)^2\}a_2A^i\Big]$$

and since the constant term in δ_1v_2 is zero

$$\frac{\delta_1h_2}{a_2^2} - 2n_2\mathrm{b}_0 = 0$$

The equation determining the constant term in u_2 is to terms of first order,

$$\frac{d^2u_2}{dt^2} - 4n_2z_2 - 3n_2^2u_2 = 2\,\frac{m_1}{\mathrm{m}}\,n_2^2a_2^2\,\frac{\partial F_0}{\partial a_2}$$

This gives

$$\mathrm{b}_0 = \tfrac{1}{2}\,m_1a_2g_2 = \tfrac{1}{6}\,m_1\left[a_1a_2\,\frac{\partial A^0}{\partial a_1} + a_2A^0\right]$$

Differential Equations Including Second Order Terms.

Having now solved the differential equations as far as the terms proportional to the masses we are prepared to push our approximation still further and include all terms proportional to the squares and products of the masses. It is well known that the form of the solution remains unchanged in all the successive approximations of including the squares, cubes, and higher powers of the masses, and hence our differential equations preserve the same form, and are solved precisely in the same way as before.

1. *For Jupiter.—a) Radius Vector Equation.* When we extend the radius vector equation to terms of the second order, and omit all terms of the first order, we get

$$\frac{d^2}{dt^2}\,\delta u_1 + n_1^2\delta u_1 = \tfrac{3}{4}\,n_1^2u_1^2 + 2\,\frac{\mathrm{m}_2}{\mathrm{m}}\,n_1^2a_1\left[\delta\left(r_1\,\frac{\partial F}{\partial r_1}\right) + 2\delta\int d'F + a_1\,\frac{\partial F_1}{\partial a_1} + 2\int d'F_1 + 2\mathrm{m}\,\delta g_1\right]$$

In this equation F_1 has the value given above, and $\mathrm{m}\,\delta g_1$ is the constant of integration attached to $\delta\int d'F$, and is of the second order. We shall proceed to express fully the right member of this equation.

Since

$$r_1 = a_1(1 + \tfrac{1}{2}\,u_1 + \tfrac{1}{2}\,\delta u_1 - \tfrac{1}{8}\,u_1^2)$$

we have

$$\frac{\delta_1r_1}{a_1} = \tfrac{1}{2}\,u_1\quad;\quad\text{also}\quad\frac{\delta_1r_2}{a_2} = \tfrac{1}{2}\,u_2$$

Also since F_0 is a function of $l_2 - l_1$ we have

$$\frac{\partial F_0}{\partial l_2} = -\frac{\partial F_0}{\partial l_1}$$

With these relations and that given above, with reference to $a_2\,\partial F_0/\partial a_2$ we may easily express $\delta(r_1\,\partial F/\partial r_1)$. We also have

$$\delta\int d'F = \int\delta\left[\frac{\partial F}{\partial r_1}\,\frac{dr_1}{dt} + \frac{\partial F}{\partial v_1}\,\frac{dv_1}{dt}\right]dt$$

in which

$$\delta\left(\frac{dr_1}{dt}\right) = \frac{d}{dt}\,(\delta_1r_1) = \frac{a_1}{2}\,\frac{du_1}{dt}$$

$$\delta\left(\frac{dv_1}{dt}\right) = \frac{d}{dt}\,(\delta_1v_1) = z_1$$

Then the above differential equation in its expanded form is

(9)

$$\frac{d^2}{dt^2}(\delta u_1)+n_1^2\delta u_1 = n_1^2\Bigg[\tfrac{3}{4}u_1^2+2\frac{\mathrm{m}_2}{\mathrm{m}}a_1\Big\{\Big(a_1\frac{\partial F_0}{\partial a_1}+a_1^2\frac{\partial^2 F_0}{\partial a_1^2}\Big)\frac{u_1}{2}$$
$$-\Big(2a_1\frac{\partial F_0}{\partial a_1}+a_1^2\frac{\partial^2 F_0}{\partial a_1^2}\Big)\frac{u_2}{2}+a_1\frac{\partial^2 F_0}{\partial l_1\partial a_1}(\delta_1 v_1-\delta_1 v_2)+a_1\frac{\partial F_1}{\partial a_1}$$
$$+2\int d'F_1+2\mathrm{m}\,\delta g_1$$
$$+2n_1\int\Big[\frac{a_1\partial^2 F_0}{\partial a_1\partial l_1}\Big(\frac{u_1}{2}-\frac{u_2}{2}\Big)-\frac{\partial F_0}{\partial l_1}\frac{u_2}{2}+\frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1-\delta_1 v_2)$$
$$+\frac{1}{n_1}\Big(\frac{a_1}{2}\frac{\partial F_0}{\partial a_1}\frac{du_1}{dt}+\frac{\partial F_0}{\partial l_1}z_1\Big)\Big]dt\Big\}\Bigg]$$

We see immediately that the right member is composed of products of series, either cosine by cosine, sine by sine, or, underneath the integral sign, cosine by sine. In every case we get, after multiplication, and integration of the last mentioned products, a cosine series. If now we attach the factor $2\mathrm{m}_2 a_1/\mathrm{m}$ above to F_0, every coefficient in each factor is of the order of the masses. We shall designate the coefficients of cosine series by Roman letters, of sine series by Greek letters. In each series the subscript i has every integral value from $-\infty$ to $+\infty$ including zero. For cosine series we may put $\mathrm{a}_i = \mathrm{a}_{-i}$, for sine series $\alpha_i = -\alpha_{-i}$. Hence we may write

$$\Sigma_i \mathrm{a}_i\cos i\,\theta_0\times\Sigma_j \mathrm{b}_j\cos j\,\theta_0 = \Sigma_i\Sigma_j \mathrm{a}_i \mathrm{b}_j\cos(i+j)\,\theta_0$$
$$\Sigma_i \mathrm{a}_i\cos i\,\theta_0\times\Sigma_j \alpha_j\sin j\,\theta_0 = \Sigma_i\Sigma_j \mathrm{a}_i\alpha_j\sin(i+j)\,\theta_0$$
$$\Sigma_i \alpha_i\sin i\,\theta_0\times\Sigma_j \beta_j\sin j\,\theta_0 = -\Sigma_i\Sigma_j \alpha_i\beta_j\cos(i+j)\,\theta_0$$

whence the equation for δu_1 becomes

$$\frac{d^2}{dt^2}(\delta u_1)+n_1^2\,\delta u_1 = n_1^2\left[\Sigma_i\Sigma_j S_{i+j}\cos(i+j)\,\theta_0+2\sigma\mathrm{m}\,\delta g_1\right]$$

the solution of which gives

$$\delta u_1 = \Sigma_i\Sigma_j\frac{S_{i+j}}{1-(i+j)^2(1-\nu)^2}\cos(i+j)\,\theta_0+2\sigma\,\mathrm{m}\,\delta g_1$$

where $\sigma = 2\mathrm{m}_2 a_1/\mathrm{m}$ and

$$S_{i+j} = 3\,\mathrm{a}_i\mathrm{a}_j+\mathrm{e}_i\mathrm{a}_j-\mathrm{f}_i\mathrm{b}_j-\epsilon_i\gamma_j+\mathrm{k}_{i+j}+\mathrm{l}_{i+j}$$
$$+\frac{2}{(i+j)(1-\nu)}\left[\epsilon_i\mathrm{c}_j-\zeta_i\mathrm{b}_j+\mathrm{g}_i\gamma_j+\mathrm{h}_i\delta_j+\zeta_i\mathrm{d}_j\right]$$

These letters express in order the coefficients of the various factors just as they occur in the right-hand member of the expanded equation (9) given above.

b) Longitude Equation. To terms of the second order this equation becomes, when we put $h_1 = h'_1+\delta_1 h_1+\delta_2 h_1$ and omit terms of the first order,

(10)

$$\delta z_1 = -u_1 z_1-n_1\delta u_1+\frac{\delta_2 h_1}{a_1^2}+\tfrac{1}{2}\sigma n_1\int\Big\{a_1\frac{\partial^2 F_0}{\partial l_1\partial a_1}\Big(\frac{u_1}{2}-\frac{u_2}{2}\Big)$$
$$-\frac{\partial F_0}{\partial l_1}\frac{u_2}{2}+\frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1-\delta_1 v_2)+\frac{\partial F_1}{\partial l_1}\Big\}dt$$
$$= n_1\Sigma_i\Sigma_j P_{i+j}\cos(i+j)\,\theta_0$$

where

$$P_{i+j} = -2\mathrm{a}_i\mathrm{d}_j-\frac{1}{1-(i+j)^2(1-\nu)^2}S_{i+j}$$
$$+\frac{1}{2(i+j)(1-\nu)}\left[\epsilon_i\mathrm{c}_j-\zeta_i\mathrm{b}_j+\mathrm{g}_i\gamma_j\right]+\tfrac{1}{2}\mathrm{l}_{i+j}-2\sigma\mathrm{m}\,\delta g_1+\frac{\delta_2 h_1}{n_1 a_1^2}$$

where the whole constant part is included in P_0, which for reasons given above must be equated to zero.

Then

$$\delta_2 v_1 = \int\delta z_1\,dt = \Sigma\Sigma\frac{P_{i+j}}{(i+j)(\nu-1)}\sin(i+j)\,\theta_0$$

c) Equation Determining Constant Part of δu_1. The first of equations (4) extended to terms of the second order is sufficient to determine the constant δg_1 which occurs in δu_1, and at the same time to verify our equations for δu_1 and δz_1. For on summing the coefficients of like cosines we shall find that they vanish identically, and only a constant term is left. If we let

$$\dot u_1 = \frac{du_1}{dt}\quad,\quad \ddot u_1 = \frac{d^2u_1}{dt^2}$$

this equation is (11)

$$\frac{\delta\ddot u_1}{n_1^2}-3\delta u_1-4\frac{\delta z_1}{n_1}-\frac{u_1\ddot u_1}{n_1^2}-\tfrac{1}{2}\frac{\dot u_1^2}{n_1^2}-2\frac{z_1^2}{n_1^2}+\tfrac{15}{4}u_1^2$$
$$= \tfrac{1}{2}\sigma\Big[-a_1\frac{\partial F_0}{\partial a_1}u_1+2a_1\delta\Big(\frac{\partial F}{\partial r_1}\Big)-2a_1\frac{\partial F_1}{\partial a_1}\Big]$$

Substituting in this equation the expressions for δu_1, $\delta\ddot u_1$, δz_1, and making use of equations (5) and (6) we arrive at the equation

$$\tfrac{3}{2}u_1^2-\tfrac{1}{2}\frac{\dot u_1^2}{n_1^2}-2\frac{z_1^2}{n_1^2}+2\sigma\mathrm{m}\,\delta g_1-4\frac{\delta_2 h_1}{n_1 a_1^2}$$
$$+2\sigma\int\Big\{a_1\frac{\partial F_0}{\partial a_1}\frac{\dot u_1}{2}+\frac{\partial F_0}{\partial l_1}z_1\Big\}dt = 0$$

We shall find a different expression for the last term. Equation (6) is

$$\frac{z_1}{n_1}+u_1 = \frac{\delta_1 h_1}{n_1 a_1^2}+\frac{\sigma}{2}n_1\int\frac{\partial F_0}{\partial l_1}dt$$

By means of this relation and its derivative we find

$$2\sigma\int z_1\frac{\partial F_0}{\partial l_1}dt = 2\frac{z_1^2}{n_1^2}+4\frac{\delta_1 h_1}{n_1 a_1^2}u_1-2u_1^2$$
$$+\int 2\sigma\,n_1 u_1\Big\{\int\frac{\partial F_0}{\partial l_1}dt\Big\}dt$$

By adding

$$2\sigma\int a_1\frac{\partial F_0}{\partial a_1}\frac{\dot u_1}{2}dt$$

to each member of this equation, the integral in the right member becomes

$$\sigma\int\Big\{a_1\frac{\partial F_0}{\partial a_1}+2n_1\int\frac{\partial F_0}{\partial l_1}dt\Big\}\dot u_1\,dt$$

which by means of equation (5) may be completely integrated. Hence we obtain

$$2\sigma\int\Big\{a_1\frac{\partial F_0}{\partial a_1}\frac{\dot u_1}{2}+\frac{\partial F_0}{\partial l_1}z_1\Big\}dt = \Big[2\frac{z_1^2}{n_1^2}-\ \ldots\ +\ \ldots\Big]$$

where $[\ ]_c$ means that the constant term is absent. The equation under consideration then gives for the constant in δu_1

$$\sigma\mathrm{m}\,\delta g_1 = 2\,\frac{\delta_2 h_1}{n_1 a_1^2} + \left[\frac{z_1^2}{n_1^2} - \tfrac{3}{4}\,u_1^2 + \tfrac{1}{4}\,\frac{\dot u_1^2}{n_1^2}\right]_0$$

Here $[\ ]_0$ means that only the constant part is present. Thus it is seen that all periodic terms identically vanish, and the equations for δu_1 and δz_1 are verified. We shall use this same equation to verify the numerical work.

Since the constant term in δz_1 is zero we have

$$\frac{\delta_2 h_1}{n_1 a_1^2} = 2\sigma\mathrm{m}\,\delta g_1 + 2\,[\mathrm{a}_i\mathrm{d}_j]_0 + S_0$$

Hence

$$-3\sigma\mathrm{m}\,\delta g_1 = 4\,[\mathrm{a}_i\mathrm{d}_j]_0 + 2\,S_0 + \left[\frac{z_1^2}{n_1^2} - \tfrac{3}{4}\,u_1^2 + \tfrac{1}{4}\,\frac{\dot u_1^2}{n_1^2}\right]_0$$

2. *For Saturn.* — *a*) *Radius Vector Equation.* The equations for *Saturn*, being formed in a manner exactly similar to that pursued in forming *Jupiter's* equations, may simply be written down. The first is

$$(12)\qquad \frac{d^2}{dt^2}(\delta u_2) + n_2^2\,\delta u_2$$
$$= n_2^2\Bigg[\tfrac{3}{4}\,u_2^2 - 2\,\frac{m_1}{\mathrm{m}}\,a_2\,\Bigg\{\left(2a_1\frac{\partial F_0}{\partial a_1} + a_1^2\frac{\partial^2 F_0}{\partial a_1^2}\right)\left(\frac{u_1}{2} - \frac{u_2}{2}\right)$$
$$-\left(F_0 + a_1\frac{\partial F_0}{\partial a_1}\right)\frac{u_2}{2} + \left(\frac{\partial F_0}{\partial l_1} + a_1\frac{\partial^2 F_0}{\partial l_1\partial a_1}\right)(\delta_1 v_1 - \delta_1 v_2)$$
$$- a_2\frac{\partial F_1}{\partial a_2} - 2\int d'' F_1 - 2\mathrm{m}\,\delta g_2$$
$$+ 2n_2\int\left[a_1\frac{\partial^2 F_0}{\partial l_1\partial a_1}\left(\frac{u_1}{2} - \frac{u_2}{2}\right) - \frac{\partial F_0}{\partial l_1}\frac{u_2}{2} + \frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1 - \delta_1 v_2)\right.$$
$$\left. + \left(F_0 + a_1\frac{\partial F_0}{\partial a_1}\right)\frac{\dot u_2}{2n_2} + \frac{\partial F_0}{\partial l_1}\frac{z_2}{n_2}\right]dt\,\Bigg\}\Bigg]$$

We see that, as in *Jupiter's* radius vector equation, the right member is composed of the products of series, all of which result in cosine series. Many of the individual series are the same as those entering *Jupiter's* equation, except for the constant factor $2m_1 a_2/\mathrm{m}$. Denoting this constant by ω, we can put

$$\omega = \frac{\omega}{\sigma}\cdot\sigma$$

and we can then use the same letters as before to denote the same coefficients here. New letters will be used where we have new coefficients, and arranging them in exactly the order in which they occur above, we may write the equation for δu_2.

$$\frac{d^2}{dt^2}(\delta u_2) + n_2^2\,\delta u_2 = n_2^2\left[\Sigma_i\Sigma_j\,R_{i+j}\cos(i+j)\,\theta_0 + 2\omega\mathrm{m}\,\delta g_2\right]$$

the solution of which is

$$\delta u_2 = \Sigma_i\Sigma_j\,\frac{\nu^2}{\nu^2-(i+j)^2(1-\nu)^2}\,R_{i+j}\cos(i+j)\,\theta_0 + 2\omega\mathrm{m}\,\delta g_2$$

where

$$R_{i+j} = 3\mathrm{b}_i\mathrm{b}_j - \frac{\omega}{\sigma}\Big[\mathrm{f}_i\mathrm{c}_j - \mathrm{q}_i\mathrm{b}_j - \theta_i\gamma_j - \mathrm{m}_{i+j} - 2\mathrm{n}_{i+j}$$
$$+ \frac{2\nu}{(i+j)(1-\nu)}\Big\{\epsilon_i\mathrm{c}_j - \zeta_i\mathrm{b}_j + \mathrm{g}_i\gamma_j + \mathrm{q}_i\eta_j + \zeta_i\mathrm{p}_j\Big\}\Big]$$

b) *Longitude Equation.* To terms of the second order this is (13)

$$\delta z_2 = -u_2 z_2 - n_2\,\delta u_2 + \frac{\delta_2 h_2}{a_2^2}$$
$$+ \tfrac{1}{2}\,\omega\int d'' F_0 - \tfrac{1}{2}\,n_2\,\omega\int\Big\{a_1\frac{\partial^2 F_0}{\partial l_1\partial a_1}\left(\frac{u_1}{2} - \frac{u_2}{2}\right)$$
$$- \frac{\partial F_0}{\partial l_1}\frac{u_2}{2} + \frac{\partial^2 F_0}{\partial l_1^2}(\delta_1 v_1 - \delta_1 v_2)\Big\}\,dt$$
$$= n_2\,\Sigma_i\Sigma_j\,K_{i+j}\cos(i+j)\,\theta_0$$

where

$$K_{i+j} = -2\mathrm{b}_i\mathrm{p}_j - \frac{\nu^2}{\nu^2-(i+j)^2(1-\nu)^2}\,R_{i+j} - 2\omega\mathrm{m}\,\delta g_2 + \frac{\delta_2 h_2}{n_2 a_2^2}$$
$$+ \tfrac{1}{2}\,\frac{\omega}{\sigma}\,n_{i+j} - \tfrac{1}{2}\,\frac{\omega}{\sigma}\,\frac{\nu}{(i+j)(1-\nu)}\left[\epsilon_i\mathrm{c}_j - \zeta_i\mathrm{b}_j + \mathrm{g}_i\gamma_j\right]$$

Then

$$\delta_2 v_2 = \int\delta z_2\,dt = \Sigma_i\Sigma_j\,\frac{\nu}{(i+j)(\nu-1)}\,K_{i+j}\sin(i+j)\,\theta_0$$

c) *Equation Determining Constant Part of δu_2.* The second of equations (4), expressed to terms of the second order, is

$$\frac{\delta\ddot u_2}{n_2^2} - 3\,\delta u_2 - 4\,\frac{\delta z_2}{n_2} - \frac{u_2\ddot u_2}{n_2^2} - \tfrac{1}{2}\,\frac{\dot u_2^2}{n_2^2} - 2\,\frac{z_2^2}{n_2^2} + \tfrac{15}{4}\,u_2^2 \qquad (14)$$
$$= -\omega\left[A\left(\frac{u_1}{2} - \frac{u_2}{2}\right) - 3B\,\frac{u_2}{2} + \frac{\partial B}{\partial l_1}(\delta_1 v_1 - \delta_1 v_2) - a_2\frac{\partial F_1}{\partial a_2}\right]$$

where

$$A = 2a_1\frac{\partial F_0}{\partial a_1} + a_1^2\frac{\partial^2 F_0}{\partial a_1^2}\quad;\qquad B = F_0 + a_1\frac{\partial F_0}{\partial a_1}$$

From this equation we get, exactly as in the equation for *Jupiter*,

$$-3\omega\mathrm{m}\,\delta g_2 = 4\,[\mathrm{b}_i\mathrm{p}_j]_0 + 2R_0 + \left[\frac{z_2^2}{n_2^2} - \tfrac{3}{4}\,u_2^2 + \tfrac{1}{4}\,\frac{\dot u_2^2}{n_2^2}\right]_0$$

which is exactly similar to the expression for δg_1.

Reference of Coordinates of *Saturn* to Center of *Sun*.

Let r_2', v_2' be the polar coordinates of *Saturn* referred to the center of the *Sun* as origin. Then in the triangle of *Sun*, *Saturn*, mass-center of *Sun* and *Jupiter*, the angles are respectively $v_2' - v_1$, q, and $\pi - (v_2 - v_1)$, and the sides opposite r_2, $\kappa_1\mathrm{r}_1$, and r_2'. If we put

$$v_2 - v_1 = l_2 - l_1 + \delta_1 v_2 - \delta_1 v_1 + \ldots = \theta_0 + \theta_1 + \ldots .$$

we get $\qquad \mathrm{r}_2' = \left[\mathrm{r}_2^2 + \kappa_1^2\mathrm{r}_1^2 + 2\kappa_1\mathrm{r}_1\mathrm{r}_2\cos(v_2 - v_1)\right]^{\frac{1}{2}}$

or, approximately

$$\mathrm{r}_2' = \mathrm{r}_2 + \kappa_1\mathrm{r}_1\cos(\theta_0+\theta_1) + \tfrac{1}{4}\,\kappa_1^2\frac{\mathrm{r}_1^2}{\mathrm{r}_2}\left[1 - \cos 2(\theta_0+\theta_1)\right]$$
$$= \mathrm{a}_2\Bigg[1 + \tfrac{1}{2}\,u_2 + \tfrac{1}{2}\,\delta u_2 - \tfrac{1}{8}\,u_2^2 + \tfrac{1}{4}\,\kappa_1^2\frac{\mathrm{a}_1^2}{\mathrm{a}_2^2}(1 - \cos 2\theta_0)$$
$$+ \kappa_1\frac{\mathrm{a}_1}{\mathrm{a}_2}\Big\{(1 + \tfrac{1}{2}\,u_1)\cos\theta_0 - \theta_1\sin\theta_0\Big\}\Bigg]$$

since $\theta_1 = \delta_1 v_2 - \delta_1 v_1$ is a very small angle. As far as to terms of the first order

$$r_2' = a_2\left[1+\tfrac{1}{2}\,u_2+\kappa_1\,\frac{a_1}{a_2}\cos\theta_0\right]$$

so that to terms of this order r_2' differs from r_2 only in the term of argument θ_0. It is also seen that r_2' has the same mean value as has r_2. When terms of the second order are included this ceases to be true.

In the same triangle as mentioned above we have

$$v_2' = v_2 - q$$

and
$$\frac{\sin q}{\sin(v_2-v_1)} = \frac{\kappa_1 r_1}{r_2'} = \frac{1}{2000}, \quad \text{approximately.}$$

Hence

$$\sin q = q = \frac{\kappa_1 r_1}{r_2'}\sin(v_2-v_1)$$

and therefore

$$v_2' = v_2 - \kappa_1\frac{a_1}{a_2}\left[1+\frac{u_1}{2}-\frac{u_2}{2}-\kappa_1\frac{a_1}{a_2}\cos\theta_0\right]\left[\sin\theta_0+\theta_1\cos\theta_0\right]$$

$$= v_2 - \kappa_1\frac{a_1}{a_2}\left[\sin\theta_0+\theta_1\cos\theta_0 + \left(\frac{u_1}{2}-\frac{u_2}{2}\right)\sin\theta_0 - \tfrac{1}{2}\,\kappa_1\frac{a_1}{a_2}\sin 2\theta_0\right]$$

It is seen that v_2' has the same mean rate of increase, n_2, as has v_2, being as much less than the latter in the first and second quadrants as greater in the third and fourth.

Computation of First-Order Terms.

It is necessary first to obtain the values of the functions A^i entering into the perturbative function. Let

$$\left[1-2\alpha\cos\theta_0+\alpha^2\right]^{-\frac{1}{2}} = \tfrac{1}{2}\sum_{-\infty}^{+\infty} b^i\cos i\,\theta_0$$

Hence if we compute $b^i,\ \alpha\dfrac{db^i}{d\alpha},\ \alpha^2\dfrac{d^2b^i}{d\alpha^2}$ we can obtain from them $A^i,\ a_1\dfrac{\partial A^i}{\partial a_1},\ a_1^2\dfrac{\partial^2 A^i}{\partial a_1^2}$ by well known relations.

These quantities may be computed in several ways, all well known, and it is unnecessary here to reproduce the formulas. By glancing at the perturbations under consideration as given by DePontécoulant, "*Théorie Analytique du Système du Monde*," it is seen that several coefficients are quite large; for instance, 196″ is the coefficient of $\sin 2\theta_0$ in $\delta_1 v_1$. For this and similar terms nine-place logarithms are necessary, but only a few terms demand so many figures. In general seven-place logarithms suffice for terms of the first order, while five-, and for one or two terms, six-place logarithms, will give the same accuracy for the second-order terms. The b^i and their derivatives have been computed for $\log\alpha = 9.736327557$, and the computations were checked twice, and in some cases, three times by recomputation.

The values found for $b^i,\ \alpha\dfrac{db^i}{d\alpha},\ \alpha^2\dfrac{d^2b^i}{d\alpha^2}$ are

i	b	$\alpha\dfrac{db}{d\alpha}$	$\alpha^2\dfrac{d^2b}{d\alpha^2}$
0	0.338138916	9.613539018	9.930590
1	9.792423038	9.907211161	9.878787
2	9.410262287	9.779191774	0.018692
3	9.07072175	9.59673039	0.020155
4	8.7510906	9.3914979	9.948196
5	8.4130357	9.173599	9.833742
6	8.1125680	8.947617	9.691950
7	7.817163	8.715983	9.53118
8	7.556353	8.480187	9.35651
9	7.268330	8.24120	9.17124
10	6.98277	7.99967	8.97758
11	6.69922	7.75609	8.7772
12	6.4174	7.5105	8.5716

From these data we immediately compute the first-order terms of *Jupiter* given below. The coefficients are expressed in abstract numbers for $\delta_1 r_1/a_1$, in seconds of arc for $\delta_1 v_1$.

$$\frac{\delta_1 r_1}{a_1} = \left\{\begin{array}{rl}
-0{,}00001\,11252 & \\
+0{,}00012\,15121 & \cos\theta \\
-0{,}00053\,33873 & \cos 2\theta \\
0{,}00005\,55968 & \cos 3\theta \\
-0{,}00001\,13951 & \cos 4\theta \\
-0{,}00000\,47600 & \cos 5\theta \\
-0{,}00000\,17772 & \cos 6\theta \\
-0{,}00000\,07141 & \cos 7\theta \\
0{,}00000\,03016 & \cos 8\theta \\
0{,}00000\,01320 & \cos 9\theta \\
0{,}00000\,00593 & \cos 10\theta \\
0{,}00000\,00273 & \cos 11\theta \\
-0{,}00000\,00127 & \cos 12\theta
\end{array}\right.$$

$$\delta_1 v = \left\{\begin{array}{rl}
+\ 79.21829 & \sin\theta \\
-195.77043 & \sin 2\theta \\
-\ 16.33180 & \sin 3\theta \\
3.75436 & \sin 4\theta \\
1.15702 & \sin 5\theta \\
0.41297 & \sin 6\theta \\
-\ 0.16100 & \sin 7\theta \\
0.06656 & \sin 8\theta \\
-\ 0.02868 & \sin 9\theta \\
-\ 0.01275 & \sin 10\theta \\
0.00581 & \sin 11\theta \\
0.00269 & \sin 12\theta
\end{array}\right.$$

The corresponding values for *Saturn* are

$$\frac{\delta_1 r_2}{a_2} = \left\{\begin{array}{l} +0.00041\,67147 \\ +0.00031\,91670 \ \cos\theta_0 \\ +0.00011\,71618 \ \cos 2\theta_0 \\ +0.00003\,40816 \ \cos 3\theta_0 \\ +0.00001\,05662 \ \cos 4\theta_0 \\ +0.00000\,37863 \ \cos 5\theta_0 \\ +0.00000\,14791 \ \cos 6\theta_0 \\ +0.00000\,06123 \ \cos 7\theta_0 \\ +0.00000\,02639 \ \cos 8\theta_0 \\ +0.00000\,01173 \ \cos 9\theta_0 \\ +0.00000\,00534 \ \cos 10\theta_0 \\ +0.00000\,00247 \ \cos 11\theta_0 \\ +0.00000\,00116 \ \cos 12\theta_0 \end{array}\right\}$$

$$\delta_1 v_2 = \left\{\begin{array}{ll} +103''.82924 & \sin\theta_0 \\ +\ \ 32.01024 & \sin 2\theta_0 \\ +\ \ \ \ 6.66903 & \sin 3\theta_0 \\ +\ \ \ \ 1.99553 & \sin 4\theta_0 \\ +\ \ \ \ 0.70687 & \sin 5\theta_0 \\ +\ \ \ \ 0.27562 & \sin 6\theta_0 \\ +\ \ \ \ 0.11428 & \sin 7\theta_0 \\ +\ \ \ \ 0.04914 & \sin 8\theta_0 \\ +\ \ \ \ 0.02206 & \sin 9\theta_0 \\ +\ \ \ \ 0.01008 & \sin 10\theta_0 \\ +\ \ \ \ 0.00469 & \sin 11\theta_0 \\ +\ \ \ \ 0.00222 & \sin 12\theta_0 \end{array}\right\}$$

We have shown that in order to reduce $\delta_1 r_2 / a_2$ and $\delta_1 v_2$ to $\delta_1 \mathrm{r}_2' / \mathrm{a}_2$ and $\delta_1 v_2'$ respectively, it is necessary to change the coefficient of argument θ_0 only, adding $\kappa_1 \mathrm{a}_1 / \mathrm{a}_2$ in the first case, and subtracting it in the second. This amounts to

$$\text{Red. to } \frac{\delta_1 \mathrm{r}_2'}{\mathrm{a}_2} = +0.00052\,00157 \qquad \text{Red. to } \delta_1 v_2' = -107''.26093$$

Computation of Second-Order Terms.

With the values obtained for A', $a_1 \dfrac{\partial A'}{\partial a_1}$, $a_1^2 \dfrac{\partial^2 A'}{\partial a_1^2}$ were computed the coefficients $\mathrm{a}_i, \ldots, \mathrm{q}_i$ and $\alpha_i, \ldots, \theta_i$. In order then to find the numerical values of δu_1, δz_1, δu_2, δz_2 it was necessary to multiply together series having the above as coefficients. This multiplication was performed by the method of special values as set forth in HANSEN's "*Auseinandersetzung*," pp. 159–164, or in TISSERAND's "*Mécanique Céleste*," Tome IV. The semi-circumference was divided into twelve equal parts, and to θ_0 were given the thirteen equidistant values 0°, 15°, 30°, ... 180°. It is important in these computations to take advantage of any checks that may present themselves. When no checks were available the computations were repeated. After all the products had been computed equation (11), determining the constant part of the radius-vector, was employed as a partial verification of the work.

1. *Computation of δu_1 and $\delta z_1 / u_1$.* The numerical values of the coefficients entering into δu_1 and δz_1 are tabulated below in terms of their logarithms. It will be denoted whether the series (which is a product of two other series) is a cosine or a sine series, and by the numbers $i+j$ at the left what is the multiple of the argument θ_0 whose coefficient is opposite. By multiplying by two each of the coefficients $\mathrm{a}_i, \ldots, \alpha_i, \ldots$ except when $i = 0$, we may regard $i+j$ as always positive.

$i+j$	cosine $\mathrm{a}_i\mathrm{a}_j$	cosine $\mathrm{e}_i\mathrm{a}_j$	cosine $\mathrm{f}_i\mathrm{b}_j$	cosine $-\varepsilon_i\zeta_j$
0	3.18127	3.13068n	3.39200	3.37675n
1	2.588179n	3.260253n	3.681298	3.506168n
2	2.322029	3.261197n	3.711587	3.331918n
3	2.80822n	3.10171n	3.70378	2.97340n
4	3.13372	3.21001n	3.61656	3.07604
5	2.41947	3.15498n	3.48959	3.22051
6	1.9421	3.05398n	3.33996	3.19793
7	1.501	2.9239n	3.17452	3.10977
8	1.098	2.7740n	2.99747	2.98594
9	0.718	2.6055n	2.8098	2.8358
10	0.35	2.4241n	2.6133	2.6738
11	9.95	2.1817n	2.3949	2.5558
12	9.7	2.0110n	2.1544	2.3660

$i+j$	sine $\varepsilon_i\mathrm{c}_j$	sine $\zeta_i\mathrm{b}_j$	sine $\mathrm{g}_i\gamma_j$	sine $\mathrm{h}_i\mathrm{d}_j$
1	3.228778n	2.624453	3.264226n	1.597713
2	3.502550n	2.976050	3.256116n	2.420552n
3	3.54078n	2.95902	3.04924n	2.19131n
4	3.59282n	2.84670	2.96747	2.73957n
5	3.52926n	2.67205	3.15861	2.64359n
6	3.42139n	2.4729	3.15214	2.48659n
7	3.28530n	2.2601	3.07239	2.3024n
8	3.12894n	2.0379	2.9536	2.1014n
9	2.9597n	1.812	2.8096	1.8935n
10	2.7762n	1.577	2.7506	1.680n
11	2.5576n	1.33	2.5291	1.472n
12	2.3456n	1.06	2.3477	1.249n

$i+j$	sine $\zeta_i\mathrm{d}_j$	cosine $\mathrm{a}_i\mathrm{d}_j$	cosine k_{i-j}	cosine l_{i+j}
0	. . . . .	3.50648n	2.64482	. . . .
1	2.349305	2.796439	. . . . .	. . . . .
2	2.340550	2.514531n	3.121940	3.044708
3	2.04846	3.10381	. . . .	. . . .
4	2.95404	3.46114n	. . . .	. . . .
5	2.85854	2.81880n	. . . .	. . . .
6	2.69805	2.35268n	. . . .	. . . .
7	2.5099	1.9517n	. . . .	. . .
8	2.3051	1.5856n	. . .	. . . .
9	2.0929	1.245n	. . . .	. . . .
10	1.872	0.926n	. . . .	. . . .
11	1.613	0.573n	. . . .	. . . .
12	1.395	0.28n	. . . .	. . .

In order to find the constant δg_1 which enters into δu_1, and at the same time verify the preceding calculations, it is necessary to compute the additional products in equation (11), namely,

$$\frac{\dot{u}_1^2}{4 n_1^2}, \ \frac{\dot{z}_1^2}{n_1^2}, \ \frac{u_1 \ddot{u}_1}{4 n_1^2}, \ \text{and } \sigma a_1 \frac{\partial F_0}{\partial a_1} \frac{u_1}{2}$$

the numerical values of which are tabulated below. The same nomenclature is used as before, and the tabulation is in the same order in which the terms are here written.

$i+j$	cosine $-\delta\delta_i$	cosine $d_i d_j$	cosine $a_i o_j$	cosine $a_i r_j$
0	3.32522	3.83271	3.32522*n*	2.55100*n*
1	2.30343	2.96510*n*	2.12056*n*	2.97100*n*
2	2.12794	2.66130	2.45334*n*	2.89591*n*
3	2.73772	3.39792*n*	2.68638	2.88204*n*
4	3.28384*n*	3.78892	3.29218*n*	2.76675*n*
5	2.77321*n*	3.18525	2.82099*n*	2.82824*n*
6	2.39858*n*	2.75328	2.49363*n*	2.80472*n*
7	2.0656*n*	2.38169	2.20678*n*	2.72159*n*
8	1.7530*n*	2.0415	1.9372*n*	2.60879*n*
9	1.451*n*	1.7187	1.673*n*	2.4579*n*
10	1.158*n*	1.407	1.412*n*	2.301*n*
11	0.886*n*	1.087	1.147*n*	2.068*n*
12	0.60*n*	0.80	0.89*n*	1.911*n*

It was found that the last three or four coefficients (except the twelfth) obtained by the method of special values did not satisfy the checks, whereas the same coefficients computed by direct multiplication of series did. Hence all these coefficients were thus recomputed. In this way were obtained the twelfth coefficients in the sine series, which are not given by the method of special values. In the cosine series the twelfth coefficient is the same for both ways of computing.

From the above data we get for δu_1 and $\delta z/n$.

$i+j$	cosine δu_1	cosine $\delta z/n$
0	3.78191	. . . .
1	1.559802*n*	1.501189
2	1.554736	1.569894
3	3.84237	4.04265*n*
4	2.84305	3.63491
5	2.6374	2.6812
6	2.2726	1.713
7	1.909	1.125*n*
8	1.555	1.232*n*
9	1.211	1.076*n*
10	0.86	0.78*n*
11	0.09	0.46
12	9.4	0.45

We have

$$\frac{\delta_2 r_1}{a_1} = \tfrac{1}{2}\left(\delta u_1 - \tfrac{1}{4}\, u_1^2\right) \quad . \quad \delta_2 v_1 = \int \delta z \; dt$$

The numerical values of these quantities are here given.

$$\frac{\delta_2 r_1}{a_1} = \left\{\begin{array}{l} +0.00000\,02267 \\ -0.00000\,17952 \ \cos\theta \\ +0.00000\,17830 \ \cos 2\theta \\ +0.00000\,03800 \ \cos 3\theta \\ -0.00000\,00332 \ \cos 4\theta \\ +0.00000\,00076 \ \cos 5\theta \\ +0.00000\,00050 \ \cos 6\theta \\ +0.00000\,00025 \ \cos 7\theta \\ +0.00000\,00012 \ \cos 8\theta \\ +0.00000\,00006 \ \cos 9\theta \\ +0.00000\,00003 \ \cos 10\theta \end{array}\right.$$

$$\delta_2 v_1 = \left\{\begin{array}{l} -1.09575 \ \sin\theta \\ +0.61131 \ \sin 2\theta \\ +0.12699 \ \sin 3\theta \\ -0.03725 \ \sin 4\theta \\ -0.00332 \ \sin 5\theta \\ -0.00030 \ \sin 6\theta \\ +0.00007 \ \sin 7\theta \\ +0.00007 \ \sin 8\theta \\ +0.00005 \ \sin 9\theta \\ +0.00002 \ \sin 10\theta \\ -0.00001 \ \sin 11\theta \\ -0.00001 \ \sin 12\theta \end{array}\right.$$

These values of $\delta_2 r_1 / a_1$ and $\delta_2 v_1$ constitute the solution of the problem for *Jupiter's* coordinates. but, that the expressions for $\frac{r_1}{a_1}$ and v_1 may be complete, we add the first- and second-order terms, thus forming the tables

$$\frac{r_1}{a_1} = \left\{\begin{array}{l} 1-0.00001\,11985 \\ \quad +0.00012\,27470 \ \cos\theta \\ \quad -0.00053\,16043 \ \cos 2\theta \\ \quad -0.00005\,52168 \ \cos 3\theta \\ \quad -0.00001\,44266 \ \cos 4\theta \\ \quad -0.00000\,47521 \ \cos 5\theta \\ \quad -0.00000\,17722 \ \cos 6\theta \\ \quad -0.00000\,07116 \ \cos 7\theta \\ \quad -0.00000\,03001 \ \cos 8\theta \\ \quad -0.00000\,01314 \ \cos 9\theta \\ \quad -0.00000\,00591 \ \cos 10\theta \\ \quad -0.00000\,00272 \ \cos 11\theta \\ \quad -0.00000\,00127 \ \cos 12\theta \end{array}\right.$$

$$v_1 = nt + \left\{\begin{array}{l} +\ 78.15254 \ \sin\theta \\ -195.12909 \ \sin 2\theta \\ \quad 16.20481 \ \sin 3\theta \\ \quad 3.79461 \ \sin 4\theta \\ \quad 1.16033 \ \sin 5\theta \\ -\ 0.41327 \ \sin 6\theta \\ \quad 0.16093 \ \sin 7\theta \\ \quad 0.06649 \ \sin 8\theta \\ \quad 0.02863 \ \sin 9\theta \\ \quad 0.01273 \ \sin 10\theta \\ \quad 0.00582 \ \sin 11\theta \\ -\ 0.00270 \ \sin 12\theta \end{array}\right.$$

2. *Computation of δu_2 and $\delta z_2 / n_2$.* Several of the series entering into δu_2 and $\delta z_2 / n_2$ have already been computed as they enter also into δu_1 and $\delta z_1 / n_1$. The remaining coefficients, in terms of their logarithms, are tabulated below.

$i+j$	cosine $b_i b_j$	cosine $f_i e_j$	cosine $q_i b_j$	cosine $-\theta_i \gamma_j$
0	3.39116	3.63196*n*	3.32535	3.52518*n*
1	3.541189	3.849053*n*	3.415321	3.622736*n*
2	3.29549	3.89548*n*	3.36101	3.42146*n*
3	2.92530	3.81759*n*	3.19688	3.01129*n*
4	2.52038	3.79771*n*	3.02462	3.29467
5	2.09649	3.68966*n*	2.84428	3.36085
6	1.69460	3.55232*n*	2.59122	3.30380
7	1.3130	3.39590*n*	2.36144	3.19518
8	0.948	3.22480*n*	2.12730	3.05590
9	0.595	3.0439*n*	1.8916	2.89760
10	0.25	2.8505*n*	1.647	2.7281
11	9.91	2.6250*n*	1.395	2.5953
12	9.60	2.4080*n*	1.119	2.4079

$i+j$	sine $q_i \eta_j$	sine $\zeta_i p_j$	cosine $m_i \tau_j$	cosine $n_i \tau_j$
0	. . . .	. . . . .	2.82091*n*	. . .
1	3.169687	2.848613*n*	. . .	. . .
2	3.19792	2.79111*n*	3.29803*n*	2.64967*n*
3	3.09005	3.00085*n*	. . .	. . .
4	3.05805	3.00635*n*	. . .	. . .
5	2.91879	2.88634*n*	. . .	. . .
6	2.74412	2.72391*n*	. . .	. . .
7	2.55063	2.53883*n*	. . .	. . .
8	2.34727	2.3401*n*	. . .	. . .
9	2.1352	2.1324*n*	. . .	. . .
10	1.9187	1.9153*n*	. . .	. . .
11	1.7057	1.684*n*	. . .	. . .
12	1.503	1.428*n*	. . .	. . .

There is one additional product needed for $\delta z_2 / n_2$ and three for the numerical expression of equation (14). These are, respectively,

$$\frac{u_2 z_2}{2n_2}, \quad \frac{\dot{u}_2^2}{4n_2^2}, \quad \frac{z_2^2}{n_2^2}, \quad \frac{u_2 \ddot{u}_2}{4n_2^2}$$

The coefficients of these products are given below in the same order in which they occur here. Then δu_2 and $\delta z_2 / n_2$ have the values given.

$i+j$	cosine $b_i p_j$	cosine $-\gamma_i \zeta_j$	cosine $p_i p_j$	cosine $s_i b_j$
0	3.22296*n*	3.38700	3.59894	3.38700*n*
1	3.669376*n*	3.48381	3.62334	3.85723*n*
2	3.56529*n*	2.35849*n*	3.62027	3.92742*n*
3	3.32728*n*	3.25359*n*	3.60254	3.81581*n*
4	3.01265*n*	3.18388*n*	3.37820	3.63415*n*
5	2.66516*n*	2.94751*n*	3.08329	3.40628*n*
6	2.33235*n*	2.68587*n*	2.78977	3.16908*n*
7	2.00986*n*	2.41787*n*	2.50202	2.92815*n*
8	1.7018*n*	2.1475*n*	2.2185	2.6852*n*
9	1.3983*n*	1.8775*n*	1.9386	2.4122*n*
10	1.106*n*	1.6090*n*	1.6602	2.192*n*
11	0.80*n*	1.3185*n*	1.378	1.937*n*
12	0.46*n*	1.0995*n*	1.080	1.655*n*

$i+j$	cosine δu_2	cosine $\delta z_2/n_2$
0	1.20364*n*	. . .
1	5.005758*n*	5.074954
2	4.04903*n*	4.36740
3	3.59051*n*	4.07674
4	3.08096*n*	3.71344
5	2.65093*n*	3.37455
6	2.25643*n*	3.0540
7	1.8850*n*	2.7147
8	1.5291*n*	2.4142
9	1.1877*n*	2.150
10	0.835*n*	1.852
11	0.13*n*	1.34
12	9.13*n*	0.89

As in the case of r_1 and v_1 we have

$$\frac{\delta_2 r_2}{a_2} = \tfrac{1}{2}\left[\delta u_2 - \tfrac{1}{4}\, u_2^2\right] \quad , \quad \delta_2 v_2 = \int \delta z_2\, dt$$

In order to determine $\frac{\delta_2 r_2'}{a_2}$ and $\delta_2 v_2'$, the second-order perturbations of *Saturn's* coordinates when referred to the center of the sun, we must apply to the former the following reductions respectively: —

$$+\tfrac{1}{4}\,\kappa_1^2 \frac{a_1^2}{a_2^2}(1-\cos 2\theta_0) + \kappa_1 \frac{a_1}{a_2}\left[\frac{u_1}{2}\cos\theta_0 + (\delta_1 v_1 - \delta_1 v_2)\sin\theta_0\right]$$

and

$$+\tfrac{1}{2}\,\kappa_1^2 \frac{a_1^2}{a_2^2}\sin 2\theta_0 - \kappa_1 \frac{a_1}{a_2}\left[\left(\frac{u_1}{2} - \frac{u_2}{2}\right)\sin\theta_0 - (\delta_1 v_1 - \delta_1 v_2)\cos\theta_0\right]$$

In order to show the amount of these reductions the values of $\frac{\delta_2 r_2}{a_2}$ and $\delta_2 v_2$ are placed beside them below. In $\frac{\delta_2 r_2}{a_2}$ and its reduction the numbers are expressed in units of the tenth decimal.

$i+j$	cosine $\frac{\delta_2 r_2}{a_2}$	Reduction	sine $\delta_2 v_2$	Reduction
			″	″
0	− 9222	+ 690	. . . .	. . . .
1	−52407	−1318	−1.65252	−0.04982
2	− 6585	− 477	−0.16202	+0.02275
3	− 2368	+1374	−0.05531	−0.02554
4	− 768	+ 109	−0.01797	−0.00211
5	− 286	+ 22	−0.00659	−0.00051
6	− 115	+ 6	−0.00262	−0.00017
7	− 49	+ 2	−0.00110	−0.00007
8	− 21	0	−0.00048	−0.00003
9	− 10	0	−0.00022	−0.00001
10	− 4	0	−0.00010	0.00000
11	− 1	0	−0.00003	0.00000
12	0	0	−0.00001	0.00000

We can now form the tables for r_2' / a_2 and v_2'.

$$\frac{r_2'}{a_2} = \left\{ \begin{array}{l} 1+0.00041\,58615 \\ +0.00086\,35112 \cos\theta \\ +0.00041\,67556 \cos 2\theta \\ +0.00003\,39822 \cos 3\theta \\ +0.00001\,05003 \cos 4\theta \\ +0.00000\,37599 \cos 5\theta \\ +0.00000\,14685 \cos 6\theta \\ +0.00000\,06076 \cos 7\theta \\ +0.00000\,02618 \cos 8\theta \\ +0.00000\,01163 \cos 9\theta \\ +0.00000\,00530 \cos 10\theta \\ +0.00000\,00246 \cos 11\theta \\ +0.00000\,00116 \cos 12\theta \end{array} \right.$$

$$v_2' = n_2 t + \left\{ \begin{array}{l} -\ \ 5.13402 \sin\theta \\ +31.87097 \sin 2\theta \\ +\ \ 6.58817 \sin 3\theta \\ +\ \ 1.97545 \sin 4\theta \\ +\ \ 0.69977 \sin 5\theta \\ +\ \ 0.27283 \sin 6\theta \\ +\ \ 0.11311 \sin 7\theta \\ +\ \ 0.04893 \sin 8\theta \\ +\ \ 0.02183 \sin 9\theta \\ +\ \ 0.00998 \sin 10\theta \\ +\ \ 0.00466 \sin 11\theta \\ +\ \ 0.00221 \sin 12\theta \end{array} \right.$$

Thus we get

$$\frac{r_1}{a_1} = 1 + \sum_1^\infty A_i \cos i\,(l_2 - l_1)$$

$$v_1 = l_1 + \sum_1^\infty B_i \sin i\,(l_2 - l_1)$$

$$\frac{r_2'}{a_2} = 1 + \sum_1^\infty A_i' \cos i\,(l_2 - l_1)$$

$$v_2' = l_2 + \sum_1^\infty B_i' \sin i\,(l_2 - l_1)$$

Leander McCormick Observatory, 1903 *May* 15.

If we refer the coordinates to axes intersecting in the *Sun*, and rotating in the direction of motion with the uniform velocity n_1, it is evident that we may write

$$v_1 - l_1 = w_1 = \sum_1^\infty B_i \sin ikt$$

$$v_2' - l_1 = w_2' = kt + \sum_1^\infty B_i' \sin ikt$$

where $k = n_2 - n_1$. Then all the coordinates are periodic with respect to the time.

WHITE SPOT ON *SATURN*.

By E. E. BARNARD.

I always make a habit of looking at *Saturn* frequently during the time the planet is visible each year. In all the observations I have ever made of it, I had never seen any marking that could be used for determining the rotation period. On June 15th, at $15^h\ 0^m$, while examining the planet with the 40-inch, I noticed a decided bright spot half way from the central meridian of the planet and the following limb. The sky immediately clouded over, and no opportunity occurred to see the spot again until this morning (a.m. of June 24).

On this occasion the spot was very noticeable, and was preceded by a smaller white spot, which was separated from the main spot by a small dusky patch extending north and south. This dusky patch was estimated to be in transit at $2^h\ 59^m$.

At $15^h\ 3^m$ the following measures were made:

Center of spot fol. p. limb	11.7 (2)
Center of spot pre. f. limb	6.4 (2)

At $15^h\ 7^m$ its center was

Dist. from S. limb	11.8 (2)
Dist. from N. limb	5.7 (2)

When on the central meridian the east and west diameter was 2″.6, from one setting of the wires. The spot was then slightly elongated, and very luminous, though not definitely defined.

In transit the position was

From S. limb	11.51 (2)
From N. limb	6.00 (2)

This would make it 2″.75 north of the center of the disc.

The rotational motion of the spot was rapid.

The following careful records were made:

$15^h\ 37^m$ the large white spot is now in transit
15 42 I think it is now in transit.
15 50 it is certainly past transit.
15 55 it is noticeably past transit.

I think the second observation is perhaps the best, and have adopted it for the time of transit, viz.:

June 23 $15^h\ 42^m$ Central St'd Time ($6^h\ 0^m$ slow of G. M. T.).

At $16^h\ 0^m$ the spot was strikingly distinct, and a little elongated. At this time no other spot was visible, so it will be easy to definitely identify it for determination of the rotation period. The belt south of the spot was very heavy, and was extended out to the south.

The spot was observed again last night with the 12-inch and a power of about 300 diameters.

It was decidedly conspicuous, and when in transit was somewhat elongated.

The following observations of its transit were made:

June 24 $12^h\ 48^m$ not quite in transit.
12 52 " " "
12 54 " " "
12 55 transit, or not quite.
12 58 transit.
13 0 transit.
13 2 past transit.

Yerkes Observatory, Williams Bay, Wis., 1903 *June* 25.

June 24 $13^h\ 4^m$ perhaps past.
13 7 certainly past transit.
13 11 noticeably past.
13 13 conspicuously past.

At transit there was no other spot visible. The central transit was perhaps close to

June $24^d\ 12^h\ 58^m$ Central Standard Time.

COMET *c* 1903.

[From RITCHIE'S Circular of July 6, No. 135.]

A message received from Dr. KREUTZ, via Harvard College Observatory, on June 22, announced the discovery of a comet with nucleus by BORRELLY, of Marseilles, on June 21. Subsequent positions were received from Lick Observatory and from Professor PAYNE, which are given below.

From observations of June 22, 23 and 24, Mr. BELLAMY has computed the following approximate

ELEMENTS.

T = 1903 August 21.707 Greenw. M.T.

π = 67° 55′ ⎫
☊ = 292 37 ⎬ 1903.0
i = 92 11 ⎭

q = 0.2375

EPHEMERIS.

Gr. Midnight	α	δ	Br.
1903 July 3	21h 32m 47s	+ 8° 51′	3.9
7	21 9 16	+21 42	
11	20 24 13	+43 42	
15	17 53 41	+66 24	16.8

Brightness June 21 = 1.

POSITIONS.

Greenw. M.T.	α	δ	Observer
1903 June 21.469	21h 52m 52s.	− 8° 10′ – ″	Borrelly
22.8742	21 51 38.8	−7 0 48	Aitken
23.8278	21 50 51.4	−6 9 26	Aitken
23.8613	21 50 50.1	−6 7 38	Wilson
24.8745	21 49 52.0	−5 8 48	Aitken

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE*,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

BY W. W. DINWIDDIE.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

All of the following measures were made by double distances, five comparisons on each side of coincidence; and ten position angles. The center of *Neptune* was bisected in each case. A magnifying power of six hundred diameters was used. The measures have been corrected for refraction. An unusual amount of cloudy weather this season prevented a greater number of observations being secured. The seeing has been uniformly bad, except on Oct. 30, Nov. 1, and Dec. 5.

Washington M.T.	p	Wash. M.T.	s	Washington M.T.	p	Wash. M.T.	s
1902 Oct. 21 16h 28m 45s	101°.99	16h 28m 45s	16″.28	1903 Jan. 6 11h 8m 30s	263°.97	11h 8m 45s	16″.48
28 16 14 45	235.03	16 14 15	14.39	22 10 5 30	19.61	10 6 30	10.86
29 16 41 15	154.10	16 41 0	11.39	Feb. 20 8 2 36	52.00	8 3 0	13.57
30 16 14 30	96.33	16 15 15	16.05	22 8 30 0	269.53	8 29 40	16.55
31 16 44 0	50.96	16 43 45	13.68	25 8 37 30	88.07	8 38 0	16.88
Nov. 1 16 3 0	327.27	16 2 45	11.88	26 8 5 15	43.24	8 6 30	13.17
2 16 4 30	272.43	16 4 15	16.51	Mar. 3 7 34 45	84.92	7 35 15	16.77
21 14 43 0	205.87	14 44 45	11.74	12 7 27 58	257.50	7 28 17	15.97
Dec. 1 14 6 30	291.17	14 7 45	15.34	17 7 32 58	295.24	7 32 43	13.46
5 14 14 30	69.20	14 16 0	15.93	18 7 51 35	251.43	7 51 17	16.45
7 13 33 15	286.28	13 34 45	15.62	26 7 41 28	108.42	7 42 7	14.79

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A 4½-INCH REFRACTOR,

BY A. W. QUIMBY.

1903		Time	New Grs.	Total Grs.	Total Spots	Fac Grs.	Def.
Jan.	*1	1	–	–	–	–	poor
	*3	11	–	–	–	–	poor
	6	1	1	1	2	1	fair
	7	7	–	–	–	–	poor
	8	9	–	–	–	1	fair
	9	10	–	–	–	1	fair
	10	9	–	–	–	1	fair
	12	1	–	–	–	–	fair
	13	9	–	–	–	–	fair
	14	11	–	–	–	–	poor
	15	3	–	–	–	–	fair
	16	3	–	–	–	–	fair
	17	2	–	–	–	–	fair
	18	8	1	1	1	1	fair
	19	10	–	1	5	1	fair
	20	11	–	1	3	1	poor
	21	11	–	1	17	–	v. good
	22	9	1	2	11	1	poor
	23	8	–	2	11	1	poor
	24	10	–	1	6	1	poor
	26	9	–	–	–	–	poor
	27	3	–	–	–	1	fair
	28	11	2	2	7	1	fair
	29	9	–	1	8	1	poor
	30	9	–	1	8	–	poor
	31	10	–	1	8	–	poor
Feb.	2	10	–	1	2	1	poor
	3	3	–	1	2	1	poor
	4	1	–	–	–	–	poor
	5	8	1	1	2	–	fair
	6	8	–	1	2		fair
	7	9	–	–	–	–	poor
	8	5	–	–	–	–	poor
	9	8	2	2	3	1	fair
	10	9	1	3	10	1	fair
	12	8		3	6	1	fair
	13	9	–	2	4	–	fair
	14	3		1	2		poor
	17	8		1	2	–	fair
	18	8	1	2	3	–	poor
	19	8	1	3	7	–	fair
	20	3	–	1	8	3	fair
	21	8	–	1	1	2	poor
	22	8	–	1	2		poor
	23	8		1	3		fair
	24	8	2	3	7	1	fair
	25	8	–	3	6	2	fair
	26	8		2	7	2	fair
	27	8	–	2	5	–	fair
	28	2	–	2	3		poor
Mar.	1	8		1	1	1	poor
	2	8		1	3	1	fair
	3	8	–	1	2	2	fair
	4	8		1	2	3	fair

1903		Time	New Grs.	Total Grs.	Total Spots	Fac Grs.	Def.
Mar.	5	8	–	–	–	1	fair
	6	8	–	–	–	1	good
	10	11	–	–	–	–	poor
	11	5	–	–	–	–	poor
	12	8	1	1	1	1	fair
	13	8	–	1	1	–	fair
	14	8	–	1	7	–	fair
	15	8	–			1	fair
	17	8	–	–	–	–	fair
	18	8	–	–	–	–	fair
	19	8	–	–		2	good
	20	10	–	–	–	–	poor
	21	8	–	–	–	–	poor
	23	1	1	1	1	–	poor
	24	8	1	2	5	1	fair
	25	8	–	2	6	1	fair
	26	8	1	2	3	1	fair
	27	8	–	2	7	1	fair
	28	8	–	2	12	1	good
	29	8	1	3	24	1	fair
	31	1		3	12	1	good
Apr.	1	8	–	3	23	1	fair
	2	8	1	3	7	2	poor
	3	8		2	3	1	poor
	4	8	–	1	3	1	fair
	5	9	–	1	3	1	fair
	6	8		1	8	2	fair
	7	5	–	1	1		poor
	9	8	1	2	7	1	fair
	10	8	–	2	7	1	fair
	11	7	–	2	6	1	fair
	13	10	–	1	2	1	poor
	16	2	–	–	–		poor
	17	8		–			fair
	18	7	–			2	fair
	19	8	–	–	–	–	fair
	20	8	–	–		1	fair
	21	7				1	fair
	22	4	1	1	1	2	fair
	23	7	–	1	1	2	fair
	24	8	2	3	9	3	fair
	25	11		2	3	1	poor
	26	5	1	3	10	1	fair
	27	8	1	1	24	2	good
	28	8		1	27	2	fair
	29	10		3	12	2	good
	30	8		3	80	2	good
May	1	8		3	24	2	fair
	2	6	1	3	18	2	fair
	3	8		3	16	2	fair
	4	8	–	3	10	3	fair
	5	4				1	good
	6	8				1	fair
	7	8				1	fair

1903		Time	New Grs.	Total Grs.	Total Spots	Fac Grs.	Def.
May	8	8	–		–	1	fair
	9	8	1	1	1	1	good
	10	8		–	–	1	fair
	11	7	–		–	1	fair
	12	6	1	1	1	2	fair
	13	8	–	1	1	2	fair
	14	8	–	1	1	2	fair
	15	8	–	–	–	2	fair
	16	8	1	1	1	2	fair
	17	8	–	1	2	1	fair
	18	7	–	1	2	1	fair
	19	8	–	1	1	1	fair
	20	8	1	1	2	1	fair
	21	7	1	2	10	2	fair
	22	7	–	2	4		poor
	23	10	–	1	2	–	fair
	24	6	–	1	1	–	poor
	25	7	–	1	1	–	poor
	26	7	1	2	13	1	good
	27	7	–	1	1		poor
	28	8		1	2	2	fair
	29	3		–	–	1	good
	30	8	–		–		poor
	31	6		–	–	1	fair
June	1	8	–		–	1	good
	2	6			–	–	poor
	3	8		–	–		fair
	4	7	–			–	fair
	5	7	–				poor
	6	7	–		–		poor
	7	6	–				poor
	8	8					fair
	9	8			–		fair
	10	8	–				fair
	11	8					fair
	12	10					poor
	13	4	1	1	1	2	fair
	14	2		1	1	1	poor
	15	8		1	1	2	poor
	16	8	1	2	6	2	fair
	17	10		1	1	1	poor
	18	10	–	1	3	2	fair
	19	8		1	2	1	poor
	20	4	2	3	30		poor
	21	8		3	50		good
	22	8		3	16	1	poor
	24	4		2	5	2	fair
	25	7		2	3		poor
	26	7	–	2	5	1	fair
	27	7		1	1	1	poor
	28	7		1	1	1	fair
	29	4		1	1	1	poor
	30	8	1	1	1		[illegible]

*2¼-inch refractor.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY.

By J. C. HAMMOND

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1903 Wash. M.T.	∗	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				(387) *Aquitania.*						
	h m s		m s	′ ″	h m s	° ′ ″			s	″
Feb. 4 10 16 33	1	15 , 4	−1 21.06	− 1 37.9	9 35 39.88	+19 11 3.6	n9.451	0.527	+1.99	−15.7
4 10 29 35	2	20 , 5	−1 11.42	− 6 56.7	9 35 39.60	+19 11 8.9	n9.414	0.517	+1.99	−15.7
5 11 48 22	3	19 , 4	−1 27.76	− 2 51.9	9 34 15.83	+19 20 17.3	n8.982	0.472	+2.00	−15.7
				(102) *Chloe.*						
Feb. 21 10 0 22	4	29 , 6	+0 30.81	+ 0 0.3	9 45 8.71	+20 3 11.3	n9.319	0.483	+2.10	−15.7
22 9 14 48	5	30 , 10	−0 11.02	− 1 2.8	9 44 21.07	+20 12 52.7	n9.452	0.510	+2.11	−15.6
23 9 57 46	6	19 , 1	−2 56.36	−13 55.4	9 43 31.36	+20 23 0.8	n9.291	0.472	+2.11	−15.6
25 10 8 19	7	25 , 5	+1 6.16	−13 39.6	9 41 56.05	+20 42 15.1	n9.193	0.451	+2.12	−15.4
				(11) *Parthenope.*						
Jan. 30 11 0 6	8	24 , 5	+1 55.53	− 6 38.4	10 8 22.70	+13 43 10.4	n9.461	0.608	+1.84	−15.2
Feb. 5 10 13 55	9	30 , 6	+1 10.08	− 1 48.8	10 3 21.70	+14 21 58.9	n9.430	0.594	+1.94	−15.5
9 10 15 13	10	23 , 6	+2 22.84	+ 0 21.2	9 59 45.86	+14 48 34.8	n9.368	0.579	+2.00	−15.8
12 10 58 51	11	23 , 6	−0 16.47	+ 3 19.8	9 56 58.32	+15 8 38.0	n9.258	0.563	+2.03	−15.9
				(79) *Eurynome.*						
Feb. 19 9 56 18	12	30 , 6	−2 34.51	+ 1 24.4	10 20 31.87	+ 3 19 48.3	n9.442	0.709	+2.08	−15.7
20 9 58 41	13	29 , 10	−0 0.09	− 4 43.2	10 19 35.35	+ 3 56 25.1	n9.424	0.708	+2.09	−15.9
21 11 3 39	14	35 , 8	+0 24.96	− 4 53.1	10 18 36.27	+ 4 3 21.3	n9.147	0.701	+2.10	−16.0
22 10 13 10	15	29 , 6	+2 10.61	+ 3 29.8	10 17 41.84	+ 4 9 52.4	n9.351	0.703	+2.10	−16.1
				(18) *Melpomene.*						
Mar. 27 10 19 32	16	10 , 2	−1 39.04	− 1 2.1	10 29 34.99	+13 47 12.5	8.885	0.571	+2.06	−15.0
Apr. 1 9 46 24	17	27 , 7	−1 15.32	− 7 22.2	10 26 53.60	+14 13 32.4	n7.840	0.562	+2.02	−14.6
4 9 37 56	18	30 , 6	+0 54.62	−11 28.4	10 25 33.51	+14 26 15.8	7.427	0.558	+1.98	−14.4
5 9 8 28	18	28 , 6	+0 31.38	− 7 32.3	10 25 10.26	+14 30 12.0	n8.692	0.558	+1.97	−14.3
				(29) *Amphitrite.*						
Mar. 26 11 39 25	19	30 , 6	+0 10.07	− 1 18.8	13 1 32.61	− 9 33 25.6	n9.131	0.814	+2.38	−11.7
Apr. 1 10 35 14	20	30 , 6	+0 43.67	+10 32.0	12 53 11.06	− 9 3 40.8	n9.243	0.808	+2.44	−12.8
4 11 9 41	21	20 , 4	−3 29.94	+ 3 11.9	12 53 9.62	− 9 3 37.3	n9.036	0.811	+2.41	−12.5
5 10 45 24	22	25 , 5	+2 52.93	+ 0 51.6	12 52 13.82	− 9 0 5.4	n9.166	0.809	+2.41	−13.0
8 9 54 42	22	30 , 6	+0 5.11	+11 46.0	12 49 26.31	− 8 49 10.1	n9.330	0.804	+2.45	−12.4

Mean Places of Comparison-Stars for the beginning of the year.

∗	α	δ	Authority
	h m s	° ′ ″	
1	9 36 58.95	+19 12 57.2	Berlin (A) A.G. 3903
2	9 36 49.03	+19 18 21.3	" " " 3900
3	9 36 11.59	+19 23 27.9	" " " 3895
4	9 44 35.80	+20 3 26.7	Berlin (B) A.G. 3854
5	9 44 29.98	+20 11 11.1	" " " 3853
6	9 46 25.61	+20 37 11.8	" " " 3860
7	9 40 47.77	+20 56 10.1	" " " 3840
8	10 6 25.33	+13 50 4.0	Leipzig I. A.G. 3962
9	10 2 9.68	+14 24 3.2	" " " 3948
10	9 57 21.02	+14 48 29.4	" " " 3928
11	9 57 12.76	+15 5 34.1	Berlin (A) A.G. 4017
12	10 23 4.30	+ 3 48 39.6	Albany, A.G. 4017
13	10 19 33.35	+ 4 1 24.2	" " 4030
14	10 18 9.21	+ 4 8 30.4	" " 4021
15	10 15 29.13	+ 4 6 38.7	" " 4007
16	10 31 11.97	+13 48 59.9	Bonn VI. +14°2267
17	10 28 6.90	+14 21 9.2	Leipzig I, A.G. 4053
18	10 24 36.91	+14 38 28.6	" " " 4042
19	13 1 20.19	− 9 31 55.1	Schjellerup 4728 [140
20	12 52 24.95	− 9 14 0.0	Wien. A.G. Zones 122 &
21	12 56 37.12	− 9 7 6.7	" " Zone 131
22	12 49 18.45	− 9 0 44.0	Newcomb's Fund.Catal.

OBSERVATIONS OF COMET *d* 1902 (*GIACOBINI*),

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By W. W. DINWIDDIE.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1902–3 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Dec. 18 12 26 28	1	t15 , 2	−0 18.39	+ 1 35.4	7 11 45.14	+ 0 44 20.3	n9.053	0.733	+4.65	−15.1
26 10 44 23	2	10 , 10	+0 3.00	+ 3 22.8	7 6 27.16	+ 2 33 14.6	n9.370	0.718	+4.81	16.1
Jan. 17 12 42 30	3	10 , 10	+0 6.91	+ 0 40.9	6 50 39.21	+ 9 0 33.8	9.129	0.641	+1.91	12.6
18 8 17 49	4	10 , 10	−0 21.82	+10 6.7	6 50 4.19	+ 9 16 59.6	n9.181	0.662	+1.93	−12.6
22 11 27 30	5	10 , 10	−0 37.19	− 3 34.9	6 47 17.18	+10 38 45.7	8.963	0.619	+1.94	12.8
Feb. 4 9 36 5	6	t18 , 8	−0 37.13	+ 0 57.8	6 40 12.42	+11 56 34.3	n8.451	0.550	+1.90	12.3
5 8 28 12	7	t39 , 10	−0 49.44	− 1 40.1	6 39 49.28	+15 15 20.4	n9.163	0.555	+1.90	12.3
22 7 55 39	8	t29 , 6	+2 16.08	− 3 10.8	6 36 31.16	+20 31 5.8	n8.856	0.443	+1.80	10.8
23 8 4 14	9	10 , 10	−0 5.99	+ 0 19.1	6 36 33.63	+20 48 27.3	n8.658	0.431	+1.80	10.8
26 8 33 44	10	10 , 10	−0 20.89	− 1 37.6	6 36 51.07	+21 39 24.9	8.627	0.413	+1.75	−10.4
Mar. 1 9 0 18	11	10 , 10	−0 7.51	− 6 38.8	6 37 23.69	+22 28 40.2	9.091	0.405	+1.70	10.0
3 10 31 39	12	t32 , 6	−1 11.79	+ 5 21.4	6 37 51.55	+23 1 20.9	9.496	0.471	+1.67	9.9
17 9 45 19	13	10 , 10	−0 24.83	− 3 36.1	6 44 23.86	+26 20 56.0	9.434	0.371	+1.48	8.6
18 9 56 31	14	t25 , 5	−0 49.08	+ 3 25.8	6 45 4.72	+26 34 15.9	9.545	0.430	+1.47	8.6
Apr. 17 8 34 15	15	t12 , 6	+1 11.47	+ 4 4.7	7 15 14.85	+31 36 43.0	9.577	0.328	+1.04	7.0
18 8 45 10	16	t11 , 6	−1 17.51	+ 6 41.9	7 16 34.31	+31 44 17.5	9.602	0.356	+1.04	7.0
27 9 33 32	17	10 , 10	+0 20.54	− 6 43.0	7 29 8.01	+32 44 51.3	9.696	0.494	+0.89	6.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	7 11 58.88	+ 0 43 0.3	Nicolajew, A.G. 2097	9	6 36 37.82	+20 48 19.0	Berlin B, A.G. 2535
2	7 6 19.35	+ 2 30 7.9	Albany, A.G. 2677	10	6 37 10.21	+21 41 12.9	Berlin B, A.G. 2543
				11	6 37 29.50	+22 35 29.0	Berlin B, A.G. 2545
3	6 50 30.39	+ 9 0 5.5	Leipzig II, A.G. 3353	12	6 39 1.67	+22 56 9.4	Berlin B, A.G. 2561
4	6 50 24.08	+ 9 7 5.5	Leipzig II, A.G. 3348	13	6 44 47.21	+26 24 40.7	Camb., Eng., A.G. 3532
5	6 47 52.43	+10 42 32.4	Leipzig I, A.G. 2557	14	6 45 52.33	+26 30 58.7	Camb., Eng., A.G. 3547
6	6 40 47.65	+11 55 48.8	Leipzig I, A.G. 2477	15	7 14 2.34	+31 32 45.3	Leiden, A.G. 3076
7	6 40 36.82	+15 17 12.8	Berlin A, A.G. 2369	16	7 17 50.84	+31 37 42.6	Leiden, A.G. 3113
8	6 34 13.28	+20 34 27.4	Berlin B, A.G. 2511	17	7 28 46.58	+32 51 40.8	Leiden, A.G. 3192

Comparisons in α were made by transits when marked *t*, otherwise $\Delta\alpha$ was determined by the micrometer.

OBSERVATIONS OF COMET *b* 1902 (*PERRINE*),

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By C. W. FREDERICK.

[Communicated by Captain C. M. CHESTER, U.S.N., Superintendent.]

1903 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 5 11 0 8	1	23 , 5	+1 37.17	5 21.2	8 3 39.73	33 23 12.4	n7.862	0.922	+2.27	17.4
5 11 15 17	1	20 , 4	+1 32.89	1 32.7	8 3 35.45	33 22 20.9	8.483	0.922	+2.27	17.4
20 9 15 57	2	21 , 5	+1 37.64	2 22.2	6 54 39.38	18 9 39.2	9.023	0.864	+1.70	20.7
21 10 50 12	3	d10 , 10	0 10.45	+6 31.7	6 52 0.15	17 20 1.5	9.385	0.846	+1.69	20.7
22 8 59 15	4	d10 , 10	−0 26.32	+0 33.5	6 49 18.93	16 37 52.0	8.538	0.859	+1.67	20.7
23 10 33 11	5	d10 , 10	+0 10.96	7 35.9	6 47 27.49	15 50 46.5	9.366	0.840	+1.64	20.6
25 9 33 30	6	27 , 6	+1 24.29	3 31.5	6 43 33.66	14 29 8.8	9.446	0.843	+1.58	20.4
26 9 5 50	7	d10 , 10	+0 9.89	5 52.0	6 41 48.59	13 50 22.5	8.982	0.842	+1.56	[illegible]
Mar. 1 7 59 51	8	28 , 6	+0 14.27	+1 12.9	6 37 12.53	12 1 37.7	7.746	0.833	+1.47	[illegible]
3 8 58 42	9	d10 , 10	0 28.72	1 55.4	6 34 34.44	10 53 1.6	9.126	0.822	+1.43	[illegible]
4 8 44 47	10	24 , 6	+2 12.10	+1 46.8	6 33 25.78	10 24 28.8	9.065	0.820	[illegible]	[illegible]
17 8 33 12	11	20 , 6	+1 49.37	1 53.0	6 24 54.16	4 54 30.4	9.344	0.778	+1.15	[illegible]
18 8 39 10	12	30 , 6	+0 48.12	3 17.5	6 24 34.39	4 34 37.5	9.350	0.775	+1.14	[illegible]
20 8 47 59	15	26 , 6	1 23.35	2 44.6	6 24 7.97	3 57 13.2	9.301	0.772	+1.12	[illegible]
25 7 49 29	16	d10 , 5	0 20.49	5 44.4	6 23 39.73	2 32 18.8	9.266	0.761	+1.03	17.8
26 8 23 8	18	d10 , 10	+0 11.91	0 24.4	6 23 39.79	2 17 0.4	9.398	0.757	+1.01	17.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s				h m s		
1	8 2 0.29	−33 17 30.8	C.G.C. 10728	10	6 31 11.99	−10 22 55.7	Camb., U.S., A.G.Zones
2	6 53 0.04	18 6 56.3	Washington, A.G.Zones	11	6 23 0.64	− 4 52 18.8	Strassburg, A.G. Zones
3	6 52 8.91	−17 26 15.5	Washington, A.G.Zones	12	6 23 45.43	4 31 1.1	Strassburg, A.G. Zones
4	6 50 13.58	16 38 4.8	Washington, A.G.Zones	13	6 25 57.57	− 3 38 45.9	Strassburg, A.G. Zones
5	6 47 14.89	15 42 50.0	Washington, A.G.Zones	14	6 25 45.21	− 3 46 56.6	Mic. Comp. with *13
6	6 42 7.79	−14 25 13.9	Washington, A.G.Zones	15	6 25 30.20	− 3 54 40.1	Mic. Comp. with *14
7	6 41 37.14	−13 41 40.2	Camb., U.S., A.G.Zones	16	6 23 58.89	− 2 26 49.6	Strassburg, A.G. Zones
8	6 36 26.79	12 5 30.5	Camb., U.S., A.G.Zones	17	6 22 41.55	− 2 9 2.6	Strassburg, A.G. Zones; Nicolaiew, A.G. 1691
9	6 35 1.40	−10 47 46.3	Camb., U.S., A.G.Zones	18	6 23 26.87	− 2 16 18.2	Mic. Comp. with *17.

The second observation by W. W. Dinwiddie. Comparisons in *α* were determined by the micrometer when marked *d*, otherwise by transits. The comet was best seen at the last observation. Continued cloudy weather prevented further observations.

SEARCHING EPHEMERIS FOR APPEARANCE IN 1903 OF COMET 1896 V.

Continued from *A.J.* 534, as Abridged from M. Ebell's Communication in *A.N.* 3881.

Boston M.T.	Assumed Per. Pass. June 6.5		Assumed Per. Pass. June 22.5			Assumed Per. Pass. July 8.5	
	α	δ	α	δ	Br.	α	δ
1903	h m s	° ′	h m s	° ′		h m s	° ′
July 16.5	2 32 34	+17 30.6	1 59 36	+17 33.9	2.55	1 18 42	+17 32.9
20.5			2 10 22	+17 57.4			
24.5	2 53 9	+18 2.9	2 20 51	+18 16.6	2.60	1 40 13	+18 29.4
28.5			2 31 1	+18 31.3			
Aug. 1.5	3 12 29	+18 19.7	2 40 51	+18 41.6	2.64	2 0 32	+19 4.7
5.5			2 50 19	+18 47.5			
9.5	3 30 23	+18 21.9	2 59 22	+18 49.1	2.67	2 19 19	+19 19.1
13.5			3 7 59	+18 46.5			
17.5	3 46 40	+18 9.9	3 16 7	+18 39.7	2.69	2 36 12	+19 12.0
21.5			3 23 45	+18 28.8			
25.5	4 1 6	+17 44.9	3 30 50	+18 11.0	2.70	2 50 52	+18 44.3
29.5			3 37 20	+17 55.4			
Sept. 2.5	4 13 30	+17 7.7	3 43 15	+17 33.2	2.70	3 2 55	+17 56.2
6.5			3 48 32	+17 7.4			
10.5	4 23 42	+16 19.6	3 53 10	+16 38.3	2.70	3 12 11	+16 49.6
14.5			3 57 8	+16 6.0			
18.5	4 31 30	+15 21.7	4 0 23	+15 30.8	2.69	3 18 18	+15 25.7
22.5			4 2 55	+14 52.8			
26.5	4 36 44	+14 15.4	4 4 43	+14 12.2	2.66	3 21 24	+13 47.4

Unit of brightness assumed as for 1897 Jan. 4, when last seen. At discovery in 1896 it was 11M–12M (Br. = 2.93.)

CONTENTS.

Published at 5 Craigie St., Cambridge (Boston Postal District), Mass., semi-monthly, by S. C. Chandler. Address, Cambridge, Mass.
Associate Editors, Asaph Hall and Lewis Boss. Price, $5 00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. *Closed July 20.*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 544.

VOL. XXIII. BOSTON, 1903 AUGUST 8. NO. 16

ON THE FIFTH SATELLITE OF *JUPITER*.

By E. E. BARNARD.

The large southern declination of *Jupiter*, and the consequent low altitude of the planet, even when on the meridian, have made it impossible to secure any measures of the fifth satellite from the spring of 1899, until the past summer and fall.

Though it was looked for at different times in the interval, when the conditions were favorable, and the opportunity occurred, it could not be seen.

In the latter part of July, and in August and September of the past year, the satellite was again observed, but it was at all times difficult to measure. Fortunately the elongations occurred when the planet was near the meridian; otherwise it would have been impossible to secure the measures. Even when the seeing was best the satellite was difficult from the low altitude.

The following west elongation times were observed in 1902. They are $6^h\,0^m$ slow of G.M.T.

	d	h	m
July	28	13	59.5
Aug.	25	11	28.7
Sept.	9	10	6.3

Reducing these to the *Sun* of 1892 Oct. 9, we have

Julian Day	5, 959.590750
	5, 987.490542
	6, 002.134626

The following west elongations were observed in 1892. They are $8^h\,0^m\,0^s$ slow of G.M.T.

	d	h	m		d	h	m
Oct.	9	16	3	Nov.	4	13	41
	17	15	17		11	13	6
	23	14	18		18	12	25
	28	14	26				

These reduced to the elongation of 1892 Oct. 9 give

Julian Day	2, 381.669584	Julian Day	2, 381.670025
	.668656		.669880
	.668628		.664901
	.672912		
		Mean	2, 381.669231

From the three elongations of 1902, above, and the normal elongation time for 1892 Oct. 9, we get the following three values of the periodic time:

	$0^d.49817899$	7182 periods
	0.49817923	7238 "
	0.49817906	7268 "
Mean	0.49817909	$\pm 0^d.00000006$

The sidereal period of the satellite is, therefore,

$$0^d\ 11^h\ 57^m\ 22^s.6698 \quad \pm 0^s.0052$$

The probable error of a single determination is $\pm 0^s.007$.
The mean daily motion in the orbit is, $722°.631636$.

The following are comparisons of the predicted times of west elongation from the "*Connaissance des Temps*" for 1902, and the observed times. They are all in Paris mean time.

	d	h	m		d	h	m	C − O
Pred. 1902 July	28	20	11.4	Obs'd July	28	20	8.9	+2.5
Aug.	25	17	38.4	Aug.	25	17	38.1	+0.3
Sept.	9	16	17.4	Sept.	9	16	15.7	+1.7
							Mean	+1.5

It would seem, therefore, that the satellite was about one and a half minutes ahead of its predicted time, which rested upon the periodic time derived by Dr. COHN.

As in the previous observations of this object, a piece of smoked mica covering one-half of the field was placed in front of the field-lens of the eye-piece — between it and the wires. This dulled the light of *Jupiter* so that the planet could be distinctly seen and measured, the eye at the same time not being blinded to the feeble light of the satellite which was visible in the unobscured part of the field. It is by this means only that this object can be referred direct to *Jupiter*. I have previously called attention to the fact that my measures of the diameters of *Jupiter* with the 36-inch of the Lick Observatory were made through similar smoked mica. These diameters are therefore specially suited for reducing these observations of the satellite to the center of the planet.

Miss E. E. DOBBIN, of the University of Chicago, has recently undertaken a new determination of the orbit of

this satellite, under the supervision of Dr. KURT LAVES. Miss DOBBIN will use all the measures obtained here with the 40-inch in the years 1898, 1899 and 1902, and will base her work on Dr. COHN's orbit, *A.N.* 3404. Until her important investigation is finished, it has seemed desirable in sending the present list of measures to the *Astronomical Journal*, to make some effort to approximately determine some of the elements of the orbit (by another method than that which she will use), in which the elongation distances alone are taken into account.

In 1894, TISSERAND, from the observations made up to December of 1893, during which time the satellite had been under observation, but little more than a year, computed certain elements of the orbit from the observed elongation distances (*C.R.*, CXIX, No. 15, for 1894 Oct. 8).

The great polar compression of *Jupiter* must produce a large motion in the orbit of this satellite. From the known compression of *Jupiter* TISSERAND computed the amount of this motion, which he found to be 882° a year in a positive direction, or a complete revolution of the orbit in about five months. With this motion, and the elongation distances measured by the writer at the Lick Observatory, he derived the mean distance, the eccentricity and the longitude of the perijove of the satellite's orbit. These elements represented the observations with great exactness.

His elements were

$$a = 47''.906 \quad , \quad e = 0.0073$$

ω_0 (the longitude of the perijove) $= -4°$ epoch 1892 Nov. 1. The daily motion of the orbit used by TISSERAND was $\omega_1 = +2°.42$

No matter how well the orbit of this satellite may be determined, it is impossible, on account of the rapid motion of the line of apsides, to get a fair representation of the observations unless the motion of the perijove has been well determined. This very fact also militates against a satisfactory determination of the orbit from observations extending over any considerable period of time.

The values of the polar compression of *Jupiter* from the various measures of his diameters are very discordant. There is, furthermore, a characteristic difference between the compression derived from filar-micrometer and from heliometer measures amounting to about one unit in its value.

Not being satisfied with the way in which my measures with the 40-inch refractor were represented in 1898 and 1899 by TISSERAND's elements, I varied his motion of the perijove until these observations were satisfied. At the same time the measures of 1892 and 1893 were, if anything, better represented than with TISSERAND's motion (see *A.J.* 472, Vol. XX, p. 126). It was then shown that by accelerating the motion of the line of apsides to 900° a year a very close agreement was obtained between observation and theory over the entire six or seven years. The observed elongation distances up to 1902 July 28 are represented with great exactness with this combination of TISSERAND's elements and my value of the apsidal motion. But for the other measures of 1902, with the exception of Oct. 7, which is closely represented, the results are disappointing, and I believe the discordances are not warranted by the character of the observations.

A comparison of various ephemerides (computed from different elements) with the observations, shows that TISSERAND's elements with his motion of the line of apsides, give a fair representation of all the observations. The agreement between observation and calculation is remarkable, with the exception of the measures of 1899. But the observations of 1899 are, I believe, very exact, especially the elongation distance of May 1, which is combined with that of April 25, with which it is accordant, to form the normal of April 28.

In *A.N.* 3403–4, Dr. FRITZ COHN, using the observations made by HERMANN STRUVE at Pulkowa, and by the writer at the Lick Observatory up to the end of 1894, made a thorough investigation of the orbit of the satellite. He did not derive the mean distance from the elongation distances, as was done by TISSERAND, but determined it from a modification of KEPLER's third law in which he took into account the motion of the line of apsides.

Of these elements, the values with which we are concerned, are

$$a = 48''.065$$
$$e = 0.00501$$
$$\omega_0 = 207°.2 \quad \text{Epoch 1892 Nov. 1}$$

The motion of the perijove determined by him was 911°.7 a year, or $+2°.4961$ daily.

This motion differed from TISSERAND's value by 30° a year, while the longitude of the perijove was 149° less than TISSERAND's. It is evident that one or the other of these computations must be very badly out.

I have shown that with an increase of 18° annually in TISSERAND's motion his other elements closely represented the place of the satellite up to the end of the observations in 1899, while those of COHN gave badly discordant results. A comparison of the residuals (O—C) from an ephemeris of each is given in the following table:

	TISSERAND	COHN	COHN, using TISSERAND's ω
	"	"	"
1892 Sept. 12	+0.03	+0.25	−0.07
Oct. 8	−0.11	+0.27	−0.16
19	−0.11	−0.66	−0.36
26	+0.07	+0.15	−0.02
Nov. 10	+0.08	−0.22	−0.08
12	+0.02	+0.05	−0.13
1893 Sept. 27	−0.07	−0.65	−0.25
Nov. 12	+0.03	−0.29	−0.23
Dec. 10	0.00	+0.29	−0.13
1894 Dec. 3	+0.15	−0.03	+0.12
1898 Mar. 15	0.00	−0.14	+0.14
1899 May 6	+0.09	+0.30	+0.08
June 16	+0.05	−0.25	+0.18

It would seem from this comparison that TISSERAND's results are nearer the truth. If, however, we reject COHN's longitude of the perijove and substitute, instead, TISSERAND's value, there is a great improvement in the results, as will be seen from the residuals in last column above.

This would seem to show that COHN's longitude of the perijove is wrong, and this is further proved to be true later on.

In trying to get a satisfactory agreement between calculation and observation, I have completed nearly twenty ephemerides from different elements, several sets of which have been computed new, or by varying portions of them.

The results in general would almost leave one in doubt as to what the motion of the orbit really is.

Some of these comparisons may be of interest. A few of them are therefore incorporated in the following table:

TABLE OF RESIDUALS FROM VARIOUS ELEMENTS.

	1	2	3	4	5	6	7	8	9	10	11
	″	″	″	″	″	″	″	″	″	″	″
1892 Sept. 12	+0.03	+0.12	0.00	+0.25	+0.02	+0.01	−0.01	+0.01	−0.01	−0.06	−0.02
Oct. 8	−0.11	+0.02	−0.11	+0.27	−0.11	−0.01	−0.15	−0.04	−0.03	−0.13	−0.67
19	−0.11	−0.27	−0.36	−0.65	−0.11	−0.31	−0.11	−0.25	−0.23	−0.16	−0.17
26	+0.06	+0.11	−0.02	+0.15	+0.07	+0.14	+0.05	+0.11	+0.17	+0.11	+0.15
Nov. 10	+0.09	0.00	−0.12	−0.22	+0.07	+0.07	+0.07	+0.09	+0.14	+0.13	+0.15
12	+0.02	+0.05	−0.06	+0.06	+0.03	−0.05	0.00	−0.03	−0.09	0.06	0.03
1893 Sept. 27	−0.07	−0.23	−0.34	−0.62	−0.16	−0.21	−0.16	−0.17	−0.08	0.24	−0.21
Nov. 12	+0.03	−0.08	−0.17	−0.29	+0.11	−0.35	+0.10	−0.01	−0.01	+0.15	+0.13
Dec. 10	0.00	+0.10	−0.02	+0.29	+0.09	+0.26	+0.07	+0.07	+0.01	+0.15	+0.16
1894 Dec. 3	+0.15	+0.13	0.00	−0.07	−0.02	+0.06	−0.04	+0.05	−0.02	−0.02	+0.03
1898 Mar. 4	−0.10	0.00	−0.13	−0.04	+0.34	+0.07	+0.18	+0.03	−0.01	+0.01	+0.03
Apr. 5	+0.32	+0.18	+0.07	−0.17	−0.12	−0.03	+0.16	−0.09	−0.06	−0.19	−0.13
1899 Apr. 28	+0.16	+0.28	+0.16	+0.38	+0.72	+0.54	+0.12	+0.58	+0.57	+0.06	+0.10
May 23	+0.07	+0.19	+0.06	+0.12	+0.12	+0.36	+0.39	+0.35	+0.30	+0.02	+0.07
June 16	+0.05	+0.01	−0.12	−0.28	−0.17	−0.10	0.20	−0.06	−0.15	−0.19	−0.19
1902 July 28	−0.03	−0.08	−0.20	−0.32	0.00	−0.01	+0.01	+0.03	−0.08	+0.03	+0.04
Aug. 5	+0.22	+0.11	0.00	+0.13	+0.01	+0.05	+0.17	+0.02	−0.01	+0.01	+0.06
25	+0.58	+0.36	+0.27	+0.20	−0.07	+0.03	+0.22	0.00	−0.01	−0.08	−0.03
Sept. 9	+0.61	+0.40	+0.31	+0.31	−0.05	+0.18	+0.21	0.00	+0.01	−0.10	−0.05
Oct. 7	+0.07	+0.05	−0.06	+0.04	+0.04	−0.01	+0.03	0.00	+0.10	−0.03	0.00

Elements used for the ephemerides from which the above table of residuals was determined:

	a	e	ω	ω_1	Epoch
	″		° ′	°	
1	17.906	0.0073	− 1 –	+2.166	1892 Nov. 1
2	47.945	0.0036	−18 30	+2.466	" "
3	48.060	0.0041	−18 30	+2.166	" "
4	48.065	0.00501	207 12	+2.1961	" "
5	47.906	0.0073	− 1 –	+2.12	" "
6	47.958	0.00426	7 27	+2.1148	" "
7	47.921	0.00769	20 58	+2.12	1897 Oct. 1
8	47.932	0.00542	31 44	+2.1175	1902 Aug. 27
9	47.917	0.00578	48 54	+2.42052	" "
10	47.921	0.00768	20 57	+2.414784	1897 Oct. 1
11	47.903	0.00687	22 12	+2.414784	" "

Of these elements 1 and 5 are those of COHN and TISSERAND, respectively. No. 9 was determined by computing corrections to 8 from the residuals given by 8. No. 3 was computed from the same observations as for 2, with the semi-diameter of *Jupiter* increased by 0″.12, in an endeavor to get an agreement between theory and observation in the mean distance.

I have adopted 11 as final, so far as the present observations are concerned.

In a delicate problem of this kind, it is useless to combine observations by different observers, for some of the quantities sought will be masked by the personalities of the observers. It is therefore best to depend on the work of one observer alone, in the hope that the consistency of his measures may in the end more nearly attain to the truth.

With the exception of the measures of HERMANN STRUVE, there are almost no other observations but my own. To avoid the effects of personal equation, it has appeared best to use only my measures. These seem, at least, to have the merit of consistency.

It seems probable that by this time the motion of the orbit can be determined with great exactness from the observations of elongation alone. The measures of 1892 are sufficiently numerous and exact to determine the longitude of the perijove during that year with a small amount of error. In the last few years the low altitude of the planet, for one thing, has not permitted a sufficient number of observations of elongation distances to very exactly locate the perijove. It is probable that the observations which will be made at the coming opposition of [illegible] will be enough when combined with those of last year, to determine its position very closely, and this combined with the position in 1892 will give a very exact value for the motion. I have thought, however, that after all [illegible]

to determine the apsidal motion closely with the material already on hand. With this point in view the following investigations have been made.

TISSERAND has given the following formulas which were used by him, in "*Comptes Rendus*," Tome CXIX, p. 583.

For the computation of the elongation distances:

$$(1)\qquad \begin{aligned} r_0 &= a - ae \sin(l-\omega_0-\omega_1 t) \\ r_e &= a + ae \sin(l-\omega_0-\omega_1 t) \end{aligned}$$

For the eccentricity and the longitude of the perijove:

$$(2)\qquad ae \cos\omega_0 = x \qquad , \qquad ae \sin\omega_0 = y$$

For the equations of condition:

$$(3)\qquad \begin{aligned} r_0 &= a - x \sin(l-\omega_1 t) + y \cos(l-\omega_1 t) \\ r_e &= a + x \sin(l-\omega_1 t) - y \cos(l-\omega_1 t) \end{aligned}$$

In these r_0 and r_e are the west and east elongation distances, and

l = the geocentric longitude of *Jupiter*.
ω_1 = the daily motion of the perijove.
ω_0 = the longitude of the perijove.
t = the time interval from the epoch.

From the measures of 1902 I have deduced the following elongation distances. In all these the satellite was to the west of the planet, that is, preceding. These are reduced to the distance 5.20.

	Observed	Computed	O—C
	"	"	"
1902 July 28	47.94	47.96	−0.02
Aug. 5	48.07	48.05	+0.02
25	48.18	48.20	−0.02
Sept. 9	48.17	48.18	−0.01
Oct. 7	47.92	47.92	0.00

Though these measures do not cover a revolution of the orbit of the satellite, I have thought they might give some idea of the orbit, and especially of the position of the perijove. Adopting the epoch 1902 Aug. 27, and with a daily motion of $+2°.42$ the following equations of condition were formed by the aid of eq. (3):

$$\begin{aligned} 47''.94 &= a - 0.44\,x + 0.90\,y \\ 48.07 &= a - 0.09\,x + 1.00\,y \\ 48.18 &= a + 0.79\,x + 0.61\,y \\ 48.17 &= a + 0.99\,x + 0.14\,y \\ 47.92 &= a + 0.47\,x - 0.88\,y \end{aligned}$$

These give the following normal equations:

$$\begin{aligned} 240.28 &= 5.00\,a + 1.72\,x + 1.77\,y \\ 82.85 &= 1.72\,a + 2.02\,x - 0.28\,y \\ 85.18 &= 1.77\,a - 0.28\,x + 2.97\,y \end{aligned}$$

and these give:

$$\begin{aligned} x &= +\ 0.221 \\ y &= +\ 0.137 \\ a &= 47''.932 \end{aligned}$$

From these

$$\omega_0 = 31°\,44' \quad \text{Epoch 1902 August 27}$$
$$e = 0.00542$$

These elements closely represent the observations used, as will be seen by the residuals (O−C) in the above table.

These also closely represent all the other observations back to 1892, with the exception of those of 1899, as will be seen by the following table of residuals:

	"		"
1892 Sept. 12	+0.01	1893 Dec. 10	+0.07
Oct. 8	−0.04	1894 Dec. 3	+0.05
19	−0.25	1898 Mar. 1	+0.03
26	+0.14	Apr. 5	−0.09
Nov. 10	+0.09	1899 Apr. 28	+0.58
12	−0.03	May 23	+0.35
1893 Sept. 27	−0.17	June 16	−0.06
Nov. 12	−0.01		

The position of the perijove as given by this last orbit ought to enable one to determine the motion of the line of apsides very closely; for, combining it with TISSERAND's position in 1892, there will be an interval of ten years, which ought to reduce any error in the motion so determined to a small quantity.

The longitude of the perijove from this orbit is $+31°.7$. If we combine this with TISSERAND's value we shall get a daily motion of the orbit of $+2°.4200$, which is exactly the value derived by TISSERAND. It is equal to $883°.9$ a year.

At the suggestion of Mr. W. S. ADAMS, who kindly supplied me with the formula for the purpose, equations of condition were formed from the residuals given by this last orbit for a determination of corrections to that orbit. In this manner the elements (No. 9) of the preceding table of elements were obtained. They show a slight improvement over the first one, but I do not feel satisfied with the results.

With the above value of the apsidal motion ($+2°.4200$) and the following observations, assuming the epoch 1897 Oct. 1, a new set of elements was computed.

	"	
1892 Sept. 12	48.11 (4)	East
Oct. 8	48.14 (4)	East
19	47.51 (4)	West
26	48.19 (3)	East
Nov. 10	47.98 (3)	East
12	47.97 (3)	West
1893 Sept. 27	47.67 (3)	East
Nov. 12	47.74 (2)	East
Dec. 10	48.12 (1)	East
1894 Dec. 3	48.17 (1)	East
1898 Mar. 15	48.12 (3)	East
1899 May 6	48.31 (3)	East
June 16	48.03 (1)	East
1902 Aug. 1	47.96 (2)	West
Sept. 1	48.17 (2)	West
Oct. 7	47.92 (1)	West

Equations of condition were formed from these observations, which by the method of least-squares gave the following normal equations:

$$\begin{aligned} 768''.11 &= +16.00\,a + 2.68\,x + 3.39\,y \\ 130.74 &= +\ 2.68\,a + 7.09\,x - 0.99\,y \\ 163.28 &= +\ 3.39\,a - 0.99\,x + 8.86\,y \end{aligned}$$

The solution of these gave

$$x = +0.3141$$
$$y = +0.1318$$
$$a = 47''.921$$

From these, by eq. (2),

$$\omega_0 = +20°\,58' \quad \text{Epoch 1897 Oct. 1}$$
$$e = 0.00769$$

If we combine the longitudes of the perijove derived from the orbits of 1897 Oct. 1 and 1902 Aug. 27, we get a daily motion of $+2°.4194$ or $883°.7$ annually.

Reducing the two values of the longitude of the perijove to 1892 Nov. 1, using a motion of $+2°.4200$, for comparison with TISSERAND's, we have

Tisserand.	1892 Nov. 1	$\omega_0 = -1$
Orbit of 1897 Oct. 1	"	$\omega_0 = -3$
Orbit of 1902 Aug. 27	"	$\omega_0 = -4$

The close agreement of the middle one of these with the others is doubtless accidental. The first and last should agree of course, for the motion was derived from them.

Several different sets of elements were computed from the above observed elongation distances. These in general gave somewhat different results, depending on the assumed epoch and motion of the orbit.

Forming equations of condition from the same set of observations with a motion of 882° a year, and the epoch 1892 Nov. 1, the following normal equations resulted:

$$768''.11 = +16.00\,a + 3.48\,x + 2.57\,y$$
$$168.19 = +\ 3.48\,a + 6.43\,x - 0.28\,y$$
$$123.45 = +\ 2.57\,a - 0.28\,x + 9.57\,y$$

These gave

$$a = 47''.958$$
$$x = +\ 0.2024$$
$$y = +\ 0.0265$$
$$\omega_0 = +7°.4 \quad \text{Epoch 1892 Nov. 1}$$
$$e = 0.00426$$

With $\omega_1 = 900°$ a year, and the epoch 1897 Oct. 1, equations of condition were formed from the preceding observations, from which resulted the following normal equations:

$$768''.11 = +16.00\,a + 0.45\,x - 5.79\,y$$
$$21.61 = +\ 0.45\,a + 10.51\,x - 0.18\,y$$
$$-278.51 = -\ 5.79\,a - 1.28\,x + 5.48\,y$$

The solution of these gave

$$a = 47''.941$$
$$x = -\ 0.0046$$
$$y = -\ 0.173$$
$$\omega_0 = -88°.5 \quad \text{Epoch 1897 Oct. 1}$$
$$e = 0.00365$$

If this value of ω_0 were carried back to 1892 Nov. 1 with the motion of 900° a year its longitude would be $-18°.5$, which is 14° from TISSERAND's value.

As will be seen later this last set of elements is [illegible] influenced by the large motion of the perijove [illegible] used.

After much experimenting I decided to separate some of the normal observations because the interval between them was too great for a simple mean to be taken. This gave twenty observations of elongation distance. The observations of 1898 and 1899 were also corrected for the final value of the micrometer screw.

I also decided to reject my motion of the perijove, and to adopt one nearly in accord with the value derived from a comparison with TISSERAND's elements, and my elements from the observations of 1902 alone.

These observations, all of which were made with either the 36-inch or the 40-inch refractors, are given in the following table:

Date	Observations		Computed Elong. Dist.	Residuals O—C
1892 Sept. 12	48.11 (1)	E	48.129	−0.02
Oct. 8	48.14 (1)	E	48.214	−0.07
19	47.51 (1)	W	47.680	−0.17
26	48.19 (3)	E	48.044	+0.15
Nov. 10	47.98 (3)	E	47.829	+0.15
12	47.97 (3)	W	48.005	−0.03
1893 Sept. 27	47.67 (3)	E	47.885	−0.21
Nov. 12	47.74 (2)	E	47.614	+0.13
Dec. 10	48.12 (1)	E	47.958	+0.16
1894 Dec. 3	48.17 (1)	E	48.140	+0.03
1898 Mar. 4	48.08 (2)	E	48.048	+0.03
Apr. 5	48.08 (1)	E	48.210	−0.13
1899 Apr. 28	48.27 (2)	E	48.167	+0.10
May 23	48.27 (1)	E	48.197	+0.07
June 16	47.99 (1)	E	48.176	−0.19
1902 July 28	47.94 (1)	W	47.896	+0.04
Aug 5	48.07 (1)	W	48.012	+0.06
25	48.18 (1)	W	48.21[illegible]	−0.03
Sept. 9	48.17 (1)	W	48.216	−0.05
Oct. 7	47.92 (1)	W	47.924	0.00

Assuming the epoch 1897 Oct. 1, which falls near the middle of the series, and adopting a motion of the perijove of $+2.411784$ daily, which I have found by experiment to be close to the true motion, we have with formulae [illegible] the following equations of condition:

$48.11 = a + 0.36\,x + 0.93\,y$	$48.08 = a + 0.07\,x + 1.00\,y$
$48.14 = a + 1.00\,x + 0.05\,y$	$48.08 = a + 1.00\,x + 0.08\,y$
$47.51 = a - 0.91\,x + 0.42\,y$	$48.27 = a + 0.52\,x + 0.86\,y$
$48.19 = a + 0.73\,x - 0.68\,y$	$48.27 = a + 1.00\,x - 0.08\,y$
$47.98 = a + 0.16\,x - 0.99\,y$	$47.99 = a + 0.5[illegible]\,x + 0.8[illegible]\,y$
$47.97 = a - 0.07\,x + 1.00\,y$	$47.94 = a - 0.40\,x + 0.92\,y$
$47.67 = a + 0.33\,x - 0.95\,y$	$48.07 = a - 0.05\,x + 1.00\,y$
$47.74 = a - 0.99\,x + 0.11\,y$	$48.18 = a + 0.74\,x + 0.67\,y$
$48.12 = a - 0.22\,x + 0.98\,y$	$48.17 = a + 1.00\,x + 0.08\,y$
$48.17 = a + 0.93\,x - 0.37\,y$	$47.92 = a + 0.44\,x + 0.00\,y$

NORMAL EQUATIONS.

$$960.57 = +20.00\,a + 6.20\,x + 1.96\,y$$
$$299.62 = +\ 6.20\,a + 9.00\,x - 0.97\,y$$
$$238.68 = +\ 1.96\,a - 0.97\,x + 11.[illegible]\,y$$

Solving these normal equations we have

$$x = +0.30447$$
$$y = +0.12432$$
$$a = 47''.9033$$
$$\omega_0 = 22^\circ\ 12' \quad \text{Epoch 1897 Oct. 1}$$
$$e = 0.0068655$$

The smallness of the residuals and the nearly equal distribution of the signs, viz.: -0.87 and $+0.82$ are very satisfactory, and would seem to show that both the orbit and its motion are closely determined.

I would therefore take the following elements and motion as the finally adopted values for the orbit of the satellite from the elongation distances observed by me.

ELEMENTS.

Mean distance $= a$	$= 47''.903$ (at Δ 5.20)
Eccentricity $= e$	$= 0.006866$
Longitude of perijove $= \omega_0$	$= 22^\circ.0$ Epoch 1897 Oct. 1
Daily motion of the line of apsides $= \omega_1$	$= +\ 2^\circ.414784$
Annual " " " "	$= +82^\circ.0$
Sidereal period of the satellite	$= 0^d\ 11^h\ 22^s.6698 \pm 0^s.0052$
Mean daily motion in the orbit	$= 722^\circ.631636$
Mean distance from theory	$= 48''.066$

The above values for the mean distance of the satellite would give

From the observed elongations = 112300 miles
From the theoretical value = 112670 "

It has not been deemed necessary to give the time of the epoch closer than to the nearest day.

In the early observations of the satellite I pointed out the fact that the measured elongation distances showed the orbit to be eccentric. Though this eccentricity is small, it is clearly indicated in the observations; from a mere glance at these one can form some idea of the amount of the eccentricity.

The various determinations of the mean distance from my observations of elongation do not vary much from $47''.92$. Assuming this value we can deduce a close approximation to the eccentricity of the orbit from a simple inspection of the measures; taking the mean of all the elongation distances that fall below $47''.80$, and of all those that fall above $48''.10$ as containing the least and greatest distances of the satellite, the eccentricity will be determined by the formulas,

$$e = 1 - \frac{r}{a} \quad , \quad e = \frac{r'}{a} - 1$$

The observations give

$$r = 47''.606 \text{ (8 obs.)} \quad , \quad r' = 48''.215 \text{ (17 obs.)}$$

From the formulas the first gives $e = 0.00655$ (wt. 8)
the second gives $e = 0.00610$ (wt. 17)
The mean of these is $e = 0.00625$

This is in good accord with the adopted value.

In deriving the mean distance of the satellite from the measures, the personality of the observer enters, and no matter how consistent his measures may be there will nevertheless be a small error introduced into the mean distance. The combination of measures by different observers may lessen this uncertainty, and if there are very few observers, as in the present case, it may increase the error.

It therefore needs some method to determine this quantity in which the personality of the observer does not enter.

Dr. COHN has shown (*A.N.* 3404, p. 322) that the semi-major axis of the orbit of the satellite can be better determined theoretically, from the periodic time, by the aid of KEPLER's third law. He gives the following formula:

$$\sin^3(\Delta) = \frac{1}{\mu}\left(\frac{N}{n}\right)^2 \frac{1}{(\rho)^3}\left(1 + \frac{\delta P}{n}\right)(1 - \Sigma m)$$

where (Δ) is the semi-major axis of the satellite's orbit.

$\frac{1}{\mu}$ = the mass of *Jupiter* = $\frac{1}{1047.35}$ (NEWCOMB; *A.N.* 136, pp. 133–134).
$(\rho) = 5.20280$, The mean distance of *Jupiter* from the sun.
$N = 59'.14$, The mean daily motion of the earth.
$n = 43356'$, The mean daily motion of the satellite.
δP = the annual motion of the line of apsides of the orbit in minutes of arc.
$\Sigma m = 0.00017$, The sum of the masses of the four old satellites, according to TISSERAND (*Méc. Cél.*, T. IV, p. 81).

n in the second parenthesis of the above formula is multiplied by 365.25.

From this formula, using $\delta P = 914^\circ.7$, his value for the motion of the perijove, Dr. COHN gets for the semi-major axis of the orbit, $(\Delta) = 48''.065$.

(An actual computation seems to give $48''.068$ instead of the above quantity.)

If the annual motion is assumed to be 900° this formula would give $(\Delta) = 48''.067$; or, using a value of 884°, $(\Delta) = 48''.066$.

From these it will be seen that a considerable variation in δP has very little effect on the mean distance. Hence any uncertainty in the motion of the line of apsides will not materially affect the resulting mean distance.

The mean distance derived from this formula is doubtless very exact, and is to be preferred to the value derived from the elongation distances. The difference between theory and observation in this case, considering the difficulty of the object, is small. This difference is in part due to errors of observation, and in part to a small error, perhaps, in the diameter used to reduce the measures.

I have long ago called attention (*A.J.* 325) to the discordance between micrometer and heliometer measures of the diameters of *Jupiter*, and have shown that the former are uniformly about one second of arc the greater. The values for the diameters of the planets adopted in the almanacs have in the main depended on heliometer measures. In the case of *Jupiter* it would be fatal to use such

"standard values" for the reduction of my measures of this satellite, for they would make a discordance some four or five times as great as those mentioned above. No other diameters, therefore, but those determined by me should ever be used in reducing my observations of this satellite. This statement should emphasize the necessity of the greatest caution in the reduction of such observations by the indiscriminate use of any adopted value of the diameter.

The inference drawn from the preceding investigations would lead to the following conclusion.

The daily motion of the line of apsides is close to + 882° a year, which is the value derived by Tisserand from theoretical considerations.

Dr. Cohn's motion of the orbit is evidently very much too large.

The longitude of the perijove for 1892 Nov. 1 was within a few degrees of 360°, which is also very near the value assigned by Tisserand. The position assigned to it by Cohn must therefore be in error pretty nearly a half revolution of the orbit.

If we give preference to the theoretical determination of the mean distance, it will be very close to 18″.07 at the mean distance of *Jupiter*, while the value from my measures will be 17″.90, which is almost identical with the value derived by Tisserand.

From the close representation of all the observations, extending through ten years, it would seem that the orbit has suffered no noticeable change in the interval.

In conclusion, I am indebted to Mr. W. S. Adams for valuable advice, and for a kindly interest in the subject.

Note: An elongation distance of the satellite observed 1903 July 21 gives a residual from elements No. 11 of −0″.07.

Observations of the Fifth Satellite of Jupiter in 1902.

July 21.

	Dist. from pr. limb	Dist from center	Comp.
h m s	″	″	
13 39 43	28.18	52.69	3
13 43 17	29.83	54.33	3
13 47 40	79.90*	55.40	3
13 53 12	31.76	56.26	3

Position angle of the wires = 161°.6.
* From following limb.

July 28.

	From pr. limb.	From center	Comp.
h m s	″	″	
12 41 47	72.62*	47.98	3
12 44 27	73.09*	48.44	3
12 47 15	24.66	49.31	3
12 49 39	25.37	50.02	3
12 51 49	25.86	50.51	3
12 54 34	26.86	51.51	3
12 56 52	26.80	51.45	2
12 58 22	28.54	53.19	2
13 0 19	76.95*	52.30	2
13 3 3	78.77*	54.12	3
13 5 51	29.86	54.50	3
13 7 39	30.34	54.99	2
13 14 20	32.19	56.83	3
13 20 11	33.06	57.71	3
13 24 43	33.81	58.46	3

Position angle of the wires at the longitude measures = 161°.5.
* Measured from following limb.

Latitude Measures.

From South Limb.

		Latitude	
h m s	″	″	
13 30 7	23.30	+0.19	1

From North Limb.

13 33 59	22.88	+0.23	5

	From pr. limb.	From center.	Comp.
13 38 55	35.75	60.40	3
13 41 3	36.01	60.66	3
13 43 17	36.10	60.75	3
13 45 32	36.26	60.90	3
13 47 24	86.22*	61.57	2
13 50 24	85.98*	61.33	3
13 52 0	36.68	61.33	3
13 54 25	36.81	61.46	3

Latitude Measures — Cont.

	From pr. limb.	From center.	Comp.
h m s	″	″	
13 56 25	36.64	61.29	3
13 59 9	36.72	61.37	3
14 1 45	36.15	60.80	3
14 3 58	36.59	61.24	3
14 7 20	36.52	61.17	3
14 10 26	36.23	60.87	3
14 14 9	36.44	61.09	3
14 17 19	36.02	60.67	3
14 20 27	35.07	59.71	3
14 23 54	35.35	59.99	3
14 27 25	35.39	60.04	3
14 30 19	34.26	58.91	3
14 33 10	34.05	58.70	3
14 36 32	33.75	58.30	3
14 39 34	32.93	57.58	2
14 44 39	32.12	56.77	2

Position angle of the wires at the latitude measures = 71°.5.
* Measured from following limb.

Times of Elongation Computed from the Observations.

Before Elongation.	After Elongation.
13 58.8	13 59.9
13 59.8	14 0.7
13 59.7	13 59.7
13 59.9	13 57.5
Mean 13 59.3	Mean 13 59.4

Aug. 5.

	From pr. limb.	From center.	Comp.
h m s	″	″	
12 51 18	35.54	60.21	2
12 56 16	35.29	59.99	2
12 59 20	36.11	60.81	2
13 9 36	36.74	61.44	3
13 13 1	87.25*	62.54	3
13 15 6	87.46*	62.76	2
13 17 13	36.74	62.03	3
13 20 6	36.94	61.64	3
13 22 43	36.80	61.51	3
13 25 41	36.40	61.10	3
13 27 28	36.19	60.89	2
13 28 53	36.36	61.06	2
13 31 26	36.79	61.49	3
13 34 51	35.85	60.55	3
13 38 32	35.88	60.58	3
13 41 23	35.87	60.57	4

Position angle of the wires = 161°.0.
* From following limb.

Aug. 21.

	From pr. limb.	From center.	Comp.
h m s	″	″	
10 43 58	27.15	51.63	3
10 48 25	28.35	52.82	3
10 52 30	29.56	54.03	3
10 55 25	30.84	55.32	3
10 58 29	31.73	55.21	3
11 8 59	33.22	57.70	3
11 12 11	33.27	57.74	3
11 15 10	35.16	59.64	3
11 18 15	34.51	58.99	3
11 20 32	34.62	59.10	3
11 23 43	35.31	59.78	3
11 28 2	35.36	59.83	3
11 31 43	36.19	60.63	3
11 34 11	36.60	61.08	3
11 38 0	36.46	[illegible]	3
11 41 31	36.63	61.11	1
11 44 17	36.87	61.35	3
11 47 30	37.11	61.59	2

Position angle of the wires = 162°.8.

In these observations there is possibly an uncertainty of one minute in the recorded times caused by the stopping of the watch before comparison was made with standard clock. It is probable the error will be only a few seconds at most.

Aug. 25.

	From pr. limb.	From center	Comp.
h m s			
10 38 12	31.01	55.36	3
10 41 26	31.67	56.03	3
10 44 39	31.92	56.28	3
10 47 16	32.52	56.88	3
10 49 17	32.84	57.20	3
10 53 52	33.37	57.73	3
10 56 12	34.27	58.63	3
10 59 18	34.75	59.10	3
11 2 17	34.66	59.02	3
11 5 16	34.76	59.12	3
11 8 32	35.24	59.60	3
11 11 29	35.61	59.97	3
11 15 22	35.57	59.92	3
11 18 19	36.33	60.69	3
11 22 30	36.52	60.88	3
11 27 27	36.89	61.25	3
11 30 11	36.35	60.71	3
11 34 12	36.58	60.94	3
11 37 17	36.15	60.51	3
11 40 11	36.10	60.46	3
11 43 21	35.92	60.27	3

Aug. 25 — Cont.

h m s	From pr. limb. ″	From center. ″	Comp.
11 45 16	35.60	59.96	3
11 48 6	35.36	59.71	3
11 51 32	34.47	58.83	3
11 54 11	34.83	59.19	3
11 57 11	34.08	58.44	3
12 0 44	33.97	58.33	3
12 1 54	33.73	58.09	1

Position angle of the wires = 162°.1.

Sept. 9.

h m s	From pr. limb. ″	From center. ″	Comp.
9 24 27	31.04	54.73	3
9 24 52	31.06	54.75	3
9 27 45	32.22	55.92	3
9 30 43	32.43	56.12	3
9 32 46	32.74	56.43	3
9 34 18	33.55	57.24	3
9 35 48	33.33	57.02	3
9 37 31	33.37	57.06	3
9 39 34	34.42	58.11	3
9 42 18	34.56	58.25	3
9 45 7	34.48	58.17	3
9 47 36	35.18	58.87	3
9 49 17	34.85	58.54	3
9 50 50	35.24	58.93	3
9 52 17	34.85	58.54	3
9 54 21	35.38	59.08	4
9 58 8	35.72	59.41	3
9 59 44	35.22	58.91	3
10 1 20	35.28	58.97	3
10 2 44	35.67	59.37	3
10 4 25	35.65	59.34	3
10 6 7	35.65	59.35	3

Sept. 9 — Cont.

h m s	From pr. limb. ″	From center. ″	Comp.
10 7 26	35.06	58.75	3
10 8 52	35.12	58.81	3
10 10 48	36.12	59.81	3
10 12 33	35.18	58.87	3
10 14 5	35.71	59.40	3
10 15 35	35.16	58.85	3
10 17 17	35.33	59.02	3
10 18 48	35.32	59.01	3
10 20 10	35.45	59.14	3
10 21 29	35.09	58.79	3
10 23 11	35.08	58.77	3
10 24 52	34.62	58.31	3
10 26 28	34.61	58.30	3
10 27 55	34.60	58.29	3
10 30 7	34.38	58.07	3
10 31 34	33.76	57.45	3
10 37 1	33.16	56.85	3
10 38 44	33.24	56.93	3
10 39 28	33.29	56.98	3
10 40 50	33.17	56.86	3
10 42 53	32.31	56.00	4
10 48 10	31.76	55.45	3
10 50 23	31.42	55.11	3
10 52 32	30.71	54.40	3
10 55 20	30.19	53.89	3
10 57 17	30.01	53.70	3
10 59 23	29.47	53.16	4

Position angle of the wires at the longitude measures = 162°.7.

LATITUDE MEASURES.

From South Limb.

h m s	″	Latitude. ″	
11 5 45	22.02	−0.19	5

From North Limb.

h m s	″	″	
11 40 57	22.28	0.07	1
11 42 45	23.05	0.84	3
11 44 12	22.85	−0.64	3

From South Limb.

11 46 43	21.50	−0.70	3
11 49 38	22.43	+0.22	3

Position angle of the wires at the latitude measures = 72°.8.

The observations were recorded by Mr. W. S. ADAMS.

This date (Sept. 9) was the tenth anniversary of the discovery of the Fifth Satellite.

Oct. 7.

Satellite and pr. limb.

h m s	From pr. limb. ″	From center. ″	Comp.
7 30 45	32.53	54.47	3
7 33 20	32.98	54.92	3
7 35 49	32.94	54.88	3
7 38 27	32.24	54.18	3
7 41 21	32.38	54.32	3
7 45 48	32.70	54.64	3
7 51 28	32.38	54.32	3
7 54 54	31.79	53.73	3
7 58 17	32.39	54.33	3
8 2 31	31.88	54.81	3
8 5 4	31.49	53.43	3
8 7 38	30.86	52.80	3
8 10 21	30.49	52.43	3
8 13 35	30.93	52.87	3
8 16 57	29.99	51.93	3
8 20 18	30.22	52.16	2

Position angle of the wires = 163°.3

The observations were carefully plotted, and the following elongation distances determined from the curves thus obtained:

	Apparent ″	Δ 5.20 ″
1902 July 28	61.35	47.94
Aug. 5	61.65	48.07
25	60.92	48.18
Sept. 9	59.25	48.17
Oct. 7	54.58	47.92

In all the observations the given times are $6^h\ 0^m$ slow of Greenwich M.T.

The satellite was preceding in every case.

The measures all depend upon a value for the micrometer screw of 9″.665.

The following apparent semi-diameters of *Jupiter* were used to reduce the observations:

	Equatorial ″	Polar ″
1902 July 21	24.503	
28	24.649	23.107
Aug. 5	24.704	
21	24.476	
25	24.359	
Sept. 9	23.691	22.209
Oct. 7	21.939	

They are derived from the diameters given in *A.J.* 325.

The following are corrections to the paper in *A.J.*, No. 472, Vol. XX.

p. 126, in the table, *for* 1892 Nov. 11 *read* Nov. 12
p. 127, *for* 1898 March 6 $11^h\ 56^m\ 5^s$ *read* $11^h\ 55^m\ 5^s$.
p. 128, 1898 April 26 in the three last distances,

	″		″
for	47.88	*read*	48.00
	47.47		47.59
	46.14		46.26

p. 128, 1899 May 1, first observation *for* 43″.38 *read* 42″.38.
p. 129, 1899 May 23, at $10^h\ 53^m\ 2^s$ *for* 43″.76 *read* 32″.76.
p. 128, 1898 April 26, the latitude observations are one measure each. The same for 1899 April 18, except the first observation. Same for April 20 for the latitude observations.

In the measures of 1898 and 1899 a preliminary value of the micrometer screw was used, which is a little larger than the final value. The measures of those years should all be reduced slightly to the amount of 0″.07 to 60″ of measured distance. As the greatest distance measured in the observations was about 32″ the largest corrections will not amount to 0″.04.

Yerkes Observatory, Williams Bay, Wis., 1903 *June*.

CONTENTS.

PUBLISHED AT 9 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed August 7.*
Entered ... at Boston, Mass., as second-class mail matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 545.

VOL. XXIII. BOSTON, 1903 AUGUST 20. NO. 17

ON THE SYSTEMATIC DIFFERENCE IN DECLINATION BETWEEN BRADLEY (AUWERS) AND THE CATALOGUE OF 627 STANDARD STARS (*A.J.* 531–2).

By LEWIS BOSS.

Systematically the Catalogue of 627 Standard Stars (*A.J.* 531–2) is independent of BRADLEY's determinations. These did not enter into the computations until it became possible to ascertain and apply with care the approximate systematic corrections necessary to reduce them to the system, B. So far as the right-ascensions are concerned, this proved to be a matter of no special difficulty; but the curve of systematic correction for the declinations, as they appear in the Catalogue of AUWERS, appears to be tortuous and uncertain, in relation to the material available for its determination.

In extending the system, B, to the computation of star-positions upon which the determinations of BRADLEY must exert important effects, on account of the relatively small weight of computed μ arising from the observations of the nineteenth century alone, it is essential to know the systematic difference, B − BRADLEY, with as much certainty as possible; otherwise, the system of proper motions for the additional stars, in certain restricted zones, might sensibly differ from that defined by the proper motions of the 627 primary standards. This consideration has led me to pay far more attention to the determination of $\Delta\delta$ for BRADLEY's declinations than to that for any of the later catalogues.

In his reduction AUWERS assumed that the errors of graduation were materially different for the two positions of the quadrant, apparently owing to suspected increase of deformations in its plane when adjusted for observations south of the zenith. Apparently these deformations were detected by means of the transits recorded in the use of the quadrant. In my preliminary researches upon $\Delta\delta$ for BRADLEY's declinations, as derived from comparison with B for stars south of the zenith, my attention was naturally directed to the supposed deformation at 8° of zenith-distance south. Comparison with the Standard Catalogue did not confirm this, but did indicate a very marked positive maximum of graduation error at 18° of zenith-distance south. Other anomalies of lesser moment appeared. For illustration, we have the following mean differences (from the curve, — the observed differences are larger), $\Delta\delta$, from the comparison, B − BRADLEY.

At +14° of declination, −0".15
At +36° of declination, +2".60
At +27° of declination, +0".60

It does not seem possible that any very large portion of the anomaly at +36° can be attributed either to relative systematic error in B, or to accumulation of casual errors in BRADLEY's declinations. It seemed desirable, therefore, to investigate whether the observations with BRADLEY's quadrant are consistent with the hypothesis that the graduation error remained substantially the same in the two positions of the quadrant, as tested by the Standard Catalogue.

In the second volume of *Neue Reduction der Bradley'schen Beobachtungen* (pp. 253–410) Dr. AUWERS has published the results which he obtained for the zenith-distances measured with BRADLEY's quadrant in the two positions, North and South. If we combine with these the latitude, 51° 28' 38".72, adopted by AUWERS, we shall obtain the declinations as they result without the application of any correction for error of graduation, such as AUWERS has adopted (p. 252 of the *Neue Reduction*, Bd. II). This course was adopted in the present investigation, and the declinations so obtained were compared with the Catalogue of Standard Stars, B. The results of this comparison are exhibited in Table I, for quadrant North under ΔA_N in the first section of the table, and for quadrant South, in the second section, ΔA_S. The individual results have been collected in zones of 5° in width; and the signs of the differences, B − BRADLEY, are given as applicable to altitudes, considered as positive, whether north or south. Therefore, for quadrant south and for quadrant north, lower culmination, these signs represent assumed corrections, $\Delta\delta$, to BRADLEY's declinations, and they correspond to $-\Delta\delta$ for declinations observed at upper culmination.

The weights here, as throughout, are computed so as to correspond to ±0".50 as the probable error of [illegible]. It

general my computations confirm the weights computed by Dr. AUWERS, except for zenith-distances greater than 75°, for which my adopted weights are less.

From inspection of these values of ΔA_N and ΔA_S it becomes evident that there is a very decided systematic difference between them; though the sinuosities in the trend of the two sets of numbers present a degree of resemblance which is, perhaps, quite as good as could have been anticipated, in view of the paucity of observations north of the zenith. This led to the examination of the hypothesis that the graduation-error, proper, remained practically invariable, and that the systematic difference in question is due to other causes. It seems very natural to suppose that the eccentricity of the quadrant might have been materially different in the two positions, either through wear in the pivot, or bearing, upon which the telescope turned, or through differences of strain in the fastenings for the two positions. This difference of eccentricity would give rise to a systematic difference of the form, $x \sin\zeta + y \cos\zeta$. If the wear was in the bearing, rather than in the pivot, we should expect y to be much smaller than x. It is also very probable that there may have been an appreciable alteration in the flexure of the telescope when the quadrant was set up in the position, south.

The researches of OLUFSEN (*Astr. Nach.*, Bd. 9, pp. 86–106), and those of SAFFORD (*Astr. Papers, Am. Eph.*, Vol. II, Pt. II) indicate that the declinations of the quadrant after BRADLEY's time require large and increasing corrections. From SAFFORD's results (Table X, of the work cited), I infer that, for stars south of the zenith, the part of this correction which is variable with the zenith-distance, and which he regarded as directly proportional to the zenith-distance in degrees, was roughly $+6'' \sin\zeta$ in 1767, and $+12'' \sin\zeta$ in 1787. These corrections are large enough so that no great part of them can possibly be attributed to error in the system, B_1, (declinations of the *American Ephemeris*, 1881–1900), which was employed as standard. As will be seen, further on, the progression of this error is remarkably consistent with the results found in the present investigation. From these various considerations, as well as from inspection of the differences themselves, it seemed best to assume that the systematic difference, $\Delta A_S - \Delta A_N$, is of the form, $k + x \sin\zeta$. This resulted in the following expression which has been adopted.

$$\Delta A_S - \Delta A_N = +0''.42 + 1''.20 \sin\zeta$$

The determination of this quantity was strengthened by means of a comparison, Lower *minus* Upper Culmination. Thus there is an alteration in the sine coefficient of $+1''.2$ in approximately four years; while from SAFFORD's researches the alteration appeared to be about $+6''$ in twenty years.

Modifying the values of ΔA_N by the amount of this correction, we have the numbers in the first section of Table I, under the caption, $\Delta' A_N$.

TABLE I. OBSERVED CORRECTIONS, ΔA, APPLICABLE TO BRADLEY'S OBSERVED ALTITUDES.

Quadrant North					Quadrant South				Mean			
δ	n	p	ΔA_N	$\Delta' A_N$	δ	n	p	ΔA_S	δ	n	p	ΔA
°			"	"	°			"				"
+51.2	16	6.6	+0.13	+0.51	+51.7	8	2.6	+0.07	+51.8	24	9.2	+0.41
55.2	4	1.1	+0.82	+1.32	48.1	9	2.9	−0.05	48.2	13	4.0	+0.33
57.4	7	2.1	+0.92	+1.47	15.2	9	2.8	+1.14	15.4	16	4.9	+1.28
60.4	11	2.2	+0.89	+1.51	42.4	8	1.5	+1.20	12.5	19	3.7	+1.38
63.6	6	1.6	+0.95	+1.62	39.5	12	3.5	+2.12	39.6	18	5.1	+1.96
66.4	4	0.8	+0.82	+1.55	36.9	8	1.3	+2.11	36.8	21	2.1	+1.89
69.5	6	1.6	+1.73	+2.52	33.2	9	1.9	+2.64	33.3	15	3.5	+2.59
72.1	5	1.0	+0.70	+1.54	30.3	9	2.0	+0.78	30.5	14	3.0	+1.04
76.4	5	0.8	+0.88	+1.91	27.8	21	5.5	+1.20	27.7	26	6.3	+1.29
78.8	2	0.3	+0.03	+1.00	24.3	10	2.8	+1.49	21.2	12	3.1	+1.44
81.8	2	0.1	+0.68	+1.71	21.5	21	7.3	+1.41	21.6	23	7.7	+1.42
81.8	2	0.8	+0.17	+1.25	18.7	11	2.8	+1.76	18.6	13	3.6	+1.64
87.6	3	1.1	+1.06	+2.19	15.8	14	3.6	+2.15	15.8	17	4.7	+2.16
90.6	1	1.0	+0.44	+1.62	12.7	16	5.0	+1.81	12.6	20	6.0	+1.78
93.5	1	0.5	−1.11	+0.11	9.2	17	5.6	+1.06	9.2	18	6.1	+0.98
95.8	3	1.1	−0.32	+0.91	6.5	18	5.9	+1.30	6.7	21	7.3	+1.23
98.1	1	0.1	−0.92	−0.37	3.8	21	5.8	+1.11	3.9	22	6.2	+1.29
100.2	6	1.2	−0.03	+1.29	+ 0.5	7	2.6	+0.87	+ 0.6	13	3.8	+1.00
104.9	1	0.1	−0.26	+1.12	− 2.1	12	3.7	+1.92	− 2.1	13	4.1	+1.84
108.1	7	1.9	+0.11	+1.53	5.4	14	4.3	+0.92	5.2	21	6.2	+1.12
111.2	5	1.8	−0.26	+1.19	8.6	23	7.6	+0.86	8.5	28	9.4	+0.92
114.9	4	1.0	−0.07	+1.43	11.1	15	3.8	+0.98	11.3	19	4.8	+1.07
117.8	7	2.1	+0.21	+1.76	14.8	22	6.2	+1.35	11.9	29	8.3	+1.45
120.4	8	2.8	+0.08	+1.62	17.7	22	6.2	+1.37	17.6	30	9.0	+1.45
123.4	9	2.3	−0.47	+1.09	20.5	18	4.9	+0.85	20.4	27	7.2	+0.93
127.3	6	1.7	−0.92	+0.67	23.3	17	2.9	+0.60	23.7	23	4.6	+0.63
129.5	7	1.9	−1.25	+0.35	26.3	14	2.5	+0.38	26.4	21	4.4	+0.37
132.1	11	1.9	−1.16	+0.45	29.6	15	1.5	−0.21	29.3	26	3.4	+0.16
+135.1	11	. .	−1.58	+0.04	−32.0	9	. .	−1.55	−32.1	20	. .	−0.56

The quantities, ΓA_N and ΔA_S, are now comparable, according to the hypotheses made. In fact, the differences, $\Delta A_S - \Gamma A_N$, show rather suspicious accumulations of negative signs near the zenith and near 65° of zenith-distance, and of positive signs near 45° of zenith-distance. An analysis of the comparison, Lower − Upper Culmination, does not confirm these apparent systematic deviations in all cases, however; so that it may be doubted whether they are wholly real. Furthermore, taking the differences with their weights, the probable error of the unit comes out, ±0″.32, against ±0″.30, the predicted probable error. Considering all of the difficulties of the case this is probably an agreement as close as ought to have been expected. The representation would have been somewhat better through the employment of the full formula, $k + x\sin\zeta$, $+ y\cos\zeta$ to express the difference between the two positions. Yet, with material so scanty, it might prove dangerous to attempt the derivation of such a formula of correction for observations extending over less than a quadrant, since a chance distribution of accidental errors at unlucky points might lead to an illusory result.

Accordingly, the means by weight of ΓA_N and ΔA_S were computed as they appear in the last section of Table I. From these the curve of correction, ΔA, was drawn, as it appears under ΔA in the first section of Table II. These mean values of ΔA correspond to those of "k," computed by Dr. AUWERS, as published in the second volume of the *Neue Reduction* (p. 252). The correction to the catalogue declinations (Band III) is found from the equation:

$$\Delta\delta = \Delta A - k$$

This result is found in Table II under the caption $\Delta\delta$; and these are the finally adopted corrections for the declinations of AUWERS' Catalogue. It is scarcely necessary to remark that the values of ΔA for stars, quadrant north, at upper culmination, are found by subtracting the values of ΔA, for corresponding zenith-distance south, from the formula, $+0''.42 + 1''.20 \sin\zeta$; and for those at lower culmination this formula is subtracted from corresponding values of ΔA, south.

For the zenithal arc, +49° to +54° of declination, as well as for the combination of lower with upper culminations, the values of $\Delta\delta$ applicable to the catalogue places must be computed by a combination of the several values of $\Delta\delta$, applicable for the different circumstances of observation, in proportion to the weights given in the two tables, pp. 22–33, in the introduction to the Catalogue.

In drawing the curve, from which the results for ΔA in Table II were derived, some slight modifications were introduced on account of the differences, Lower − Upper Culmination, taken from the introduction to the Catalogue (pp. 22–33), k having been first removed and approximate values of ΔA introduced.

TABLE II. CORRECTIONS, ΔA AND $\Delta\delta$.

Quadrant South.

δ	ΔA	Δδ	δ	ΔA	Δδ	δ	ΔA	Δδ	δ	ΔA	Δδ	δ	ΔA	Δδ
°	″	″	°	″	″	°	″	″	°	″	″	°	″	″
+53	+0.26	+0.18	+37	+1.93	+2.53	+19	+1.62	+1.14	+1	+1.36	+0.77	−17	+1.13	+1.35
52	+0.28	+0.18	36	+2.00	+2.65	18	+1.70	+1.19	0	+1.37	+0.60	18	+1.33	+1.63
51	+0.30	+0.48	35	+2.06	+2.32	17	+1.79	+0.93	−1	+1.37	+0.42	19	+1.18	+1.69
50	+0.33	+0.18	34	+2.13	+2.09	16	+1.88	+1.01	2	+1.36	+0.36	20	+1.02	+1.38
49	+0.40	+0.52	33	+2.17	+1.99	15	+1.90	+1.26	3	+1.32	+0.53	21	+0.88	+1.16
48	+0.51	+0.54	32	+2.07	+1.97	14	+1.80	+1.36	4	+1.28	+0.96	22	+0.78	+1.05
47	+0.79	+0.63	31	+1.65	+1.71	13	+1.68	+1.41	5	+1.22	+1.24	23	+0.69	+0.93
46	+1.01	+0.56	30	+1.32	+1.44	12	+1.52	+1.37	6	+1.14	+1.28	24	+0.59	+0.73
45	+1.17	+0.30	29	+1.25	+1.14	11	+1.33	+1.35	7	+1.03	+1.19	25	+0.50	+0.60
44.5	+1.24	−0.03	28	+1.24	+0.76	10	+1.09	+1.29	8	+0.96	+1.07	26	+0.42	+0.51
44	+1.30	−0.53	27	+1.27	+0.52	9	+1.00	+1.32	9	+0.94	+1.00	27	+0.33	+0.59
43.5	+1.36	−0.53	26	+1.31	+0.74	8	+1.02	+1.22	10	+0.98	+0.98	28	+0.26	+0.87
43	+1.42	+0.05	25	+1.36	+1.03	7	+1.06	+1.06	11	+1.05	+0.99	29	+0.18	+1.38
42	+1.52	+0.92	24	+1.41	+1.45	6	+1.12	+0.92	12	+1.14	+1.04	30	+0.10	+2.22
41	+1.63	+1.31	23	+1.45	+1.69	5	+1.17	+0.72	13	+1.23	+1.11	31	+0.03	+3.02
40	+1.72	+1.59	22	+1.48	+1.61	4	+1.24	+0.61	14	+1.34	+1.19	32	0.05	+3.30
39	+1.79	+1.86	21	+1.52	+1.52	3	+1.29	+0.67	15	+1.42	+1.24			
+38	+1.86	+2.21	+20	+1.56	+1.48	+2	+1.34	+0.76	16	+1.45	+1.25			

Quadrant North. Above Pole.

δ	ΔA	Δδ	δ	ΔA	Δδ	δ	ΔA	Δδ	δ	ΔA	Δδ	δ	ΔA	Δδ
+49	+0.13	−0.02	+58	0.62	0.10	+67	−1.26	1.14	+76	0.35	+0.25	+85	0.62	0.37
50	+0.13	0.01	59	0.72	+0.30	68	1.30	0.94	77	0.37	+0.10	86	0.69	0.26
51	+0.13	+0.01	60	0.82	0.03	69	1.35	0.81	78	0.40	0.08	87	0.71	0.33
52	+0.13	+0.03	61	0.90	0.17	70	1.37	0.75	79	0.44	0.35	88	0.77	0.14
53	+0.12	+0.05	62	0.99	0.70	71	1.25	0.70	80	0.46	0.48	89	0.75	0.11
54	+0.07	+0.03	63	1.06	0.78	72	0.81	0.39	81	0.47	0.45	+90	0.73	0.10
55	−0.05	0.02	64	1.11	0.86	73	0.46	0.13	82	0.49	0.43			
56	−0.28	−0.15	65	1.16	1.00	74	0.37	0.01	83	0.51	0.44			
+57	−0.18	0.19	+66	1.21	1.11	+75	0.31	+0.15	+84	0.55	0.45			

Quadrant South — Below Pole.

δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$	δ	ΔA_s	$\Delta\delta_s$
°	″	″		″	″	°	″	″	°	″	″		″	″
+45	−1.67	+0.10	+55	−0.79	+0.38	+64	−0.27	−0.67	+73	−0.13	−0.61	+82	−0.12	−0.30
47	−1.51	−0.01	56	−0.69	+0.37	65	−0.36	−0.81	74	−0.08	−0.73	83	−0.16	−0.17
48	−1.13	−0.10	57	−0.51	+0.11	66	−0.44	−0.92	75	−0.02	−0.73	84	−0.20	−0.06
49	−1.31	−0.05	58	−0.37	+0.44	67	−0.50	−0.98	76	.00	−0.64	85	−0.23	+0.05
50	−1.27	.00	59	−0.21	+0.27	68	−0.53	−1.00	77	+0.01	−0.52	86	−0.23	+0.13
51	−1.17	+0.10	60	−0.11	−0.03	69	−0.49	−0.91	78	+0.01	−0.40	87	−0.13	+0.17
52	−1.09	+0.20	61	−0.08	−0.23	70	−0.41	−0.82	79	+0.01	−0.36	88	+0.13	+0.28
53	−0.99	+0.28	62	−0.10	−0.39	71	−0.29	−0.66	80	−0.03	−0.37	89	+0.43	+0.42
+54	−0.89	+0.36	+63	−0.17	−0.53	+72	−0.20	−0.57	+81	−0.06	−0.37	+90	+0.51	+0.40

In fact, the drawing of the curve was a process of trial and error, in successive approximations, until what was deemed a fairly satisfactory representation of the various classes of material was obtained.

The curve for ΔA was not usually pushed to the indicated maxima and minima of the residuals, since it was felt that some of these might be due to unlucky combinations of residuals having like signs.

In the subjoined statement appears a digest of the comparison, Lower *minus* Upper Culmination. In the first column, following the argument, number of stars concerned, and the weight, are given the means of "$u-o$" taken from the tables of AUWERS (pp. 24–33, Bd. III), and in the last column those which arise from the present discussion after taking into account $\Delta\delta_s$, as it appears in Table II.

LOWER *minus* UPPER CULMINATION.

δ	**	p	"$u-o$"	L−U
			″	″
+46.5	44	(3.8)	−0.14	−0.64
49.5	54	6.4	−0.05	−0.36
52.5	31	5.4	−0.06	.00
55.5	45	5.2	−0.10	+0.37
58.5	49	5.6	−0.48	−0.17
61.5	37	1.5	−0.19	+0.02
64.5	33	3.9	+0.11	+0.31
67.5	20	2.4	+0.18	+0.54
70.5	19	2.3	−0.23	−0.28
73.5	11	1.6	+0.39	−0.08
76.5	15	1.5	+0.37	−0.39
79.5	12	1.2	+0.11	+0.02
82.5	13	1.1	−0.44	−0.32
85.5	10	1.3	+0.62	+0.95
+88.5	4	0.6	−1.13	−0.40

It will be seen that the amended results indicate an agreement between the declinations from Lower and Upper Culmination, respectively, as good as could have been anticipated, perhaps, when all the difficulties are considered. An intimate study of the details of this comparison confirms me in the opinion that the graduation-error of the quadrant was substantially the same in the two positions; though it is not at all improbable that very small differences might have existed.

Before leaving this part of the subject, certain remarks may not be out of place.

It should be recalled that, in the first use of the quadrant, north, the observed zenith-distances may have been systematically different from those determined after the new balancing of the quadrant, Dec. 2, 1750.

Some part of the differences, $\Delta A_s - \Delta' A_N$, which are expressed in the formula $+0''.12 + 1''.20 \sin\zeta$, may be due to systematic errors in the system, B; but I should be greatly surprised if it should turn out that half of this difference shall hereafter be found chargeable to this source.

There are evidences of outstanding systematic differences between the corrections for zenith-distances, north and south, as reconciled in this discussion, which are rather larger than the weights would lead one to anticipate. Allusion to these has already been made. The difference at large zenith-distances may easily be due to a real difference in the refractions at low altitudes, north and south. Between declinations, 57° s.p. and 73° s.p., also, there is a considerable discordance between observed and computed ΔA_N. This amounts to $+0''.28$ (weight, 9.6), in the sense of a further correction for the declinations corrected according to Table II. But if the curve of ΔA had been deflected at this point to produce a better agreement between computed, ΔA, and observed, ΔA_N, then a larger discrepancy in L−U would have resulted. The mean value of L−U for the zone in question is only $+0''.05$ (weight, 18.7), to be sure, and this might readily admit of some increase; but in the zone, +60° s.p. to 69° s.p., where this increase could most naturally be made, the corresponding value of L−U is already $+0''.24$ (weight, 10.8), and this would be increased by any further reduction of the discordance in question. It thus appears that the standard stars for this zone do not very well represent the generality of BRADLEY's quadrant observations within those limits.

In order to show clearly what outstanding discrepancies exist when the assumption is made, as in this discussion, that the error of graduation remained practically the same for the two positions of the quadrant, we have the following schedule. The residuals are gathered in general means, including both Lower and Upper Culminations. If, now, we suppose that the adopted corrections of Table II have been applied to the catalogue declinations, comparison of

the Standard Catalogue with the observations of quadrant north, alone, indicates that further mean corrections to the catalogue declinations are required as follows:

OUTSTANDING DISCREPANCIES.

δ	p	$\Delta'\delta$
°		″
+51.0	8.5	−0.04
55.7	7.2	−0.16
61.1	8.7	+0.17
67.9	5.2	+0.09
73.1	4.4	+0.01
80.3	2.3	−0.13
+86.6	4.8	−0.13

So far as these quantities are concerned, they do not appear to interpose any very practical objection to the adoption of the corrections contained in Table II.

For quadrant south the adopted mean curve of correction represents the observations substantially as well as would a curve based upon southern observations alone.

This investigation was undertaken more with the object of ascertaining the best practicable systematic corrections for the BRADLEY–AUWERS star positions, rather than for the purpose of deciding upon the peculiarities of BRADLEY's quadrant. Yet it seems very probable that the deductions set forth in the foregoing brief abstract have some resemblance to the actual facts concerning the graduation-error of the BIRD quadrant. The somewhat regular recurrence of large positive corrections at zenith-distances of approximately 18°, 36°, 54° and 67°, wears rather a suspicious aspect, as pointing to some systematic source in the method of marking the graduations (or in fastening the quadrant to the pier). These maximum points seem to be quite consistently indicated in the observations both of quadrant north and quadrant south, as exhibited in Table I. If, after removal of such errors, as collimation, eccentricity, flexure and error in adopted refraction, the quantities, $\Delta\Delta$, are to be considered as mainly due to errors of graduation, it can scarcely be objected to them that they are exceptionally large. It may be doubted whether a single quadrant, under a single microscope, of many of the modern transit-circles would show much more perfect graduation.

Adopting the values of $\Delta\Delta$, as exhibited in the foregoing, the declinations were next tested for systematic errors of the form, $\Delta\delta_\alpha$. For this purpose the declinations, quadrant north, were divided into two series, the first containing all observations at zenith-distances less than 58° .5 (decl 70° s.p.), and the second, all observations from declination, 70° s.p. to 47° s.p. In a similar way the observations, quadrant south, were divided into two series, of which the common boundary is the equator. Table III represents the results, together with the means for quadrant north and south, respectively.

TABLE III. OBSERVED CORRECTIONS FOR $\Delta\delta_\alpha$.

−30° to 0°			0° to +74°			[illegible] +74°		
α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$
h		″			″			″
0.5	2.1	0.53	0.5	5.1	+0.37	0.5	5.2	+0.01
1.4	0.7	+0.14	1.5	4.2	+0.07	1.5	4.9	+0.08
2.4	1.0	0.20	2.6	1.2	0.01	2.5	5.2	0.05
3.6	1.7	−0.38	3.5	5.3	0.28	3.5	7.0	0.50
4.7	1.3	+0.30	4.6	3.8	+0.33	4.6	5.1	+0.32
5.3	2.2	+0.04	5.4	2.1	−0.11	5.3	4.3	0.04
6.6	1.0	−0.09	6.3	3.2	0.06	6.4	4.2	0.07
7.9	0.2	+1.12	7.4	2.5	+0.41	7.4	2.7	+0.18
...	...	....	8.5	2.4	−0.01	8.5	2.4	−0.03
9.6	0.5	+0.94	9.6	3.2	+0.11	9.6	3.7	+0.22
10.7	1.0	+0.55	10.1	2.8	+0.10	10.5	3.8	+0.17
11.5	1.0	+0.57	11.4	4.1	+0.29	11.4	5.4	+0.31
12.6	1.6	−0.39	12.4	2.0	−0.56	12.5	3.6	−0.18
13.4	1.1	0.13	13.5	1.7	0.16	13.5	3.1	−0.31
14.6	2.8	−0.74	14.5	1.8	−0.57	14.6	4.6	0.68
15.5	3.7	−0.21	15.1	3.0	−0.19	15.5	6.7	0.21
16.4	2.3	+0.15	16.5	1.7	−0.05	16.4	4.0	+0.03
17.5	2.4	+0.09	17.5	4.0	0.36	17.5	6.4	0.19
18.6	1.7	+0.08	18.7	2.1	−0.02	18.7	3.8	−0.06
19.5	1.8	+0.60	19.5	4.0	+0.05	19.5	5.8	+0.22
20.4	3.7	+0.67	20.5	3.7	+0.29	20.5	7.4	+0.48
21.5	3.7	−0.12	21.4	2.0	+0.16	21.5	5.7	+0.08
22.5	3.0	−0.14	22.6	2.5	.00	22.5	5.5	+0.06
23.5	3.4	−0.10	23.5	2.2	+0.09	23.5	5.6	0.03

+49° to 70° s.p.			70° s.p. to 46° s.p.			49° to 46° s.p.		
α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$	α	p	$\Delta\delta_\alpha$
h		″	h		″			″
0.3	1.2	−0.12	0.0	0.7	+0.26	0.3	1.9	−0.09
1.9	1.8	+0.39	1.5	0.6	−0.35	1.8	2.4	+0.20
3.6	0.9	−0.56	3.8	1.0	−0.77	3.7	1.9	−0.67
5.9	1.9	+0.01	6.0	2.9	+0.20	6.0	4.8	+0.13
8.1	1.4	+0.69	7.8	2.3	+0.11	7.9	3.7	+0.33
9.8	2.1	+0.51	9.7	1.3	+0.47	9.8	3.4	+0.50
12.2	3.7	+0.09	12.2	0.8	+0.38	12.2	4.5	+0.13
13.9	2.2	+0.51	13.8	1.3	0.38	13.9	3.5	+0.18
16.0	1.3	+0.11	16.2	0.7	0.26	16.0	2.0	+0.03
17.9	2.7	+0.06	.		. .	17.9	2.7	+0.04
19.7	3.1	0.10	20.4	0.5	−0.47	19.8	3.9	−0.14
21.8	3.1	0.69	22.2	1.5	0.71	21.9	4.6	0.70

For the declinations south of the zenith the general agreement of the mean values of $\Delta\delta_\alpha$ in the two divisions seems to warrant the consolidation of them into a single series. For the representation of this series of observed corrections I adopt,

$$+0''.08 \sin\alpha + 0''.07 \cos\alpha - 0''.25 \sin 2\alpha - 0''.05 \cos 2\alpha$$

For the observations, quadrant north, the material is much more scanty than for quadrant south, and there seem to be some anomalies in the observations at great zenith-distances, to which AUWERS has made allusion in the introduction to his Catalogue (pp. 34–35). On this point Dr. AUWERS remarks (p. 35): "Die letzte Columne der obigen Tafel, welche die mittleren δ − δ [illegible] Sterne enthält, würde einen Fehler von [illegible] Periode andeuten, aber die näheren Circumpolarsterne schliessen

eine solche Annahme direct an; die Störung, auf welche noch bei der Ableitung der Polhöhe zurückzukommen sein wird, scheint vielmehr auf einzelne Abschnitte der Beobachtungen bei Quadrant Nord beschränkt gewesen zu sein." This may serve to account for some of the other anomalies already noticed. I have, therefore, considered it advisable to apply a uniform correction, $\Delta\delta_\alpha$, to all observations, quadrant north, derived from the combined result of the two divisions. I have adopted the correction,

$$+0''.08 \sin\alpha - 0''.30 \cos\alpha,$$

employing the argument, true right-ascension, whether the star was observed above, or below the pole. In a somewhat summary analysis of the material I have also found as an expression for this correction, $+0''.08 \sin\alpha - 0''.20 \cos\alpha + (0''.01 \sin\alpha - 0''.12 \cos\alpha) \operatorname{tg}\delta$; but this improves the representation very little, and does not wholly remove the anomalies found in the observations beyond 75° of zenith-distance. In the course of this discussion I compared the declinations observed, quadrant south, with those which were observed, quadrant north, below the pole, from +15° to 51° 23′ of declination. The column next the last in the subjoined table contains the difference, L—U, uncorrected for $\Delta\delta_\alpha$, and the last column contains the same corrected for the combined values of $\Delta\delta_\alpha$, quadrant north and south.

h	p	L—U ″	L—U (corr'd.) ″
0.0	0.6	+1.17	+1.22
2.0	1.2	+0.64	+0.54
3.8	1.9	+0.16	+0.15
5.7	0.7	−0.17	−0.20
6.5	0.5	−2.37	−2.11
9.9	0.2	+0.13	+0.27
12.0	0.2	−0.26	+0.15
13.7	0.3	−0.81	−0.31
17.3	0.9	−1.03	−0.97
20.0	1.8	+0.35	−0.05
21.8	0.7	+1.29	+0.80

This shows the nature of the anomalies, and exhibits the improvement produced by the adoption of the formulas of correction $\Delta\delta_\alpha$. Probably it would be better to assign weight zero for observations having zenith-distance greater than 80°. There appears to be no particular trace of these anomalies for zenith-distances less than 75°.

The corrections contained in Table II, together with the adopted values of $\Delta\delta_\alpha$, are now in use at this observatory, in the computations for extension of the Catalogue of 627 Standard Stars to include a much greater number of what might be termed secondary standard stars.

ELEMENTS AND EPHEMERIS OF COMET *c* 1903 (*BORRELLY*),*

By H. R. MORGAN and ELEANOR A. LAMSON.

[Communicated by Capt. C. M. CHESTER, U.S.N., Superintendent Naval Observatory.]

The following elements were deduced from three normal places derived from observations made at Lick and Washington Observatories, on June 22, 23, 24, 30, July 1, 2, 7, 8 and 9:

ELEMENTS.

T = 1903 August 27.60110 Gr. M.T.

$$\pi = 60°\ 52'\ 31''$$
$$\Omega = 293\ 32\ 53$$
$$i = 81\ 59\ 50$$

$$q = 0.32966$$

Residuals (O—C): $\cos\beta\,\Delta\lambda = +3''.7$, $\Delta\beta = +2''.1$

HELIOCENTRIC COORDINATES.

$$x = r[9.610061] \sin(206°\ 0'\ 16'' + v)$$
$$y = r[9.962176] \sin(13\ 53\ 11 + v)$$
$$z = r[9.998661] \sin(105\ 51\ 40 + v)$$

EPHEMERIS.

1903 G.M.T.	α h m s	δ ° ′	log Δ	Light
July 28.5	12 17 36	+57 11.2	9.6146	8.8
Aug. 1.5	11 43 42	51 18.5	9.7264	7.4
5.5	11 22 55	46 10.1	9.7991	6.7
9.5	11 7 32	42 50.7	9.8635	6.4
13.5	10 51 25	39 21.5	9.9208	6.7
17.5	10 42 3	35 58.0	9.9722	7.3
21.5	10 29 56	32 6.8	0.0179	8.2
25.5	10 18 31	27 27.8	0.0570	8.6
29.5	10 9 18	21 54.3	0.0878	7.4
Sept. 2.5	10 3 34	15 49.4	0.1098	5.4
6.5	10 1 8	9 42.9	0.1254	3.7
10.5	10 1 3	+ 3 51.1	0.1372	2.5
14.5	10 2 28	− 1 12.3	0.1471	1.8
18.5	10 4 52	6 58.0	0.1562	1.3
22.5	10 7 54	11 57.7	0.1648	1.0
26.5	10 11 20	16 12.9	0.1735	0.8
30.5	10 15 2	−21 15.1	0.1821	0.6

Brightness on June 22 is taken as the unit.

* *From Supplement to No. 544.*

DEFINITIVE ORBIT OF COMET 1891 IV.

By HENRY A. PECK.

This comet was discovered by BARNARD October 2, 1891. It is described as moderately bright, about one minute in diameter, with scarcely any nucleus. It was far south, and was always observed with difficulty by its discoverer, disappearing from his sight in a week. It was observed at no other northern observatory, but was followed, however, for nearly two months at Cordoba. These two series of observations, together with a short one from Sydney, are all that have appeared. In *A.N.* 3237 HIND has published an orbit based upon the Cordoba observations of October 19, November 12 and December 3. As will be seen from the following, these elements satisfy the southern observations with a fair degree of approximation, but leave much to be desired for those of BARNARD.

The HIND elements are as follows:

$$T = 1891 \text{ Nov. } 13.54555 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 269^\circ\ 34'\ 59''.5 \\ \Omega &= 218\ \ 0\ 13.4 \\ i &= 77\ 59\ 54.7 \end{aligned}\right\} 1891.0$$

$$\log q = 9.9872737$$

$$\begin{aligned} x &= [9.9021683]\, r \sin [188^\circ\ 48'\ 43''.03 + v] \\ y &= [9.8927485]\, r \sin [135\ 53\ 30.45 + v] \\ z &= [9.9382463]\, r \sin [253\ 10\ 25.45 + v] \end{aligned}$$

The ephemeris positions are

		α apparent	δ apparent	ab. t	log Δ
		h m s	° ′ ″		
Oct.	1	7 20 9.91	−24 53 51.8	0.00554	9.982
	2	25 36.67	26 21 33.9	551	980
	3	31 12.81	27 49 17.8	548	978
	4	36 58.63	29 16 49.3	546	977
	5	42 54.40	30 43 54.0	545	976
	6	49 0.36	32 10 16.9	544	975
	7	55 16.77	33 35 42.9	544	975
	8	8 1 43.79	34 59 56.8	544	975
	9	8 21.63	36 22 43.9	545	976
	10	15 10.38	37 43 49.8	547	977
	11	22 10.12	39 3 0.4	548	978
	12	29 20.86	40 20 2.8	550	980
	13	36 42.56	41 34 44.6	553	982
	14	44 15.06	42 46 54.6	556	985
	15	51 58.15	43 56 21.8	560	987
	16	59 51.51	45 2 57.5	564	990
	17	9 7 54.75	46 6 33.2	568	993
	18	16 7.33	47 7 2.2	573	997
	19	24 28.67	48 4 18.6	578	0.001
	20	32 58.00	58 18.5	583	005
	21	41 34.54	49 48 58.4	589	009
	22	50 17.35	50 36 16.7	595	014
	23	59 5.42	51 20 12.6	601	018
	24	10 7 57.66	52 0 46.3	608	023
	25	16 52.92	37 59.4	615	028
	26	25 49.99	53 11 54.7	622	033
	27	34 47.63	42 35.7	629	038
	28	43 44.58	54 10 6.7	637	043
	29	10 52 39.63	−54 34 33.1	0.00645	0.048
Oct.	30	11 1 31.55	−54 56 0.7	0.00653	0.054
	31	10 19.17	55 14 35.9	661	059
Nov.	1	19 1.40	30 25.7	669	065
	2	27 37.16	43 37.1	677	070
	3	36 5.50	54 18.0	686	075
	4	44 25.59	56 2 35.9	694	080
	5	52 36.63	8 38.8	703	086
	6	12 0 37.99	12 34.4	711	091
	7	8 29.10	14 30.4	720	096
	8	16 9.55	34.7	728	101
	9	23 38.97	12 54.3	737	107
	10	30 57.12	9 36.9	745	112
	11	38 3.83	4 49.5	754	117
	12	44 59.01	55 58 38.8	763	122
	13	51 42.67	51 11.1	772	127
	14	58 14.84	42 32.7	780	132
	15	13 4 35.65	32 48.8	789	136
	16	10 45.23	22 5.7	797	141
	17	16 43.79	10 27.8	806	146
	18	22 31.55	54 58 0.4	814	151
	19	28 8.78	44 47.7	823	155
	20	33 35.74	30 54.0	831	159
	21	38 52.72	16 23.0	839	163
	22	44 0.00	1 18.5	847	167
	23	48 57.91	53 45 43.4	855	171
	24	53 46.76	29 41.3	863	175
	25	58 26.87	13 14.6	871	179
	26	14 2 58.55	52 56 26.2	879	183
	27	7 22.09	39 18.3	886	187
	28	11 37.81	21 53.1	893	191
	29	15 45.99	4 12.5	900	194
	30	19 46.91	51 46 18.4	907	197
Dec.	1	23 40.83	28 12.3	914	200
	2	27 28.04	9 55.8	921	203
	3	31 8.78	50 51 30.2	928	207
	4	34 43.29	32 56.8	934	210
	5	38 11.84	14 16.7	940	213
	6	41 34.62	49 55 31.2	947	215
	7	44 51.88	49 36 40.7	0.00953	0.218

The following star-positions referred to the mean equinox and ecliptic of 1891.0 have been used:

No.	α	δ	Authority
	h m s	° ′ ″	
1	7 30 55.65	−27 52 29.4	Wash. M. C. Z. S. L. N. 103
2	7 36 56.55	29 17 43.9	Arg. General Cat.
3	7 44 18.72	30 29 4.8	Arg. Gen. Catal. [illegible]
4	7 50 15.24	32 12 16.0	[illegible]
5	7 54 55.50	33 36 56.5	[illegible]
6	8 5 2.95	35 8 9.0	[illegible]
7	8 8 24.47	36 39 44.4	[illegible]
8	8 8 31.08	36 20 24.5	[illegible]
9	8 10 0.19	36 37 16.9	Arg. General Cat.
10	8 15 7.56	37 50 8.6	[illegible]
11	8 17 27.55	39 16 25.5	[illegible]
12	8 21 30.87	39 31 6.7	Arg. General Cat.
13	9 32 55.48	48 51 59.9	[illegible]
14	9 38 55.67	49 38 37.5	Arg. Zone Cat.
15	9 40 55.25	49 30 29.3	Arg. General Catal.
16	9 49 50.45	50 37 55.7	A. G. C., Stone, Cape 10.750

No.	α h m s	δ ° ′ ″	Authority
17	9 55 49.11	51 19 37.3	Arg. General Catal.
18	10 1 12.46	51 52 32.4	Arg. General Catal.
19	10 3 6.12	52 0 11.4	Arg. G.C.; Stone; Cape '50
20	10 32 21.70	53 10 22.4	Arg. General Catal.
21	10 47 26.95	54 33 35.5	Arg. G.C.; Stone; Cape '50
22	12 25 20.41	56 22 22.0	3 comp. with γ *Crucis*
23	12 31 9.12	56 11 15.7	Arg. General Catal.
24	12 40 7.61	55 53 31.6	A.G.C.; Stone; Cape '40, '50
25	12 50 57.99	55 12 59.5	A.G.C.; Madras Gen. C.
26	13 12 11.67	55 24 11.8	4 comp. with No. 28
27	13 13 29.41	55 11 29.4	Arg. General Catal.
28	13 14 3.27	55 13 11.2	Arg. General Catal.; Stone
29	13 17 2.95	54 19 1.0	Arg. General Catal.
30	13 55 20.15	53 27 58.7	Arg. Zone Catal.
31	11 2 39.92	52 55 6.9	Arg. General Catal.; Stone
32	11 20 5.28	52 1 57.8	Arg. General Catal.
33	14 25 2.57	51 29 12.5	Arg. General Catal.; Stone
34	14 36 30.13	50 12 19.6	Arg. Zone Catal.
35	14 37 0.58	50 13 37.3	Arg. General Catal.
36	14 43 25.23	49 38 27.2	Arg. Zone Catal.

The following new proper motions have been used:

No.	$\mu\alpha$	$\mu\delta$
21	$-0^s.005$	$0''.000$
24	$+0.002$	-0.014

In comparing the observations with the ephemeris, the time of observation has been corrected for aberration, and then reduced to the meridian of Greenwich.

Date	Place	α apparent	π	O−C Δα cos δ	δ apparent	π	O−C Δδ	∗
		h m s		s	° ′ ″	″	″	
Oct. 3.03695	Mt. Hamilton	7 31 25.20	−0.26	− 6.3	−27 52 18.5	+8.0	+21.7	1
4.00155	"	37 0.36	.36	3.0	29 17 3.5	7.7	17.3	2
5.00022	"	12 54.02	.38	10.8	30 43 52.3	7.7	10.1	3
5.99582	"	18 58.91	.41	5.7	32 15 28.3	7.7	(−5′25″.3)	4
7.00977	"	55 20.62	.42	− 3.7	33 36 16.0	7.9	− 5.5	5
8.01554	"	. . . . .	. .	. .	35 1 12.7	8.2	+10.0	6
8.03347	"	8 1 57.58	.34	+ 3.7	. . . . .	.	. .	6
9.00940	"	8 26.27	.39	+ 5.6	36 23 34.6	+7.9	+ 3.4	8
9.15258	Sydney	9 23.28	.63	− 7.1	34 52.9	−3.1	17.1	7
9.25637	"	10 0.09	.45	− 0.7	42 18.3	−0.8	15.0	9
10.01027	Mt. Hamilton	15 15.76	.41	+ 8.8	37 44 12.7	+7.9	4.4	10
11.15711	Sydney	23 17.39	.65	− 3.7	39 14 56.5	−3.2	15.8	11
11.23089	"	18.70	.51	− 4.8	20 49.1	0.8	9.6	12
Oct. 19.76897	Cordoba	9 31 0.51	.74	+ 1.6	48 46 2.6	2.0	+ 3.0	13
19.78790	"	9.51	.72	− 5.4	47 10.6	1.3	− 4.1	13
20.79807	"	39 49.73	.72	− 7.2	49 38 56.6	1.0	+ 3.1	15
20.82112	"	40 2.20	.68	− 1.2	40 3.7	−0.3	+ 5.5	14
20.83521	"	9.03	.64	− 5.5	46.2	+0.2	+ 6.4	14
21.81367	"	48 11.18	.70	+ 9.5	50 27 42.0	−0.5	+ 0.7	16
22.84073	"	57 11.21	.65	− 4.2	51 13 37.3	+0.3	−10.8	17
23.81578	"	10 6 20.11	.72	+ 1.3	53 32.5	−0.5	0.0	18
23.83821	"	32.72	.67	+ 6.9	54 24.7	+0.2	1.7	19
26.81974	"	33 11.00	.73	− 3.9	53 37 17.9	−0.6	− 0.1	20
28.82513	"	51 7.12	.72	+ 1.3	54 30 28.9	−0.5	+ 0.4	21
Nov. 8.82389	Cordoba	12 22 20.91	.70	− 6.8	56 13 10.5	−1.3	+ 7.6	22
9.82341	"	29 11.73	.70	+ 3.7	10 21.8	1.4	− 4.6	23
10.82869	"	36 52.35	.69	+ 1.8	5 49.5	1.3	6.4	24
11.81539	"	43 44.08	.68	+ 1.5	55 59 51.7	1.7	0.2	24
12.80694	"	50 25.90	.67	− 3.4	52 46.2	2.0	5.0	25
15.80859	"	13 9 34.66	.64	−11.3	24 13.7	2.1	2.6	28
15.82961	"	43.96	.64	+ 2.3	23 1.8	1.5	1.0	26
16.80983	"	15 36.98	.63	− 0.8	12 14.2	2.1	1.7	27
17.81480	"	21 28.07	.62	− 4.2	0 20.3	2.0	− 0.1	29
Nov. 23.81328	Cordoba	52 54.34	.57	+ 2.1	53 32 34.4	2.1	+ 6.4	30
25.81480	"	11 2 9.58	.55	+ 1.6	52 59 29.2	2.0	+ 3.4	31
28.80785	"	14 59.46	.52	+ 0.8	7 31.5	2.2	+ 3.7	32
30.80338	"	22 55.84	.51	− 0.2	51 31 45.8	2.2	− 1.3	33
Dec. 3.80628	"	34 2.75	.49	+ 0.6	50 36 30.6	2.1	+ 0.3	35
4.80494	"	37 31.63	.48	− 4.6	17 57.2	2.1	− 3.7	34
6.79378	"	41 11.66	.46	− 2.7	49 40 34.7	2.3	− 0.4	36
6.81202	"	15.46	−0.47	− 0.4	32.7	−1.9	(+1′40″.9)	36

The observations have in general received the weight unity. Those in ( ) have been rejected entirely, and three others have had their weight reduced. The hour-angle of the second Sydney observation on Oct. 9 has been corrected by subtracting twenty minutes from the time of observation. For the first observation on Dec. 6 the difference ∗ − [illegible] in declination has been made to read −2′ 4″.6, instead of −24″.56, as printed.

Combining in the usual manner, the following normal places result, referred to the mean obliquity and equinox of 1891.0.

	α	δ	O—C $\Delta\alpha\cos\delta$	O—C $\Delta\delta$
	h m s	° ′ ″	″	″
Oct. 8.0	2 1 42.67	−34 59 45.79	−2.33	+10.81
23.0	9 59 4.61	51 20 5.64	0.62	+ 0.74
Nov.13.0	12 51 41.03	55 51 0.89	1.35	− 1.89
Dec. 1.0	14 23 38.60	51 27 58.20	0.31	+ 1.20

By a very rough preliminary computation it was ascertained that the large residual in declination for Oct. 8 could be made to practically vanish by slight changes in the elements most intimately connected with the perihelion passage. The further work is therefore based upon the elements

$$T = \text{Nov. } 13.54150 \text{ G.M.T.}$$
$$\omega = 269^\circ\ 34'\ 31''.6$$
$$\log q = 9.9872612$$

and these corrected elements leave the residuals

$\Delta\alpha\cos\delta$	$\Delta\delta$
−0.12	−0.28
−0.56	−1.68
−2.05	1.09
+0.09	+3.61

Using SCHÖNFELD's notation the equations of condition are

$$\begin{array}{rrrrrcrl}
+650\,\partial\kappa & -803\,\kappa\sqrt{2}\,\partial T & +\ 65\,\partial q & +169\,\partial\lambda & -182\,\partial r & = & -\ 12 & +227\,\partial\epsilon \quad 3 \\
559 & 701 & -247 & -204 & +118 & = & 56 & +111 \quad 3 \\
263 & 278 & 414 & 539 & +\ 7 & = & -205 & +\ 1 \quad 2 \\
+132 & -\ 63 & 351 & -501 & -230 & = & +\ 9 & +\ 1 \quad 2 \\
-549 & +377 & 605 & +741 & 800 & = & -\ 28 & -\ 53 \quad 3 \\
+\ 23 & -\ 87 & 207 & 832 & 479 & = & -168 & +\ 20 \quad 2.5 \\
430 & 152 & 347 & 471 & -\ 7 & = & -109 & +\ 2 \quad 2 \\
535 & 459 & 549 & 236 & +108 & = & +361 & -\ 10 \quad 2
\end{array}$$

(Wt. in last column.)

the units being the third decimal place for the coefficients and the hundredths of a second for the absolute terms.

From these equations of condition the following normal equations are deduced:

$$\begin{array}{rrrrrcrl}
+4221\,\partial\kappa & -4406\,\kappa\sqrt{2}\,\partial T & -\ 500\,\partial q & -\ 944\,\partial\lambda & +1188\,\partial r & = & +106 & +667\,\partial\epsilon \\
-4406 & +4826 & +\ 816 & +\ 400 & -\ 682 & = & +\ 33 & -\ 811 \\
-\ 500 & +\ 816 & +2830 & -1379 & +1619 & = & +\ 21 & +\ 92 \\
-\ 944 & +\ 400 & -1379 & +5230 & -2676 & = & -105 & \ \ 19 \\
+1188 & -\ 682 & +1619 & -2676 & +2764 & = & +340 & +\ 12
\end{array}$$

The elimination equations are

$$\begin{array}{rrrrrcrl}
\partial\kappa & -1044\,\kappa\sqrt{2}\,\partial T & -\ 118\,\partial q & -\ 221\,\partial\lambda & +\ 281\,\partial r & = & +\ 25 & +160\,\partial\epsilon \\
& \kappa\sqrt{2}\,\partial T & +1258\,\partial q & -2187\,\partial\lambda & +2360\,\partial r & = & +606 & -149\,\partial\epsilon \\
& & \partial q & -\ 315\,\partial\lambda & +\ 446\,\partial r & = & -\ 61 & +127\,\partial\epsilon \\
& & & \partial\lambda & -\ 207\,\partial r & = & +\ 69 & -\ 20\,\partial\epsilon \\
& & & & \partial r & = & +169 & -155\,\partial\epsilon
\end{array}$$

The most probable values of the unknown quantities are

$$\begin{aligned}
\partial\kappa &= +6''.09 & -0.239\,\partial\epsilon \\
\kappa\sqrt{2}\,\partial T &= +5.94 & -0.431 \\
\partial q &= -1.03 & +0.180 \\
\partial\lambda &= +1.01 & -0.052 \\
\partial r &= +1.69 & -0.155
\end{aligned}$$

and the corrections to the elements are

$$\begin{aligned}
\partial T &= +0.00118 & -[5.9310]\,\partial\epsilon \\
\partial\omega &= +6''.45 & -\ 0.271\,\partial\epsilon \\
\partial\Omega &= -1.71 & +\ 0.158\,\partial\epsilon \\
\partial i &= -1.05 & +\ 0.053\,\partial\epsilon \\
\partial q &= -0.0000050 & +[3.9109]\,\partial\epsilon
\end{aligned}$$

where $\partial\epsilon$ is understood as expressed in seconds of arc. If a parabolic form is assumed for the orbit, the resulting elements and their probable errors are

$$T = \text{Nov. } 13.54268 \pm 0.00179 \text{ G.M.T.}$$

$$\left.\begin{aligned}
\omega &= 269^\circ\ 31'\ 38''.0 & \pm 7''.0 \\
i &= 77\ 59\ 53.6 & \pm 1.4 \\
\Omega &= 218\ \ 0\ 11.7 & \pm 3.2
\end{aligned}\right\}\ 1891.0$$

$$\log q = 9.9872590 \pm 42 \text{ units of 7th place}$$

$$\begin{aligned}
x &= [9.9021712]\,r \sin(188^\circ\ 18'\ 21''.80 + v) \\
y &= [9.8927475]\,r \sin(135\ 53\ \ 7.40 + v) \\
z &= [9.9382445]\,r \sin(253\ 10\ \ 4\ 3.3 + v)
\end{aligned}$$

A comparison of the residuals obtained by computing an ephemeris for the dates of the normal places, with those that result from direct substitution in the equations of condition is given as proof of the numerical accuracy of the solution.

$\Delta\alpha\cos\delta$ Eq. of Cond.	$\Delta\alpha\cos\delta$ Elements.	$\Delta\delta$ Eq. of Cond.	$\Delta\delta$ Elements.
$''$	$''$	$''$	$''$
+0.89	+0.86	+0.78	+0.86
−0.04	−0.10	−1.57	−1.57
−1.88	1.85	−1.87	−1.88
+0.21	+0.19	+2.08	+2.06

Several characteristics of these residuals suggest that the parabola may not be the true form of the orbit. Certainly the one represented by these elements is little, if any, better than that produced by the supposition that the corrections to the longitude of the node and the inclination are zero. It will be noticed that $\Sigma(pvv)$ has only dropped from 45″.1 to 33″.1 and that the probable errors of the elements in all cases except that of $\log q$ exceed the corrections themselves. A direct solution of the equations, retaining the eccentricity as an unknown quantity indicates a tendency toward an ellipse. The coefficient of δe in the last equation becomes so small, however, that its theoretical weight is practically at the vanishing point.

If the results given above are substituted in the weighted equations, the sum of the squares of the residuals becomes a minimum for $\delta e = -139''.5$ and $\Sigma(pvv)$ becomes only 9″.7, the individual residuals being

$\Delta\alpha\cos\delta$	$\Delta\delta$
$''$	$''$
−0.16	−0.58
−0.74	+0.73
+0.73	+1.18
+0.79	+0.45

The range of uncertainty is quite large so that this value of δe can not by any possibility be considered as fixed within a limit of ±10″. The ellipse above indicated has an eccentricity of 0.999324 and the other elements are

$$T = \text{Nov. } 13.5517 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 269^\circ\ 35'\ 16'' \\ i &= \ \ 77\ \ 59\ \ 46 \\ \Omega &= 217\ \ 59\ \ 50 \end{aligned}\right\}\ 1891.0$$

$$\log q = 9.987201$$

Syracuse University, June 19, 1903.

ON THE APPARENT ELLIPTICITY OF *MARS*,

By E. E. BARNARD.

At the opposition of *Mars* in 1894 the disc of the planet appeared decidedly elliptical with the 36-inch of the Lick Observatory. On two dates I made settings for the position angle of the apparent equation.

	d h m		°
1894 Oct.	15 14 30	P.A. =	62.6
Nov.	4 11 50		43.8
			53.2

These are discordant. It is, however, difficult to measure the position angle of an ellipticity of this kind. I was not looking for ellipticity of the disc, and was surprised when it was noticed. Thinking there might be some deception in the matter, I have never referred to the observations.

At the opposition of this spring. I was again struck with this peculiarity, and on four dates made settings for the position of the apparent equator.

	d h m		°
1903 Mar.	25 10 0	P.A. =	122.9
	30 11 0		133.2
Apr.	1 10 0		121.0
	6 9 30		122.8
			125.0

From the ephemerides of *Mars* printed in the *Monthly Notices, R.A.S.*, for April 1894, and June 1902, by Mr. MARTH and Mr. CROMMELIN, respectively, I find the position of the Martian equator for the approximate dates of observation, and residuals C−O.

	Comp.	C−O
	°	°
In 1894	54.5	$+1\frac{1}{2}$
In 1903	120.0	−5

The accordances with the known position of the equator leads me to think the apparent ellipticity may be real. If it is real, the ellipticity of the planet must be decidedly greater than the theoretical value. It is a well known fact that observations differ greatly in respect to the polar compression of *Mars*. Some of the larger values obtained would make it readily apparent to the eye.

In the observations the eyes were placed in different positions with respect to the direction of the ellipticity, which remained unchanged and decided in character.

Yerkes Observatory, Williams Bay, Wis., 1903 April 25.

SUSPECTED VARIABLE NEAR *R CYGNI*,

By ORMOND STONE.

July 15, 1903, a star of 10^M.8, 14^s preceding and 0′.8 north of 7045 *R Cygni*, was seen by Mr. CHAS. P. OLIVIER and Mr. G. F. PADDOCK with the 26-inch equatorial of this Observatory. No star is shown in this place on HAGEN'S chart. July 18 it was estimated at 14^M.5, and July 19 at 14^M.7.

CONTENTS

PUBLISHED AT 5 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed August 14.*
Entered February 2, 1903 at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 546.

VOL. XXIII. BOSTON, 1903 SEPTEMBER 7. NO. 18

DOUBLE-STAR MEASURES,

BY JOHN A. MILLER AND W. A. COGSHALL.

The stars in the following list are those which were noted as double by the Berlin observers while making the observations for the Catalog of the *Astronomische Gesellschaft* (Zone 20°–25°), no measures of which have hitherto been published. The list was prepared by Professor S. W. BURNHAM. The measures were made with the 12-inch refractor of Kirkwood Observatory, to which is attached a micrometer by Warner and Swasey.

The position angle for each night is the mean of four settings, and the distance the mean of three settings.

The stars which have been noted as double by the Berlin observers, but which seem to us single, have been examined on two or more nights, and pronounced single after having been observed at least once when seeing was steady. The magnitude given is the average of estimates made at the time the stars were measured. The letter C. follows the measures made by Mr COGSHALL, and the letter M. those made by myself. The positions given are for 1900.

JOHN A. MILLER.

DM. 20°18. A.G. 56. $8^M.9$: $9^M.1$.

$\alpha = 0^h\ 11^m\ 49^s.99$: $\delta = +21°\ 13'\ 11''.9$

t	θ_0	ρ_0	
1902.793	135.4	1.73	
1902.804	133.8	1.63	
1903.030	133.7	1.92	
1902.876	134.3	1.79	M.

DM. 23°135. A.G. 305. $9^M.0$: $10^M.0$.

$\alpha = 0^h\ 53^m\ 21^s.73$: $\delta = +24°\ 8'\ 36''.6$

1902.785	109.4	3.92	
1902.804	112.1	3.80	
.845	112.5	3.91	
1902.811	111.3	3.88	C.

DM. 25°139. A.G. 314.

$\alpha = 0^h\ 56^m\ 27^s.77$: $\delta = +23°\ 15'\ 21''.7$

1902.785	211.5	1.61	
.804	213.3	1.18	
.826	242.3	1.66	
1902.805	213.4	1.48	M.

DM. 20°154. A.G. 330. $9^M.8$: $9^M.9$.

$\alpha = 1^h\ 0^m\ 11^s.43$; $\delta = +20°\ 31'\ 8''.9$

1902.785	201.9	0.83	
.804	206.2	0.85	
1902.795	201.1	0.84	C.

DM. 20°410. A.G. 757. $9^M.4$.

$\alpha = 2^h\ 25^m\ 17^s.44$: $\delta = +21°\ 3'\ 40''.5$

1902.017	217.9	1.41	
.115	246.9	1.50	
.131	246.9	1.65	
1902.088	217.2	1.52	C.

DM. 20°507. A.G. 907.

$\alpha = 3^h\ 0^m\ 8^s.72$: $\delta = +20°\ 28'\ 34''.2$

t	θ_0	ρ_0	
1902.801	20.2	0.80	
.826	22.7	0.89	
1903.227	31.1	0.93	
1902.922	24.7	0.87	M.

DM. 21°412. A.G. 989. 9^M : 11^M.

$\alpha = 3^h\ 14^m\ 59^s.59$: $\delta = +21°\ 17'\ 42''.4$

1902.131	287.9	3.91	
.826	284.0	4.00	
1903.162	283.4	3.54	
1902.706	285.3	3.82	M.

DM. 22°620. A.G. 1298. 9^M : 10^M.

$\alpha = 3^h\ 56^m\ 7^s.79$: $\delta = +23°\ 3'\ 57''.4$

1902.017	171.2	1.69	
.151	170.7	1.60	
1903.068	170.4	1.35	
1902.412	171.7	1.55	M.

DM. 24°772. A.G. 1662.

$\alpha = 5^h\ 4^m\ 36^s.57$: $\delta = +25°\ 1'\ 18''.5$

1902.151 Not double.

Marked "obl." in one zone in A.G. Catal.

DM. 22°978. A.G. 1856. 8^M : $9^M.5$.

$\alpha = 5^h\ 31^m\ 0^s.32$. $\delta = +22°\ 28'\ 46''.3$

1902.017	112.8	7.10	
1903.022	111.0	7.07	
1903.068	112.5	6.93	
1902.702	113.1	7.03	C.

DM. 21°1008. A.G. 1992.

$\alpha = 5^h\ 43^m\ 37^s.35$: $\delta = +21°\ 47'\ 47''.9$

1902.195 Not double.

Marked "dupl. seq. maj." in A.G. Catal.

DM. 20°1216. A.G. 2113. $8^M.8$.

$\alpha = 5^h\ 55^m\ 6^s.40$. $\delta = +20°\ 13'\ 48''.5$

t	θ_0	ρ_0	
1902.115	100.4	11.08	
.131	103.4	11.55	
.151	96.2	11.91	
1903.023	100.7	11.14	
1902.355	100.2	11.37	C.

DM. 20°1259. A.G. 2175. $8^M.7$: 10^M.

$\alpha = 6^h\ 0^m\ 46^s.04$: $\delta = +20°\ 6'\ 50''.9$

1902.195	198.5	1.50	
.214	199.4	1.53	
1902.205	199.0	1.51	M.

DM. 24°1161. A.G. 2242. 9^M : $9^M.2$

$\alpha = 6^h\ 6^m\ 41^s.64$: $\delta = +24°\ 27'\ 2''.7$

1902.151	181.2	1.85	
.195	179.4	2.07	
1903.068	182.5	1.63	
1902.471	181.0	1.85	M.

DM. 22°1280. A.G. 2308. $8^M.7$

$\alpha = 6^h\ 13^m\ 43^s.79$: $\delta = +22°\ 9'\ 5''.5$

1902.151	48.8	1.73	
.195	50.2	1.51	
.208	49.4	1.66	
1902.185	49.4	1.63	C.

DM. 24°1270. A.G. 2392. 9ᴹ ; 9ᴹ.1.

$\alpha = 6^h\,21^m\,21^s.90 \quad : \quad \delta = +24°\,35'\,32''.0$

t	θ_0	ρ	
1902.151	209.5	2.81	
.208	206.8	2.52	
1903.151	210.0	2.18	
1902.503	208.8	2.50	M.

DM. 23°1480. A.G. 2559. 8ᴹ.5 ; 9ᴹ.

$\alpha = 6^h\,38^m\,33^s.02 \quad : \quad \delta = +23°\,32'\,51''.1$

1902.151	76.4	1.45	
.195	77.7	1.46	
.208	76.2	1.60	
1902.188	76.8	1.50	C.

DM. 21°1445. A.G. 2696.

$\alpha = 6^h\,51^m\,46^s.95 \quad : \quad \delta = +21°\,8'\,57''.1$

1902.195 Not double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 24°1508. A.G. 2739. 9ᴹ ; 9ᴹ.2.

$\alpha = 6^h\,56^m\,52^s.57 \quad : \quad \delta = +24°\,36'\,5''.6$

1902.208	23.3	1.64	
.244	20.6	1.28	
1903.151	21.5	1.29	
.162	19.9	1.69	
1902.681	21.3	1.48	M.

DM. 22°1655. A.G. 2911.

$\alpha = 7^h\,16^m\,12^s.30 \quad : \quad \delta = +22°\,50'\,5''.6$

1902.195 Not double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 22°1797. A.G. 3152. 9ᴹ ; 10ᴹ.5.

$\alpha = 7^h\,45^m\,57^s.99 \quad : \quad \delta = +22°\,30'\,44''.6$

1902.195	329.8	10.92	
.208	329.5	11.36	
.211	331.6	11.01	
1902.206	330.3	11.10	C.

DM. 22°1678. A.G. 2941. 8ᴹ.7 ; 10ᴹ.3.

$\alpha = 7^h\,20^m\,0^s.95 \quad : \quad \delta = +22°\,17'\,8''.1$

1902.211	176.1	1.55	
.271	175.6	1.59	
1903.022	174.1	1.58	
1902.502	175.3	1.57	C.

DM. 20°2095. A.G. 3396.

$\alpha = 8^h\,23^m\,40^s.46 \quad : \quad \delta = +20°\,45'\,41''.1$

1903.208 Not double.

Marked "dupl.?" in one zone in A.G. Cata..

DM. 23°1978. A.G. 3449. 9ᴹ.2 ; 10ᴹ.2.

$\alpha = 8^h\,31^m\,4^s.14 \quad : \quad \delta = +23°\,35'\,49''.3$

1902.291	7.9	1.61	
1903.151	5.8	1.52	
1903.162	7.5	1.65	
1902.868	7.1	1.59	M.

DM. 25°1997. A.G. 3558.

$\alpha = 8^h\,42^m\,21^s.26 \quad : \quad \delta = +24°\,57'\,10''.5$

1902.291 Not double. W.A.C.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 23°2001. A.G. 3559. 9ᴹ ; 9ᴹ.1.

$\alpha = 8^h\,44^m\,37^s.28 \quad : \quad \delta = +23°\,30'\,8''.7$

t	θ_0	ρ_0	
1903.153	71.4	1.71	
.208	73.7	1.61	
.227	75.7	1.78	
1903.196	73.6	1.70	M.

DM. 24°2053. A.G. 3688. 9ᴹ ; 9ᴹ.5.

$\alpha = 9^h\,7^m\,47^s.23 \quad : \quad \delta = +24°\,28'\,10''.5$

1902.227	318.5	4.91	
.271	316.7	4.66	
.307	318.8	4.18	
1902.268	318.0	4.59	M.

DM. 24°2089. A.G. 3766.

$\alpha = 9^h\,22^m\,59^s.25 \quad : \quad \delta = +24°\,14'\,25''.7$

1902.271. Not double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 21°2128. A.G. 3876.

$\alpha = 9^h\,51^m\,22^s.52 \quad : \quad \delta = +21°\,15'\,12''.6$

1902.304 Uncertain.

.373 Probably elongated.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 23°2288. A.G. 4150.

$\alpha = 10^h\,54^m\,27^s.48 \quad : \quad \delta = +23°\,45'\,2''.2$

1902.271 Not double.

Marked "dupl., med.," in one zone, and single in three zones in A.G. Catal.

DM. 22°2387. A.G. 4323.

$\alpha = 11^h\,34^m\,19^s.28 \quad : \quad \delta = +21°\,52'\,0''.9$

1902.271 Not double.

Marked "comp. 9ᴹ.5, 1–2 sec.?" in one zone in A.G. Catal.

DM. 23°2471. A.G. 4544. 8ᴹ.8 ; 9ᴹ.4.

$\alpha = 12^h\,27^m\,45^s.41 \quad : \quad \delta = +23°\,33'\,47''.3$

1902.271	312.2	0.88	
.307	320.4	0.76	
1903.206	311.8	0.99	
1902.595	314.8	0.88	M.

DM. 21°2434. A.G. 4563.

$\alpha = 12^h\,31^m\,49^s.01 \quad : \quad \delta = +20°\,47'\,14''.8$

1902.373 Probably double.

Marked "dupl.?" in one zone in A.G. Catal.

DM. 23°2528. A.G. 4672.

$\alpha = 12^h\,57^m\,42^s.56 \quad : \quad \delta = +23°\,29'\,4''.2$

Single. Marked "dupl. 2-3 sec. comp. less than 9ᴹ" in A.G. Catal.

DM. 23°2530. A.G. 4674.

$\alpha = 12^h\,58^m\,7^s.23 \quad : \quad \delta = +23°\,10'\,32''.4$

Single. Marked "dupl. pr." in A.G. Catal.

DM. 24°2532. A.G. 4681. 9ᴹ ; 9ᴹ.5.

$\alpha = 12^h\,58^m\,48^s.54 \quad : \quad \delta = +24°\,10'\,50''.3$

t	θ	ρ_0	
1902.307	127.8	3.17	
.373	. .	2.04	
.411	129.5	2.25	
1903.206	127.7	2.51	
.266	128.4	2.59	
1902.713	128.3	2.52	C.

DM. 24°2542. A.G. 4702.

$\alpha = 13^h\,3^m\,11^s.85 \quad : \quad \delta = +23°\,56'\,48''.7$

Not double. Marked "dupl. maj.?" in two zones in A.G. Catal.

DM. 21°2531. A.G. 4789. 9ᴹ.1 ; 9ᴹ.5.

$\alpha = 13^h\,22^m\,21^s.71 \quad : \quad \delta = +20°\,58'\,52''.6$

1902.307	305.3	1.16	
.441	305.7	1.31	
.504	300.2	1.16	
1903.206	305.2	1.50	
1902.615	304.1	1.28	M.

DM. 24°2588. A.G. 4798. 8ᴹ.8 ; 12ᴹ.

$\alpha = 13^h\,23^m\,55^s.70 \quad : \quad \delta = +24°\,5'\,55''.3$

1902.307	245.4	3.69	
.444	248.3	2.51	
.515	248.7	2.43	
1902.422	247.5	2.88	C.

DM. 23°2682. A.G. 5046.

$\alpha = 14^h\,15^m\,33^s.21 \quad : \quad \delta = +23°\,30'\,49''.5.$

1902.504 Double, but too close to meas.!

Marked "dupl.?" in one zone.

DM. 21°2798. A.G. 5383. 9ᴹ ; 10ᴹ.

$\alpha = 15^h\,35^m\,22^s.90 \quad : \quad \delta = +21°\,36'\,8''.3.$

1902.444	129.5	2.80	
.520	131.7	2.47	
1903.227	128.4	2.68	
1902.730	129.9	2.65	M.

DM. 20°3216. A.G. 5528.

$\alpha = 16^h\,4^m\,19^s.89 \quad : \quad \delta = +20°\,40'\,10''.6.$

1902.518 Not double.

Marked "dupl.?" in one zone.

DM. 24°3048. A.G. 5715.

$\alpha = 16^h\,40^m\,18^s.09 \quad : \quad \delta = +23°\,59'\,7''.3.$

This is a nebula. Marked "multiple" in Catal.

DM. 24°3080. A.G. 5764. 9ᴹ ; 11ᴹ.5.

$\alpha = 16^h\,50^m\,28^s.76 \quad : \quad \delta = +24°\,41'\,22''.3.$

1902.411	209.6	2.20	
.502	213.9	2.12	
.504	209.4	1.84	
.695	210.9	2.09	
1902.528	210.9	2.06	M.

DM.23°3151. A.G.6056. 9M ; 9M.3.

$\alpha = 17^h\,33^m\,39^s.01 \quad ; \quad \delta = +22^\circ\,58'\,11''.2.$

t	θ_0	ρ_0	
1902.411	172.4	2.65	
.504	170.5	2.81	
.698	173.4	3.02	
1902.538	172.1	2.83	M.

DM.20°3540. A.G.6078. 9M ; 9M.5.

$\alpha = 17^h\,36^m\,19^s.02 \quad ; \quad \delta = +20^\circ\,19'\,49''.3.$

1902.411	127.7	2.33	
.504	128.5	2.46	
.531	133.0	2.30	
1902.482	129.7	2.36	C.

DM.21°3386. A.G.6410. 9M ; 9M.2.

$\alpha = 18^h\,15^m\,46^s.91 \quad ; \quad \delta = +21^\circ\,17'\,20''.7.$

1902.515	191.6	1.56	
.698	197.0	1.34	
1903.266	197.0	1.43	
1902.400	193.9	1.43	
1902.970	194.9	1.41	M.

DM.24°3423. A.G.6481.

$\alpha = 18^h\,23^m\,27^s.24 \quad ; \quad \delta = +24^\circ\,18'\,58''.7.$

1902 Not double.

Marked "dupl. pr. med." in Catal.

DM.24°3798. A.G.7129.

$\alpha = 19^h\,31^m\,53^s.62 \quad ; \quad \delta = +24^\circ\,30'\,24''.6.$

1902.559 Not double.

Marked "obl.?" in one zone in A.G. Catal.

DM.21°3991. A.G.7387. 9M ; 10M.1.

$\alpha = 19^h\,53^m\,57^s.56 \quad ; \quad \delta = +21^\circ\,52'\,4''.8.$

1902.515	278.2	1.21	
.824	273.3	0.96	
.848	275.2	1.31	
1902.729	275.6	1.16	M.

Indiana University, Bloomington, Ind.

DM.21°4017. A.G.7501.

$\alpha = 20^h\,4^m\,41^s.12 \quad ; \quad \delta = +25^\circ\,2'\,41''.6.$

1902.583 Uncertain.

Marked "dupl.??" in one zone in Catal.

DM.21°4202. A.G.7824. 8M.6 ; 10M.2.

$\alpha = 20^h\,33^m\,50^s.64 \quad ; \quad \delta = +24^\circ\,49'\,52''.4.$

t	θ	ρ	
1902.693	218.7	10.11	
.706	218.3	10.30	
.766	217.5	10.66	
1902.722	218.2	10.46	M.

DM.24°4235. A.G.7921. 9M ; 9M.1.

$\alpha = 20^h\,41^m\,16^s.66 \quad ; \quad \delta = +24^\circ\,19'\,55''.5.$

1902.695	355.7	1.43	
.766	357.5	1.81	
.818	359.3	1.80	
1902.760	357.5	1.68	M.

DM.20°4822. A.G.8079. 8M.8 ; 10M.1.

$\alpha = 21^h\,0^m\,5^s.62 \quad ; \quad \delta = +20^\circ\,29'\,6''.2.$

1902.695	173.4	7.64	
.766	172.9	7.38	
.818	176.8	7.63	
1902.760	174.4	7.55	M.

DM.22°4455. A.G.8332. 9M ; 9M.5.

$\alpha = 21^h\,35^m\,0^s.29 \quad ; \quad \delta = +22^\circ\,54'\,27''.4.$

1902.695	153.2	8.74	
.818	153.1	8.76	
.826	154.1	8.53	
1902.780	153.5	8.68	M.

DM.21°4611. A.G.8379. 9M.2 ; 9M.5.

$\alpha = 21^h\,40^m\,53^s.11 \quad ; \quad \delta = +21^\circ\,28'\,46''.8.$

1902.695	356.4	1.88	
.818	358.1	1.66	
.826	359.2	1.69	
1902.780	357.9	1.71	M.

DM.20°5007. A.G.858[illegible]

$\alpha = 21^h\,41^m\,25^s.38 \quad ; \quad \delta = +20^\circ\,42$ [illegible]

t	θ	ρ	
1902.695	233.4	2.05	
.766	232.6	2.19	
.826	234.4	2.05	
1902.762	233.5	2.10	M.

DM.21°4711. A.G.8560.

$\alpha = 22^h\,8^m\,26^s.61 \quad ; \quad \delta = +21^\circ\,18'\,1$[illegible]

1902.785	230.6	5.05	
.826	230.6	5.00	
1903.088	232.4	4.82	
1902.900	231.2	4.96	M.

DM.21°4718. A.G.8568. 8M.8 ; 9M.8.

$\alpha = 22^h\,10^m\,6^s.15 \quad ; \quad \delta = +21^\circ\,27'\,15''.0.$

1902.785	19.1	1.68	
.801	20.6	2.26	
.826	19.4	1.94	
1903.088	26.1	1.64	
1902.876	21.3	1.88	C.

DM.23°4600. A.G.8733.

$\alpha = 22^h\,40^m\,2^s.83 \quad ; \quad \delta = +23^\circ\,51'\,12''.0.$

1902 Single.

Marked "obl?" in one zone in Catal.

DM.22°4769. A.G.8840. 9M ; 9M.5.

$\alpha = 22^h\,58^m\,54^s.80 \quad ; \quad \delta = +22^\circ\,37'\,2''.1.$

1902.785	228.3	2.04	
.807	224.8	1.80	
1902.796	226.6	1.92	C.

DM.22°4936. A.G.9171.

$\alpha = 23^h\,54^m\,1^s.71 \quad ; \quad \delta = +22^\circ\,53'\,5$[illegible]$.2.$

1902.766 Single.

Marked "dupl.?" in Cata[illegible]

OBSERVATIONS OF THE STAR *KRUEGER* 60.

MADE WITH THE 40-INCH TELESCOPE.

By E. E. BARNARD.

In 1890 Professor BURNHAM, at the Lick Observatory, measured a list of stars noted by KRUEGER as double in his catalogue of the *Astronomische Gesellschaft*.

In the *Astronomical Journal*, 186, p. 17, Mr. ERIC DOOLITTLE gives a list of measures of No. 60 of this list, and shows that the stars *A* and *B* had greatly changed their places when compared with the measures of Professor BURNHAM in 1890.

In concluding, Mr. DOOLITTLE says: "Though the measures are so few, they indicate that there is here a faint star with a large proper motion, attended by a minute companion, which either has a large proper motion of its own, or else is in rapid revolution about the primary."

When Mr. DOOLITTLE's measures were published, it appeared to me wholly improbable that two faint stars, so near each other, should have such motions without being physically connected. In *A.J.* 488, p. 64, I have printed some measures which I obtained in 1900 of these objects. These clearly showed the motion assigned to the stars by Mr. DOOLITTLE.

Acting upon the assumption that *A* and *B* must be a binary system, I have since carefully measured these stars, and the measures of the present year clearly show that the motion of *B* with reference to *A* cannot be rectilinear even when compared with my measures of 1900, 1901 and 1902 alone. If compared with the measures of Professor

BURNHAM in 1890 it will be seen that his distance AB must be nearly 2″ too small to even suggest rectilinear motion.

Following are a continuation of the measures of 1900 printed in *A.J.* 488, p. 64.

OBSERVATIONS IN 1901.

A and *B*.

			″		
1901.729	Sept. 23	130.05	3.37		
.731	24	129.11	3.39	9.5	10.5
.748	30	134.50	3.37		
.751	Oct. 1	131.65	3.25		
.783	13	129.69	3.31		
.805	21	130.59	3.26	9.2	10.5
.827	29	130.59	3.22		
1901.768		130.45	3.31	9.3	10.5

A and *C*.

		°	″	
1901.729	Sept. 23	59.65	37.17	
.731	24	59.84	37.19	9.5
.748	30	59.86	37.29	
.751	Oct. 1	59.52	37.14	
.783	13	59.63	37.42	
.805	21	59.48	37.35	9.7
.827	29	59.49	37.33	
1901.768		59.64	37.27	9.6

A and *D*.

		°	″	
1901.783	Oct. 13	23.56	21.53	15
.805	21	23.04	21.71	15.3
.827	29	23.07	21.78	
1901.805		23.22	21.67	15.1

A and *E*.

		°	″	
1901.729	Sept. 23	98.93	68.18	12
.731	24	99.14	68.47	12
.748	30	99.06	68.62	Single distances. ½ weight.
.751	Oct. 1	98.92	68.20	
.783	13	99.01	68.30	
.805	21	98.91	68.37	11.3
.827	29	98.83	68.61	
1901.768		98.90	68.39	11.8

A and *F*.

		°	″	
1901.729	Sept. 23	275.36	39.60	14
.731	24	274.78	39.52	14
.751	Oct. 1	275.20	39.60	
.783	13	275.33	39.51	15
.805	21	275.34	39.52	15
.827	29	275.53	39.55	
1901.771		275.26	39.55	14.5

OBSERVATIONS IN 1902.

A and *B*.

		°	″
1902.744	Sept. 29	127.25	3.33
.761	Oct. 6	128.17	3.43
.766	7	126.73	3.35
.786	14	126.06	3.35
1902.765		127.30	3.37

A and *C*.

1902.744	Sept. 29	59.64	38.27
.761	Oct. 6	59.69	38.09
.766	7	59.64	38.34
.786	14	59.60	38.49
1902.765		59.64	38.29

OBSERVATIONS IN 1903.

A and *B*.

			″		
1903.380	May 19	122.36	3.32		
.418	June 2	122.34	3.12		
.437	9	123.85	3.40	9.5	11.0
.454	15	123.49	3.37		
.473	22	123.47	3.33		
.492	29	124.90	3.36		
.495	30	125.96	3.26		
.511	July 6	121.31	3.51	Very bad seeing. Difficult.	
.514	7	123.18	3.30		
1903.464		123.43	3.36	9.5	11.0

A and *C*.

		°	″	
1903.380	May 19	59.55	38.59	
.396	25	59.57	38.67	
.418	June 2	58.89	38.83	Excessively bad seeing.
.437	9	59.19	38.75	
.454	15	59.35	38.70	
.473	22	59.59	38.73	
.492	29	59.57	38.73	
.495	30	59.22	38.73	
.511	July 6	59.54	38.83	
.514	7	59.70	38.70	
1903.457		59.42	38.73	

A and *E*.

		°	″	
1903.437	June 9	97.89	69.14	13
.454	15	97.99	69.35	
.473	22	97.87	69.18	
.495	30	97.96	69.40	
.511	July 6	98.07	69.45	
.514	7	97.78	69.49	
1903.481		97.93	69.34	13

A and *F*.

		°	″	
1903.473	June 22	277.24	38.46	13
.495	30	276.84	38.45	12.5
.511	July 6	277.06	38.33	12
.514	7	277.09	38.50	
1903.498		277.06	38.43	12.5

A and *D*.

		°	″	
1903.511	July 6	24.68	22.74	
.514	7	24.09	22.82	15
1903.512		24.39	22.78	15

Following is a complete list of all the measures of these stars that are known to me. Professor BURNHAM has kindly supplied me with his yet unpublished measures.

A and *B*.

	°	″	M	M		
1890.79	178.8	2.32	9.0	12.0	1*n*	β
1898.45	140.7	3.19	9.1	10.5	5*n*	Doolittle
1900.74	134.0	3.18	9.1	11.1	4*n*	Doolittle
1900.94	133.4	3.25	9.1	10.5	4*n*	Barnard
1901.37	131.4	3.35	. .	. .	4*n*	β
1901.77	130.4	3.31	. .	. .	7*n*	Barnard
1902.76	127.4	3.37	. .	. .	3*n*	Barnard
1902.81	126.5	3.36	. .	. .	4*n*	β
1903.43	123.5	3.31	. .	. .	5*n*	β
1903.46	123.4	3.36	9.5	11.0	9*n*	Barnard

A and *C*.

	°	″		M		
1890.79	56.3	26.82	. .	9.3	1*n*	β
1898.45	58.7	34.39	. .	9.4	5*n*	Doolittle
1900.74	59.2	36.18	. .	9.4	1*n*	Doolittle
1900.94	59.3	36.71	. .	9.4	4*n*	Barnard
1901.37	58.6	36.55	. .	. .	4*n*	β
1901.77	59.6	37.27	. .	. .	7*n*	Barnard
1902.76	59.7	38.23	. .	. .	3*n*	Barnard
1902.81	59.4	38.29	. .	. .	4*n*	β
1903.43	58.5	38.61	. .	. .	5*n*	β
1903.46	59.4	38.73	. .	. .	10*n*	Barnard

A and *D*.

	°	″		M		
1900.94	21.0	21.29	. .	15.5	2*n*	Barnard
1901.80	23.2	21.67	. .	. .	3*n*	Barnard
1903.51	24.1	22.78	. .	15.0	2*n*	Barnard

A and *E*.

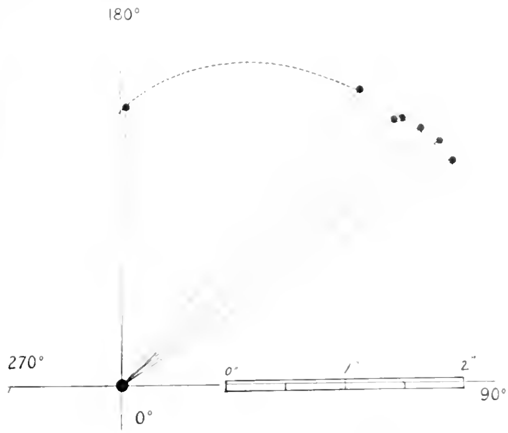
	°	″		M		
1900.94	99.0	67.83	. .	13.0	3*n*	Barnard
1901.34	98.5	67.08	. .	. .	3*n*	β
1901.77	98.9	68.39	. .	. .	7*n*	Barnard
1903.48	97.9	69.34	. .	13.0	6*n*	Barnard

A and *F*.

	°	″		M		
1901.77	275.3	39.55	. .	. .	6*n*	Barnard
1903.50	277.1	38.43	. .	12.5	4*n*	Barnard

In the inclosed diagram I have plotted the observations of the stars *AB*. The measures will be easily identified on the diagram, and to prevent confusion I have not inserted the dates. Professor Burnham's measures of 1901, 1902 and 1903 were received after the diagram was made, and have not been inserted. They would not materially change the results. I have roughly drawn a curve through the measures to give some rough idea of what may be the form of the path of *B*.

In inspecting this diagram, it might be said that if we allow a very large error in Burnham's distance of 1890 — which is improbable — that the observations made since 1898 can be represented by a straight line. It will be seen, however, that the position angle is diminishing at a uniform rate, while the distance is essentially stationary. This could only approximately occur for a short time in rectilinear motion when the stars were at their nearest approach. Assuming this to be the case, then to reconcile my last observation with this idea, Burnham's distance of 1890 would have to be increased as much [illegible] or 1″, which is asking too much.



The mean angular motion previous to 1898 was about 5°. Since then it has been between 3° and 4°. The change of angle between 1902 and 1903 seems too large, but the measures appear to be good in both years, and they are verified by Burnham's measures.

From the observations of *AC* Mr. Doolittle determined the motion *A* to be 0″.93 in the direction 247°.9. Using my measures of this year and Burnham's of 1890, I make the motion to be 0″.951 in the direction of 246°.3. Reduced to rectilinear coordinates this motion is

$$\begin{aligned} &\text{In } \alpha \quad -0''.871 \text{ or } -1^s.606 \\ &\text{In } \delta \quad -0''.382 \end{aligned}$$

From the foregoing observations and remarks it would appear that we have here a very rare case of a binary system, where the components are of the 9th and 11th magnitudes, with an apparent distance of over three seconds of arc, which in all probability has a period well within one hundred years, and which has a large apparent proper motion through space.

These considerations would lead one to think that this star may possibly be relatively near to our solar system. With this last idea in view, I have made observations of it with reference to some of the surrounding stars in the hope of getting some evidence of parallax. Some of these measures are included in the present paper. The measures so far made, show that the parallax can not be [illegible]

On May 25, 1903, the position of *A* was [illegible] with reference to the star Helsingfors Gotha A G C 13177

$\Delta\alpha = -0^m\ 11^s.13$ from 10 transits

$\Delta\delta = -2'\ 11''.26$ from 2 measures

A was north preceding.

This gives for the position of A (corrected for motion).

$$1903.0 \quad \alpha = 22^h\ 24^m\ 32^s.69$$
$$1903.0 \quad \delta = +57°\ 12'\ 38''.5$$

In my paper on this object in *A.J.* 488, p. 64, the last distance for AB is printed 2″.23. It should have been 3″.23. In this same paper I have erroneously called the system of AB, β1291. Professor BURNHAM has already assigned this number to another star.

While observing these stars in 1901 I noticed a rather wide double, very much like AB, and whose position for 1855.0 is

$$\alpha = 22^h\ 19^m\ 42^s.8 \quad , \quad \delta = +57°\ 6'.3$$

Yerkes Observatory, Williams Bay, Wis., 1903 July 10.

The following measures have been made of this object:

			″	M	M
1901.731	Sept. 24	247.04	2.99	9.5	12.0
.805	Oct. 21	247.19	3.11	9.0	11.0
1901.768		247.11	3.05	9.2	11.5
1903.473	June 22	248.79	3.09		
.511	July 6	247.62	3.04		
.514	7	246.68	3.25		
1903.499		247.70	3.13		

There seems to be no change in this star.

There is also another small double in this region which I have so far only located as being 15′± preceding Krueger 60.

			″	M	M
1902	Sept. 29	260.15	1.37	10	11
1903	June 15	262.50	1.41		

ON THE RELATIVE VALUES OF THE MICROMETERS AND THEIR TEMPERATURE-COEFFICIENTS AT THE SIX INTERNATIONAL LATITUDE STATIONS.

BY H. KIMURA.

The large mean deviation of the reduction to the group mean at Tschardjui, given on page 120 of "*Resultate des Internationalen Breitendienstes*," Bd. I, by Prof. ALBRECHT, arouses my doubt that it partly comes from the error of the adopted value of the micrometer. The existence of such errors for all the stations has been proved by the calculation of Mr. NAKANO, who had earlier noticed the same matter. His results given below, as A in Table I, are obtained by the following process: First, for each pair were formed the differences between the individual values of the reduction to the group-mean and the mean of all, the data being taken from pp. 114–118 of the above-named report. From the differences thus formed for the six pairs in each group belonging to each station by the method of least-squares, the following values A were derived, the coefficients of the unknown quantity A being $(\Delta z - \Delta z_m)$, where Δz is the semi-difference of the zenith-distances of two stars in each pair, and Δz_m, the mean of all Δz in the group.

TABLE I.
(unit 0″.0001).

Group	Mizusawa		Tschardjui		Carloforte		Gaithersburg		Cincinnati		Ukiah	
	A	ΔA	A	ΔA	A	ΔA	A	ΔA	A	ΔA	A	ΔA
I	− 23	− 7	−186	+ 44	+170	+ 15	− 83	− 40	+ 28	−48	+ 80	+ 9
II	− 95	− 79	− 50	+180	+ 64	− 91	− 86	− 43	+ 88	+12	+ 86	+15
III	− 35	− 19	−146	+ 84	+130	− 25	−112	− 69	+ 66	−10	+137	+66
IV	−155	−139	− 62	+168	+330	+175	−283	−240	+ 71	− 5	+ 90	+19
V	− 37	− 21	− 8	+222	+102	− 53	−171	−131	+ 22	−54	+ 74	+ 3
VI	+ 28	+ 44	−181	+ 49	+ 66	− 89	− 38	+ 5	+ 51	−25	+ 90	+19
VII	+ 21	+ 37	−280	− 50	+231	+ 76	− 33	+ 10	+ 59	−17	+ 70	− 1
VIII	+ 36	+ 52	−293	− 63	+ 51	−104	+ 53	+ 96	+111	+35	+ 48	−23
IX	+ 18	+ 34	−319	− 89	+ 96	− 59	+ 48	+ 91	+ 66	−10	+ 90	+19
X	+ 30	+ 46	−582	−352	+259	+104	+ 99	+142	+104	+28	+115	+44
XI	+ 15	+ 31	−376	−146	+146	− 9	+109	+152	+162	+86	− 16	−87
XII	+ 11	+ 27	−282	− 52	+222	+ 67	− 16	+ 27	+ 83	+ 7	− 16	−87
Mean	−0″.0016		−0″.0230		+0″.0155		−0″.0043		+0″.0076		+0″.0071	

The mean values in the last line are the relative constant errors of the micrometers.

Now, from the above results, I have tried to find the temperature-coefficients of the micrometers for the six stations. It is, however, a quite difficult matter to determine their absolute values, because the general conditions of the temperature-variations at all the stations resemble each other, and hence their mutual dependencies cannot be wholly avoided. Thus I have aimed here to find the relative amounts of the coefficients which have been

brought as near to the absolute as possible. The mathematical form of Δ is $\Delta M - \left(\Delta\alpha.t - \frac{\Sigma\,\Delta\alpha.t}{6}\right)$, where ΔM is the relative constant correction of the value of the micrometer; $\Delta\alpha$, the correction of the temperature-coefficient; t, the mean temperature for a group; and $\frac{\Sigma\,\Delta\alpha.t}{6}$, the mean of $\Delta\alpha.t$ for all the stations in each group. $\Delta\Delta$, the difference from the mean given in Table I, is therefore in the expression of $\Delta\alpha.(t-t_0) - \frac{\Sigma\,\Delta\alpha.(t-t_0)}{6}$, where t_0 is the mean of t for each station.

For finding $\Delta\alpha$ from these differences, I have, first of all, assumed that, in the three stations, Mizusawa, Gaithersburg and Cincinnati, where the temperature-coefficients have already been applied, the employed coefficients are correct, namely, $\Delta\alpha$ = zero. Then the mean of $\Delta\Delta$ for these three stations will give nothing more than $-\frac{\Sigma\,\Delta\alpha.(t-t_0)}{6}$. Apply this value of $\frac{\Sigma\,\Delta\alpha.(t-t_0)}{6}$ to all $\Delta\Delta$'s, and find $\Delta\alpha$ for all the stations by the method of least-squares. Next form $\frac{\Sigma\,\Delta\alpha.(t-t_0)}{6}$ with $\Delta\alpha$ newly found, and add them to the original $\Delta\Delta$. From these sums, by least-squares, the following values of the temperature-coefficients are obtained:

TABLE II.

For 1′	Mizusawa	Tschardjui	Carloforte	Gaithersburg	Cincinnati	Ukiah
$\Delta\alpha$	−0.00005	−0.00192	−0.00066	+0.00063	−0.00018	−0.00173
Adopted α	−0.00205	0	0	−0.00142	−0.00141	0
Corrected α	−0.00210	−0.00192	−0.00066	−0.00079	−0.00159	−0.00173

For the comparison, I give here the calculated Δ with the above numbers.

TABLE III.

Calculated Δ (unit 0″.0001).

Group	Mizusawa	Tschardjui	Carloforte	Gaithersburg	Cincinnati	Ukiah
I	−25	−182	+132	−50	+66	+76
II	−45	−126	+139	−125	+58	+114
III	−59	−44	+132	−153	+46	+93
IV	−61	+19	+138	−164	+41	+77
V	−45	−85	+151	−137	+65	+65
VI	−22	−203	+169	−100	+77	+90
VII	−14	−280	+172	−49	+71	+111
VIII	+5	−355	+177	+7	+87	+95
IX	+24	−406	+177	+55	+101	+66
X	+34	−415	+172	+91	+110	+24
XI	+31	−386	+162	+88	+107	+13
XII	+3	−285	+143	+32	+84	+38

Now, after correcting the reductions to the group-mean by the calculated Δ, as the relative errors of the values of the micrometers, I have found the mean deviations for the six stations to be

Mizusawa	±0.033	Gaithersburg	±0.028
Tschardjui	±0.045	Cincinnati	±0.034
Carloforte	±0.046	Ukiah	±0.026

and those for the order of the differences of the zenith-distances to be

0 — ±1.0	±0.032
±1.0 — ±2.0	±0.038
±2.0 — ±3.0	±0.035
±3.0 — ±4.0	±0.033
±4.0 — ±5.0	±0.033
±6.0 — ±7.0	±0.036
±7.0 — ±8.0	±0.061

These numbers show that the corrected individual value of the reduction to the group-mean has no considerable dependence either upon the stations or upon the differences of the zenith-distances.

It will however be noted that this discussion would fail if there is any abrupt displacement or readjustment of the focus. At my station, in the autumn of 1901, the readjustment of the focus was made, but fortunately the scale-reading on it remained sensibly unchanged.

The corrections for the errors of the values of the micrometer upon the final mean values from the six stations, of the reductions to the group-mean, of the group-differences, and of the corrections to the aberration-constant, are slight in the present case, rarely rising to ±0″.02. But at those stations in which the errors of the values of the micrometers are pretty large, the final latitudes may be thereby affected directly and considerably, because, although $\Delta\alpha$'s were chosen so as to be correct at a certain epoch, they will become pretty considerable as time elapses, on account of the precession. Thus I dare to say that, for the latitude-work of the high precision of the present time, such systematic errors of the values of the micrometers ought not to be wholly neglected.

There exists still another kind of corrections to the calculation of the latitude-variation. They arise from some terms of the nutation of short period which are neglected in the *Jahrbuch*, the maximum amount of their sum being 0″.06. These might of course have a slight effect upon the mean latitude of a group, especially in cases when the group was observed on the days for which the corrections of this kind have mostly the same sign.

Mizusawa International Latitude Station, 1903 June.

OBSERVATIONS OF COMET *c* 1903 (*BORRELLY*),

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By THEO I. KING.

[Communicated by Rear Admiral C. M. CHESTER, U.S.N., Superintendent.]

Washington M.T. 1903 d h m s	*	Comp.	$\Delta\alpha$ m s	$\Delta\delta$ ′ ″	App. α h m s	App. δ ° ′ ″	log $p\Delta$		Red. to App. Pl. s	″
June 29 15 2 52.3	1	29 , 6	+1 40.09	+ 6 33.4	21 42 52.63	+ 0 55 6.5	*n*8.3458	0.7315	+2.65	+15.7
30 14 53 16.7	2	30 , 6	+0 56.88	+ 0 54.8	21 40 53.07	+ 2 33 58.7	*n*8.4703	0.7148	+2.69	+15.4
July 1 15 5 8.1	3	30 , 6	+1 16.18	− 4 27.6	21 38 35.79	+ 1 19 22.1	7.8216	0.6960	+2.73	+15.1
2 11 59 28.5	4	30 , 6	+3 4.92	+ 2 46.7	21 36 1.35	+ 6 13 58.6	7.9222	0.6741	+2.77	+14.9
6 15 32 42.0	5	30 , 6	+2 42.58	+ 7 9.6	21 21 19.32	+16 12 16.5	9.1429	0.5379	+2.94	+13.8
7 11 40 7.9	6	30 , 6	+0 40.32	− 1 6.2	21 16 21.76	+19 15 39.7	8.7066	0.4686	+2.99	+13.5
7 15 2 25.3	7	30 , 6	−1 22.40	− 4 49.1	21 16 16.62	+19 18 45.8	8.9901	0.4721	+2.99	+13.4
8 11 57 50.9	8	30 , 6	+2 39.44	+ 6 52.8	21 10 16.43	+22 18 8.3	9.0468	0.3913	+3.05	+13.2
9 14 28 35.4	9	30 , 6	+1 6.94	+ 1 42.3	21 3 20.32	+26 34 4.3	8.8714	0.2751	+3.10	+12.9
13 13 1 32.1	10	14 , 3	−0 23.00	+ 6 34.6	20 18 23.04	+44 45 26.7	8.2874	*n*9.9639	+3.40	+13.0
15 11 50 1.6	11	30 , 6	+1 38.90	+ 2 40.8	19 37 31.33	+54 27 10.6	*n*8.7406	*n*0.3742	+3.52	+14.6
17 9 49 8.5	12	20 , 4	−2 15.62	+13 16.9	18 34 28.85	+62 39 50.0	*n*9.4535	*n*0.5199	+3.38	+17.0
21 9 32 35.5	13	24 , 5	−4 48.89	+ 5 20.9	15 14 4.51	+68 30 34.0	9.8358	*n*0.5207	+0.74	+17.6
23 9 59 29.1	14	29 , 6	−0 11.19	+ 9 28.2	13 53 3.68	+65 59 44.4	9.9976	*n*9.8306	−0.15	+13.1
28 8 44 49.2	15	30 , 6	−3 35.45	−11 17.1	12 16 49.37	+57 7 44.1	9.9049	0.0996	−0.20	+ 5.1
Aug. 5 9 0 24.1	16	30 , 6	−1 25.91	+ 0 55.0	11 22 36.37	+46 35 49.7	9.8189	0.6495	+0.14	− 1.7
11 8 43 0.6	17	20 , 4	+2 58.13	+ 1 51.8	11 0 37.11	+41 3 22.0	9.7677	0.7247	+0.31	− 5.0

Mean Places of Comparison-Stars for the beginning of the year.

*	α h m s	δ ° ′ ″	Authority	*	α h m s	δ ° ′ ″	Authority
1	21 41 9.89	+ 0 48 17.4	Nicolajew, A.G. 5509	10	20 18 42.64	+44 38 39.1	Bonn, A.G. 14116
2	21 39 53.50	+ 2 32 48.5	Albany, A.G. 7593	11	19 35 48.91	+54 24 15.2	Camb.(U.S.), A.G. 6147
3	21 37 16.88	+ 4 23 34.6	Albany, A.G. 7580	12	18 36 41.09	+62 26 16.1	Hels.-Gotha, A.G. 9906
4	21 32 53.66	+ 6 10 57.0	Leipzig II, A.G. 10845	13	15 18 52.66	+68 24 55.5	Christiania, A.G. 2298
5	21 18 33.80	+16 4 53.1	Berlin A, A.G. 8723	14	13 53 45.02	+65 50 3.1	Christiania, A.G. 2079
6	21 15 38.45	+19 16 32.4	Berlin A, A.G. 8695	15	12 20 25.02	+57 18 56.1	Hels.-Gotha, A.G. 7156
7	21 17 36.03	+19 23 21.5	1 Peg., Newc. Fund. Cat.	16	11 24 2.14	+46 34 56.4	Bonn, A.G. 8060
8	21 7 33.94	+22 41 2.3	Berlin B, A.G. 8126	17	10 57 38.67	+41 1 35.2	Bonn, A.G. 7881
9	21 2 10.28	+26 32 9.1	Camb.(Eng.) A.G. 12121				

BROOKS'S PERIODICAL COMET, *NOVA GEMINORUM*.

A dispatch from Prof. CAMPBELL, received Aug. 19, at Harvard College Observatory, states that BROOKS's periodical comet was found by AITKEN at the Lick Observatory, on Aug. 18, in the following position:

1903 Aug. 18.8500 Gr. M.T., $\alpha = 21^h\ 10^m\ 11^s.3$, $\delta = -27°\ 4'\ 19''$

Also, that the spectrum of *Nova Geminorum* was observed on Aug. 17, by CURTIS, to be of the nebular type.

CORRIGENDUM.

No. 544, p. 154, col. 1, line 19 from top: *for* +82°.0 *put* +88°.0.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed Sept. 5*
Entered February 2, 1903 at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 547.

VOL. XXIII. BOSTON, 1903 SEPTEMBER 30. NO. 19

MEAN RESULTS OF THE MEASURES OF 227 DOUBLE STARS.

By ERIC DOOLITTLE.

The double star work at the Flower Observatory during the past three years has consisted of the measurement of somewhat more than 1000 pairs. These pairs were selected as follows: (1), about 550 Burnham stars, including those which are in rapid motion, or on which there are no recent measures: (2), about 300 Hough stars, the intention being to re-measure during the next few years all of the stars discovered by Dr. Hough; and (3), about 150 miscellaneous pairs, including the binaries of which Dr. See has computed the orbits, and several neglected stars of Stone, the Washburn Observatory, Berlin, etc. Each star was measured on an average of from three to four nights, each night's observation consisting of at least four measures of double distance, and four of position angle.

These measures will in time be published as Vol. II, Part II, of the Publications of the Flower Observatory: as this volume will probably not appear for some time, it is thought best to publish now a few of the results.

In the following table, the stars are arranged in the order of their right-ascensions, and to save space the right-ascensions and declinations are not written. The fifth column refers to the notes at the foot of the table, and the sixth shows the number of nights of observation on each star. Stars for which orbits have been computed are marked with an asterisk.

Star	Date	θ	ρ	Notes	n
		°	″		
β 484	1902.11	156.4	1.80	a, o	3
β 253	1901.76	46.5	0.60	a, o	6
β 485	1901.81	300.3	0.41	e	5
β 1026	1900.80	336.6	0.42	a, k	2
β 998	1900.78	115.2	1.18	k	3
β 1015	1900.82	119.4	0.58	f	3
β 779	1901.07	254.1	1.05	e	1
β 1157	1902.71	86.9	1.73	o	3
β 391	1901.29	278.9	1.12	a, o	1
β 107	1900.78	355.8	5.96	g, h	3
β 1158	1902.81	150.1	0.41	b, d	3
β 780	1902.71	111.3	2.70	a, o	2
β 395	1902.80	113.6	0.57	t	1
β 257	1900.80	235.0	0.62	k	2
β 866	1900.80	70.8	1.55	a, o	2
β 1160	1901.86	117.1	1.28	a, o	2
β 232 AB	1900.87	336.2	0.30	r	1
AC	1900.84	291.2	28.37	h	4
β 781	1902.86	28.8	1.10	a, o	4
β 496	1901.82	2.7	5.44	a, o	2
β 498	1902.76	154.3	2.82	a, o	5
β 1028	1900.80	260.5	2.27	t	2
β 499	1900.80	347.3	52.77	t	3
β 302	1900.82	105.5	0.61	h	3
Ho. 493	1902.34	20.2	35.11	h, y	3
β 396	1902.71	66.9	1.27	a, o	3
Ho. 213	1901.86	203.0	0.34	t, y	2
β 1228	1900.83	278.1	0.80	f, y	2
β 235 Aa	1900.84	95.9	0.80	h	2
β 1162	1900.90	148.8	0.40	a, b	2
β 1100	1900.86	11.2	0.49	t	2
β 503	1900.78	132.2	5.81	g	3
β 1229	1900.81	291.7	1.20		2
β 1163	1901.85	215.1	0.34	c, y	2
β 783	1902.73	318.1	0.86	a, o	3
β 870	1901.85	53.6	1.11	e	2
β 509	1900.90	251.7	0.68	k	3
β 1016	1900.90	207.8	0.58	z	3
β 1001	1900.80	4.3	1.16	z	1
β 260	1902.78	237.9	0.77	b, d	3
β 515	1902.71	241.0	1.50	a, o	2
β 873	1902.77	21.3	2.13	a, o	3
β 1172	1900.82	241.0	1.71	t, y	2
β 518	1902.78	112.0	1.55	a, o	1
β 519	1900.95	17.4	0.80	t	2
β 306	1901.75	20.1	3.17	a, o	1
β 83	1901.98	101.7	1.03	c, e	3
β 307	No trace of duplicity				
β 525	1902.38	143.3	0.32	r	5
β 100	1899.76	52.8	23.00	h, y	8
ΟΣ 52	1901.01	117.1	0.73	r	3
β 81	1901.90	23.1	0.69	t	3
β 878	1901.99	69.4	1.51	y	1
β 533	1901.88	47.5	0.61	e	3
β 1181	1901.09	86.7	0.45	o	3
β 538	1900.84	132.0	1.79	y	3
β 1184	1900.96	269.1	0.53	z	3
β 743	1901.07	249.5	0.71	i	3
β 512	1900.82	188.8	1.42	g, y	3
β 1004	1900.80	138.2	1.76	e	3
ΟΣ 80	1901.63	178.0	0.71	e	3
*ΟΣ 82	1901.57	117.7	0.66		3
β 789	1901.79	320.9	1.21	t, o	2
β 882	1900.85	225.8	2.35	g	1
β 1011	1900.92	227.8	0.83	t, y	1
*β 883	1901.98	70.3	0.22		2
β 552	1901.98	203.3	0.56	e	2
β 313	No trace of duplicity				1
ΟΣ 92	1901.00	255.7	2.97	h	3
β 1238	1900.91	6.5	1.30	z	2
β 1047	1900.87	46.3	0.30	r	1
β 886	1900.97	255.5	0.99	h	2
β 191	1901.07	21.3	3.55	c, s	1
β 1018	1901.05	352.8	2.33	z	3
β 1019	1900.89	297.6	0.68	z	2

Star	Date	θ	ρ	Notes	n
β 892	1901.61	273.3	1.25	a, o	4
β 1055	1903.13	336.2	1.52	z	4
β 1058	1901.97	260.2	0.34	a,e,g	1
β 1242	1901.85	120.1	0.42	a, z	3
Ho.513	1901.41	3.3	1.51	t	3
β 1021	1900.92	85.0	0.63	z	3
β 191	1901.08	275.6	1.14	a, o	4
OΣ 149	1902.01	271.4	0.91	e	3
OΣ 151	1900.96	123.0	26.45		4
*Sirius	1903.42	127.8	6.31		1
OΣ 157	1902.05	336.4	0.71	e	2
β 899	1900.93	270.1	0.75	f	2
β 900	1900.95	276.2	1.60	a, z	2
β 1279	1901.10	13.4	1.08	a, z	3
OΣ 170	1901.77	108.7	1.51		3
β 330	1901.10	216.0	1.29	a, o	4
β 1024	1901.08	96.9	1.34	t, g	3
β 578	1902.10	45.5	2.28	c, e	7
β 332	1901.64	171.4	0.90	f, h	4
*β 101	1901.13	301.7	0.61		4
β 902	1901.10	243.1	1.25	a, g	4
β 581 AB	1900.91	289.7	0.44	b, e	3
AC	1900.96	195.5	4.72	b, e	4
*Σ 1196	1901.05	361.9	1.17		7
	1903.18	354.0	1.21		4
β 205	1903.04	230.5	0.70	e	6
β 587	1903.21	141.1	0.83	e	3
β 589	1901.48	216.3	3.22	h	4
*Σ 1356	1901.09	113.1	0.79		6
	1903.24	116.9	0.80		3
β 909	1901.25	89.6	6.28	h	3
OΣ 215	1902.53	210.2	0.88	e	4
β 1281	1901.19	67.9	0.96	t	3
β 1073	1902.36	41.9	3.31	h	3
β 1074	1901.27	203.9	2.62	t	3
β 915	1901.30	230.2	1.47	h	4
β 597	1901.64	40.6	0.75	g	4
*Σ 1523	1901.21	148.7	2.37		6
	1903.31	142.2	2.42		6
OΣ 237	1901.15	263.3	1.28	c, d	3
β 602	1901.37	81.7	0.55	t	1
β 603	1901.27	313.2	0.80	e	3
β 794	1902.79	184.1	0.43	v	4
β 919	1901.35	12.9	4.46	t	3
β 28	1901.86	7.6	2.14	b	4
Ho.537	1902.34	174.3	1.21	t, y	4
β 607	1901.68	315.7	1.02	z	4
*Σ 1670	1901.19	328.8	5.99		6
	1903.29	328.0	6.05		4
OΣ 256	1901.87	81.1	0.68	b	3
β 1082	1902.79	93.7	1.41	h	4
β 929	1901.31	223.8	0.61	m	3
β 930	1901.40	119.5	2.90	t	3
β 799	1901.37	248.1	0.76	b, d	3

Star	Date	θ	ρ	Notes	n
*Σ 1728	1903.31	190.0	0.68		8
β 609	1901.83	359.1	0.81	a, z	3
β 221	1901.24	18.9	1.62	k	3
β 800	1901.34	113.2	2.76	d	3
β 610	1902.07	15.9	4.04	a, z	8
Ho.260	1902.34	224.3	0.70	b	4
β 114	1902.21	144.4	1.46	a, b	4
*Σ 1768	1903.49	133.2	1.28		8
β 612	1901.35	237.0	0.36	r	2
β 115	1901.34	229.2	1.70	a, t	3
Ho.512	1902.34	263.6	0.50	z, y	3
β 807	1903.05	240.1	1.11	a, z	4
β 346	1902.51	250.1	1.40	b, y	5
*Σ 1888	1903.44	186.3	2.49		14
β 350	1901.51	158.8	1.19	k	3
Ho.60	1902.35	35.7	0.37	m	4
β 227	1903.25	174.8	2.25	c, h	3
β 228	1903.26	221.8	1.05	e	3
*Σ 1937	1901.57	7.0	0.87		4
	1903.34	15.2	0.99		7
*Σ 1938	1903.40	67.4	1.01		7
*OΣ 298	1903.40	186.0	1.27		10
*Σ 1967	1903.45	115.1	0.74		10
β 620	1901.15	163.0	0.67	a, k	3
β 946	1902.40	147.9	1.64	a, g	5
β 621	1901.36	56.8	0.54	v	3
*Σ 2032	1903.39	214.4	4.51		8
β 813	1901.44	168.2	1.04	a, z	3
β 815	1901.34	339.3	9.15	b, d	3
β 817	1901.52	328.8	1.16	a, z	4
*Σ 2084	1903.42	205.5	1.23		7
β 821	1901.44	313.2	1.28	a, z	4
β 823	1903.46	11.8	1.03	b, e	3
β 1118	1901.43	247.2	0.52	r	2
β 628	1903.30	352.7	0.49	e	4
β 128	1902.25	324.3	4.05	m	1
β 1250	1901.47	68.3	2.02	b	4
β 1089	1901.38	346.9	0.79	e	2
*Σ 2173	1903.41	326.8	1.19		8
β 961	1901.58	139.8	8.15	a, z	4
β 1251	1901.48	65.1	1.30	e	4
Ho.560	1902.34	89.7	0.47	t	3
Ho.70	1902.41	106.5	0.43	t	3
*A.C. 7	1901.49	59.8	1.77		5
	1902.46	63.0	1.86		3
	1903.35	65.4	1.78		5
β 358	1902.91	204.1	4.49	a, z	4
*Σ 2262	1903.45	257.3	2.13		6
Ho.565	1902.42	66.4	0.37	a, z	4
*A.C. 15	1903.37	332.2	1.54		7
β 1091	1900.52	29.4	0.36	i	3
β 641	1900.56	344.1	1.08	e	3
β 648	1901.57	221.7	1.23	r	5
β 1204	1901.59	13.8	0.42	t	2

Star	Date	θ	ρ	Notes	n
β 1129	1900.55	337.5	0.40	z	2
β 142	1900.56	336.5	1.61	b, d	3
β 827	1900.55	266.0	0.89	k	2
β 658	1901.60	302.5	0.51	t	2
β 439	1901.61	240.1	3.12	e	4
β 986	1901.61	238.5	4.53	t	1
β 670	1901.60	44.2	0.55	c, e	3
*β 151	1900.82	13.5	0.63		2
	1901.80	14.7	0.63		3
	1902.76	16.0	0.51		5
β 1209	1901.67	292.9	0.47	z	2
β 152	1901.56	97.5	0.58	c	3
β 367	1901.79	140.3	0.52	e	3
β 68	1901.57	152.4	1.91	o	3
β 368	1901.59	87.1	0.70	c, h	3
β 251	1901.61	232.4	3.05	z	3
β 271	1901.76	238.8	3.12	b, d	3
β 164	1901.67	244.1	0.72	m	3
β 684	1901.65	123.6	1.05	c	3
β 1212	1901.79	273.6	0.60	f, h	3
Ho.608	1902.15	119.2	0.52	z	5
β 1213	1901.44	310.7	0.85	t	3
Ho.610	1902.15	239.9	0.67	z	5
β 842	1902.74	119.7	1.29	e, z	3
β 375	1902.71	308.5	0.81	a, z	3
β 1215	1902.76	271.9	1.68	a, z	3
Ho.179	1902.74	258.0	0.51	n	5
β 376	1902.40	150.7	3.67	a, z	3
Ho.180	1902.73	227.6	0.68	t	4
β 1216	1901.79	319.2	0.57	m	3
β 379	1902.22	333.7	1.27	a, z	4
β 172	1902.30	7.4	0.73	c	6
β 291	1901.81	176.9	0.44	b	2
β 844	1902.31	317.2	3.14	a, z	3
β 175	1902.37	318.6	1.39	a, z	3
Ho.295	1902.74	327.7	0.32	t	2
Ho.296	1902.74	73.9	0.25 ±	t	2
β 710	1901.80	241.0	0.42	p	3
β 711	1902.27	44.0	0.95	v	6
β 851	1902.24	161.1	2.32	a, h	2
OΣ 489	1901.99	49.5	1.04	v	3
Ho.197	1902.70	103.2	0.43	t	3
β 716	1901.77	205.3	1.72	z	3
β 79	1901.65	85.1	0.93	c, e	3
β 80	1901.84	12.1	0.34	r	3
Ho.301	1901.75	351.2	1.49	c	4
β 1221	1902.77	145.8	1.93	z	6
β 1149	1901.79	305.2	0.60	z	2
β 720	1901.84	170.3	0.43	r	3
OΣ 500	1902.09	333.0	0.69	b	3
β 723	1902.79	168.1	3.64	z	5
β 858	1901.80	262.1	0.66	e, d	2
β 1223	1902.81	298.0	1.28	z	6
β 1152	1901.83	100.3	0.74	z	3
β 861	1902.74	177.2	1.39	a, z	6

NOTES.

(a) No recent measures; (b) The angle has certainly increased; (c) The angle has certainly decreased; (d) The distance has increased; (e) The distance has decreased; (f) The angle has probably increased; (g) The angle has probably decreased; (h) Probable increase of distance; (i) Probable decrease of distance; (k) The suspected change is not confirmed by these measures. (m) Observations very discordant: probably fixed. (n) Observations very discordant; motion is probable. (o) Fixed; (t) The character of the motion is uncertain; (v) Rapid motion; (y) Further measures are much needed; (z) The pair is probably fixed.

The Flower Observatory, 1903 Aug. 20.

MEASURES OF *SIRIUS*, *ξ BOOTIS* AND F. 70 *OPHIUCHI*.

By ERIC DOOLITTLE.

The following measures were made with the 18-inch refractor of the Flower Observatory; the fourth column states whether, while making the measures, the line joining the eyes was Parallel to (*P*), or at Right-angles to (*R*), the line joining the stars, and the fifth column gives the number of measures of position angle and double distance on each night.

Sirius.

On only one night during the past year was the companion seen with perfect distinctness. The diffraction rings were then clear and steady, and such satisfactory measures could be secured that I do not think it probable that the resulting angle can be in error by more than five degrees.

	°	″	
1903.123	121.8	6.51	*P* (*a*)
	127.2	6.55	*P* (*a*)
	127.9	6.30	*P* (*a*)
	129.3	6.16	*P* (*a*)
	130.9	6.14	*R* (*b*)
	129.2	6.27	*R* (*b*)
	127.9	6.30	*R* (*b*)
	128.6	6.22	*R* (*b*)
1903.123	127.85	6.31	1*n*

(*a*), Lamp to the right. (*b*), Lamp to the left. Seeing not quite so good as at first.

The position for this date computed from Dr. SEE's elements* is

$$127°.76 \qquad 6''.46$$

The agreement is thus very exact.

ξ Bootis.

	°	″		
1903.269	187.11	2.53	*R*	1
1903.291	187.05	2.35	*R*	1
1903.334	184.75	2.39	*R*	6
1903.414	187.22	2.47	*P*	4
1903.416	184.83	2.65	*R*	4
1903.433	186.14	2.48	*R*	4
1903.463	185.65	2.45	*R*	4
1903.485	186.63	2.51	*R*	4
1903.490	186.30	2.57	*R*	6
1903.496	187.35	2.33	*R*	4
1903.499	185.95	2.39	*R*	1
1903.504	187.24	2.47	*P*	1
1903.534	186.35	2.65	*R*	4
1903.542	186.01	2.56	*R*	1
1903.441	186.33	2.49		14*n*

* There is a misprint in Dr. SEE's article on this star published in *A.J.* 418. The value of *a* should be 8″.0316, instead of 8″.0136. This error does not occur in the "*Evolution of the Stellar Systems.*"

The residuals, (O−C), from Dr. SEE's ephemeris are

$$1903.44 \qquad +30°.5 \qquad +1''.22$$

It is thus evident that the companion is departing from the orbit in a remarkable manner. A trial orbit indicates that the angles obtained by HERSCHEL in 1780 and 1792 are considerably too large, and that the period will largely exceed the value of 128.0 years assigned by Dr. SEE. According to Dr. SEE's elements, the companion should pass the periastron this year, but the above measures indicate that the periastron passage will not occur before 1905.7. It should be noticed that measures during the next four years will be of the utmost value in fixing the values of the elements.

F. 70 *Ophiuchi.*

	°	″		
1903.441	203.92	1.81	*R*	1
1903.446	202.86	1.84	*R*	8
1903.433	202.24	1.87	*R*	8
1903.463	201.73	1.83	*R*	4
1903.485	199.67	2.00	*P*	1
1903.490	197.35	1.82	*R*	4
1903.534	196.73	2.05	*P*	6
1903.551	198.48	1.86	*R*	1
1903.638	200.33	1.79	*R*	4
1903.668	195.64	1.89	*R*	4
1903.674	199.36	1.91	*P*	4
1903.675	198.05	1.93	*R*	1
1903.537	199.70	1.88		12*n*

The residuals, (O−C), from the orbits of Dr. SEE and Dr. SCHUR, and from my own orbit, (*A.J.* 400), together with the residuals from my measures of 1897–1900 are as follows:

	SEE	SCHUR		SEE	SCHUR		
1897.51	−8.44	−10.56	+3.13	+0.03	−0.28	+0.11	13
1899.45	−5.21	−11.59	+2.79	+0.08	−0.30	+0.02	3
1900.57	6.81	−15.95	+1.12	+0.21	+0.06	+0.11	6
1903.56	2.62	−17.21	−0.27	+0.25	+0.24	+0.03	12

The steady departure from SCHUR's orbit still continues; it is remarkable that the residuals from this orbit are still increasing, though it is now seven years since the companion passed periastron. While the measures now agree reasonably well with the positions predicted by Dr. SEE and myself, neither of these ephemerides satisfy the later observations with the exactness which might reasonably be expected in a star of this character.

It may be added that during the past three years I have examined the pair on every night when the definition was unusually good. Even when stars of 0″.2 were [illegible] the components of F. 70 (0″.5?) appeared round with all powers.

The Flower Observatory, 1903 Sept. 6.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(20) *Massalia.*						
d h m s			m s	′ ″	h m s	° ′ ″			s	″
May 17 10 29 30	1	30 , 6	+0 8.11	+9 17.7	14 49 17.73	−15 50 8.3	n8.929	0.853	+2.92	− 6.2
17 10 52 21	2	30 , 6	−2 20.96	+5 14.3	14 49 16.76	−15 50 1.4	n8.578	0.854	+2.92	− 6.0
19 9 59 41	3	17 , 4	+1 56.55	−3 22.1	14 47 30.08	−15 41 18.5	n9.099	0.850	+2.91	− 6.5
21 10 11 39	3	30 , 6	+0 10.82	+1 59.6	14 45 44.36	−15 33 26.7	n8.913	0.852	+2.92	− 6.4
21 10 18 11	4	30 , 6	+0 22.04	+2 19.4	14 45 44.15	−15 33 25.7	n8.836	0.852	+2.92	− 6.5
				(68) *Leto.*						
May 9 10 51 7	5	29 , 6	+0 7.64	+0 7.4	15 32 14.76	−21 2 30.5	n9.293	0.868	+3.00	− 2.5
13 10 21 49	6	30 , 6	+0 22.17	−0 39.9	15 28 56.83	−20 59 48.0	n9.333	0.866	+3.04	− 3.0
21 11 15 38	7	30 , 6	−1 36.61	+0 15.3	15 21 7.90	−20 52 5.6	n8.371	0.879	+3.11	− 3.8
28 10 28 58	8	30 , 6	+1 22.04	+0 55.6	15 14 33.20	−20 44 5.8	n8.701	0.878	+3.13	− 4.7
June 1 9 49 15	9	30 , 6	+0 46.39	−4 33.2	15 11 1.68	−20 39 26.0	n8.971	0.876	+3.13	− 5.1
				(69) *Hesperia.*						
June 2 10 29 7	10	30 , 6	+0 10.62	−5 17.0	15 40 45.98	− 9 29 23.4	n8.781	0.816	+2.94	− 1.4
3 10 8 2	10	29 , 6	−0 31.77	−2 45.8	15 40 3.59	− 9 26 52.2	n8.970	0.815	+2.94	− 1.4
14 10 11 29	11	30 , 6	+0 47.88	−9 28.2	15 32 59.84	− 9 5 23.6	8.176	0.811	+2.94	− 1.2
15 10 37 57	11	27 , 6	+0 14.13	−8 9.2	15 32 26.09	− 9 4 4.5	8.888	0.812	+2.94	− 1.1
				(39) *Laetitia.*						
June 2 9 39 40	12	30 , 6	+1 6.62	−4 57.0	15 42 34.73	− 3 50 24.5	n9.202	0.772	+2.84	− 0.8
3 9 24 5	12	30 , 6	+0 20.66	−3 31.3	15 41 48.77	− 3 48 58.8	n9.254	0.771	+2.84	− 0.8
8 9 39 42	13	30 , 6	−0 17.06	−6 24.3	15 38 6.04	− 3 44 13.3	n9.022	0.772	+2.85	− 0.3
8 10 1 44	12	25 , 5	−3 22.83	+1 13.5	15 38 5.30	− 3 44 13.3	n8.792	0.773	+2.86	− 0.1
14 11 34 26	14	30 , 6	+0 48.95	+1 0.1	15 34 3.89	− 3 43 45.5	9.242	0.771	+2.84	− 0.2
				(129) *Antigone.*						
June 3 10 48 59	15	30 , 6	−0 46.82	+2 2.2	16 33 29.84	− 1 31 56.2	n9.065	0.754	+2.87	+ 2.7
8 10 56 14	16	30 , 6	+0 48.30	+0 38.2	16 29 33.34	− 1 43 20.9	n8.745	0.756	+2.90	+ 3.0
14 12 20 22	17	30 , 6	−0 43.74	+0 27.7	16 25 5.31	− 2 5 9.1	9.218	0.758	+2.94	+ 3.4
14 12 37 30	18	30 , 6	−1 40.36	−2 10.4	16 25 4.89	− 2 5 11.4	9.293	0.757	+2.94	+ 3.4
15 11 10 49	19	30 , 6	+0 51.69	−3 42.3	16 24 26.17	− 2 9 20.3	8.588	0.760	+2.94	+ 3.3
				(17) *Thetis.*						
May 21 12 13 10	20	30 , 6	−1 2.68	−0 40.6	17 6 26.55	−14 13 49.6	n9.077	0.843	+2.90	+ 4.2
28 12 57 52	21	30 , 6	+0 12.03	+2 37.0	17 0 32.09	−14 11 12.3	8.608	0.845	+3.02	+ 4.2
28 12 58 10	22	30 , 6	−0 3.75	+1 13.4	17 0 32.03	−14 11 11.6	8.614	0.845	+3.02	+ 4.2
June 2 11 19 12	23	30 , 6	−2 24.39	−3 15.4	16 56 0.78	−14 11 55.1	n9.049	0.843	+3.08	+ 4.1
3 11 20 8	24	30 , 6	+2 21.72	+0 57.3	16 55 4.46	−14 12 18.5	n9.002	0.844	+3.10	+ 3.8
				(43) *Ariadne.*						
June 18 10 41 5	25	30 , 6	−2 15.62	+1 24.9	16 48 44.75	−22 58 18.8	n8.694	0.888	+3.44	+ 3.6
21 9 23 22	26	20 , 8	+0 0.30	+0 40.7	16 46 15.71	−22 43 54.4	n9.262	0.877	+3.45	+ 3.2
				(432) *Pythia.*						
June 18 11 32 39	27	27 , 5	−1 52.50	+4 25.8	16 55 48.22	−19 16 40.1	8.662	0.872	+3.35	+ 4.4
21 10 9 49	28	24 , 8	+0 38.51	−1 38.3	16 52 52.88	−19 39 56.8	n8.991	0.871	+3.37	+ 4.0
				(405) *Thia.*						
June 21 12 2 14	29	30 , 6	+3 55.37	+3 22.3	17 23 44.24	−19 35 42.0	8.872	0.872	+3.39	+ 6.4
30 11 31 43	30	40 , 8	+0 11.12	+2 13.8	17 16 10.20	−18 31 19.9	8.999	0.866	+3.41	+ 6.2
30 11 39 1	31	29 , 6	−0 34.61	+1 29.6	17 16 9.94	−18 31 17.9	9.060	0.865	+3.42	+ 6.3
July 1 10 8 24	32	25 , 5	−3 30.72	−3 55.1	17 15 28.75	−18 25 9.7	n8.814	0.867	+3.42	+ 6.5

1903 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				(57) *Mnemosyne.*						
July 18 10 15 9	33	30 , 6	−2 51.88	−0 52.1	19 51 38.13	+ 1 6 8.2	n9.211	0.730	+3.26	+17.7
21 10 55 4	34	30 , 6	+0 20.46	+2 35.5	19 49 23.66	+ 0 59 21.3	n9.069	0.731	+3.29	+18.1
22 10 1 30	34	30 , 6	−0 22.56	+0 7.1	19 18 10.65	+ 0 56 56.0	n9.319	0.732	+3.30	+18.2
24 10 17 10	35	30 , 6	−0 24.18	+5 38.4	19 47 11.05	+ 0 51 20.1	n9.211	0.733	+3.31	+18.5
25 10 56 26	36	20 , 4	+2 13.93	+2 59.2	19 46 25.28	+ 0 48 13.6	n8.892	0.733	+3.31	+18.5
				(270) *Anahita.*						
July 18 11 39 3	37	29 , 6	+2 12.50	+7 50.3	20 38 21.58	−14 13 10.5	n9.188	0.843	+3.34	+19.9
21 11 33 14	37	29 , 6	+0 1.98	+3 19.7	20 35 11.10	−14 47 10.8	n9.137	0.844	+3.38	+20.2
22 10 41 13	38	30 , 6	+1 24.22	+4 10.8	20 34 47.92	−14 48 37.3	n9.356	0.836	+3.39	+20.2
23 10 54 40	38	24 , 8	+0 27.43	+3 2.4	20 33 51.14	−14 50 15.7	n9.286	0.840	+3.40	+20.2
24 11 14 50	39	25 , 5	+3 29.35	−5 50.6	20 32 53.22	−14 51 58.8	n9.160	0.844	+3.43	+20.2
Aug. 7 10 7 6	40	30 , 6	−1 9.86	−3 35.5	20 19 31.35	−15 21 1.6	n9.158	0.847	+3.55	+20.3
7 10 28 43	41	30 , 6	+0 50.06	−3 33.8	20 19 30.58	−15 21 3.0	n9.002	0.850	+3.55	+20.2
9 10 15 26	42	30 , 6	+1 42.91	−7 1.2	20 17 44.34	−15 25 35.3	n9.032	0.850	+3.56	+20.0
9 10 33 23	43	30 , 6	−0 1.70	+7 39.8	20 17 43.70	−15 25 36.7	n8.852	0.851	+3.56	+20.1
11 10 59 44	44	30 , 6	+1 17.72	−7 2.1	20 16 0.42	−15 30 11.7	7.483	0.853	+3.56	+19.9

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	14 49 6.70	−15 59 49.8	W., A.G.Z.52, 114, 209	23	16 58 22.09	−14 8 13.8	Wash. A.G.Z. 43, 122
2	14 51 34.80	−15 55 12.7	" " 51, 114, 209	24	16 52 39.64	−14 13 19.6	Rad. 1890, 4407
3	14 45 30.62	−15 38 19.9	Newcomb's Fund.Catal.	25	16 50 56.93	−22 59 47.3	Cape 1885, 1184
4	14 45 19.19	−15 35 38.6	" " "	26	16 46 11.96	−22 44 38.3	Rad. 1890, 4380
5	15 32 34.12	−21 2 35.4	Cincinnati 1885, 2632	27	16 57 37.37	−19 21 10.3	Cincinnati 1885, 2797
6	15 28 31.62	−20 59 5.1	" " 2626	28	16 52 11.00	−19 38 22.5	" " 2789
7	15 22 41.40	−20 52 17.1	" " 2613	29	17 19 45.48	−19 39 10.7	" " 2842
8	15 13 8.03	−20 44 56.7	" " 2591	30	17 15 55.67	−18 33 39.9	U.S.N. Obs. Tr.Cir.Pos.
9	15 10 12.16	−20 34 47.7	" " 2586	31	17 16 11.13	−18 32 53.8	" " "
10	15 40 32.42	− 9 24 5.0	Wien, A.G.Z. 57, 111	32	17 18 56.05	−18 21 21.1	Rad. 1890, 4530
11	15 32 9.02	− 8 55 54.2	" " 258, 326	33	19 54 26.75	+ 1 6 42.9	Nicolajew, A.G. 5027
12	15 41 25.27	− 3 45 26.7	U.S.N. Obs. Tr.Cir.Pos.	34	19 48 59.91	+ 0 56 30.7	" " 5006
13	15 38 20.25	− 3 37 48.7	" " "	35	19 47 31.92	+ 0 45 23.2	Newcomb's Fund.Catal.
14	15 33 12.10	− 3 44 45.4	" " "	36	19 41 8.04	+ 0 44 55.9	Nicolajew, A.G. 4985
15	16 34 13.79	− 1 31 1.1	Nicolajew, A.G. 4184	37	20 35 35.74	−14 51 20.7	Rad. 1890, 5564
16	16 28 42.14	− 1 44 2.1	" " 4166	38	20 33 20.31	−14 53 38.3	Wash., A.G.Z. 68, 129
17	16 25 46.11	− 2 5 40.2	" " 4151	39	20 29 20.44	−14 46 28.4	" " 72, 129
18	16 26 42.31	− 2 3 4.4	" " 4153	40	20 20 37.66	−15 17 46.4	Rad. 1890, 5487
19	16 23 31.54	− 2 5 11.3	" " 4140	41	20 18 36.97	−15 17 49.4	Wash., A.G.Z 66, 131
20	17 7 26.33	−14 13 13.2	Wash., A G.Z. 46, 120	42	20 15 57.81	−15 18 54.1	" " 61, 131
21	17 0 17.01	−14 13 53.5	" " 43, 120	43	20 17 11.84	−15 33 36.6	" " 63, 131, 219
22	17 0 32.76	−14 12 29.2	" " 43, 120	44	20 14 39.11	−15 23 29.5	" " 64, 131

OBSERVATIONS OF BROOKS'S COMET (1881 V),

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By C. W. FREDERICK.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
Aug. 20 11 37 39	1	27 , 6	−1 22.66	−4 55.4	21 1 27.31	−27 4 26.5	8.820	0.903	+3.73	+21.5
21 11 20 21	2	23 , 5	+1 55.95	+2 38.7	21 0 43.53	−27 4 12.9	8.576	0.903	+3.75	+20.9

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	21 2 46.24	−26 59 52.3	C.G.C. 28982	2	20 58 43.83	−27 7 12.5	C.G.C. 28872

THE WHITE SPOT ON *SATURN*.

By E. E. BARNARD.

This object, observations of which were given in *A.J.* 512-513, promises to be of unusual interest, because its period of rotation is much longer than that previously attributed to *Saturn*.

It is not often that the average observer has the chance to determine the rotation period of *Saturn*, for conspicuous spots on the planet are very rare — the last one, a conspicuous white spot, was observed by HALL at Washington in December, 1876. The observations of that spot gave a period of $10^h\ 14^m\ 23^s.8 \pm 2^s.30$, according to Professor HALL (*A.N.* 2146).

It would seem that the only determination previous to HALL's, was by Sir WM. HERSCHEL in 1794, when he found the period to be $10^h\ 16^m\ 0^s.4$, which might be in error by 2^m. This period was determined from some peculiarity of the belts, and not from a definite spot.

Since HALL's observations in 1876, no conspicuous spot has appeared on the planet, though faint spots have been reported by one or two observers, but they could not be seen by others.

The present opportunity, therefore, is a rare one, and the period of the planet (or of the spot, since the spots of *Saturn* doubtless have different proper motions) ought to be well determined this time, for the object is both conspicuous and distinct.

At the second observation of this spot — that of June 24 — it was seen that the period of $10^h\ 14^m$ would not fit the observations, and that the rotation time must be decidedly longer. It was later found to be about $10^h\ 39^m$.

In *A.N.* 3883, K. GRAFF of the Hamburg Observatory, by combining the observation here of June 23 with an observation at Hamburg on June 26, and one at Bamberg, by HARTWIG, on the same date, found the period to be $10^h\ 39^m.01$.

This period is some 25 minutes longer than HALL's. It is therefore important that the spot be observed as carefully as possible, so that its exact period may be determined.

It is well known that the different spots on *Jupiter* have different rotation periods in general, but there has never been observed any such great difference as that indicated above for *Saturn*.

At an observation here of this spot on Aug. 2, with the 12-inch, it was very distinct and easy. The probability is, therefore, that it may last out the season of *Saturn's* present apparition.

There seem to be several spots, and it is well to avoid confusion in their identification.

Following is a continuation of the observations of the original white spot. The observations of June 23 and 24 will be found in *A.J.* 542-543.

1903 *July* 6 (transit $12^h\ 41^m$).

A note says: "The observation refers to the following of two spots."

July 13 $12^h\ 30^m$ no spots visible at this time.
13 20 there is a luminous spot following the center. It is long and irregular.
14 5 the main body not quite in transit.
14 9 in transit.
14 14 in transit.
14 16 I think it is past transit.
14 17 certainly past transit. There is a smaller, and not so distinct a spot, joining this, following and separate from the larger by a dark patch.

Adopted time of transit $14^h\ 11^m$.

July 14 the spot above (observed on the 13th) identified with certainty. Following are observations of its transit:
$11^h\ 19^m$ not quite in transit.
11 21 not quite in transit.
11 25 in transit.
11 27 in transit.
11 29 I think it is past transit.
11 30 uncertain yet.
11 32 a little uncertain yet.
11 34 it is certainly past transit.

Adopted time of transit $11^h\ 26^m$.

There is a small spot following. No other spots visible on the disc. At $11^h\ 41^m$ the preceding spot was conspicuously past transit. Distance between the spots $= 3''.5$.

At $12^h\ 30^m$ the two spots are past transit, and no others visible.

Aug. 2 $13^h\ 46^m$ a luminous spot very nearly in transit.
13 49 perhaps not yet in transit.
13 52 in transit.
13 55 in transit.
13 56 in transit, fairly well seen; small and distinct.
13 58 perhaps past transit.
14 0 perhaps past transit; a little uncertain.
14 2 it seems to be past now, but a little uncertain.
14 8 past, but not decidedly so.
14 10 decidedly past transit.

The time of transit would be close to $13^h\ 57^m$.

It was slightly elongated.

No other spot visible on the disc.

I believe that these are all observations of the original white spot, though there may be some uncertainty on account of there being two spots at this point.

The following observations are also of white spots, but they do not seem to be of the original spot.

June 30 13^h 19^m a definite white spot, quite well defined at its following end; in transit at the above time. "If this is the spot seen before, it is not near so conspicuous as it was."

July 7 13 20 a small, luminous, round spot, 1½″ in diameter, in transit. It is on a cold steel blue or bluish gray narrow belt. No other spot [illegible]

July 20 11^h 55^m no other spots v[illegible]e

July 21 12 35 there seems to be a w[illegible]t [illegible]

July 27 12 30 can see no spots.

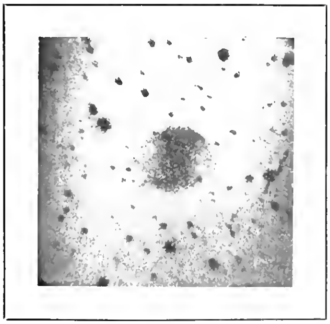
If the observations of Aug. 2 are of th[illegible] white spot, a rough approximation w[illegible]d make the pe[illegible]

10 [illegible] 38 [illegible] 8

ASTRONOMICAL JOURNAL No. [illegible]

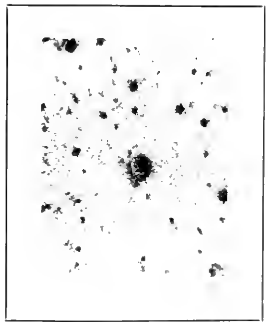
DUMB BELL NEBULA IN VULPECULA AS A GREAT SPIRAL

Photographed with [illegible] inch Reflector of [illegible]-inch Fo[illegible] July [illegible] 19[illegible] Exposure 20 minutes. Enlarged [illegible] diameters



RING NEBULA IN LYRA AS A GREAT SPIRAL

Ph[illegible] [illegible] Re[illegible] F[illegible]cus August [illegible] Ex[illegible] [illegible]tes. Enlarged [illegible] [illegible]eters



* That each of any two fluid or gaseous masses at the instant of separation should break up into parts having various radial velocities would seem to be much more probable than that the whole mass of each half should have a common velocity.

† Two superposed gaseous spirals having a common origin, but different initial velocities, would produce a spiral structure composed of arcs of greater density where these spirals overlap, connected by the less dense areas formed by the separated individual streams.

ne[illegible] formed [illegible] a comparat[illegible]y small cent[illegible] part

The enlargement of the negative which [illegible] of 35', sent with this paper, may not be [illegible] reproduction. On this particular occasion a [illegible] exposure might well have been given with better results

* See [illegible] No. 3200. Th[illegible] nebula is [illegible] negatives having an exposure of one minute

The general curvature of these exterior streams is unmistakably that of a clock-wise spiral whose maximum visible diameter is about one-quarter of a degree. One gets the impression that if these exterior streams were brighter a structure similar to the Whirlpool nebula (M. 51) would result; the nucleus of the latter being regarded as a ring nebula on a small scale.

As a result of the photographic examination of the Dumb-bell nebula, mentioned above, the very first series of negatives, taken several weeks ago, revealed, unmistakably, that this object is a great *counter* clock-wise spiral, at least half a degree in diameter, the well-known nebula occupying the central area. This central area (plainly elliptical in outline on the photographs) seems to be formed in much the same way as the above-described Ring nebula, except that the several streams into which the main branches divide are much more divergent near the origin. The several exterior streams, which are very regular, and like the Ring-nebula, studded with faint stars, appear to make but little more than a complete revolution up to points 15′ distant from the origin,* while the inner streams probably make several revolutions before reaching the bright exterior boundary of the Dumb-bell only about 1′ from the center.

I have been waiting for a good night to make a long exposure on this nebula, but as this opportunity may not soon be available, a photograph, which may possibly be reproduced so as to show the spirals, is sent herewith. It is a 15-diameter enlargement of a negative exposed for 20 minutes on July 22, 1903.

Additional interest will attach to planetary nebulas in general if it shall be found that the well-known objects of this class are but parts of structures similar to the two considered in the present article.

*The streams probably extend much farther from the center, but in my instrument the aberrational effects become so large that the results are uncertain at greater distances from the optical axis.

Ann Arbor, 1903 *Sept.* 1.

OBSERVATIONS OF 7793 *SS CYGNI*.

By ZACCHEUS DANIEL.

SS Cygni has recently acted in a very strange manner. The maximum in February was anomalous in form, and similar to that of December, 1899, a curve of which was published by J. A. Parkhurst in *Popular Astronomy*, V. 8, p. 46, and the *Astrophysical Journal*, V. 12, p. 268. The observations indicate a stand-still at a point where the rise is usually very rapid. This maximum was also observed by Hartwig (*A.N.* 3866). After a short period of normal brightness following the April maximum, I found it on May 10, apparently, somewhat brighter, but in a few days it returned to normal. On May 25, however, it was certainly above normal, but a few days later it had become faint again. That was probably the end of a short maximum. The next maximum was also short. The rise began on June 21. On the decrease, a stand-still occurred at 10^{M}.5. The maximum in July is noteworthy in that the decrease was somewhat checked toward the end, the approach to normal brightness being comparatively slow. This has also been observed at several previous maxima.

Following are my observations this year to date. They were made by Argelander's method, with apertures ranging from 12.7 cm. to 58.4 cm. The magnitudes are on the visual scale given in the *Astrophysical Journal*, V. 12, p. 260. The decimals of a day are in Greenwich M.T.

1903	J.D.	Mag.	1903	J.D.	Mag.
Jan. 1	6116.553	8.73	June 29	6295.744	9.49
6	121.533	10.30	30	296.691	9.87
10	125.526	11.32	July 1	297.776	10.02
18	133.521	11.28	3	299.816	10.52
Feb. 5	151.515	10.29	4	300.802	10.59
6	152.524	10.13	6	302.743	11.32
9	155.517	9.35	9	305.656	11.37
10	156.506	9.35	10	306.737	11.37
12	158.507	8.76	15	311.657	11.27
14	160.503	8.86	16	312.684	11.27
Apr. 8	213.771	8.66	19	315.716	11.17
9	214.889	8.76	21	317.839	11.27
22	227.864	11.22	27	323.858	8.36
May 10	245.776	11.05	28	324.708	8.46
13	248.673	11.12	29	325.828	8.66
15	250.712	11.17	31	327.833	8.66
17	252.721	11.22	Aug. 1	328.665	8.76
25	260.802	9.74	2	329.831	8.96
28	263.780	10.92	5	332.618	9.70
June 2	268.742	11.37	7	334.653	10.26
3	269.767	11.37	9	336.625	10.98
12	278.641	11.42	10	337.583	11.02
13	279.724	11.32	11	338.826	11.12
21	287.708	10.98	12	339.805	11.27
26	292.781	8.76	14	341.790	11.22
27	6293.660	8.96	17	6344.810	11.32

The Observatory, Princeton, N.J., 1903 *Aug.* 28.

CONTENTS.

Published at 10 Craigie St., Cambridge (Boston Postal District), Mass., semi-monthly, by S. C. Chandler. Address, Cambridge, Mass.
Associate Editors, Asaph Hall and Lewis Boss. Price, $5.00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. *Closed Sept* 24.
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 548.

VOL. XXIII. BOSTON, 1903 OCTOBER 31. NO. 20

ORBIT OF COMET 1900 II.

By J. M. POOR.

1. Introduction.

The definitive orbit of this comet has already been published as it was determined by Manoel Soares de Mello e Simas from a very thorough discussion of nearly all the published observations.* The following elements of the same comet had been deduced, and were nearly ready for the printer when Dr Mello's paper was received. Publication has been delayed that the discussion of the perturbations might be carried considerably farther than was at first intended, though most of the computations were completed by the writer while Fellow at Princeton University. The writer congratulates himself as well as the astronomical public in general on the close agreement of the results as assuring confidence in the numerical accuracy of the computations, though this work would not have been undertaken had occupation of the same field by another been known. The following paper is abridged in order to avoid repetition so far as is consistent with the satisfactory statement of results.

This comet, independently discovered on July 23, 1900, by Borrelly, at Marseilles, and Brooks of Geneva, U.S.A., was observed for a little more than three months, while an attempted observation by Aitken at Mt. Hamilton, five months after discovery, though interrupted by rising fog, showed the comet to be nearly in its predicted position, and indicated that the orbit was nearly parabolic.

2. Elements, Ephemeris, and Stars.

The elements here employed for comparison and correction are based on those computed by Schiller and Wedemeyer (*A.N.* 3660). They are

$$T = 1900 \text{ August } 3.19930 \text{ Gr. M.T.}$$

$$\left.\begin{aligned} \omega &= 12^\circ\ 25'\ 34''.80 \\ \Omega &= 328\ \ \ 0\ \ 26.20 \\ i &= \ \ 62\ \ 30\ \ 11.00 \end{aligned}\right\} \text{Ecliptic 1900.0}$$

$$\log q = 0.0063830$$

which give [illegible] = 9.9457964 [illegible] 86 2[illegible] [illegible] 7 +
[illegible] 9.6867100 [illegible] 28[illegible] [illegible] 27 0[illegible] +
[illegible] = 9.9966354 [illegible] 0 [illegible] 27 35 +

Through[illegible] [illegible] Greenwich [illegible] been [illegible] the solar co-ordinates [illegible] taken from the [illegible] *[illegible]* [illegible] N[illegible]

Comparisons with the [illegible] placements fro[illegible] ephemeris were made by interpolation for the [illegible] correction corrected for aberration-time from positions [illegible] at intervals of twelve or twenty-four hours, [illegible] right [illegible] except [illegible] between August 20.5 and August 29.5, [illegible] declination between August 23.5 and August 27.5, when the [illegible] the comet near the pole made it seem best [illegible] the place for the time of each observation directly [illegible] the elements.

As far as possible A.G. star places have been employed, [illegible] for polar stars *[illegible]* has been used. Several star places are the result of comparison with A.G. stars or Berlin J.B. stars, and others are the result of meridian observations published with the comet observations. D.M., Cincinnati, Nos. 12 and 13, Berlin J.B. 1900, Gr. Obs. 1896, Romberg, and Hamburg zones have been used, [illegible] positions have been modified by Pulkova Obs., Y[illegible], Greenwich Ten-Year Cat., Radcliffe (1860.0), [illegible] Obs. 1898. Sixteen A.G. Dorpat places were kindly [illegible] in manuscript by Profs. Levitzky and Schwarz [illegible] Dorpat Observatory. Romberg was consulted for me by Mr. D. F. Wilson at Cincinnati, and *[illegible]* by Prof. Searle at Harvard, though the latter positions were [illegible]. Proper motions have been applied whenever [illegible]. The position of the star 100 of Dr Mello's [illegible] calls attention, was found to be R.A. [illegible] 54, Decl. [illegible] 71 58 12.1 [illegible] from [illegible] from Dorpat. The reduction of observations [illegible] on the star *[illegible]* 685, Dr Mello's star 120, [illegible] a proper motion in declination, and its [illegible] with Nos. 708 and 720 of the same Catalogue [illegible] Princeton

* *Astronomische Abhandlungen als Ergänzungshefte zu den Astronomischen Nachrichten herausgegeben* von Prof. Dr. H. Kreutz, Nr. 4, 1 Teil.

by the computer, yielded as a result an annual proper motion in R.A. of $0^s.0057$, and in Decl. of $-0''.1338$ (equator 1903.0). Its position for 1900.0, as deduced from these observations, was found to be R.A. = $1^h\,16^m\,37^s.92$; Decl. = $84°\,19'\,57''.5$.

In reducing stars to the epoch 1900.0 the constants of STRUVE and PETERS have been uniformly employed, and above 75° declination rigorous formulas were used.

3. OBSERVATIONS AND WEIGHTS.

It was intended to collect all observations, and a request for such as had not been published was inserted in *A.N.* 3753. The usual journals were examined, but by an oversight all observations published in *Comptes Rendus* only remained unknown to the computer until the appearance of DE MELLO's work. These included three observations made at Algiers, the valuable series of Bordeaux and Lyons, with the less numerous series made at Marseilles, Paris and Toulouse. To those omitted must also be added the series made at Washington, and published in *A.J.* 535, too late for consideration in this discussion, and likewise omitted by DE MELLO, as were also two unpublished observations made at Pulkowa, which were kindly furnished by the observer, A. SOKOLOW. For completeness the latter are here inserted.

1900 Gr. M.T.	Comet—Star R.A.	Comet—Star Decl.	Parallax R.A.	Parallax Decl.	R.A.	Decl.	Comp. Star
	m s	′ ″	s	″	h m s	° ′ ″	
September 6.29000	+2 27.65	−5 27.3	+1.67	+2.5	13 24 3.06	78 58 32.7	A.G. Kasan 2333
11.33524	+0 35.58	−2 7.2	+1.11	+3.8	13 46 53.67	76 2 46.7	" " 2428

In reducing observations parallax factors have all been recomputed by means of BAUSCHINGER's Tables and reductions from mean to apparent place have been made by the use of constants of the *American Ephemeris and Nautical Almanac*.

For convenience in weighting, observations have been divided into three groups. Group 1 consists of those cases where more than five observations were made with a filar micrometer by one observer. Group 2 is made up of those series more than five in number made by a single observer with a ring or bar micrometer; and Group 3 includes all series less than six in number by a single observer. The only apparent exceptions to these rules are two cases in Group 3 in which the series numbered six, but these were subsequently reduced to five or less by rejected observations. Such rejections were determined as follows:

First. Because recent or accurate catalogue-places of companion stars were not found, or because no catalogue place was found: Arcetri, Aug. 19 (comp.-star *Fedorenko*); Hamburg (Sch.), Sept. 19; Hamburg (M), Sept. 19; Kiel; July 28; Königsberg (S), Aug. 23; Paris, July 24; Pola (H), July 25; Pola (M), July 25. Letters in parenthesis indicate the observer as explained in the list below.

Second. Because of serious discrepancy with other observations made at nearly the same time: Denver (L), July 28, Decl.; Denver (H), July 30 and all other observations in declination; Heidelberg (C), July 25, Decl.; Heidelberg (V), Aug. 18, Decl.; Kiel, July 25, Decl.; Kremsmünster, Aug. 19, Decl.; Lemberg, Aug. 10 and all other observations in declination; Leipzig, July 25, Decl.; Göttingen, July 25; Utrecht (V), Aug. 16, Decl. and Sept. 5, R.A.; Geneva, July 26, Decl.; Padua, Aug. 1, second obs. Decl. In no case was the attempt made to harmonize by changing the date of an observation.

In determining weights each remaining residual was first given unit weight, and curves, one for each coordinate, were constructed by means of normals at convenient intervals. These curves were assumed to represent the true path of the comet with which the residuals of each observer were compared, thus giving "secondary" residuals from the first powers of which the computed weight of an observation was found by the formulas

$$r = 0.8453\,\frac{[v]}{m} \quad \text{and} \quad p = \frac{e^2}{r^2}$$

in which e, the mean value of r for all observers whose observations numbered more than 5, was found to be $0^s.16$ in R.A., and $1''.9$ in Decl.

Observations of Group 1 were then given the integral weights 2, 3, or 4, according to the numerical value of the weight as computed. In the same way, those of Group 2 received weights ½ or 1, while the weights of Group 3 were assigned arbitrarily because of the small number in each series.

From the results of comparisons with the curves it was thought best to apply corrections to observations as follows:

CORRECTION.

Observatory	R.A.	Decl.
	s	″
Arcetri,	−0.25	. .
Geneva,	+0.20	+2.3
Kiel,	. .	+2.9
Kremsmünster,	−0.34	. .

The following table contains details outlined above with references to journals where observations were found.

Observatory	Observer		Group	R.A. Obs.	R.A. p	Decl. Obs.	Decl. p	Journal
Algiers	Sy		1	8	1	8	3	*B.A.*, Vol. 18
Arcetri	Abetti		1	12	2	12	2	*A.N.* 3674, 3687. Arcetri Pub., No. 15
Besançon	Chofardet	= C	1	12	1	12	1	*A.N.* 3656, 3691. *B.A.*, Vol. 18
	Sailet	= S	3	2	1	2	1	*A.N.* 3656
Copenhagen	Pechule		3	3	1	3	1	*A.N.* 3654, 3655
Denver	Heller	= H	3	6	1	6	0	*A.J.* 508
	Ling	= L	1	22	3	22	3	*A.J.* 492, 503
Geneva	Pidoux		1	8	2	8	2	*A.N.* 3660
Göttingen	Schur		3	1	0	1	0	*A.N.* 3655
Hamburg	Scheller	= Sl	1	13	3	13	4	*A.N.* 3655, 3724
	Schorr	= Sch.	3	3	1	3	1	*A.N.* 3724
	Messow	= M	3	6	1	6	1	*A.N.* 3724
Heidelberg	Courvoisier	= C	3	4	1	4	1	*A.N.* 3654, 3720
	Valentiner	= V	3	4	1	4	1	*A.N.* 3720
Jena	Knopf		2	6	1	6	1	*A.N.* 3692
Kiel	Ristenpart		2	8	1	8	1	*A.N.* 3654, 3655
Königsberg	Cohn	= C	3	1	1	1	1	*A.N.* 3655
	Struve	= S	1	12	4	12	4	*A.N.* 3760
Kremsmünster,	Schwab		2	8	1	8	1	*A.N.* 3760
Leipzig	Hayes		3	1	1	1	0	*A.N.* 3654
Lick	Crawford	= C	3	4	1	4	1	*A.J.* 484
	Aitken	= A	1	10	3	10	2	*A.J.* 490
	Perrine	= P	3	3	2	3	2	*A.J.* 494
Lemberg	Ernst		2	15	½	15	0	*A.N.* 3655, 3740
Nicolaeff	Kortazzi		1	11	2	11	3	*A.N.* 3677
Northampton	Byrd		3	2	1	2	1	*A.J.* 495
Paris	Bigourdan		3	1	0	1	0	*A.N.* 3654
Padua	Autoniazzi		1	13	3	13	3	*A.N.* 3678
Pola	Hohl	= H	3	3	½	3	½	*A.N.* 3661
	Marchetti	= M	3	1	0	1	0	*A.N.* 3661
Poughkeepsie	Whitney		3	2	1	2	1	*A.J.* 495
Pulkowa	Sokolow		3	2	2	2	2	Correspondence.
Rome	Millosevich		3	4	2	4	2	*A.N.* 3655, 3745
Strassburg	Kobold		1	10	1	10	1	*A.N.* 3654, 3726
Utrecht	Nijland	= N	1	12	3	13	3	*A.N.* 3654, 3719
	Veenstra	= V	1	11	2	11	2	*A.N.* 3719
Vienna	Palisa	= P	3	2	½	2	½	*A.N.* 3655, 3718
	Holetschek	= H	3	3	½	3	½	*A.N.* 3713

4. PERTURBATIONS.

Perturbations taking into account the action of *Mercury*, *Venus*, *Earth*, *Mars*, *Jupiter* and *Saturn* were computed in rectangular coordinates referred to the ecliptic, with July 29.0 as the date of osculation. A uniform interval of 10 days was adopted for *Mars*, *Jupiter* and *Saturn*, while for *Mercury*, *Venus* and *Earth*, the interval was 10 days until September 17, after which it was 20 days. Referred to the equator as fundamental plane the perturbations in rectangular coordinates deduced were as follows:

PERTURBATIONS (Seventh decimal place).

Date		δx	δy	δz
July	19	− 1	0	0
	29	0	0	0
Aug.	8	− 11	+ 0	− 3
	18	30	+ 1	12
	28	55	+ 10	− 25
Sept.	7	− 86	+ 20	− 43
	17	121	+ 33	67
Sept.	27	−160	+ 49	96
Oct.	7	−201	+ 67	131
	17	−244	+ 88	172
	27	−288	+110	−219

5. NORMAL PLACES AND LEAST-SQUARE SOLUTION.

For the construction of normal places the list of observations was divided into eight sections at the following dates: July 29.0, Aug. 3.0, Aug. 11.0, Aug. 20.0, Sept. 1.0, Sept. 11.0, and Oct. 10.0. To find a normal place with its corresponding time and weight, the following equations were applied to the observations of each section:

$$t_0 = \frac{[pt]}{[p]}, \quad \cos\delta_0 . l\alpha_0 = \frac{[p.\cos\delta . l\alpha]}{[p]},$$

$$l\delta_0 = \frac{[p.l\delta]}{[p]} \text{ and } p_0 = [p]$$

The normal places thus found, including corrections for perturbations, are as follows

No.	Date	R.A.	(cos δ . Δα)	p	Decl.	(Δδ)	p
		h m s	s		° ′ ″	″	
1	July 26.7	11 37 49.81	−3.98	115	20 51 8.27	− 0.01	114
2	31.7	13 9 30.24	−2.21	116	36 9 55.38	2.88	113.5
3	Aug. 8.1	16 51 52.18	− 2.90	76.5	57 11 19.99	− 2.26	70
4	16.1	56 36 8.51	−3.28	72	75 11 17.31	+ 3.50	65
5	24.84	116 19 3.02	+5.46	61	85 42 33.68	+ 3.52	65
6	Sept. 8.62	204 2 46.37	+8.68	50	77 33 18.50	− 9.82	51
7	22.0	213 22 18.83	+8.37	61	71 23 36.87	−13.42	60
8	Oct. 19.1	224 5 18.06	+4.11	41	65 58 25.52	−16.37	41

For the least-square solution the elements were referred to the mean equator of 1900.0 and differential coefficients computed by means of the formulas given in KLINKERFUES, *Theoretische Astronomie, Zweite Auflage.*

The elements so referred are

$T = 1900$ Aug. 3.19930 Gr. M.T.

$$\left.\begin{aligned} \omega' &= 0^\circ\ 9'\ 27''.35 \\ \Omega' &= 331\ 43\ 41.17 \\ i' &= 82\ 52\ 37.55 \end{aligned}\right\}\text{Equator 1900.0}$$

$\log q = 0.0063830$

Equations of the form

$$\cos\delta . \frac{\partial\alpha}{\partial\Omega'} d\Omega' + \cos\delta . \frac{\partial\alpha}{\partial i'} di' + \cos\delta \frac{\partial\alpha}{\partial\omega'} d\omega' + \cos\delta . \frac{\partial\alpha}{\partial T} dT + \cos\delta . \frac{\partial\alpha}{\partial q} dq + \cos\delta . \frac{\partial\alpha}{\partial e} de = \Delta\alpha . \cos\delta$$

$$\text{and} \quad \frac{\partial\delta}{\partial\Omega'} d\Omega' + \frac{\partial\delta}{\partial i'} di' + \frac{\partial\delta}{\partial\omega'} d\omega' + \frac{\partial\delta}{\partial T} dT + \frac{\partial\delta}{\partial q} dq + \frac{\partial\delta}{\partial e} de = \Delta\delta$$

each made homogeneous and multiplied by the square root of its weight, after putting $x = d\Omega'$, $y = di'$, $z = d\omega'$, $t = 10^4 dT$, $u = 10^6 dq$ and $w = 10^4 de$, become the following:

EQUATIONS OF CONDITION.

Right-Ascension.

1	$+7.7683\,x$	$+1.4477\,y$	$-2.9631\,z$	$+0.4528\,t$	$-4.6772\,u$	$-1.0195\,w$	=	$-42''.6807$
2	$+7.7522$	$+0.4159$	-0.3558	-0.1698	-4.7957	-0.3213	=	-24.1255
3	$+5.0272$	-0.5654	$+2.7374$	-0.8271	-3.6921	$+0.3836$	=	-25.3646
4	$+1.8913$	-0.4116	$+1.7915$	-1.2005	-3.2057	$+0.2169$	=	-27.8317
5	-8.1736	$+1.6175$	$+2.2463$	-0.2434	-1.4365	-3.1798	=	$+12.6440$
6	-4.4486	$+3.7781$	-5.1038	$+1.2800$	$+1.6025$	-1.1228	=	$+61.3769$
7	-3.1791	$+3.5780$	-6.9729	$+1.4305$	$+1.8471$	$+1.3630$	=	$+66.9600$
8	-1.3642	$+2.0688$	-6.6233	$+1.0666$	$+1.5173$	$+6.6962$	=	$+26.2627$

Declination.

1	$-\ 8.0679\,x$	$-1.8922\,y$	$+20.6923\,z$	$-10.2847\,t$	$-0.2164\,u$	$-19.6231\,w$	=	$-\ \ 0''.4271$
2	-13.7908	-0.9145	$+17.7338$	$-\ 8.7714$	-0.8230	$-\ 5.5256$	=	$-\ 30.6825$
3	-14.1487	$+1.8631$	$+\ 8.0771$	$-\ 3.8545$	-0.8764	$+\ 4.5982$	=	$-\ 18.9085$
4	-13.4567	$+1.3590$	$+\ 2.2198$	$-\ 1.0104$	-0.3191	$+\ 3.0084$	=	$+\ 28.2479$
5	$-\ 6.8807$	$+3.4364$	$-\ 4.7621$	$+\ 1.0979$	$+2.1503$	$+\ 0.3760$	=	$+\ 28.3792$
6	$+\ 4.5611$	-1.7897	$-\ 1.8458$	$-\ 0.2072$	$+1.0876$	$+11.6799$	=	$-\ 72.1620$
7	$+\ 3.5866$	-5.7932	$-\ 1.2971$	$-\ 0.4515$	$+0.7335$	$+19.0539$	=	-103.9509
8	$+\ 1.3023$	-5.4001	$-\ 0.9393$	$-\ 0.3970$	$+0.3366$	$+21.4661$	=	-104.8192

in which coefficients are given in natural numbers.

These differential coefficients were checked according to the method given in the above reference with very slight modification. To check

$$\frac{\partial\alpha}{\partial T}, \frac{\partial\alpha}{\partial q}, \frac{\partial\alpha}{\partial e}, \frac{\partial\delta}{\partial T}, \frac{\partial\delta}{\partial q} \text{ and } \frac{\partial\delta}{\partial e}$$

as well as the computation of the positions to which residuals had been applied in finding the normal places, increments appropriate for each date were successively applied to T, q, and e, and new positions computed from the slightly changed elements. The remaining coefficients were checked by finding $d\log\xi$, $d\log\eta$ and $d\log\zeta$ where the notation is that of the reference.

To illustrate (KLINKERFUES, page 710).

$dx = -y\,d\Omega'$ or $d\log\xi = -\dfrac{m}{206265}\,\dfrac{y}{\xi}\,(d\Omega')''$ where $(d\Omega')''$ is given in seconds of arc. This in units of the seventh place becomes

$$d\log\xi = [1.82336_n]\,\frac{y}{\xi}\,(d\Omega')''$$

where numbers are expressed in logarithms. Similar expressions for

$$d \log \xi, \quad d \log \eta \text{ and } d \log \zeta$$

were found in terms of $(d\Omega')''$, $(di')''$, and $(d\omega')''$.

Whether the numerical values of $\log \xi$, $\log \eta$ and $\log \zeta$ were to be increased or decreased in a positive [illegible] easily determined by inspection of the equations.

From the equations of condition normal and elimination equations given below were obtained. As a [illegible] computing machine was employed, coefficients are given in natural numbers.

Normal Equations.

$$
\begin{array}{rrrrrrcr}
+1605.4289\,w & -580.7418\,z & +287.9879\,x & +216.6062\,t & -235.4222\,y & +59.0753\,u & = & -5097.5541 \\
-580.7418 & +1007.7049 & -506.0848 & -439.4582 & -82.1140 & -85.1039 & = & -1361.9109 \\
+287.9879 & -506.0848 & +975.6906 & +248.6591 & -188.6705 & -78.1787 & = & -2284.0715 \\
+216.6062 & -439.4582 & +248.6591 & +207.4111 & +37.9990 & +27.0762 & = & +681.7583 \\
-235.4222 & -82.1140 & -188.6705 & +37.9990 & +179.8256 & -2.0807 & = & +2406.0835 \\
+59.0753 & -85.1039 & -78.1787 & +27.0762 & -2.0807 & +87.1847 & = & +604.2810
\end{array}
$$

Elimination Equations.

$$
\begin{array}{rrrrrrcr}
w & -0.362187\,z & +0.179608\,x & +0.135089\,t & -0.116824\,y & +0.036843\,u & = & -3.179158 \\
 & z & -0.503882\,x & -0.452748\,t & -0.209917\,y & -0.079897\,u & = & -4.023466 \\
 & & x & +0.038600\,t & -0.319781\,y & -0.167550\,u & = & -4.137199 \\
 & & & t & +0.214254\,y & -0.371981\,u & = & +2.643181 \\
 & & & & y & -1.241788\,u & = & +0.612815 \\
 & & & & & u & = & +28.268782
\end{array}
$$

Whence

$$
\begin{array}{ll}
u = +28.269 & x = +11.81 \\
y = +35.72 & z = +14.18 \\
t = +\ 5.5065 & w = +\ 3.2929 \text{ and } [nn.6] = 807.6
\end{array}
$$

Reversed elimination gave identical results and also quantities by means of which were obtained

$$
\begin{array}{ll}
r_u = \pm 1.0484 & r_t = \pm 1.6997 \\
r_z = \pm 2.60 & r_y = \pm 4.70 \\
r_x = \pm 2.21 & r_w = \pm 3.691
\end{array}
$$

Substitution in the equations of condition gave as check $[vv] = 807.6$. We have, therefore, the following corrections to the equatorial elements

$$
\begin{array}{llll}
d\Omega' = +11''.81 & \pm 2''.21 & dT = +0^d.000551 & \pm 0^d.000170 \\
di' = +35.72 & \pm 4.70 & d\eta = +0.0000283 & \pm 0.0000037 \\
d\omega' = +14.18 & \pm 2.60 & d\varepsilon = +0.0003293 & \pm 0.0001048
\end{array}
$$

Collecting results the definitive equatorial elements are

Epoch of osculation, July 29.0, 1900

$T = 1900$ August 3.129851 Gr. M. T.

$$
\left.
\begin{array}{l}
\omega' = \ \ 0^\circ\ \ 9'\ 11''.53 \\
\Omega' = 331\ 43\ 52.98 \\
i' = \ \ 82\ 53\ 13.27
\end{array}
\right\}
\text{Equator 1900.0}
$$

$$
\begin{array}{l}
\log q = 0.0063951 \\
e = 1.0003293 \\
\log e = 0.0001430
\end{array}
$$

from which are found

$$
\begin{array}{l}
x = r\,[9.9458067]\sin(86^\circ\ 21'\ \ 7''.40 + v) \\
y = r\,[9.6866365]\sin(283\ \ 7\ 44.81 + v) \\
z = r\,[9.9966447]\sin(\ \ 0\ \ 9\ 11.53 + v)
\end{array}
$$

and also

$$
\left.
\begin{array}{l}
\omega = \ \ 12^\circ\ 25'\ 10''.55 \\
\Omega = 328\ \ 0\ 47.62 \\
i = \ \ 62\ 31\ 16.38
\end{array}
\right\}
\text{Ecliptic of 1900.0}
$$

As a final check on the accuracy of the computation the residuals due to elements and those due to equations were determined for each normal place. They are as follows:

	Elements		Equations	
	$\Delta\alpha \cos\delta$	$\Delta\delta$	$\Delta\alpha \cos\delta$	$\Delta\delta$
July 26.7	+1.1	+0.3	+1.0	+0.3
" 31.7	−1.1	−0.3	1.0	0.3
Aug. 8.1	−0.1	0.1	0.5	0.2
" 16.1	+1.1	−0.7	+0.8	0.5
" 24.81	+0.8	+1.7	+0.7	+1.7
Sept. 8.62	1.0	0.4	1.0	0.4
" 22.0	−1.4	+0.2	1.1	+0.3
Oct. 19.4	+1.1	0.3	+1.1	+0.3

From the equations [illegible] — 11.2

Collecting the definitive elements referred to the ecliptic are

Epoch of osculation, July 29.0, 1900

$T = 1900$ August 3.129851 Gr. M. T.

ω = 12 25 10.55

Ω = 328 0 47.62 Ecliptic 1900.0

i = 62 31 16.38

[illegible] 0.006[illegible]

[illegible] 0.0001[illegible]

[illegible]

[illegible]

Accordingly ω^2X, ω^2Y, and ω^2Z (WATSON, page 155) were computed for *Venus*, *Earth*, *Mars*, *Jupiter*, and *Saturn* at intervals of 40/3 days. These quantities were directly computed for *Mars* and *Saturn* at intervals of 40 days from June 4.0, 1899, to Aug. 18.0, 1900. Likewise they were computed for *Jupiter* until May 30, 1900, after which date they were directly computed at intervals of 40/3 days. All remaining intermediate values were found by interpolation. For *Venus* and the *Earth* the quantities were directly computed at intervals of 40/3 days throughout the whole period. The quantities

$$\Sigma(\omega^2X),\ \Sigma(\omega^2Y)\ \text{and}\ \Sigma(\omega^2Z)$$

were obtained from which mechanical quadrature gave the perturbations referred to the ecliptic in the table below. The computing machine was used whenever possible. In this table perturbations at intervals of 40 days only are included.

PERTURBATIONS. (Seventh decimal place).

Date	δx	δy	δz
1899 June 4	− 35	+291	+5413
July 14	−305	+244	+4259
Aug. 23	−483	+206	+3257
Oct. 2	−569	+173	+2401
Nov. 11	−575	+156	+1689
Dec. 21	−528	+154	+1116
1900 Jan. 30	−457	+151	+ 677
Mar. 11	−367	+126	+ 360
Apr. 20	−257	+ 75	+ 154
May 30	−129	+ 21	+ 41
July 9	− 21	+ 0	+ 1

The disturbances in the components of the velocity on June 4.0 were found to be $\delta\frac{dx}{dt} = -7.9$, $\delta\frac{dy}{dt} = -1.4$, $\delta\frac{dz}{dt} = -30.8$, in the same units.

Following the method of WATSON, Sec. 168, the definitive elements gave for this date the undisturbed coordinates

$x_0 = -3.4959772$; $y_0 = -0.6369125$; $z_0 = -1.2938069$;

and the velocities

$\frac{dx_0}{dt} = +0.00857336$; $\frac{dy_0}{dt} = -0.00182998$;

$\frac{dz_0}{dt} = +0.00574773$;

from which, after corrections for perturbations had been applied, the following elements were deduced.

Epoch of osculation, June 4.0 1899

$$\begin{aligned} T &= 1900\ \text{August}\ 3.24627 \\ \omega &= 12^\circ\ 25'\ 12''.5 \\ \Omega &= 328\ \ 4\ \ 16.6 \\ i &= 62\ 31\ 19.2 \\ \log q &= 0.006438 \\ \log e &= 0.000058 \\ e &= 1.000133 \end{aligned}$$

(ω, Ω, i: Ecliptic of 1900.0)

According to this computation, therefore, the eccentricity had increased during the 400 days previous to the comet's discovery.

Among the disturbances caused by the planets during this period those of *Jupiter* were the most important during the greater part of the time, while those of *Saturn* and *Mars* were much less, as were also those of *Venus* and *Earth*, except near the time of discovery. Therefore the quantities, ω^2X, ω^2Y, ω^2Z for *Jupiter* alone were computed for four dates at intervals of 100 days previous to June 4, 1899. When the position and motion of the comet during this time are considered, ω^2X, ω^2Y, ω^2Z, indicate that the effect of perturbations during this earlier period was acting in the same direction.

In the light of this preliminary study it seems not unreasonable therefore to account for this hyperbolic orbit at the time of discovery as the result of perturbations, and to conclude that a careful study sufficiently extended would show that this comet entered the solar system in a sensibly parabolic orbit.

7. CONCLUSION.

Defective though it is, this piece of work is now brought to a point where it is possible for the computer to lay it aside. The long computations involved would certainly never have reached even their present degree of perfection had it not been for the continued encouragement and many suggestions received from Prof. YOUNG of Princeton.

Besides being indebted to those already mentioned, I am also indebted to Prof. ANTONIO ABETTI of Arcetri for publications from that Observatory, to Prof. OTTO KNOPF of Jena for a correction to his observations as published, and to Prof. LOVETT and Mr. HILTEBEITEL of Princeton, for actually assisting in the computations.

Shattuck Observatory, Dartmouth College, Hanover, N.H.

OBSERVED MINIMA OF 4.1903 *DRACONIS*.

BY W. M. REED.

[Communicated by Prof. C. A. YOUNG.]

Seven minima of this star have recently been observed by Mr. ZACCHEUS DANIEL and the author with a photometer attached to the 23-inch equatorial of the Halsted Observatory. A part of the expense of the these observations is borne by the Carnegie Institution. An artificial star, caused by an electric light, was compared alternately with a neighboring comparison-star and with the variable. The light of the artificial star was diminished by means of

a "photometric wedge" photographically prepared, the gift of Prof. E. C. PICKERING. The author is indebted to Mr. E. S. KING for a provisional value of the scale of this wedge. It seems desirable to make a preliminary statement of these observations in order to call attention to certain unprecedented irregularities in this Algol-type variable. Although the minima occur with great regularity, and satisfy the elements of Mr. BLAŽKO with a correction of only 1.3 minutes, yet the magnitude at minimum is far from constant. In the seven minima observed the magnitude at minimum has ranged from 12.7 to 13.6. Also the shape of the light-curve has undergone equally pronounced changes.

On July 7, the star diminished in brightness at the rate of about $0^{M}.10$ in eight minutes, until within 30 minutes of minimum, when it decreased at the rate of $0^{M}.10$ in two minutes. When the minimum was reached the increase began almost at once, and in a manner symmetrical with the decrease.

On July 15, the star decreased in light at the rate of about $0^{M}.10$ in eight minutes, until about 20 minutes from minimum, when the rate increased to about $0^{M}.10$ in one minute. It then staid constant in brightness at minimum light for 26 minutes. The increase was symmetrical with the decrease.

August 14, the third minimum was observed. The decrease was at a fairly uniform rate until the minimum light was reached, the rate being about $0^{M}.10$ in 4 minutes. The variable staid at minimum brightness for about 20 minutes. On account of clouds only one magnitude of the increase was observed. The rate of the increase was about $0^{M}.10$ in one minute, and therefore unsymmetrical with the decrease.

Setember 21, the variable began to decrease $2^{h}\ 12^{m}$ before the time of minimum. At first the rate was about $0^{M}.10$ in 12 minutes; gradually the rate was increased to 4 minutes, and during the last magnitude of decrease the rate was $0^{M}.10$ in 2 minutes. The star remained at minimum light for 15 minutes. The curve of increase was not symmetrical with the decrease. At first the rate was $0^{M}.10$ in 4 minutes, then in 2 minutes, and finally $0^{M}.10$ in 9 minutes. Observations were discontinued before the variable reached normal light.

September 25, the decrease was at a nearly uniform rate of $0^{M}.10$ in 5 minutes. The star remained at minimum light for 32 minutes. During the first 10 minutes of the increase the star became brighter at the rate of $0^{M}.10$ in 3 minutes, but during the next 40 minutes the increase was at the rate of only $0^{M}.10$ in 13 minutes.

September 29, the increase and decrease were nearly symmetrical. The decrease occupied $1^{h}\ 45^{m}$ in changing 2.2 magnitudes, while the increase occupied $1^{h}\ 34^{m}$ for the same range. The light remained constant at minimum for 21 minutes.

Epoch	Observed Minimum	O−C	[illegible]	[illegible]	[illegible]
	d h m	m			
93	July 7 15 21.8	−0.9	13.7	[illegible]	[illegible]
99	15 18 50.2	0.8	12.8	[illegible]	[illegible]
121	Aug. 14 15 11.2	+2.7	13.2	[illegible]	
124	18 17 9.6	+1.1	13.6		[illegible]
149	Sept. 21 15 57.5	0.3	13.2	15	[illegible]
152	25 17 31.8	0.9	12.7	[illegible]	45
155	29 19 12.5	2.6	13.0	21	249

The epochs are reckoned from the [illegible] elements recently published by Mr. S. BLAŽKO in A. N. [illegible]

Min. = 1903 Mar. 3 $9^{h}\ 34^{m}$ Gr. M.T. + $1^{d}\ 8^{h}\ 31^{m}$ 4.5 E

The observed minima are given in Geocentric Gr. M.T. The moment midway in the "stationary period" has been chosen as the time of minimum. On August 18 the grouping of the observations was such that the exact length of the stationary period could not be determined. The increase and decrease were so rapid that a fairly accurate time of minimum was found. The column headed O−C gives the difference between the observed minima and those computed from the above elements. The fourth column gives the magnitude at minimum derived from a preliminary value of the magnitude of the comparison-star. While the relative value of these magnitudes are accurate the absolute value of the scale may be subject to correction. The fifth column, headed "stationary period," gives the number of minutes during which the variable was sensibly constant in brightness at minimum. The last column gives the number of settings of the instrument on both the variable and the comparison-star.

There seems to be a relation between the time during which the star is nearly constant in brightness and the magnitude at minimum. It is obvious from a comparison of the fourth and fifth columns that the [illegible] at minimum the shorter is the stationary period.

On August 18, the faintest [illegible] the duration of the "stationary period" must have been very brief, but the grouping of the observations was such as to give no positive information as to its actual length. In this case the increase and decrease were so rapid [illegible] minimum light that the time of minimum could be fairly accurately determined.

It is interesting to note that if the eclipsing body were a double star, in which the components were close together, that the phenomena of these curves could be in part explained. Such a hypothesis would require that occasionally there should be a "standstill" on the curve, when first one body should enter upon the eclipse position followed after a short interval by the other. Such irregularities are suspected from the observations now accumulated. But a more thorough set of observations must be made before this interesting feature can be regarded as proved.

Princeton University, 1903 Oct. 12.

WOLF'S "NEW STAR" IN *CYGNUS*.

BY E. E. BARNARD.

Telegraphic announcement was received on Oct. 5 of the discovery of a new star 11^{M}, by Dr. MAX WOLF, in the position,

$$1903.0 \quad 20^{h}\ 14^{m}\ 57^{s}.0 \quad +37^{\circ}\ 9'\ 19''$$

Observations of this object were made here on the same evening with the large telescope. The star was estimated as $10\frac{1}{2}^{M}$, and was very red.

The following measures were made of its position with reference to Lund A.G.C. 9237 ($8^{M}.9$).

$$\Delta\alpha\ 74''.99\ (4\ \text{Obs.}) = 0^{m}\ 6^{s}.27 \quad , \quad \Delta\delta\ 13''.72\ (4\ \text{obs.})$$

The new star was south following.

Following is the position of the comparison-star.

$$1903.0 \quad 20^{h}\ 14^{m}\ 50^{s}.75 \quad +37^{\circ}\ 10'\ 1''.1$$

From this the position of WOLF's star is

$$1903.0 \quad 20^{h}\ 14^{m}\ 57^{s}.02 \quad +37^{\circ}\ 9'\ 47''.4$$

It was also referred to the comparison-star by position angle and distance.

$$\text{P.A. } 100^{\circ}.22\ (6\ \text{obs.}) \quad , \quad \text{Dist. } 76''.11\ (5\ \text{obs. double dist.})$$

In the D.M. are two stars whose places for 1855.0 are

D.M.	+37.3875	$9^{m}.1$	$20^{h}\ 13^{m}\ 1^{s}.0$	$+37^{\circ}\ 0'.1$
D.M.	+37.3876	9.5	20 13 9.6	+37 0.4

The first of these is A.G.C. Lund 9237. The second occupies exactly the position of WOLF's object, and as there is no other star at this point, I assume that the "new star" is identical with D.M. +37°.3876 though it is at least one magnitude less than that given in D.M.

I have a trial plate, made May 8, 1902, at $11^{h}\ 15^{m}$ — $12^{h}\ 0^{m}$, with the 10-inch Brashear doublet for the Bruce telescope — then being tested here, that covers the region in question. WOLF's star is strongly shown on it. It appears to be roughly 2 magnitudes less than the Lund star preceding it. The magnitude is uncertain, but roughly it is what would be expected to result from the present red color and magnitude of the star.

The star was examined by Messrs. FROST, ADAMS and REESE through a small prism placed over the eyepiece of the micrometer. But they were unable to detect bright lines in its spectrum or to note any particular resemblance to the visual spectrum of *Nova Persei* when of about the same magnitude, although the dispersion employed renders definite statements difficult. The seeing was bad which made observations somewhat uncertain.

Mr. J. A. PARKHURST measured the brightness of the star with the photometer on the 12-inch and found it, at $9^{h}\ 15^{m}$ (Central Standard time), to be $10^{M}.6$ on the Harvard College Scale of magnitude.

Measures of the position of two small stars near WOLF's "Nova" = D.M. +37°.3876.

+37°.3876 and 13^{M} star n.f.

1903 Oct. 5 $30^{\circ}.11$ (5 obs.) $19''.19$ (8 obs.)

+37°.3876 and 14^{M} star s.p.

1903 Oct. 5 $206^{\circ}.52$ (5 obs.) $66''.17$ (8 obs.)

It has not been thought necessary to further measure these stars.

Yerkes Observatory, 1903 *Oct.* 6.

NOTE ON WOLF'S "NEW STAR" OF SEPTEMBER 21, 1903.

BY HERBERT A. HOWE.

In a recent Astronomical Bulletin of the Harvard College Observatory, the declination of WOLF's new star is given as +37° 9′ 19″ (1903.0), as per telegram from Kiel. This object is of 0′.2 south of Lund A G. Catal. 9237, by an observation made at the Chamberlin Observatory on Oct. 9. Unfortunately the catalogued declination of this star is 1′ too great; this fact has been determined at the Chamberlin Observatory by micrometrical comparison of the star with Nos. 9236, 9221, and 9211 of the same catalogue. The declination of the "new star," as determined by a rough measure, is +37° 8′ 49″ (1903.0). On Oct. 10 the "new star" seemed to be of 10^{M}, and to be quite red. It appears to be identical with one of the stars in the Birmingham Red Star Catalogue, which is called Es.-Birm. 662*a* on p. 104 of the revised edition of WEBB's *Celestial Objects for Common Telescopes*; the declination there given is 3′ too small.

CONTENTS

PUBLISHED AT 8 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed Oct.* 30.
Entered February 2, 1903 at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

Nos. 549-550.

VOL. XXIII. BOSTON, 1903 NOVEMBER 25. NOS. 21-22

WEIGHTS AND SYSTEMATIC CORRECTIONS OF MERIDIAN OBSERVATIONS IN RIGHT-ASCENSION AND DECLINATION.

By LEWIS BOSS.

The systematic corrections published in this paper were, in all cases, determined from comparison with the finally computed positions of the Standard Stars (System B., *A.J.* 531–2). There were 699 of these stars, of which 265 are south of $-22°$. Sixty-six of the latter (with six others), the motions of which have been less accurately determined, have been excluded from publication for the present.

The adopted systematic corrections for right-ascensions are contained in Tables I, II and III; those for declinations, in Tables IV and V. They do not differ materially from the corrections which were employed in computing the positions for the Catalogue, except that corrections for magnitude-equation were not employed in deriving those positions.

If Table I be employed to free the individual catalogues from the effect of magnitude-equation, the Standard Catalogue itself can be made consistent with the right-ascensions thus corrected by application of the correction,

$$-0^s.0077\ (M-3.5).$$

In this way, also, the right-ascensions of the Standard Stars would be prepared for consistent use in the reduction of transits which have been properly corrected for magnitude-equation. The Standard Catalogue, corrected in this way, has already been employed with distinct advantage in the reduction of transits for the Albany Catalogue, 1896–1900.

The derivation of the magnitude-equation, $\Delta\alpha_m$, Table I, has already been explained (*A.J.* 536). It only remains to add that the effect of the correction has been assumed to apply with sufficient accuracy down to the seventh magnitude. One may extend the corrections by extrapolation to still fainter magnitudes, but may anticipate that, in many instances, the quantities so determined may diverge materially from the truth. In some cases the material for determination of $\Delta\alpha_m$ is quite scanty. For Piazzi, Greenwich 15, Dorpat 15, Königsberg 15, Cape 30, Armagh 40, Cape 50, Santiago 55 and 60, and Pulkowa 1892, it was deemed safest to assume the mean value of the magnitude-equation, $-0^s.0077\ (M-3.5)$, to be applicable to the right-ascensions of those catalogues. For nearly all catalogues of a date earlier than 1850, $\Delta\alpha_m$ is quite uncertain, owing to small weight of material for its determination.

The adopted formula of correction in computing $\Delta\alpha_s$, Table II, and $\Delta\delta_s$, Table IV, is,

$$a \sin\alpha + b \cos\alpha + c \sin 2\alpha + d \cos 2\alpha$$

Terms in 2α have seldom been adopted, and in instances where they were employed the object is not so much to represent a known source of error (as in the case of terms of single period) as it is to use them in lieu of a graphic solution. It is especially desirable that the part of systematic correction of the form, $a \sin\alpha + b \cos\alpha$, at least, should be removed from the individual series of right-ascensions and declinations, since the equations which express the effects of various forms of sidereal rotation, of solar motion, and of correction for imperfect precession, contain terms of this form. A term of the form,

$$\Delta\alpha_s = (a' \sin\alpha + b' \cos\alpha)\tan\delta$$

is sometimes employed for the right-ascensions. In Table II the values of that part of $\Delta\alpha_s$ contained in parenthesis is given on the line below $\Delta\alpha_s$ for the catalogue in question. Therefore, in order to form $\Delta\alpha_s$, the numbers of the second line for the respective catalogues (opposite "$\times$ (tan δ)" in the margin) must be multiplied by tan δ and then added to the corresponding numbers of the first line; so that the entire correction, having the argument, right-ascension in whole or in part, is $\Delta\alpha_s + \Delta\alpha$.

The supposed equinox-corrections are always included in $\Delta\alpha_s$, Table II; since it seems to be desirable that $\Delta\alpha$ should exhibit clearly the discrepancies between the meridian of the standard and those of the individual catalogues.

In Table III, which exhibits values of $\Delta\alpha_\delta$, the values of $\Delta\alpha_\delta \cos\delta$ for 85° and 80° of declination are also given in order to facilitate interpolation. These can usually be extrapolated for declinations higher than 85°. The curves of correction were originally determined for $\Delta\alpha_\delta \cos\delta$. In

many cases it would be mechanically impracticable to determine them otherwise.

It is scarcely necessary to remark that the determinateness of the several curves of correction varies greatly,—from the uncertainty which pertains to such catalogues as those of PIAZZI, Madras, Armagh, Cape 50, and Santiago, to the definiteness which belongs to the better modern catalogues. What a simple matter the drawing of a curve of correction may become is illustrated by the following statement of observed values of $\Delta\alpha_s\cos\delta$, or $\Delta\delta_s$, for six catalogues, selected as fair representatives of a large number of the better class of modern catalogues.

OBSERVED SYSTEMATIC CORRECTIONS.

	$\Delta\alpha_s\cos\delta$			$\Delta\delta_s$		
δ	Wn. 75	Strass. 85	Lisb. 90	Paris 60	Grw. 80	Pulk. 85
	s	s	s	″	″	″
+77	−.015	+.023	−.016	−.16	+.02	+.06
70	−.006	−.015	. .	−.03	+.26	−.03
65	−.001	+.025	−.002	+.19	+.16	+.13
60	−.010	+.013	+.001	−.01	+.25	+.01
55	−.019	+.016	−.001	+.25	+.12	+.05
50	−.023	+.011	−.006	+.22	+.09	+.01
45	−.024	+.012	−.006	+.06	+.12	+.05
40	−.005	+.003	−.008	−.08	−.05	+.09
35	+.010	−.001	−.004	−.25	−.06	+.11
30	+.007	−.002	−.002	−.16	−.06	+.19
25	+.011	−.007	−.005	−.14	+.26	+.21
20	+.008	−.002	−.001	−.25	+.15	+.25
15	+.011	−.008	+.003	−.27	+.29	+.25
10	+.006	−.008	+.006	−.31	+.40	+.24
+ 5	−.009	+.005	+.002	−.24	+.12	+.29
0	−.007	+.010	+.006	−.21	−.02	+.18
− 5	−.012	+.012	+.007	−.15	+.26	+.21
10	−.011	+.005	−.008	−.17	+.37	+.20
15	−.010	+.002	−.006	−.09	+.43	+.02
20	+.007	−.003	−.008	+.11	+.50	−.03
−25	+.003	−.003	−.006	+.51	+.59	. .

The preceding table shows how small are likely to be the remaining systematic uncertainties in reducing a single star-catalogue to a given system of Standard Stars. The consequent inference would be that, in any future investigations involving the motions derived from thousands of stars, the degree of systematic accuracy attainable would practically be measured by that of the Standard Catalogue itself. Thus, aside from certain incidental reservations, connected with the magnitude-equation and similar points, the question whether systematic errors can be avoided in researches upon precession, solar motion, and the like, resolves itself into the question, what degree of freedom from systematic error can be attained in the construction of extensive standard catalogues. The obvious advantage of this rests in the probability that a general discussion of stellar motions based upon the few large catalogues of observation would practically enjoy whatever of freedom from systematic error attaches to the mean of all the best observations ever made.

In drawing the curves it is of very great importance that due regard be had for the weights of the observed quantities, and that constant errors, pertaining to comparatively broad zones, be avoided. The latter requirement was kept constantly in mind. As an aid in the fulfillment of these requirements the means by weight of successive zones were formed in the combinations: +77°, 70°, 65°; +70°, 65°, 60°; +65°, 60°, 55°, etc. Thus, when these are plotted, we have a series of points, at intervals of 5°, each representing the mean observed correction for a zone 15° in breadth. As a matter of fact, for nearly all the modern catalogues (and for the better part of the older) these means defined the curves very closely; so that very little was left in doubt as to the true location of the curve at any point. Of course the maxima and minima points were less sharply indicated by the means of 15°-zones; yet it was rarely deemed best to follow up sharper inflections apparently indicated by the 5°-means. The entire process, in its practical working, tends to inspire confidence in the reality and substantial accuracy of the principal features of curves, as indicated in Tables III and V.

It may not be superfluous to call attention again to the object of systematic correction of meridian observations. The object of first importance is to obtain positions and motions of stars, in large numbers, which shall be, in the mean, as free as possible from the effects of systematic error. A secondary object is to secure greater accuracy in the computed position and motion of an individual star. Some computers seem to look upon this secondary purpose as the only one worth considering; and they are apparently disappointed if each adopted systematic correction does not manifestly improve the accordance of the various catalogues in each individual instance. From this point of view a systematic correction of 0″.1 is certainly of no importance. But when we are considering stellar motions in large numbers for determination of the apex of solar motion, for example, a systematic error of 0″.1 in the centennial motions might mean an error of more than one degree in the determination of the apex. In all such researches the casual errors of observation can be reduced to a role of minor importance by including a sufficient number of stars and by taking advantage of all the principal series of observations already on record.

The Tables of $\Delta\alpha_s$ and $\Delta\delta_s$ apply to the indiscriminate means of observations at upper and lower culminations, as printed in the respective catalogues. Doubtless, greater precison could have been reached through the separation of the two classes of observation. Except in a few cases, however, this separation would have been either impossible or impracticable. On the other hand, some effort has usually been exerted by the observers to bring observations at lower culmination into substantial harmony with those made at upper culmination; and even where this has not

been accomplished we may still obtain a mean systematic correction by the treatment of the results indiscriminately.

For BRADLEY 1755 (AUWERS), St. Helena 1830, Cape 1833, and Madras 1875, it has been assumed that graduation errors form an important part of the systematic corrections required. The facts relative to BRADLEY'S declinations have been set forth in *A.J.* 545; and relative to Cape 1833 (HENDERSON) in *A.J.* 541. The particulars regarding Madras 1875 are given further on, in Note 17.

Pulkowa 1855 is not included in Tables I, II and III, since it does not contain observed right-ascensions of the principal standard stars. Meanwhile, the correction of its right-ascensions may be assumed to be the mean between those applicable, respectively, to Pulkowa 1845 and Pulkowa 1865, upon which it was based; though this process may not yield a very accurate result.

In general, no attempt has been made to ascertain the systematic corrections applicable to the various zodiacal catalogues, to annual results not collected in the form of a catalogue, to various catalogues of limited extent, and to certain modern catalogues that contain very few observations of the principal stars. The requisite computations for ascertaining the systematic corrections of these various classes of catalogues can be attended to with much greater precision and economy of labor at a later stage of this investigation.

Tables VI and VII contain, respectively, the computed weights for the right-ascensions and declinations of the various catalogues. In great part they remain as they were adopted in the final approximation for determination of the positions contained in the catalogue. For many of the catalogues of smaller weight, and especially for the more extensive catalogues, the weights have been revised since the computations for the catalogue. In nearly all cases the differences from the weights previously assumed were comparatively unimportant.

The adopted unit of weight in right-ascension is supposed to correspond to a probable error of $\pm 0^s.020 \sec\delta$; and in declination, to a probable error of $\pm 0''.30$.

In the preliminary stages of the work in right-ascension I have been much indebted to the valuable tables by Dr. AUWERS, *A.N.* 3615-16. In general, the weights for large numbers of observations are assumed to be less in the tables of this paper than in those of AUWERS. The theoretical factors by which the weights of AUWERS should be multiplied in order to reduce them to the units of Tables VI and VII are 0.691 and 1.111, respectively. It has been considered advisable to regard $\pm 0''.1$ as a sort of limit of precision attainable in the determination of either coordinate of an individual star. Within this limit it is assumed that minute sources of error, beyond the skill of the observer to evade, may be at work, tending to reduce all observations of high class to one level of precision, however much one may apparently excel another in recognized sources of excellence. For the catalogues of a date later than 1885 the weights of the individual catalogues are largely the result of certain approximate assumptions regarding the weights of the Standard Catalogue. To have made the computations rigorously would have cost what seemed to be an unjustifiable amount of labor.

The accuracy of the tables is greatest for the number of observations most frequently occurring. The most uncertain element is the rate of increase with number of observations. The adopted formulas for computing the weights were:

$$p_\alpha = \frac{(\pm 0^s.020)^2}{K^2r^2 + \frac{r^2}{n}} \text{ for right-ascensions, and}$$

$$p_\delta = \frac{(\pm 0''.30)^2}{K^2r^2 + \frac{r^2}{n}} \text{ for declinations.}$$

In these n represents the number of observations of a star, r represents the probable error of one observation, and Kr the probable error of an infinite number of observations upon any one star. Extensive tables, in which K has values ranging from one to 0.067, permit the values of K and r to be determined with comparative ease when the material is sufficient. In general, these formulas can be regarded merely as approximations. In catalogues where the observations have been made in a variety of conditions, over a considerable period, or by several observers, it is not practicable to represent the probable errors by any simple formula. On the whole, it is believed that the method adopted leads to results for weight which are, beyond question, superior to any which can be assigned without computation, on the basis of a general judgment alone. Furthermore, it should be remembered that these weights cannot be regarded as the weights of the catalogue positions as they stand; they are the weights of the corrected positions, and they take no account of the probable error of the corrections.

The extension of the weights in right-ascension outside the equatorial zone is a process not practically capable of a high degree of accuracy in the result. For observations of transit by "eye and ear" the polar right-ascensions are usually more precise than those of a corresponding number of observations in the equatorial zone. Sometimes this is the case when the transits have been registered on a chronograph. For catalogues in reference to which we may suppose that, either the collimation, or the polar deviation, have been badly determined, the weight may be decidedly less for stars of high declination, as in the case of the two Madras Catalogues. In Table VI the weight is usually given for the equatorial zone, "Eq." Where it is also given for higher declinations it is intended

that interpolation shall be made from ±30° (or ±20°, if preferred) to the higher declination, without extrapolation from the highest declination to the pole. In the use of these weights it has been our practice to reduce them when observations of right-ascension have been made at a zenith-distance of 72°, or greater; or of declination, at a zenith-distance of 65°, or greater, according to the following table of arbitrary factors.

FACTORS FOR WEIGHTS.

ZD	R.A.	Decl.	ZD	R.A.	Decl.
<65	1.0	0.9	74	0.8	0.5
66	1.0	0.9	75	0.8	0.5
67	1.0	0.8	76	0.7	0.4
68	1.0	0.8	77	0.7	0.4
69	1.0	0.8	78	0.6	0.3
70	1.0	0.7	79	0.5	0.2
71	1.0	0.7	80	0.4	0.1
72	0.9	0.6	81	0.25	0.0
73	0.9	0.6	82	0.1	0.0

Occasionally this amount of diminution has been inferred from statements of probable error contained in the introduction to the respective catalogues.

Some references to the peculiarities of individual catalogues are contained in the series of articles upon the Standard Catalogue. Others will be found in the notes hereto appended, to which reference is made by numbers prefixed to the designation of individual star-catalogues in the tables.

TABLE I. MAGNITUDE-EQUATION.

Magnitude	2M.0	3M.0	4M.0	5M.0	6M.0	7M.0
Br. 1755	+.017	+.006	−.006	−.017	−.028	−.040
Pi. 1800	+.012	+.004	−.004	−.012	.019	−.027
Grw. 15	+.012	+.004	.004	−.012	−.019	−.027
Dpt. 15	+.012	+.004	−.004	−.012	−.019	−.027
Kgb. 15	+.012	+.004	−.004	−.012	−.019	−.027
Kgb. 25	+.018	+.006	−.006	−.018	−.029	−.041
Dpt. 30	+.017	+.006	−.006	−.017	−.028	−.040
Cape 30	+.012	+.004	−.004	−.012	−.019	−.027
St. H. 30	+.012	+.004	−.004	−.012	−.019	−.027
Abo 30	+.017	+.006	−.006	−.017	−.029	−.041
Grw. 30	+.016	+.005	−.005	−.016	−.026	−.036
Camb. 30	+.008	+.003	−.003	−.008	−.014	−.020
Cape 33	+.014	+.005	−.005	−.014	−.024	−.033
1) Madr. 35	+.024	+.008	−.008	−.024	−.039	−.055
Arm. 40	+.012	+.004	−.004	−.012	−.019	−.027

TABLE II. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\alpha$. RIGHT-ASCENSION.

R.A.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
	s	s	s	s	s	s	s	s	s	s	s	s	s
Br. 1755	−.075	−.078	−.081	−.083	−.086	−.088	−.089	−.090	−.090	−.089	−.087	−.085	−.083
Pi. 1800	+.100	+.101	+.103	+.107	+.111	+.116	+.120	+.125	+.129	+.132	+.135	+.136	+.136
× (tan δ)	+.048	+.024	−.002	−.028	−.051	−.072	−.087	−.096	−.099	−.095	−.085	−.069	−.048
Grw. 15	−.025	−.022	−.018	−.015	−.011	−.007	−.004	−.002	.000	.000	−.001	−.002	−.005
4) Dpt. 15	+.034	+.034	+.032	+.027	+.021	+.013	+.004	−.005	−.011	−.021	−.027	−.032	−.034
Kgb. 15	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082	−.082
5) Kgb. 20	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034	−.034
Kgb. 25	+.019	+.022	+.025	+.030	+.033	+.036	+.038	+.039	+.040	+.039	+.037	+.035	+.032
Dpt. 30	−.016	−.014	−.011	−.010	−.009	−.009	−.010	−.011	−.013	−.016	−.019	−.022	−.025
Cape 30	+.027	+.035	+.041	+.046	+.048	+.047	+.045	+.040	+.033	+.025	+.016	+.007	−.001
St. H. 30	−.065	−.064	−.062	−.059	−.056	−.051	−.046	−.041	−.036	−.033	−.030	−.028	−.027
Abo 30	+.014	+.014	+.015	+.016	+.017	+.017	+.018	+.018	+.018	+.018	+.018	+.017	+.016
Grw. 30	−.061	−.063	−.065	−.067	−.069	−.070	−.071	−.071	−.070	−.069	−.068	−.066	−.063
Camb. 30	−.019	−.020	−.022	−.023	−.025	−.027	−.028	−.030	−.030	−.031	−.031	−.030	−.029
Cape 33	+.006	+.006	+.007	+.008	+.011	+.014	+.017	+.021	+.025	+.028	+.031	+.034	+.035
Madr. 35	−.062	−.057	−.052	−.047	−.042	−.038	−.035	−.033	−.032	−.033	−.035	−.038	−.042
Arm. 40	+.045	+.046	+.047	+.049	+.050	+.052	+.053	+.054	+.055	+.056	+.056	+.056	+.055
Cape 40	−.001	−.005	−.007	−.010	−.012	−.014	−.015	−.015	−.015	−.013	−.011	−.009	−.006
Grw. 40	+.094	+.086	+.078	+.070	+.063	+.058	+.054	+.052	+.052	+.054	+.058	+.064	+.071
Grw. 45	+.040	+.035	+.029	+.022	+.016	+.010	+.005	+.001	−.001	−.002	−.001	+.002	+.006
Rad. 45	+.023	+.015	+.005	−.007	−.020	−.034	−.046	−.057	−.066	−.072	−.074	−.073	−.068
Pulk. 45	+.019	+.019	+.019	+.020	+.020	+.021	+.022	+.023	+.024	+.025	+.026	+.026	+.027
Paris 45	+.027	+.025	+.023	+.021	+.019	+.018	+.017	+.017	+.017	+.018	+.019	+.021	+.023
Stgo. 50	+.012	+.009	+.007	+.004	+.001	−.001	−.004	−.006	−.007	−.007	−.007	−.005	−.004
Grw. 50	+.011	+.007	+.002	.003	−.007	−.011	−.015	−.017	−.018	−.018	−.017	−.015	−.011

TABLE I. MAGNITUDE-EQUATION. *Cont.*

Magnitude		$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$	$7^M.0$
Cape	40	+.012	+.004	.004	—.012	—.020	.027
Grw.	40	—.001	.000	.000	+.001	+.002	+.003
Grw.	45	+.001	.000	.000	—.001	—.001	.002
Rad.	45	+.024	+.008	—.008	—.024	—.040	—.056
Pulk.	45	+.008	+.003	—.003	—.008	—.014	.019
Paris	45	+.007	+.002	—.002	—.007	.012	.017
Stgo.	50	+.018	+.006	—.006	—.018	—.030	—.042
Grw.	50	+.007	+.002	—.002	—.007	—.012	—.016
Cape	50	+.012	+.004	—.004	—.012	—.019	—.027
Stgo.	55	+.012	+.004	—.004	—.012	—.019	—.027
Cape	60	+.016	+.005	—.005	—.016	—.027	.038
Wn.	60	+.009	+.003	—.003	—.009	—.015	—.021
Grw.	60	+.010	+.003	—.003	—.010	—.017	—.024
Rad.	60	+.019	+.006	—.006	—.019	—.032	—.044
Stgo.	60	+.012	+.004	—.004	—.012	—.019	—.027
Melb.	60	+.014	+.005	—.005	—.014	.023	—.032
Paris	60	+.005	+.002	—.002	—.005	—.008	—.012
Grw.	64	+.013	+.004	—.004	—.013	—.022	—.030
Cape	65	+.020	+.007	—.007	—.020	—.034	—.047
Brs.	65	+.006	+.002	—.002	—.006	—.010	—.014
Harv.	65	+.020	+.007	—.007	—.020	—.033	.046
Pulk.	65	+.014	+.005	—.005	—.014	—.024	—.034
Melb.	70	+.011	+.004	—.004	—.011	—.018	—.026

TABLE I. MAGNITUDE-EQUATION. *Cont.*

	Magnitude		$2^M.0$	$3^M.0$	$4^M.0$	$5^M.0$	$6^M.0$	$7^M.0$
	Grw.	72	+.014	+.005	—.005	.014	—.024	[illegible]
1)	Madr.	75	+.015	+.005	.005	.015	.023	.034
	Wn.	75	+.013	+.004	—.004	.013	—.021	—.029
	Pulk.	75	+.008	+.002	—.002	—.008	.012	.018
	Harv.	75	+.010	+.003	—.003	—.010	—.017	.024
	Cord.	75	+.015	+.005	.005	.015	.024	.034
	Paris	75	+.007	+.002	—.002	—.007	—.011	.016
	Cape	80	+.017	+.006	—.006	—.017	—.028	—.039
	Melb.	80	+.007	+.002	—.002	—.007	—.011	—.016
	Grw.	80	+.010	+.003	—.003	—.010	—.016	—.023
	Pulk.	85	+.012	+.004	—.004	—.012	—.019	—.027
	Cape	85	+.012	+.004	—.004	—.012	—.020	—.027
	Stbg.	85	+.009	+.003	—.003	—.009	—.016	—.022
	Rad.	90	+.011	+.004	—.004	—.011	.019	—.026
	Cape	90	+.019	+.006	—.006	—.019	—.031	—.044
	Mdn.	90	+.019	+.006	—.006	—.019	.032	—.045
	Ber.	90	+.005	+.002	—.002	—.005	—.008	—.012
	Lisb.	90	+.012	+.004	—.004	—.012	—.020	—.029
	Grw.	90	+.009	+.003	—.003	—.009	—.015	—.021
2)	Pulk.	92	+.012	+.004	—.004	—.012	—.019	—.027
	Mt. H.	95	+.016	+.005	—.005	—.016	—.026	—.037
	Ber.	95	+.011	+.004	.004	—.011	—.019	—.027
3)	Alb.	98	.000	.000	.000	.000	.000	.000

TABLE II. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\alpha$. RIGHT-ASCENSION.

	R.A.		12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20	21^h	22	23	0
	Br.	1755	—^{s}.083	—^{s}.080	—^{s}.077	—^{s}.075	—^{s}.072	—^{s}.070	—^{s}.069	^{s}.068	—^{s}.068	—^{s}.069	—^{s}.071	—^{s}.073	—^{s}.075
{	Pi.	1800	+.136	+.135	+.133	+.129	+.125	+.120	+.116	+.111	+.107	+.104	+.101	+.100	+.100
{	× (tan δ)		—.018	—.024	+.002	+.028	+.051	+.072	+.087	+.096	+.099	+.095	+.085	+.069	+.048
	Grw.	15	—.005	—.008	—.012	—.015	—.019	—.023	.026	—.028	—.030	.030	—.029	—.028	—.025
4)	Dpt.	15	—.034	—.034	—.032	—.027	—.021	—.013	—.004	+.005	+.014	+.021	+.027	+.032	+.034
	Kgb.	15	—.082	—.082	—.082	—.082	—.082	.082	.082	—.082	—.082	—.082	—.082	—.082	.082
5)	Kgb.	20	—.034	—.034	—.034	—.034	—.034	—.031	.034	.034	.034	—.034	—.031	.031	.034
	Kgb.	25	+.032	+.028	+.025	+.021	+.018	+.015	+.013	+.012	+.011	+.012	+.013	+.016	+.019
	Dpt.	30	—.025	—.027	—.030	—.031	—.032	—.032	.031	.030	—.028	.025	—.022	—.019	—.016
	Cape	30	—.001	—.009	—.015	.020	—.022	.021	.019	—.014	—.007	+.004	+.010	+.019	+.027
	St. H.	30	—.027	—.028	—.030	—.033	—.036	—.041	—.046	—.051	—.056	—.059	—.062	.064	.065
	Abo	30	+.016	+.015	+.014	+.013	+.013	+.012	+.012	+.011	+.011	+.012	+.012	+.013	+.014
	Grw.	30	—.063	—.061	—.059	.057	—.055	—.054	.053	.053	—.054	.055	.056	.058	.061
	Camb.	30	—.029	—.028	—.027	—.025	—.023	—.022	.020	.019	—.018	.018	—.018	—.018	.019
	Cape	33	+.035	+.035	+.035	+.033	+.031	+.028	+.024	+.020	+.016	+.013	+.010	+.008	+.006
	Madr.	35	—.042	—.047	—.052	.057	—.062	.066	.069	.071	.072	.071	.069	.066	.062
	Arm.	40	+.055	+.054	+.053	+.051	+.050	+.048	+.047	+.046	+.045	+.044	+.044	+.044	+.045
	Cape	40	—.006	.003	.000	+.003	+.005	+.007	+.008	+.008	+.007	+.006	+.004	+.002	.001
	Grw.	40	+.071	+.079	+.087	+.094	+.101	+.107	+.111	+.113	+.113	+.111	+.107	+.101	+.094
	Grw.	45	+.006	+.011	+.017	+.024	+.030	+.036	+.041	+.045	+.047	+.048	+.047	+.044	+.040
	Rad.	45	—.068	.060	.050	.038	.025	.012	+.001	+.012	+.021	+.026	+.029	+.028	+.023
	Pulk.	45	+.027	+.027	+.026	+.026	+.025	+.024	+.023	+.022	+.021	+.020	+.019	+.019	+.019
	Paris	45	+.023	+.025	+.028	+.029	+.031	+.034	+.033	+.034	+.033	+.032	+.031	+.029	+.027
	Stgo.	50	—.004	.001	+.001	+.004	+.007	+.010	+.012	+.014	+.015	+.015	+.015	+.014	+.012
	Grw.	50	—.011	.007	.003	+.002	+.007	+.011	+.014	+.017	+.018	+.018	+.017	+.014	+.011

TABLE II. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\delta$. RIGHT-ASCENSION. *Cont.*

R.A.	0h	1h	2h	3h	4h	5h	6h	7h	8	9h	10h	11h	12h
Cape 50	+.027s	+.026s	+.025s	+.023s	+.021s	+.019s	+.016s	+.014s	+.012s	+.010s	+.009s	+.009s	+.009s
Stgo. 55	+.065	+.057	+.048	+.039	+.030	+.023	+.016	+.012	+.009	+.009	+.012	+.016	+.023
Cape 60	+.035	+.031	+.027	+.023	+.019	+.016	+.013	+.010	+.009	+.009	+.009	+.011	+.014
Wn. 60	+.033	+.026	+.018	+.011	+.003	–.003	–.009	.013	–.014	–.014	–.012	–.008	–.002
Grw. 60	+.034	+.032	+.028	+.024	+.019	+.014	+.010	+.006	+.003	+.002	+.001	+.001	+.003
{ Rad. 60	+.033	+.029	+.024	+.021	+.017	+.015	+.014	+.014	+.016	+.018	+.021	+.025	+.030
{ ×(tan δ)	+.024	+.021	+.017	+.012	+.006	–.001	–.007	–.013	–.018	–.022	–.024	–.025	–.024
Stgo. 60	+.065	+.057	+.048	+.039	+.030	+.023	+.016	+.012	+.009	+.009	+.012	+.016	+.023
{ Melb. 60	+.062	+.053	+.045	+.037	+.029	+.022	+.018	+.015	+.014	+.016	+.020	+.025	+.032
{ ×(tan δ)	+.023	+.028	+.031	+.032	+.031	+.027	+.022	+.015	+.008	–.001	–.009	–.016	–.023
Paris 60	+.050	+.045	+.039	+.034	+.028	+.024	+.020	+.017	+.016	+.016	+.018	+.021	+.025
Grw. 64	+.042	+.039	+.036	+.033	+.030	+.026	+.024	+.022	+.020	+.020	+.020	+.021	+.023
Cape 65	–.013	–.013	–.013	–.012	–.012	–.012	–.012	–.012	–.012	–.012	–.011	–.011	–.011
{ Brs. 65	+.068	+.058	+.047	+.036	+.027	+.018	+.012	+.009	+.008	+.011	+.015	+.023	+.032
{ ×(tan δ)	–.003	+.006	+.015	+.023	+.029	+.033	+.035	+.035	+.032	+.027	+.020	+.012	+.003
Harv. 65	–.033	–.037	–.040	–.043	–.045	–.046	–.045	–.043	–.040	–.037	–.033	–.028	–.024
Pulk. 65	–.008	–.008	–.007	–.006	–.005	–.004	–.003	–.002	–.002	–.001	–.001	–.001	–.002
Melb. 70	+.050	+.041	+.032	+.023	+.014	+.007	+.002	–.001	–.001	+.001	+.005	+.012	+.020
Grw. 72	+.039	+.037	+.034	+.031	+.028	+.025	+.022	+.020	+.018	+.017	+.017	+.017	+.019
Madr. 75	+.042	+.043	+.044	+.046	+.047	+.049	+.050	+.051	+.052	+.053	+.053	+.052	+.052
Wn. 75	–.002	–.003	–.004	–.004	–.004	–.004	–.004	–.003	–.002	–.002	–.001	.000	+.001
Pulk 75	–.003	–.001	.000	+.002	+.004	+.006	+.008	+.009	+.010	+.010	+.010	+.009	+.008
Harv. 75	–.013	–.012	–.011	–.010	–.008	–.006	–.004	–.003	–.001	.000	+.001	+.002	+.002
6) Cord. 75	+.021	+.012	+.002	–.008	–.018	–.026	–.033	–.037	–.039	–.039	–.035	–.030	–.022
Paris 75	+.059	+.052	+.044	+.037	+.029	+.022	+.017	+.013	+.011	+.011	+.013	+.017	+.022
Cape 80	+.046	+.044	+.042	+.039	+.036	+.033	+.031	+.028	+.027	+.026	+.026	+.026	+.027
Melb. 80	+.055	+.052	+.047	+.042	+.036	+.030	+.024	+.019	+.015	+.013	+.012	+.012	+.014
Grw. 80	+.038	+.037	+.036	+.034	+.033	+.032	+.032	+.031	+.031	+.031	+.031	+.032	+.033
Pulk. 85	+.014	+.015	+.016	+.018	+.019	+.021	+.023	+.024	+.025	+.026	+.026	+.026	+.025
Cape 85	+.023	+.020	+.017	+.014	+.011	+.009	+.008	+.008	+.008	+.009	+.010	+.013	+.015
Stbg. 85	+.014	+.014	+.014	+.014	+.014	+.014	+.014	+.014	+.015	+.016	+.016	+.017	+.017
Rad. 90	+.025	+.024	+.023	+.021	+.019	+.017	+.014	+.012	+.010	+.009	+.008	+.008	+.008
Cape 90	+.024	+.023	+.021	+.020	+.019	+.018	+.017	+.016	+.016	+.017	+.017	+.018	+.019
Mdn. 90	+.007	+.007	+.006	+.006	+.006	+.005	+.005	+.005	+.006	+.006	+.007	+.007	+.008
Ber. 90	+.021	+.020	+.019	+.018	+.017	+.017	+.016	+.016	+.016	+.017	+.017	+.018	+.019
Lisb. 90	+.015	+.014	+.014	+.015	+.015	+0.15	+.015	+.016	+.016	+.017	+.017	+.017	+.017
Grw. 90	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045	+.045
2) Pulk. 92	+.023	+.022	+.021	+.020	+.019	+.018	+.017	+.017	+.017	+.017	+.018	+.019	+.020
Mt. H. 95	+.024	+.024	+.023	+.023	+.023	+.024	+.024	+.025	+.027	+.028	+.029	+.031	+.032
Ber. 95	+.022	+.022	+.021	+.020	+.020	+.019	+.018	+.018	+.017	+.017	+.017	+.017	+.018
3) Alb. 98	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE III. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\delta$. RIGHT-ASCENSION.

	$\Delta\alpha_\delta \cos\delta$							$\Delta\alpha_\delta$					
	+85	+80	+80°	+75	+70°	+65°	+60	+55	+50	+45°	+40°	+35	+30°
Lc. 1755	–.009s	–.017s	–.099s	–.089s	–.075s	–.059s	–.037s	–.009s	+.006s	+.012s	+.014s	+.013s	+.012s
Pi. 1800	+.038	+.040	+.230	+.175	+.161	+.178	+.187	+.167	+.132	+.100	+.093	+.087	+.079
Grw. 15	.000	.000	.000	.000	.000	–.001	–.005	–.012	–.019	–.027	–.034	–.041	–.043
4) Dpt. 15	.000	+.003	+.017	+.031	+.035	+.028	+.016	+.003	–.009	–.019	. . .	. . .	. . .
Kgh. 15	.000	–.002	–.012	–.019	–.021	–.019	–.015	–.007	+.002	+.009	+.011	+.008	+.001
5) Kgh. 25	.000	.000	.000	.000	.000	.000	–.003	–.010	–.022	–.024	–.016	–.004	+.002
Dpt. 30	+.001	+.002	+.011	+.016	+.023	+.033	+.037	+.036	+.033	+.031	+.029	+.025	+.021
Cape 30	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
St. H. 30	. . .	. . .	. . .	. . .	. . .	. . .	+.017	+.014	+.012	+.010	+.009	+.007	+.006
Abo 30	.000	+.003	+.020	+.046	+.049	+.043	+.035	+.028	+.023	+.018	+.012	+.004	–.004

TABLE II. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\alpha$. RIGHT-ASCENSION. Cont.

R.A.	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	0h
	s	s	s	s	s	s	s	s	s	s	s	s	s
Cape 50	+.009	+.010	+.011	+.013	+.015	+.017	+.020	+.022	+.024	+.026	+.027	+.027	+.027
Stgo. 55	+.023	+.031	+.040	+.049	+.058	+.065	+.072	+.076	+.079	+.079	+.076	+.072	+[illegible]
Cape 60	+.011	+.017	+.021	+.025	+.029	+.033	+.036	+.038	+.039	+.040	+.039	+.037	+[illegible]
Wn. 60	–.002	+.005	+.012	+.020	+.028	+.034	+.040	+.045	+.045	+.045	+.043	+.039	+.033
Grw. 60	+.003	+.006	+.010	+.014	+.019	+.023	+.028	+.031	+.034	+.036	+.037	+.036	+.034
{ Rad. 60	+.030	+.034	+.039	+.043	+.046	+.048	+.049	+.049	+.047	+.045	+.042	+[illegible]	+[illegible]
{ ×(tan δ)	–.024	–.021	–.017	–.012	.006	–.001	+.007	+.013	+.018	+.022	+.024	+.025	+.024
Stgo. 60	+.023	+.031	+.040	+.049	+.058	+.065	+.072	+.076	+.079	+.079	+.076	+.072	+.065
{ Melb. 60	+.032	+.041	+.049	+.058	+.065	+.072	+.076	+.079	+.080	+.078	+.074	+.069	+.062
{ ×(tan δ)	–.023	–.028	.031	–.032	.031	–.027	.022	.015	.008	+.001	+.009	+.016	+.023
Paris 60	+.025	+.030	+.036	+.041	+.047	+.051	+.055	+.058	+.059	+.059	+.057	+.054	+.050
Grw. 64	+.023	+.026	+.029	+.032	+.035	+.039	+.041	+.043	+.045	+.045	+.045	+.044	+.042
Cape 65	–.011	–.011	–.011	–.012	–.012	–.012	–.012	–.012	–.012	–.012	–.013	.013	–.013
{ Brs. 65	+.032	+.042	+.053	+.063	+.073	+.081	+.087	+.090	+.091	+.089	+.084	+.077	+.068
{ ×(tan δ)	+.003	–.006	–.015	–.023	–.029	–.033	–.035	–.035	.032	–.027	–.020	–.012	.003
Harv. 65	–.024	–.020	–.016	–.013	–.012	–.011	–.012	–.013	–.016	–.020	–.024	.028	.033
Pulk. 65	–.002	–.002	–.003	–.004	–.005	–.006	.007	–.008	.009	–.009	–.009	–.009	–.008
Melb. 70	+.020	+.029	+.038	+.047	+.056	+.063	+.068	+.071	+.071	+.069	+.065	+.058	+.050
Grw. 72	+.019	+.021	+.023	+.026	+.030	+.033	+.035	+.038	+.040	+.041	+.041	+.040	+.039
Madr. 75	+.052	+.051	+.049	+.048	+.046	+.045	+.043	+.042	+.041	+.041	+.041	+.041	+.042
Wn. 75	+.001	+.001	+.002	+.002	+.002	+.002	+.002	+.001	+.001	.000	–.001	–.002	–.002
Pulk. 75	+.008	+.006	+.005	+.003	+.001	.001	–.003	–.004	–.005	–.005	–.005	–.004	–.003
Harv. 75	+.002	+.001	.000	–.001	–.003	–.005	–.007	–.008	–.010	–.011	–.012	–.013	–.013
6) Cord. 75	–.022	–.013	–.003	+.007	+.017	+.025	+.032	+.036	+.038	+.038	+.035	+.029	+.021
Paris 75	+.022	+.029	+.037	+.044	+.052	+.059	+.064	+.068	+.070	+.070	+.068	+.064	+.059
Cape 80	+.027	+.029	+.032	+.034	+.037	+.040	+.043	+.045	+.047	+.048	+.048	+.047	+.046
Melb. 80	+.014	+.017	+.021	+.027	+.033	+.039	+.044	+.049	+.053	+.056	+.057	+.057	+.055
Grw. 80	+.033	+.034	+.035	+.036	+.037	+.038	+.039	+.040	+.040	+.040	+.039	+.039	+.038
Pulk. 85	+.025	+.024	+.023	+.022	+.020	+.018	+.017	+.016	+.014	+.014	+.013	+.014	+.014
Cape 85	+.015	+.018	+.021	+.024	+.027	+.029	+.030	+.030	+.030	+.029	+.028	+.025	+.023
Stbg. 85	+.017	+.017	+.018	+.018	+.018	+.017	+.017	+.017	+.016	+.016	+.015	+.015	+.014
Rad. 90	+.008	+.009	+.010	+.012	+.014	+.017	+.019	+.021	+.023	+.024	+.025	+.025	+.025
Cape 90	+.019	+.021	+.022	+.023	+.025	+.026	+.027	+.027	+.027	+.027	+.026	+.025	+.024
Mdn. 90	+.008	+.008	+.009	+.009	+.009	+.010	+.010	+.010	+.009	+.009	+.008	+.008	+.007
Ber. 90	+.019	+.020	+.021	+.022	+.023	+.023	+.024	+.024	+.024	+.023	+.023	+.022	+.021
Lisb. 90	+.017	+.018	+.018	+.017	+.017	+.017	+.017	+.016	+.016	+.015	+.015	+.015	+.015
Grw. 90	+.015	+.015	+.015	+.015	+.015	+.015	+.015	+.015	+.045	+.015	+.015	+.045	+.015
2) Pulk. 92	+.020	+.021	+.022	+.023	+.024	+.025	+.026	+.026	+.026	+.026	+.025	+.024	+.023
Mt. H. 95	+.032	+.033	+.033	+.033	+.033	+.033	+.032	+.031	+.030	+.028	+.027	+.026	+.024
Ber. 95	+.018	+.018	+.019	+.020	+.020	+.021	+.022	+.022	+.023	+.023	+.023	+.023	+.022
3) Alb. 98	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE III. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\delta$. RIGHT-ASCENSION.

	+25°	+20	+15	+10	+5	0	5	10	15	20	25	30
	s	s	s	s	s	s	s	s	s	s	s	s
Br. 1755	+.009	+.005	+.004	.002	.003	.001	.000	.001	.009	.018	.028	.038
Pi. 1800	+.069	+.057	+.036	+.008	.015	.027	–.036	.047	.061	.076	.095	.118
Grw. 15	–.040	–.034	.022	.006	+.009	+.021	+.028	+.031	+.034	+.039	+.046	+.058
1) Dpt. 15	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. .	. . .	. .
Kgb. 15	–.007	.013	–.011	.010	.002	+.009	+.011	+.012	+.005	.004	.017	.
5) Kgb. 25	+.005	+.005	+.003	.000	.003	.006	.009	.012	.015	–.019	.023	
Dpt. 30	+.013	+.007	+.003	.000	.003	.007	.010	.013	.016	.017	.	
Cape 30	. . .	. . .	.000	.002	.008	.017	.017	.010	.004	+.007	+.015	+.021
St. H. 30	+.006	+.004	+.004	+.003	.000	.006	.009	.008	.005	.009	.020	.036
Abo 30	.008	.007	.006	.005	.002	–.002	+.006	+.010	+.013	+.015	.	. .

TABLE III. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\delta$. RIGHT-ASCENSION. — Cont.

	$\Delta\alpha\cos\delta$		$\Delta\alpha$										
	+85	+80°	+80	+75	+70°	+65	+60	+55	+50	+45	+40	+35	+30°
Grw. 30	.000	.000	.000	+.003	+.018	+.037	+.054	+.052	+.040	+.027	+.018	+.013	+.014
Camb. 30	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	+.004
Cape 33	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.089	+.079	+.066	+.052
Madr. 35	. . .	+.060	+.313	+.230	+.173	+.110	+.117	+.102	+.090	+.079	+.067	+.054	+.041
Arm. 40	. . .	.000	.000	.017	.033	−.010	−.010	.033	.021	−.011	.016	.010	.009
Cape 40	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.059	+.046	+.035	+.024
Grw. 40	−.025	−.032	−.187	.151	.120	−.094	−.063	−.033	.011	.008	−.008	−.007	−.007
Grw. 45	.006	.014	−.081	−.093	−.088	−.076	−.060	.042	.025	−.011	−.006	−.006	−.009
Rad. 45	−.010	−.010	−.058	.036	−.016	+.006	+.027	+.035	+.031	+.021	+.013	+.011	+.017
Pulks 45	+.001	+.003	+.018	+.023	+.018	+.011	+.002	−.001	.007	−.011	−.012	−.010	−.008
Paris 45	+.008	+.008	+.016	+.031	+.023	+.026	+.027	+.022	+.015	+.001	−.003	−.007	−.008
Stgo. 50	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.021	+.023	+.023	+.022
Grw. 50	−.020	−.025	.142	.108	−.083	−.058	−.031	−.012	.000	+.002	−.003	−.010	−.013
Cape 50	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.056	+.032	+.015	+.003
Stgo. 55	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.035
Cape 60	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.021	+.015	+.009
Wn. 60	.000	.000	.000	−.003	−.007	−.014	−.017	−.016	−.010	+.001	+.010	+.013	+.010
Grw. 60	−.006	−.007	−.039	−.036	−.037	−.041	−.046	−.046	−.044	−.040	−.037	−.032	−.026
Rad. 60	−.007	−.007	−.012	−.036	−.039	−.045	−.040	−.025	−.005	+.004	+.002	.003	−.005
Stgo. 60	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.035
Melb. 60	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	−.026	−.021	−.020
Paris 60	.000	.000	.000	+.012	+.023	+.024	+.017	+.008	+.007	+.010	+.014	+.013	+.008
Grw. 64	.000	.000	.000	.000	.000	+.005	+.008	+.006	−.004	−.013	−.020	−.026	−.024
Cape 65	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.019
Brs. 65	+.005	−.010	−.055	−.093	−.117	−.111	−.095	−.067	−.034	−.015	−.008	−.007	−.009
Harv. 65	−.008	−.007	−.041	−.004	+.030	+.045	+.045	+.039	+.031	+.022	+.018	+.016	+.016
Pulk. 65	−.005	−.007	−.039	−.032	−.029	−.024	−.020	−.017	−.018	−.022	−.022	−.019	−.012
Melb. 70	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.011	+.010	+.009
Grw. 72	+.010	+.010	+.058	+.039	+.028	+.016	+.008	+.007	+.005	−.003	−.016	−.020	−.018
Madr. 75	. . .	−.047	−.273	−.141	−.075	−.035	−.007	+.013	+.024	+.027	+.022	+.014	+.006
Wn. 75	+.001	−.003	−.019	−.023	−.021	−.018	−.023	−.034	−.035	−.026	−.010	+.005	+.011
Pulk. 75	−.003	−.009	−.052	−.051	−.039	−.024	−.015	−.014	−.014	−.013	−.012	−.012	−.012
Harv. 75	−.002	−.002	−.014	−.013	−.011	−.004	+.005	+.007	+.003	+.003	+.003	+.005	+.007
6) Cord. 75	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	−.012
Paris 75	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	.000	+.017	+.022	+.017	+.009
7) Cape 80	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.010	+.006	+.003
Melb. 80	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.003	+.005	+.007
Grw. 80	+.009	+.009	+.051	+.032	+.023	+.019	+.011	+.003	−.003	−.006	−.009	−.009	−.008
Pulk. 85	.000	.000	.000	−.016	−.019	−.015	−.007	+.002	+.003	−.002	−.004	−.001	−.001
Cape 85	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	−.016	−.016	−.016	−.015
Stbg. 85	+.001	+.004	+.024	+.038	+.042	+.038	+.032	+.025	+.017	+.010	+.005	.000	−.003
Rad. 90	. . .	+.039	+.225	+.141	+.095	+.053	+.022	+.005	−.005	−.008	−.013	−.022	−.026
Cape 90	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	+.020	+.016	+.013	+.009
Mdn. 90	+.001	+.003	+.019	+.022	+.029	+.031	+.026	+.019	+.011	+.007	+.006	+.008	+.009
Ber. 90	−.006	−.006	−.036	−.027	−.023	−.022	−.022	−.023	−.025	−.026	−.026	−.023	−.018
Lisb. 90	. . .	. . .	. . .	. . .	. . .	.000	−.004	−.006	−.009	−.009	−.008	−.007	−.005
Grw. 90	−.002	+.004	+.023	+.030	+.027	+.014	−.064	−.015	−.015	−.008	−.007	−.006	−.006
2) Pulk. 92	.000	.000	.000	−.002	−.006	−.011	−.015	−.017	−.018	−.018	−.018	−.018	−.016
Mt.H. 95	+.013	+.015	+.086	+.064	+.054	. . .	. . .	. . .	. . .	. . .	−.008	.000	+.005
Ber. 95	. . .	+.012	+.069	+.041	+.020	+.005	−.004	−.010	−.016	−.017	−.015	−.013	−.011
3) Alb. 98	. . .	. . .	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE III. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\alpha_\delta$. RIGHT-ASCENSION. — Cont.

		+25°	+20°	+15	+10	+5	0	—5	—10	—15	—20°	—25	—30
	Grw. 30	+.020	+.018	+.013	+.006	—.002	—.009	—.017	—.021	.026	—.029	.031	.035
	Camb. 30	+.009	+.013	+.008	—.002	—.007	—.006	.000	+.001	—.001	—.006	—.013	—.019
	Cape 33	+.034	+.015	+.003	—.005	—.009	—.012	—.014	—.013	—.011	—.005	+.001	+.004
	Madr.2 35	+.031	+.023	+.014	+.006	—.001	—.009	—.018	—.027	.038	—.049	—.057	—.062
	Arm. 40	—.009	—.008	—.007	.000	+.005	+.001	—.002	—.001	+.005	+.010	+.012	+.014
	Cape 40	+.012	—.001	—.016	—.019	—.014	—.004	+.003	+.007	+.009	+.009	+.009	+.008
	Grw. 40	—.008	—.008	—.008	—.005	+.002	+.008	+.012	+.014	+.015	+.011	—.002	—.020
	Grw. 45	—.008	—.004	+.002	+.005	+.006	+.003	+.002	+.002	+.002	—.001	—.009	—.021
	Rad. 45	+.022	+.024	+.019	+.004	—.017	—.023	—.020	.015	—.015	—.021	—.029	. . .
	Pulk. 45	—.006	—.003	.000	+.003	+.005	+.005	+.003	.000	+.002	+.008	+.018	. . .
	Paris 45	—.006	—.001	+.003	+.004	+.003	+.001	+.001	+.002	+.001	—.004	—.004	.000
	Stgo. 50	+.022	+.019	+.013	+.005	—.004	—.012	—.018	—.021	—.023	—.025	—.027	—.033
	Grw. 50	—.012	—.008	—.002	+.001	+.002	+.004	+.007	+.008	+.008	+.006	+.005	+.004
	Cape 50	—.006	—.010	—.008	—.003	+.006	+.008	+.007	+.003	.000	.000	.000	—.006
	Stgo. 55	+.025	+.016	+.008	+.002	—.005	—.010	—.015	—.018	.020	—.021	—.028	—.044
	Cape 60	+.002	—.006	—.008	—.004	.000	+.004	+.008	+.006	—.003	—.003	+.003	+.017
	Wn. 60	+.006	+.003	+.001	.000	—.002	—.003	—.004	—.003	—.002	+.002	+.007	+.012
	Grw. 60	—.020	—.013	—.006	.000	+.006	+.011	+.013	+.014	+.012	+.007	+.004	+.002
	Rad. 60	—.004	+.002	+.011	+.017	+.018	+.013	+.001	—.008	—.017	—.028	—.040	—.052
	Stgo. 60	+.025	+.016	+.008	+.002	—.005	—.010	—.015	—.018	—.020	—.021	—.028	—.044
	Melb. 60	—.013	—.005	—.002	.000	.000	.000	+.004	+.008	+.008	+.005	.000	—.005
	Paris 60	+.003	+.003	+.003	+.003	+.001	—.003	—.006	—.007	—.008	—.012	—.019	—.023
	Grw. 64	—.018	—.008	—.001	+.003	+.008	+.011	+.011	+.010	+.004	.000	.000	.000
	Cape 65	+.009	—.002	—.008	—.013	—.014	—.012	—.007	.000	+.012	+.028	+.044	+.048
	Brs. 65	—.009	—.008	—.008	—.005	—.002	+.001	+.004	+.007	+.015	+.021	+.021	+.012
	Harv. 65	+.014	+.012	+.009	+.004	—.001	—.007	—.010	—.009	—.012	—.021	—.033	—.046
	Pulk. 65	—.004	.000	+.003	+.005	+.007	+.011	+.008	+.001	—.006	—.010	. . .	. . .
	Melb. 70	+.009	+.008	+.007	+.002	—.004	—.008	—.010	—.010	—.009	—.008	—.008	—.007
	Grw. 72	—.012	—.006	—.001	+.002	+.006	+.009	+.008	+.003	.000	.000	.000	.000
	Madr. 75	+.002	+.002	—.001	—.006	—.010	—.010	—.004	+.001	+.004	+.006	+.013	+.024
	Wn. 75	+.012	+.010	+.007	+.002	—.005	—.010	—.012	—.011	.006	—.003	—.004	—.009
	Pulk. 75	—.011	—.005	.000	+.004	+.009	+.012	+.010	+.004	.000	.000	. . .	. . .
	Harv. 75	+.006	+.004	+.002	.000	.000	—.001	—.005	.008	.006	—.004	+.007	+.017
6)	Cord. 75	—.003	+.002	+.003	+.002	—.003	—.001	+.001	+.009	+.013	+.015	+.012	+.004
	Paris 75	+.006	+.005	+.005	+.001	.000	—.004	—.009	—.010	.007	—.002	.000	. . .
7)	Cape 80	+.001	—.001	—.003	—.003	—.003	.000	+.004	+.010	+.017	+.026	+.034	+.040
	Melb. 80	+.009	+.008	+.002	—.006	—.011	—.012	.008	.000	+.014	+.029	+.040	+.049
	Grw. 80	—.006	—.003	.000	+.002	+.004	+.005	+.005	+.003	+.001	+.001	+.001	+.001
	Pulk 85	—.006	—.008	—.006	—.003	+.001	+.004	+.005	+.004	+.004	+.004	. . .	. . .
	Cape 85	—.012	—.008	—.003	—.002	.003	—.003	+.001	+.007	+.016	+.024	+.031	+.033
	Stbg. 85	—.005	.006	—.006	—.003	+.003	+.009	+.008	+.005	+.002	—.002	.007	.010
	Rad. 90	—.026	—.023	—.019	.013	.006	+.004	+.019	+.040	+.058	+.076	+.096	+.118
	Cape 90	+.004	+.001	+.001	.000	.004	—.006	.006	—.005	.000	+.008	+.019	+.028
	Mdn. 90	+.008	+.006	+.002	.000	—.002	.004	—.007	—.009	.011	. . .	.	.
	Ber. 90	—.013	—.008	—.001	.000	+.004	+.008	+.012	+.015	. . .	. . .	. . .	. . .
	Lisb. 90	—.002	.000	+.002	+.004	+.005	+.004	+.001	—.003	—.006	.007	.007	.007
	Grw. 90	—.006	—.001	—.002	.000	+.001	+.002	+.004	+.007	+.009	+.010	+.010	+.007
2)	Pulk. 92	—.011	—.004	.000	+.004	+.005	+.005	+.004	+.002	—.002	.009	. . .	. . .
	Mt. H. 95	+.007	+.008	+.007	+.004	—.001	—.005	.008	.010	.012	—.014	.012	—.009
	Ber. 95	—.009	—.007	—.004	+.002	+.006	+.008	+.009	+.011	+.013	+.016	+.017	. . .
3)	Alb. 98	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE III. $l\alpha$, SOUTH OF $-30°$ OF DECLINATION. RIGHT-ASCENSION. Cont.

	$\Delta\alpha_\delta$ -30°	-35	-40	-45	-50	-55	-60	-65°	-70°	-75°	-80	$\Delta\alpha_\delta \cos\delta$ -80	-85°
	s	s	s	s	s	s	s	s	s	s	s	s	s
Pi. 1900	-.118	-.151	-.192	-.241	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Cape 30	+.021	+.027	+.033	+.043	+.059	+.077	+.091	+.111	+.137	+.181	+.271	+.047	. . .
St.H. 30	-.036	-.045	-.052	-.060	-.070	-.082	-.098	-.116	-.143	-.189	-.282	-.049	. . .
Cape 33	+.001	+.001	000	-.005	-.009	-.010	-.008	-.003	000	000	000	.000	.000
Madr. 35	-.062	-.066	-.071	-.076	-.081	-.091	-.108	-.128	. . .	. . .	. . .	. . .	. . .
Cape 40	+.008	+.007	+.009	+.011	+.016	+.021	+.026	+.031	+.038	+.050	+.075	+.013	+.913
Stgo. 50	-.033	-.041	-.052	-.056	-.051	-.012	-.032	-.018	+.002	+.026	+.059	+.016	. . .
Cape 50	-.006	-.026	-.061	-.075	-.072	-.010	-.012	000	000	000	000	.000	.000
Stgo. 55	-.014	-.073	-.093	-.100	-.093	-.046	-.018	-.005	000	000	000	.000	.000
Cape 60	+.017	+.030	+.039	+.041	+.051	+.059	+.060	+.046	+.030	+.018	+.010	+.002	+.001
Wn. 60	+.012	+.018	+.024	000	000	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Stgo. 60	-.014	-.073	-.093	-.100	-.093	-.046	-.018	-.005	000	000	000	.000	.000
Melb. 60	-.005	-.010	-.016	-.024	-.032	-.039	-.039	-.027	+.008	+.043	+.071	+.012	+.007
Cape 65	+.018	+.016	+.030	+.021	+.025	+.016	+.082	+.109	+.123	+.113	+.102	+.018	+.010

TABLE IV. SYSTEMATIC CORRECTIONS OF THE FORM. $l\delta_\alpha$. DECLINATION.

			0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
			"	"	"	"	"	"	"	"	"	"	"	"	"
8)	Br.	1755 N.	-.30	-.27	-.22	-.16	-.08	00	+.08	+.16	+.22	+.27	+.30	+.32	+.30
		S.	+.02	-.08	-.14	-.13	-.09	+.01	+.13	+.22	+.27	+.26	+.17	+.03	-.12
	Pi.	1900	-.11	+.05	+.20	+.33	+.45	+.53	+.58	+.59	+.55	+.49	+.39	+.25	+.11
	Grw.	15	-.09	-.07	-.04	-.02	+.02	+.01	+.07	+.09	+.11	+.11	+.11	+.10	+.09
	Kgb.	20	+.05	-.02	-.09	-.16	-.21	-.25	-.27	-.27	-.26	-.23	-.18	-.12	-.05
	Bb.	25	-.08	+.07	+.22	+.34	+.45	+.53	+.57	+.57	+.53	+.46	+.35	+.22	+.08
9)	Dpt.	30	-.02	+.02	+.06	+.09	+.12	+.14	+.15	+.15	+.14	+.12	+.09	+.06	+.02
	Abo	30	-.12	-.05	+.02	+.09	+.16	+.21	+.25	+.27	+.28	+.26	+.23	+.18	+.12
	Cape	30	+.06	+.12	+.17	+.21	+.24	+.25	+.24	+.22	+.18	+.13	+.07	00	-.06
	Grw.	30	+.08	+.12	+.15	+.17	+.18	+.18	+.16	+.13	+.10	+.06	+.01	-.04	-.08
	St. H.	30	+.50	+.46	+.39	+.30	+.18	+.05	-.08	-.20	-.32	-.41	-.47	-.50	-.50
	Cape	33	-.16	-.17	-.17	-.16	-.13	-.10	-.06	-.02	+.03	+.07	+.11	+.14	+.16
	Camb.	30	-.19	-.17	-.14	-.11	-.06	-.01	+.04	+.09	+.13	+.16	+.19	+.19	+.19
10)	Madr.	35	-.08	-.11	-.16	-.20	-.24	-.25	-.21	-.13	-.02	+.11	+.23	+.32	+.36
	Cape	40	+.01	00	00	-.01	-.01	-.02	-.02	-.02	-.02	-.02	-.02	-.02	-.01
	Grw.	40	-.17	-.18	-.17	-.16	-.13	-.09	-.05	00	+.04	+.08	+.12	+.15	+.17
	Grw.	45	-.06	-.05	-.04	-.03	-.01	00	+.02	+.04	+.04	+.06	+.06	+.06	+.06
	Arm.	40	-.07	-.02	+.03	+.08	+.12	+.16	+.19	+.20	+.20	+.18	+.16	+.12	+.07
11)	Rad.	45	+.05	+.03	00	-.02	-.04	-.06	-.08	-.09	-.09	-.09	-.08	-.07	-.05
	Pulk.	45	+.07	+.06	+.05	+.03	+.01	-.01	-.03	-.05	-.06	-.07	-.08	-.08	-.07
	Paris	45	-.07	-.07	-.07	-.07	-.06	-.05	-.03	-.01	+.01	+.03	+.05	+.06	+.07
	Stgo.	50	-.21	-.15	-.08	-.01	+.07	+.14	+.20	+.25	+.28	+.29	+.28	+.26	+.21
	Grw.	50	-.07	-.08	-.08	-.07	-.06	-.05	-.03	-.01	+.01	+.03	+.05	+.06	+.07
12)	Cape	50	-.03	-.08	-.12	-.15	-.17	-.18	-.18	-.17	-.14	-.11	-.06	-.02	+.03
	Stgo.	55	-.04	-.10	-.15	-.19	-.22	-.24	-.24	-.22	-.18	-.14	-.09	-.03	+.04
	Pulk.	55	+.03	+.03	+.04	+.04	+.03	+.03	+.02	+.01	00	-.01	-.02	-.02	-.03
13)	Wn.	60 }	-.07	-.10	-.12	-.13	-.14	-.13	-.12	-.10	-.07	-.04	00	+.04	+.07
		-20° }	.00	00	00	00	00	00	00	00	00	00	00	00	00
	Grw.	60	+.01	+.02	+.03	+.04	+.05	+.05	+.05	+.04	+.04	+.03	+.02	00	-.01
	Rad.	60	+.08	+.05	+.02	-.01	-.05	-.08	-.10	-.12	-.13	-.13	-.12	-.10	-.08
	Cape	60	+.03	+.06	+.08	+.10	+.11	+.11	+.11	+.10	+.08	+.06	+.03	00	-.03
	Paris	60	-.08	-.04	00	+.04	+.08	+.11	+.14	+.16	+.16	+.16	+.14	+.11	+.08
	Stgo.	60	-.24	-.24	-.22	-.19	-.15	-.10	-.04	+.03	+.09	+.14	+.18	+.22	+.24
	Melb.	60	-.09	-.04	-.04	-.07	-.13	-.17	-.17	-.11	00	+.16	+.33	+.47	+.55
	Grw.	64	+.04	+.03	+.02	+.01	00	-.01	-.02	-.03	-.04	-.04	-.04	-.04	-.04

TABLE III. $\Delta\alpha_\delta$ SOUTH OF −30° OF DECLINATION. RIGHT-ASCENSION — Cont.

	$\Delta\alpha_\delta$ −30°	−35°	−40°	−45°	−50	−55	−60	−65	70	−75°	−80	$\Delta\alpha\cos\delta$ 80°	85
	s	s	s	s	s	s	s	s	s	s	s	s	s
Harv. 65	−.046	−.062	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Melb. 70	−.007	−.005	−.004	−.006	−.011	−.019	−.032	−.014	−.057	−.071	−.089	−.015	−.011
Madr. 75	+.024	+.034	+.039	+.038	+.031	+.017	−.007	−.038	. . .	. . .	. . .	. . .	. . .
Wn. 75	−.009	−.016	−.023	(−.030)	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Harv. 75	+.017	+.026	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
6) Cord. 75	+.004	−.010	−.020	−.024	−.025	−.026	−.027	−.028	−.023	−.010	000	.000	.000
7) Cape 80	+.040	+.045 +.014	+.024	+.034 +.053	+.058	+.065 −.029	−.025	−.018 −.003	−.003	−.003 −.018	−.026	−.005	−.005
Melb. 80	+.049	+.054	+.056	+.053	+.047	+.029	+.008	−.011	−.025	−.035	−.048	−.008	−.008
Cape 85	+.033	+.030	+.028	+.025	+.023	+.018	+.014	+.009	+.003	000	000	.000	.000
Strb. 85	−.010	−.017	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Cape 90	+.028	+.034	+.038	+.039	+.039	+.038	+.037	+.037	+.035	+.036	+.040	+.007	+.006
Mt.H. 95	−.009	−.004	+.006	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
3) Alb. 98	000	000	000	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

TABLE IV. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\alpha$. DECLINATION.

			12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	0h
			″	″	″	″	″	″	″	″	″	″	″	″	″
8)	Br.	1755 N.	+.30	+.27	+.22	+.16	+.08	.00	−.08	−.16	−.22	−.27	−.30	−.32	−.30
		S	−.12	−.25	−.34	−.35	−.29	−.18	−.03	+.11	+.21	+.24	+.21	+.13	+.02
	Pi.	1800	+.11	−.05	−.20	−.33	−.45	−.53	−.58	−.59	−.55	.49	−.39	−.25	−.11
	Grw.	15	+.09	+.07	+.04	+.02	−.02	−.04	−.07	−.09	−.11	−.11	−.11	−.10	−.09
	Kgb.	20	−.05	+.02	+.09	+.16	+.21	+.25	+.27	+.27	+.26	+.23	+.18	+.12	+.05
	Bo.	25	+.08	−.07	−.22	−.34	.45	−.53	−.57	−.57	−.53	.46	−.35	−.22	−.08
9)	Dpt.	30	+.02	−.02	−.06	−.09	−.12	−.14	−.15	−.15	.14	.12	−.09	−.06	−.02
	Abo	30	+.12	+.05	−.02	−.09	−.16	−.21	−.25	−.27	−.28	−.26	−.23	.18	−.12
	Cape	30	−.06	−.12	−.17	−.21	−.24	−.25	−.24	−.22	.18	.13	−.07	.00	+.06
	Grw.	30	−.08	−.12	−.15	−.17	.18	−.18	.16	.13	.10	−.06	01	+.01	+.08
	St. H.	30	−.50	−.46	−.39	−.30	−.18	−.05	+.08	+.21	+.32	+.11	+.47	+.50	+.50
	Cape	33	+.16	+.17	+.17	+.16	+.13	+.10	+.06	+.02	−.03	.07	.11	−.14	−.16
	Camb.	30	+.19	+.17	+.14	+.11	+.06	+.01	−.04	−.09	−.13	.16	.18	.19	.19
10)	Madr.	35	+.36	+.35	+.30	+.20	+.10	.00	−.07	−.11	−.12	.11	−.09	−.07	−.08
	Cape	40	−.01	.00	.00	+.01	+.01	+.02	+.02	+.02	+.02	+.02	+.02	+.02	+.01
	Grw.	40	+.17	+.18	+.17	+.16	+.13	+.09	+.05	.00	.01	08	.12	15	.17
	Grw.	45	+.06	+.05	+.04	+.03	+.01	.00	.02	.04	.05	06	.06	06	06
	Arm.	40	+.07	+.02	−.03	−.08	−.12	.16	.19	.20	.20	.18	.16	.12	07
11)	Rad.	45	−.05	−.03	.00	+.02	+.01	+.06	+.08	+.09	+.09	+.09	+.08	+.07	+.05
	Pulk.	45	−.07	−.06	−.05	.03	−.01	+.01	+.03	+.05	+.06	+.07	+.08	+.08	+.07
	Paris	45	+.07	+.08	+.08	+.07	+.06	+.05	+.03	+.01	.01	.03	.05	06	07
	Stgo.	50	+.21	+.15	+.08	+.01	07	.14	.20	.25	28	.29	−.28	26	21
	Grw.	50	+.07	+.08	+.08	+.07	+.06	+.05	+.03	+.01	.01	.03	.05	.06	07
12)	Cape	50	+.03	+.08	+.12	+.15	+.17	+.18	+.18	+.17	+.11	+.11	+.06	+.02	03
	Stgo.	55	+.04	+.10	+.15	+.19	+.22	+.24	+.24	+.22	+.18	+.14	+.09	+.03	04
	Pulk.	55	−.03	−.03	01	.01	03	.03	02	.01	.00	+.01	+.02	+.02	+.03
13)	Wn.	60	+.07	+.10	+.12	+.13	+.14	+.13	+.12	+.10	+.07	+.01	.00	01	07
		−20°	.00	.00	.00	00	.00	00	00	00	.00	.00	.00	.00	00
	Grw.	60	−.01	.02	−.03	.01	05	.05	.05	.01	.04	03	.02	00	+.01
	Rad.	60	−.08	.05	−.02	+.01	+.05	+.08	+.10	+.12	+.13	+.13	+.12	+.10	+.08
	Cape	60	−.03	−.06	.08	.10	.11	11	.11	.10	.08	.06	.03	00	+.03
	Paris	60	+.08	+.01	.00	.01	08	11	11	.16	.16	.16	11	.11	08
	Stgo.	60	+.21	+.24	+.22	+.19	+.15	+.10	+.01	03	09	.14	18	.22	.24
	Melb.	60	+.55	+.55	+.46	+.29	+.09	12	.29	10	.42	.38	.29	18	09
	Grw.	64	−.04	−.03	−.02	−.01	00	+.01	+.02	+.03	+.04	+.04	+.04	+.04	+.04

TABLE IV. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\alpha$. DECLINATION. — Cont.

			0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
			″	″	″	″	″	″	″	″	″	″	″	″	″
	Cape	65	–.02	–.02	–.02	–.01	–.01	–.01	00	00	+.01	+.01	+.02	+.02	+.02
	Brs.	65	–.05	–.03	00	+.02	+.04	+.06	+.08	+.08	+.09	+.09	+.08	+.07	+.05
	Pulk.	65	+.01	+.01	+.01	+.03	+.02	+.01	00	–.01	–.02	–.03	–.04	–.04	–.04
	Leid.	67	–.02	–.02	–.03	.03	–.03	–.02	–.02	–.01	–.01	00	+.01	+.01	+.02
	Melb.	70	–.11	–.10	–.09	–.07	–.05	–.02	+.01	+.04	+.06	+.08	+.10	+.11	+.11
	Grw.	72	00	00	00	–.01	–.01	–.01	–.01	–.01	–.01	–.01	00	00	00
	Madr.	75	+.17	+.25	+.31	+.35	+.36	+.35	+.32	+.26	+.19	+.11	+.04	–.08	–.17
	Wn.	75	–.02	–.01	00	+.01	+.02	+.02	+.03	+.03	+.04	+.04	+.03	+.03	+.02
	Pulk.	75	+.06	+.07	+.07	+.06	+.06	+.04	+.03	+.01	00	–.02	–.04	–.05	–.06
	Harv.	75	00	00	00	00	00	00	00	00	00	00	00	00	00
18)	Cord.	75	00	–.02	–.04	–.05	–.06	–.07	–.07	–.07	–.06	–.05	–.04	–.02	00
	Paris	75	–.13	–.06	+.01	+.08	+.15	+.21	+.25	+.28	+.28	+.27	+.24	+.19	+.13
14)	Cape	80	–.10	–.12	–.12	–.12	–.11	–.09	–.07	–.04	–.04	+.02	+.05	+.08	+.10
		–25°	+.05	+.05	+.04	+.04	+.02	+.01	00	–.01	–.02	–.04	–.04	–.05	–.05
		–35	–.23	–.21	–.17	–.13	–.07	–.01	+.05	+.11	+.16	+.20	+.22	+.24	+.23
		–45	–.16	–.16	–.14	–.12	–.09	–.05	–.01	+.03	+.07	+.11	+.13	+.15	+.16
	Melb.	80	–.07	–.08	–.09	–.09	–.09	–.08	–.06	–.04	–.02	+.01	+.03	+.05	+.07
	Grw.	80	+.06	+.04	+.02	+.02	+.02	+.04	+.06	+.07	+.07	+.04	00	–.05	–.10
	Pulk.	85	+.02	+.01	+.01	00	–.01	–.01	–.02	–.02	–.03	–.03	–.03	–.02	–.02
	Cape	85	+.12	+.11	+.08	+.06	+.02	–.01	–.04	–.07	–.10	–.11	–.12	–.13	–.12
	Stbg.	85	+.03	+.03	+.02	+.01	+.01	00	–.01	–.02	–.02	–.03	–.03	–.03	–.03
	Rad.	90	+.07	+.10	+.12	+.13	+.14	+.13	+.12	+.10	+.07	+.04	00	–.04	–.07
	Cape	90	–.06	–.06	–.06	–.05	–.04	–.03	–.01	+.01	+.02	+.04	+.05	+.06	+.06
	Grw.	90	00	+.01	+.02	+.02	+.02	+.03	+.03	+.02	+.02	+.02	+.01	00	00
	Madn.	90	–.21	–.12	–.04	+.03	+.08	+.10	+.11	+.11	+.11	+.13	+.15	+.18	+.21
	Ber.	90	+.06	+.05	+.04	+.03	+.01	00	–.02	–.04	–.05	–.06	–.06	–.06	–.06
	Mun.	92	+.01	+.01	00	00	00	–.01	–.01	–.01	–.01	–.01	–.01	–.01	–.01
	Mt. H.	95	+.05	+.06	+.07	+.07	+.07	+.06	+.05	+.04	+.02	00	–.02	–.04	–.05
	Ber.	95	+.08	+.07	+.06	+.04	+.02	00	–.03	–.05	–.06	–.08	–.08	–.09	–.08
15)	W.-Ott.	97	+.07	+.13	+.18	+.21	+.23	+.24	+.23	+.20	+.16	+.11	+.05	–.01	–.07
	Alb.	98	–.06	–.07	–.07	–.07	–.06	–.06	–.04	–.02	00	+.01	+.03	+.05	+.06

TABLE V. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\delta$. DECLINATION.

			+90°	+85°	+80°	+75°	+70°	+65°	+60°	+55°	+50°	+45°	+40°	+35°	+30°
			″	″	″	″	″	″	″	″	″	″	″	″	″
8)	Br.	1755	See *A.J.* 545.												
	Pi.	1800	.00	+.04	+.08	+.16	+.24	+.23	+.14	–.06	–.38	–.64	–.85	–1.08	–1.29
	Grw.	15	.00	–.05	–.14	–.26	–.37	–.47	–.56	–.63	–.68	–.74	–.78	–.82	–.81
	Kgb.	20	...	...	.00	.00	+.03	+.08	+.08	+.04	–.03	–.04	.00	.00	.00
	Bh.	25	...	...	...	...	...	...	...	...	...	...	...	...	...
9)	Dpt.	30	.00	–.04	–.09	–.14	–.20	–.26	–.33	–.41	–.43	–.38	–.32	–.27	–.24
	Abo	30	.00	–.04	–.08	–.09	–.06	–.05	–.05	–.06	–.09	–.13	–.18	–.24	–.31
	Cape	30	...	...	...	...	...	...	...	...	...	...	...	...	...
	Grw.	30	+.17	+.15	+.12	+.07	+.02	–.05	–.12	–.23	–.39	–.62	–.92	–1.04	–1.09
	St. H.	30	...	...	...	...	...	...	...	+1.64	+1.26	+1.06	+.98	+.99	+1.09
	Cape	33	...	...	...	...	...	...	...	...	...	...	–.08	+.14	+.29
	Camb.	30	+.11	+.03	–.06	–.16	–.28	–.35	–.40	–.46	–.54	–.62	–.63	–.59	–.48
10)	Madr.	35	...	...	.00	+.07	+.18	+.34	+.51	+.61	+.66	+.66	+.62	+.53	+.36
	Cape	40	...	...	...	...	...	...	...	...	...	–.74	–.49	–.31	–.22
	Grw.	40	.00	–.03	–.05	–.06	–.06	–.05	–.04	–.02	+.01	+.04	+.06	+.07	+.07

TABLE IV. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\alpha$. DECLINATION. Cont.

			12h	13h	14h	15h	16h	17	18	19h	20h	21h	22	23	0
			″	″	″	″	″	″	″	″	″	″			
	Cape	65	+.02	+.02	+.02	+.01	+.01	.00	.00	.00	.01	.01	.02	.02	02
	Brs.	65	+.05	+.03	.00	—.02	—.04	—.06	.08	.08	.09	—.09	.08	.07	.05
	Pulk.	65	—.04	—.04	—.04	—.03	—.02	—.01	.00	+.01	+.02	+.03	+.04	+.04	+.04
	Leid.	67	+.02	+.02	+.03	+.03	+.03	+.02	+.02	+.01	+.01	.00	01	.01	02
	Melb.	70	+.11	+.10	+.09	+.07	+.05	+.02	—.01	.04	.06	.08	.10	.11	.11
	Grw.	72	.00	.00	.00	+.01	+.01	+.01	+.01	+.01	+.01	+.01	.00	00	.00
	Madr.	75	—.17	—.25	—.31	—.35	—.36	—.35	—.32	—.26	.19	—.11	.01	+.08	+.17
	Wn.	75	+.02	+.01	.00	—.01	—.02	—.02	—.03	.03	—.04	—.01	.03	.03	02
	Pulk.	75	—.06	—.07	—.07	—.06	—.06	—.04	—.03	.01	.00	+.02	+.01	+.05	+.06
	Harv.	75	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	00	.00
13)	Cord.	75	.00	+.02	+.04	+.05	+.06	+.07	+.07	+.07	+.06	+.05	+.04	+.02	00
	Paris	75	+.13	+.06	—.01	—.08	—.15	—.21	—.25	—.28	.28	.27	.21	19	.13
14)	Cape	80	+.10	+.12	+.12	+.12	+.11	+.09	+.07	+.01	+.01	—.02	—.05	.08	.10
		—25°	—.05	—.05	—.04	—.04	—.02	—.01	.00	+.01	+.02	+.04	+.04	+.05	+.05
		—35	+.23	+.21	+.17	+.13	+.07	+.01	.05	—.11	.16	.20	.22	24	23
		—45	+.16	+.16	+.14	+.12	+.09	+.05	+.01	.03	—.07	.11	—13	—.15	16
	Melb.	80	+.07	+.08	+.09	+.09	+.09	+.08	+.06	+.04	+.02	.01	.03	.05	.07
	Grw.	80	—.10	—.14	—.16	—.16	—.13	—.08	.02	+.03	+.08	+.10	+.10	+.08	+.06
	Pulk.	85	—.02	—.01	—.01	.00	+.01	+.01	+.02	+.02	+.03	+.03	+.03	+.02	+02
	Cape	85	—.12	—.11	—.08	—.06	—.02	+.01	+.01	+.07	+.10	+.11	+.12	+.13	+12
	Stbg.	85	—.03	—.03	—.02	—.01	—.01	.00	+.01	+.02	+.02	+.03	+.03	+.03	+.03
	Rad.	90	—.07	—.10	—.12	—.13	—.11	—.13	—.12	.10	.07	.04	.00	+.04	+.07
	Cape	90	+.06	+.06	+.06	+.05	+.04	+.03	+.01	—.01	.02	.04	—.05	.06	—.06
	Grw.	90	.00	—.01	—.02	—.02	—.02	—.03	—.03	—.02	.02	.02	—.01	.00	.00
	Madn.	90	+.21	+.22	+.21	+.17	+.10	.00	—.11	.21	.29	.33	—.32	.28	—.21
	Ber.	90	—.06	—.05	—.04	—.03	—.01	.00	+.02	+04	+.05	+.06	+.06	+.06	+.06
	Mun.	92	—.01	—.01	.00	.00	.00	+.01	+.01	+.01	+.01	+01	+.01	+.01	+.01
	Mt. H.	95	—.05	—.06	—.07	—.07	—.07	.06	.05	.04	—.02	.00	+.02	+.04	+.05
	Ber.	95	—.08	—.07	—.06	—.04	—.02	.00	+.03	+.05	+.06	+.08	+.08	+09	+.08
15)	W.-Ott.	97	—.07	—.13	—.18	—.21	—.23	—.24	.23	—.20	.16	.11	.05	+.01	+.07
	Alb.	98	+.06	+.07	+.07	+.07	+.06	+.06	+.04	+.02	.00	—.01	—.03	—.05	—.06

TABLE V. SYSTEMATIC CORRECTIONS OF THE FORM, $\Delta\delta_\delta$. DECLINATION.

			+25°	+20°	+15	+10°	+5	0	—5	—10	—15	—20	—25	—30
			″	″	″	″	″	″	″		″	″		″
8)	Br.	1755	See A.J. 515.											
	Pi.	1800	—1.53	1.86	—2.06	—2.10	—1.99	1.96	2.12	2.33	—2.26	—1.79	1.26	1.16
	Grw.	15	— .77	.72	.70	.71	— .73	.78	84	.92	1.01	1.10	1 20	1 30
	Kgb.	20	.00	— .05	.12	09	.01	+ .05	+ .08	+ .05	+ .01	...	...	...
	Bb.	25	...	...	...	...	...	...	...	...	...	...	+ .29	+ 17
9)	Dpt.	30	.22	— .25	.34	.51	.68	77	83	88	.92	97	...	...
	Abo	30	.38	.43	.17	.50	.53	.55	.59	.67	.77	.91	...	...
	Cape	30	...	...	...	.20	14	.06	+ .02	+ 10	+ 18	+ 21	+ .28	+ 30
	Grw.	30	1.16	1.33	1.51	1.60	1.59	1.15	1 39	1.56	2.07	2 53	2 86	.
	St. H.	30	+1.32	+1.37	+1.26	+1.02	+ .91	+ 92	+1.01	+1.01	+ .72	03	35	.18
	Cape	33	+ .40	+ .38	.32	.22	.12	11	21	11	+ 20	+ 53	+ 72	+ 79
	Camb.	30	.30	16	14	.16	23	12	69	.95	1 22	1 49	1 76	...
10)	Madr.	35	+ .07	.17	30	.36	.36	33	.28	20	10	+ 05	+ 25	+ 17
	Cape	10	— .21	.23	.20	.12	+ .04	+ .38	+ 58	+ 56	+ 40	+ .34	+ 22	+ 10
	Grw.	10	+ .08	+ .10	+ .14	+ .18	+ .24	+ 30	+ 37	+ 12	+ 16	+ 18	+ 19	.

TABLE V. SYSTEMATIC CORRECTIONS OF THE FORM, $I\delta_\delta$. DECLINATION. — Cont.

			+90°	+85	+80	+75	+70	+65°	+60°	+55°	+50	+45	+40	+35	+30
			″	″	″	″	″	″	″	″	″	″	″	″	″
	Grw.	45	.00	+.02	+.01	+.06	+.09	+.12	+.12	+ .08	.00	− .08	−.08	+ .03	+ .11
	Arm.	40	.00	.00	.00	.00	.00	.00	.00	− .14	− .49	− .67	−.62	− .56	− .55
11)	Rad.	45	.00	+.16	+.50	+.74	+.67	+.32	+.15	+ .08	+ .01	− .12	−.31	− .53	− .70
	Pulk.	45	.00	.00	.00	.00	+.01	+.10	+.17	+ .21	+ .23	+ .26	+.32	+ .36	+ .36
	Paris	45	.00	.00	.00	.00	−.08	−.15	−.17	.13	− .01	+ .11	+.18	+ .12	+ .03
	Stgo.	50	...	...	...	...	...	...	...	...	...	...	+.17	+ .39	+ .56
	Grw.	50	−.12	−.13	−.14	−.15	−.16	−.17	−.18	− .18	− .17	− .15	−.12	− .07	− .01
12)	Cape	50	...	...	...	...	...	...	...	...	...	...	...	...	...
	Stgo.	55	...	...	...	...	...	...	...	...	...	...	...	...	...
	Pulk.	55	−.26	−.22	−.16	−.05	+.04	+.08	+.09	+ .09	+ .12	+ .21	+.30	+ .35	+ .35
	Wn.	60	.00	+.16	+.36	+.43	+.41	+.35	+.37	+ .40	+ .38	+ .32	+.25	+ .14	+ .06
	Grw.	60	.00	+.06	+.13	+.19	+.25	+.29	+.31	+ .30	+ .26	+ .18	+.08	+ .06	+ .14
	Rad.	60	.00	+.27	+.68	+.90	+.86	+.68	+.49	+ .38	+ .29	+ .06	−.30	− .70	− .90
	Cape	60	...	...	...	...	...	...	...	...	...	− .02	−.06	− .11	− .16
	Paris	60	.00	−.03	−.07	−.06	−.03	+.02	+.08	+ .15	+ .16	+ .07	−.07	− .14	− .17
	Stgo.	60	...	...	...	...	...	...	...	...	...	...	...	...	.00
16)	Melb.	60	...	...	...	...	...	...	...	...	...	...	+.38	+ .34	+ .30
	Grw.	64	.00	.00	.00	.00	−.02	−.04	−.04	.00	+ .06	+ .06	+.06	+ .06	+ .10
	Cape	65	...	...	...	...	...	...	...	...	...	...	+.20	+ .13	+ .05
	Brs.	65	+.38	+.41	+.44	+.47	+.50	+.52	+.43	+ .19	− .03	− .14	−.16	− .09	.00
	Pulk.	65	.00	.00	.00	.00	.00	.00	+.04	+ .11	+ .21	+ .26	+.27	+ .24	+ .23
	Leid.	67	.00	+.03	+.07	+.10	+.05	−.09	−.19	− .20	− .18	− .15	−.10	− .08	− .15
	Melb.	70	...	...	...	...	...	...	...	...	...	...	+.76	+ .48	+ .19
	Grw.	72	.00	−.04	−.09	−.14	−.20	−.28	−.36	− .44	− .49	− .53	−.54	− .54	− .52
17)	Madr.	75	...	...	+.26	−.04	−.64	−.70	+.60	− .13	− .23	− .22	−.26	− .17	+ .54
	Wn.	75	.00	+.01	+.03	+.06	+.09	+.12	+.17	+ .21	+ .18	+ .06	−.10	− .24	− .30
	Pulk.	75	.00	.00	.00	+.02	+.04	+.06	+.08	+ .10	+ .12	+ .12	+.10	+ .06	+ .02
	Harv.	75	.00	+.02	+.06	+.10	+.09	+.95	−.05	− .14	− .11	+ .10	+.26	+ .28	+ .24
18)	Cord.	75	...	...	...	...	...	...	...	...	...	...	...	...	− .94
	Paris	75	.00	.00	.00	−.04	−.10	−.17	−.21	− .23	− .26	− .29	−.33	− .34	− .32
14)	Cape	80	...	...	...	...	...	...	...	...	...	+ .06	−.02	− .07	− .11
	Melb.	80	...	...	...	...	...	...	...	...	...	+1.00	+.72	+ .41	+ .11
	Grw.	80	.00	+.02	+.05	+.11	+.17	+.21	+.21	+ .18	+ .10	+ .01	−.04	− .06	− .01
	Pulk.	85	.00	.00	.00	+.02	+.04	+.05	+.06	+ .04	+ .04	+ .05	+.09	+ .14	+ .18
	Cape	85	...	...	...	...	...	...	...	...	...	−1.03	−.81	− .62	− .47
	Stbg.	85	.00	.00	.00	−.02	−.06	−.07	−.07	− .06	− .07	− .09	−.08	− .06	− .05
	Rad.	90	.00	.00	.00	−.04	−.13	−.32	−.54	− .58	− .49	− .45	−.50	− .62	− .72
	Cape	90	...	...	...	...	...	...	...	...	...	− .30	−.07	+ .02	.00
	Grw.	90	−.10	−.10	−.10	−.09	−.06	−.01	.00	− .03	− .09	− .10	−.08	− .06	− .05
	Madn.	90	.00	−.06	−.10	−.13	−.14	−.14	−.14	− .12	− .08	− .02	+.10	+ .26	+ .32
	Ber.	90	.00	+.03	+.07	+.13	+.16	+.15	+.13	+ .09	+ .04	− .01	−.06	− .12	− .14
	Mun.	92	.00	−.05	−.14	−.24	−.28	−.25	−.24	− .32	− .44	− .63	−.78	...	...
	Mt. H.	95	.00	+.05	+.08	+.07	+.05	.00	...	...	...	+ .09	.00	− .17	− .25
	Ber.	95	+.06	+.06	+.06	+.06	+.07	+.11	+.19	+ .18	+ .07	− .03	−.10	− .14	− .16
15)	W.-Ott.	97	.00	.00	.00	.00	.00	+.02	+.07	+ .06	− .03	− .19	...	...	− .85
	Alb.	98	−.08	+.02	+.18	+.31	+.37	+.36	+.29	+ .17	+ .10	+ .06	+.06	+ .04	− .01

Table V. Systematic Corrections of the Form. $\Delta\delta_\delta$. Declination. — Cont.

		+25°	+20°	+15°	+10°	+5°	0	—5	—10°	—15	—20	—25	—30°
	Grw. 45	+ .17	+ .12	.00	— .04	.00	+ .09	+ .10	+ .07	.00	.07	.13	.21
	Arm. 40	— .53	— .52	.55	— .71	.86	— .88	.83	.76	.69	— .58	.31	— .22
11)	Rad. 45	— .77	— .72	— .58	.32	— .08	+ .17	+ .41	+ .56	+ .54	+ .28	.32	. . .
	Pulk. 45	+ .35	+ .34	+ .33	+ .34	+ .35	+ .38	+ .42	+ .47	+ .53	+ .60	. . .	. . .
	Paris 45	.00	.00	+ .01	— .07	— .15	— .19	— .19	— .15	.05	+ .11	+ .34	+ .60
	Stgo. 50	+ .69	+ .78	+ .84	+ .82	+ .85	+ .93	+ .99	+1.02	+1.00	+ .95	+ .82	+ .56
	Grw. 50	+ .05	+ .13	+ .15	+ .11	— .01	.13	.14	.13	— .11	— .07	.03	+ .01
12)	Cape 50	. . .	. . .	. . .	. . .	. . .	— .10	.34	.19	+ .02	+ .14	+ .14	+ .07
	Stgo. 55	+ .26	+ .20	+ .14	+ .10	+ .06	+ .04	+ .03	.01	— .11	.25	.35	.34
	Pulk. 55	+ .32	+ .33	+ .37	+ .39	+ .40	+ .40	+ .42	+ .50	+ .71	+ .99	. . .	. . .
	Wn. 60	+ .02	— .02	— .07	— .17	— .19	— .14	— .04	— .01	.00	+ .03	+ .12	+ .31
	Grw. 60	+ .26	+ .29	+ .28	+ .23	+ .12	+ .01	+ .02	+ .04	+ .10	+ .20	+ .32	+ .16
	Rad. 60	— .87	— .67	— .45	— .22	+ .01	+ .26	+ .51	+ .74	+ .82	+ .66	+ .36	+ .13
	Cape 60	— .21	— .26	— .28	— .31	— .33	— .34	— .34	— .31	— .22	— .08	.01	+ .02
	Paris 60	— .18	— .22	— .26	— .28	— .26	— .20	— .18	— .15	— .05	+ .14	+ .43	+ .78
	Stgo. 60	— .06	— .14	— .28	— .46	— .56	— .52	— .30	.12	— .14	.20	— .12	— .21
16)	Melb. 60	+ .28	+ .28	+ .35	+ .50	+ .78	+ .90	+ .90	+ .76	+ .55	+ .41	+ .41	+ .40
	Grw. 64	+ .12	+ .11	+ .09	+ .05	.00	— .01	— .05	.00	+ .11	+ .26	+ .46	+ .66
	Cape 65	— .02	— .09	— .16	— .20	— .24	— .25	— .24	— .19	.11	.00	+ .04	.03
	Brs. 65	+ .04	+ .03	.00	— .03	— .04	.04	— .02	+ .02	+ .08	+ .16	+ .27	+ .47
	Pulk. 65	+ .24	+ .31	+ .36	+ .35	+ .33	+ .35	+ .40	+ .46	+ .54	+ .64	. . .	. . .
	Leid. 67	— .29	— .36	— .37	— .32	— .22	— .05	+ .04	+ .06	+ .01	— .13	. . .	. . .
	Melb. 70	— .09	— .35	— .40	— .26	— .17	— .18	— .25	— .35	— .40	— .42	— .41	— .41
	Grw. 72	— .50	— .52	— .59	— .74	— .95	—1.15	1.26	—1.34	1.46	—1.70	2.09	2.53
17)	Madr. 75	+ .62	+ .79	+ .33	— .41	— .58	— .66	.05	.09	+ .01	.02	+ .10	+ .86
	Wn. 75	— .31	— .29	— .27	— .26	— .25	— .24	— .26	.30	— .31	— .29	.22	.08
	Pulk. 75	+ .02	+ .05	+ .09	+ .11	+ .12	+ .12	+ .17	+ .27	+ .42	+ .60	. . .	. . .
	Harv. 75	+ .23	+ .31	+ .29	+ .06	.00	+ .12	+ .25	+ .31	+ .32	+ .33	+ .36	+ .42
18)	Cord. 75	— .82	— .70	— .60	— .54	— .49	— .48	— .49	— .46	— .37	— .31	.28	.28
	Paris 75	— .28	— .26	— .25	— .24	.23	— .22	— .22	— .21	.16	.02	+ .23	+ .49
14)	Cape 80	— .11	— .16	— .17	— .18	— .15	— .07	— .03	— .02	— .03	— .07	.12	.21
	Melb. 80	— .20	— .41	— .41	.32	.28	.32	— .45	.55	.60	— .61	.65	.67
	Grw. 80	+ .11	+ .23	+ .28	+ .26	+ .15	+ .11	+ .22	+ .34	+ .44	+ .51	+ .61	+ .79
	Pulk. 85	+ .22	+ .24	+ .25	+ .25	+ .24	+ .22	+ .19	+ .15	+ .06	.14	. . .	. . .
	Cape 85	— .37	.31	— .30	— .29	— .28	.27	.26	— .26	.27	.28	.28	.24
	Stbg. 85	— .08	— .10	— .08	.02	.00	.00	.00	.00	.03	.08	.13	.18
	Rad. 90	— .76	.77	.77	.77	.76	.74	— .68	.61	.54	.47	.39	— .32
	Cape 90	— .05	— .06	.04	.01	.02	.05	.09	.11	.10	.08	.01	+ .04
	Grw. 90	— .04	— .02	.00	+ .02	+ .03	+ .04	+ .06	+ .12	+ .21	+ .34	+ .50	+ .70
	Madn. 90	+ .31	+ .28	+ .29	+ .34	+ .38	+ .38	+ .34	+ .29	+ .23	+ .18	. . .	. . .
	Ber. 90	— .11	.08	.06	.03	.00	+ .03	+ .05	+ .08	+ .10	. . .	. . .	. . .
	Mun. 92	. . .	. . .	.70	.74	.79	.84	.89	.97	1.05	. . .	. . .	. . .
	Mt. H. 95	.23	— .14	.10	.09	.10	.05	+ .04	+ .14	+ .21	+ .26	+ .32	+ .43
	Ber. 95	— .14	.12	.12	.12	.14	.09	.02	+ .08	+ .22	+ .12	+ .62	. . .
15)	W.-Ott. 97	—1.01	1.07	1.05	.98	.85	.72	.64	.59	.56	.52	.46	.35
	Alb. 98	.06	.09	.10	.09	.05	+ .01	.00	.05	.08	.06	.02	+ .06

TABLE V. SYSTEMATIC CORRECTIONS, $I\delta_0$, SOUTH OF −30°.

		−30°	−35°	−40	45	−50	−55	60	65°	−70	−75	−80	−85	−90°
		"	"	"	"	"	"	"	"	"	"	"	"	"
	Pi. 1800	−1.16	−1.36	−1.98	2.63	...	...	...	...	...	..	..	..	..
	Bb. 25	+.17	+.08	+.06	+.16	+.34	+.56	+.97	+1.20	+1.03	+.58	+.32	+.13	.00
	Cape 30	+.30	+.26	+.18	+.04	.11	.30	−.38	−.40	−.32	−.16	.00	.00	.00
	St. H. 30	−.18	+.33	+.52	+.48	+.27	+.13	+.10	+.22	+.55	+.75	+.73	..	..
	Cape 33	+.79	+.79	+.51	.06	.25	−.23	−.25	−.30	−.16	−.02	.00	.00	.00
10)	Madr. 35	+.47	+.75	+1.00	+1.15	+1.23	+1.24	+1.20	+1.13	...	..	..	..	..
	Cape 40	+.10	−.04	−.22	.38	−.52	−.64	−.74	−.79	−.71	−.42	−.21	−.07	.00
	Stgo. 50	+.56	+.25	+.13	+.27	+.36	+.36	+.19	.00	.00	.00	.00	.00	.00
12)	Cape 50	+.07	.01	−.08	.17	−.36	−.32	−.17	.23	−.37	−.31	−.12	−.03	.00
	Stgo. 55	−.34	−.19	−.07	.04	−.07	−.18	−.35	...	...	..	−.50	−.36	−.17
	Wn. 60	+.34	+.64	+.97	...	...	...	...	...	...	..	..	..	..
	Cape 60	+.02	.00	−.04	−.06	−.06	−.04	.00	+.05	+.08	+.10	+.10	+.09	+.08
	Stgo. 60	+.21	+.63	+.83	+.88	+.76	+.55	+.37	+.25	+.15	+.08	+.03	.00	.00
	Melb. 60	+.10	+.38	+.30	+.16	−.05	−.29	−.48	−.54	−.52	−.45	−.33	−.17	.00

TABLE VI. WEIGHTS IN RIGHT-ASCENSION FOR THE PRINCIPAL CATALOGUES.

	Catal.	Wts.	.05	.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
	Bradley 1755	Eq.	2	4	6	8	10	13	15	17	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
19)		+60°	1	2	3	4	5	6	7	8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
20)	Piazzi 1800		8	27	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
	Greenw. 15		..	1	2	3	..	4	5	6	8	10	13	17	..	..	..	..	..	..	..	..	..	..	..	..
	Dorpat 15			..	1	..	..	2	..	3	4	5	6	7	16	..	..	..	..	..	..	..	..	..	..	..
	Königsb. 15		1	2	3	4	6	7	8	10	13	16	21	26	..	..	..	..	..	..	..	..	..	..	..	..
	Königsb. 25			1	..	2	3	4	..	5	6	7	9	10	19	31	50	78	..	..	..	..	..	..	..	..
	Dorpat 30		..	1	2	..	3	4	5	6	7	9	10	12	26	51	..	..	..	..	..	..	..	..	..	..
	Cape 30		1	2	4	5	7	9	12	15	21	38	75	..	..	..	..	..	..	..	..	..	..	..	..	..
	St. H. 30		.	1	2	3	4	..	6	7	8	11	15	20	..	..	..	..	..	..	..	..	..	..	..	..
	Abo 30		..	..	1	..		2	..	..	3	4	5	6	10	14	20	27	35	44	63	..	..	..	..	..
	Greenw. 30		..	2	5	7	9	12	16	21	31	36	..	..	..	..	..	..	..	..	..	..	..	..	..	..
	Cambr. 30		..	1	2	3	4	5	6	8	11	15	25	65	..	..	..	..	..	..	..	..	..	..	..	..
	Cape 33		..	1	..	..	2	..	3	4	5	6	7	9	20	45	..	..	..	..	..	..	..	..	..	..
21)	Madras 35		1	2	4	6	8	11	14	19	28	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
	Armagh 40		1	2	4	6	9	13	18	27	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
	Cape 40		1	2	3	4	5	6	7	8	9	12	14	19	29	44	..	..	..	..	..	..	..	..	..	..
	Greenw. 40			1	2	3	..	4	5	6	8	10	13	15	43	..	..	..	..	..	..	..	..	..	..	..
	Greenw. 45		..	1	..	2	3	..	4	5	6	8	9	11	23	40	75	..	..	..	..	..	..	..	..	..
	Radel. 15	Eq.	1	.	2	3	4	5	6	7	10	13	18	25	..	..	..	..	..	..	..	..	..	..	..	..
		+60°	..	..	1	..	..	2	..	3	5	7	10	13	..	..	..	..	..	..	..	..	..	..	..	..
22)	Pulkowa 15	S.	..	..	..	..	1	..	..	..	2	..	3	4	6	9	13	16	21	27	37	58	..	..	..	..
		N.	..	..	..	..	..	..	..	..	..	1	..	2	..	3	5	6	7	8	10	14	19	25	35	48
	Paris 15	Eq.	..	1	2	..	3	4	5	6	7	8	10	13	23	39	64	110	..	..	..	..	..	..	..	..
		+60°	..	..	1	..	..	2	..	3	..	4	5	7	11	17	25	39	58	95	..	..	..	..	..	..
	Santiago 50		..	1	2	..	3	4		5	6	8	10	12	26	55	..	..	..	..	..	..	..	..	..	..
	Greenw. 50		.	1	..	2	..	3	.	4	5	7	8	9	17	27	41	61	91	..	..	..	..	..	..	..
	Cape 50		.	1	2	3	..	4		6	10	15	25	..	..	..	..	..	..	..	..	..	..	..	..	..
	Santiago 55		..	1	2	3	.	4		6	10	15	25	..	..	..	..	..	..	..	..	..	..	..	..	..
	Cape 60		..	.	1	..	.	2		3	..	4	5	6	11	17	25	38	57	..	..	..	..	..	..	..
	Wash'n 60		.	1	.	2	.	3	..	4	5	7	8	10	17	28	41	61	91	..	..	..	..	..	..	..
	Greenw. 60		..	..	..	1	.	..	2	..	3	..	4	5	8	13	18	24	32	42	65	..	..	..	..	..
	Radel. 60		.	1	2	3	4	..	5	6	8	10	12	17	..	..	..	..	..	..	..	..	..	..	..	..
	Santiago 60			1	2	3	..	4	.	6	10	15	25	..	..	..	..	..	..	..	..	..	..	..	..	..
	Melb. 60		..	..	1	..	..	2	..	..	3	4	5	6	10	14	20	26	34	44	..	..	..	..	..	..
	Paris 60	Eq.	1	..	2	3	4	5	..	6	8	9	11	14	23	52	71	..	..	..	150	250	..	..	..	..
		+55°	..	1	..	2	3		4	5	6	8	9	11	22	52	71	..	..	..	150	250	..	..	..	..
		+80°	..	..	1	..	2		3	..	4	5	6	8	15	52	71	..	..	..	150	250	..	..	..	..
	Greenw. 64		..			1	.		..	2	.	3	.	4	7	10	13	17	22	26	35	50	75	..	..	..

TABLE V. SYSTEMATIC CORRECTIONS, $\Delta\alpha$, SOUTH OF −30°. — Cont.

	−30	−35	40	45°	−50	55	60	65	70	−75°	80	85	90
	"	"	"	"	"	"	"	"					
Cape 65	− .03	− .03	+ .02	+ .09	+ .13	+ .11	+ .11	+ .16	+ .17	+.19	+ .21	+ .23	+ .25
Melb. 70	− .41	− .47	− .63	− .63	.54	.15	.46	.16	.12	.31	.18	.08	.00
17) Madr. 75	+ .86	+ .95	+1.11	+ .75	+ .13	+ .16	+1.60	+1.20	+2.00				
Wn. 75	− .08	+ .22	+ .71	...	...	...	...	...	...	...			
Harv. 75	+ .42	+ .54	...	...	...	...	...	...	...				
18) Cord. 75	− .28	− .26	− .23	− .18	− .14	.11	.08	.06	.04	.03	.02	.01	.00
14) Cape 80	− .21	.29	.26	− .10	+ .04	+ .12	+ .16	+ .17	+ .25	+.43	+.44	+ .26	+ .06
Melb. 80	− .67	− .69	− .74	.71	− .67	.60	.15	.28	.17	−.10	−.05	.02	.00
Cape 85	− .24	− .19	− .16	.17	.24	.30	− .29	.20	.12	.06	−.03	.01	.00
Cape 90	+ .04	+ .05	+ .02	.02	.03	.00	+ .02	+ .04	+ .04	+.03	+ .01	.00	.00
Mt. H. 95	+ .43	+ .65	+1.12	...	...	...	...	...	...	...	...	...	...
15) W-Ott. 97	− .35	− .22	...	...	...	...	...	...	...	...	...	...	...
Alb. 98	+ .06	+ .15	+ .25	...	...	...	...	...	...	...	...	...	...

TABLE VI. WEIGHTS IN RIGHT-ASCENSION FOR THE PRINCIPAL CATALOGUES. — Cont.

Catal.		Wts	.05	.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Cape	65				1			2		3	1	5	6	7	12	20	32	50								
Brussels	65			1		2		3		4		5	7	8	28											
Harvard	65				1		2			3	1	5	6	7	15											
2) Pulkowa	65	S.									1			2	3	4	6	7	9	10	11	18	28	35	56	
		N.											1		2	3	4	5	6	7	9	12	11	18	23	28
Melb.	70					1			2		3		4	5	8	12	17	22	28	35	50	78				
Greenw.	72	Eq.					1				2		3	4	6	9	11	11	18	22	30	39	51	74		
		+60°									1			2	3	5	6	7	9	11	11	20	28	37	53	77
Madras	75	Eq.	1	3	1	6	8	9	11	13	16	20	21	34	61											
		+60°	1	2	4	5	7	10	13	17	23	16														
		−50°	2	1	7	11	17	25	61																	
Wash'n	75				1			2			3	4	5	6	9	13	17	22	27	32	41	55	73	91		
Pulkowa	75	Eq.		1		2		3		4	5	6	7	8	13	18	21	30	36	43	53	69	86			
		+60°				1			2		3		4	5	8	11	14	18	21	25	30	42	54	67	84	
Harvard	75	Eq.				1			2		3	4	5	6	9	12	16	20	25	30	38	50	65	83		
		+70°							1				2	3	4	6	8	10	12	14	18	25	34	45	61	83
3) Cordoba	75					1		2	3	4	5	6	8	10	28											
Paris	75			1		2		3		4	5	6	7	10	17	29	47	80								
Cape	80			1			2		3		4	5	6	8	12	18	26	34	45	57	[illegible]					
Melb.	80			1		2		3		4	5	6	7	9	15	23	35	50	72							
Greenw.	80						1				2			3	6	8	11	11	18	23	30	45	64			
2) Pulkowa	85 }	S.									1			2	3	4	6	7	9	10	11	18	28	35	56	
	92 }	N.											1		2	3	4	5	6	7	9	12	14	18	23	28
Cape	85	Eq.			1		2			3	4	5	6	7	10	15	19	23	28	33	40	51	63	76		
		60°					1				2		3	4	6	8	11	14	17	20	27	37	50	68		
Strassb.	85									1				2	4	5	7	9	11	13	17	25	36	51	80	
Radel.	90				1		2			3	4		5	7	10	15	20	25	31	38	49	66				
Cape	90				1			2			3	4	5	6	9	13	17	22	27	32	40	54	71	91		
Madison	90												1		2	3	4	5	6	8	10	15	21	38	70	
4) Berlin	90													1		2		3	4	5	6	9	12	17	24	40
Lisbon	90													1		2	3		4	5	7	9	12	16	22	32
Greenw.	90							1			2		3	4	5	8	10	12	15	18	24	32	43	57	77	
Mt. H.	95																			10						
Berlin	95													1		2		3	4	5	6	9	12	17	24	40
5) Albany	98										1			2	3	4	6	7	9	11	14	20				

TABLE VII. WEIGHTS IN DECLINATION FOR THE PRINCIPAL CATALOGUES.

	Catal.	Wts.	.05	0.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
26)	Bradley	1755	1	2	4	5	8	11	17	28																
	Piazzi	1800	1	1	9	18																				
	Greenwich	15			1		2			3	4	5	6	8	12	18	25	35								
	Konigsberg	20	Argument, probable error.																							
	Brisbane	25	2	11																						
	Dorpat	30								1			2	3	4	7	11	17	30	69						
	Abo	30								1			2	3	4	7	11	17	30	69						
	Cape	30		1	2	4	7	10																		
	Greenwich	30	1		2		3	4	5	6	7	10	14	26												
	St. Helena	30	1	2	3	5	6	8	11	15	23															
	Cape	33			1			2			3	4	5	6	13											
	Cambridge	30				1		2			3	4	5	8	19											
	Madras	35		1	2	3	4	6	8	12	19															
	Cape	40				1			2		3	4	5	7	16											
	Greenwich	40				1		2			3	4	5	7	12	22	46									
	Greenwich	45				1				2		3	4	5	9	15	27	51								
	Armagh	40		1	2	3	4	5	6	7	9	13	18													
	Radcliffe	45				1		2		4	6	11	21													
27)	Pulkowa	45												1		2	3	4	6	7	12					
	Paris	45			1			2		3	4	5	6	8	13	23	38	70								
	Santiago	50	1		2		3	4	5	6	7	10	14	26												
	Greenwich	50				1		2			3	4	5	7	12	22	46									
	Cape	50				1				2	3	4	5	6	15											
	Santiago	55	1		2	3		4	5	7	10	13	18	25												
28)	Pulkowa	55				1				2		3	4	5	7	11	17	25	36	61						
	Washington	60		1		2		3	4	5	6	7	9	13	28	80										
	Greenwich	60								1			2	3	5	7	11	15	23	38						
	Radcliffe	60		1	2	3	4	5	6	8	12															
	Cape	60					1			2		3	4	5	7	11	17	25	39	61						
	Paris	60			1			2		3	4	5	6	8	13	23	38	70								
	Santiago	60	1		2	3		4	5	7	10	13	18	25												
	Melbourne	60			1			2		3	4	5	6	8	13	23	38	70								

NOTE 1. The magnitude-equations for the two Madras Catalogues are very uncertain. There is reason to believe that they increase more rapidly than the proportion of numerical magnitude. The ground for this suspicion is given in the chapter on Magnitude-Equation, *A.J.* 536.

NOTE 2. The corrections for Pulkowa 92 (right-ascensions of secondary standards determined with the Transit) rest on the comparison of only 148 stars with the Standard Catalogue. Many of these stars depend upon a comparatively small number of observations. For observed $\Delta\alpha_m$ we have

Mag.	p.	$\Delta\alpha_m$
1.9	8	$+0^s.011$
3.1	28	-0.005
4.0	94	$+0.005$
4.9	80	$+0.006$
6.0	17	$+0.006$

All the corrections for this catalogue in the present tables must be regarded as approximate only.

NOTE 3. The corrections for Albany 98 are intended to apply, not to the positions obtained in the preliminary reductions on quasi fundamental principles, but to the positions to be published in the final catalogue for 1900.

NOTE 4. STRUVE's right-ascensions as published in "Catalogue I," Part II, Volume I, of the Dorpat observations, were corrected (by means of the equations given for each star of the Catalogue) to the aberration, 20″.50, and nutation, 9″.224. Large corrections were also applied to the catalogue positions on account of the adopted corrections to the assumed right-ascensions of STRUVE's six standard stars, as follows:

I	*Capella*,	$+0^s.17$	IV	*α Persei*	$+0^s.21$
II	*α Lyrae*,	$+0.17$	V	*δ Cassiopeae*,	$+0.24$
III	*α Cygni*,	$+0.18$	VI	*ζ Urs. Maj.*,	$+0.18$

The corrections given in Tables II and III apply to STRUVE's right-ascensions so treated. The right-ascensions of "Catalogue II," 1814, were reduced to Catalogue I by the application of the following correction,

$$+0^s.18 - 0^s.0442\,t + [+^s.019 + ^s.031 \sin(268^\circ + \alpha)] \sec\delta$$

t is taken from Catalogue II.

TABLE VII. WEIGHTS IN DECLINATION FOR THE PRINCIPAL CATALOGUES — Cont.

Catal.	Wts.	.05	.1	.15	.2	.25	.3	.35	.4	.5	.6	.7	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Greenwich	64								1			2	3	5	7	11	15	23	38						
Cape	65					1			2		3	4	5	7	11	17	25	39	61						
Brussels	65						1			2		3	6												
Pulkowa	65												1		2		3	4	5	6	9	11	15	19	27
Leiden	67																		16	30					
Melbourne	70						1			2		3	4	7	13	28									
Greenwich	72					1			2		3	4	5	7	11	17	25	38	61						
Madras	75	1	2	3	4	5	7	8	10	14	20	30													
Washington	75				1				2		3	4	5	8	11	15	19	24	30	41	61	91			
Pulkowa	75								1			2	3	4	5	7	9	11	13	18	24	31	46	66	
Harvard	75		1		2	3		4	5	6	7	9	12	21	36	61									
Cordoba	75				1				2	3	5	7	11												
Paris	75			1			2		3	4	5	6	8	13	23	38	70								
Cape	80						1				2		3	5	7	10	14	18	25						
Melbourne	80					1			2		3	4	5	8	13	22	36								
Greenwich	80						1			2		3	4	6	8	11	15	18	23	33	52	88			
Pulkowa	85												1		2		3	4	5	6	9	11	15	19	27
Cape	85						1				2	3	4	5	8	12	16	22	30	50					
Strassburg	85						1				2		3	5	7	9	12	15	19	25	38	57	91		
Radcliffe	90					1			2		3	4	5	7	11	16	22	30	42	76					
Cape	90					1				2		3	4	6	9	12	16	21	27	40	68				
Greenwich	90						1				2		3	5	7	9	12	15	19	25	38	57	91		
Madison	90									1			2	3	5	6	8	9	11	15	20	28	38	54	78
Berlin	90												1		2		3		4	6	8	12	19	36	
Munich	92												1				15								
Mt. Hamilton	95																	10							
Berlin	95												1		2		3		4	6	8	12	19	36	
Wien-Ottak.	97				1				2		3	4	5	8	12	17	23	30	39	60					
Albany	98										1		2	3	4	5	7	10	14						

NOTE 5. Under Konigsberg 20 are designated the right-ascensions of circumpolar stars observed by BESSEL, and reduced by ARGELANDER, Volume VI, p. XV, of the Konigsberg observations. These correspond to BESSEL's equinox of 1815; but are, otherwise, supposed to be homogeneous with BESSEL's right-ascensions of time stars with the REICHENBACH circle.

NOTE 6. The corrections in right-ascension for the General Catalogue, Cordoba 1875, apply to the mean for the years, 1872–1880, inclusive. The following corrections are adopted in order to reduce the right-ascensions, clamp east, to the mean. For right-ascensions, clamp west, the opposite sign must be employed.

CORRECTIONS, CLAMP EAST.

δ	Corr.	δ	Corr.
°	s	°	s
0	.006	45	+.029
−5	−.001	50	+.030
10	+.004	55	+.030
15	+.009	60	+.030
20	+.014	65	+.030
25	+.018	70	+.030
30	+.023	75	+.030
35	+.027	80	+.030
40	+.028	85	+.030

The clamp was east in 1872, 1875.67 to 1877.0, 1878, 1880 to 1884.

NOTE 7. The values of $\Delta\alpha$ for the Cape Catalogue for 1880, are, in part, treated by zones. Thus at −35°, −45°, −55°, −65° and −75° in Table III, two values of $\Delta\alpha$ are given. The uppermost in each case pertains to the zone of lesser declination, the lowermost to that of greater declination. The great leap at −75° seems to be well established. Since, for nearly all the stars compared with the Standard Catalogue the number of observations is only three, the present results for the zones can only be regarded as approximate.

NOTE 8. The line in Table IV referring to BRADLEY 1755, and marked "N," is applicable to declinations from Quadrant North; the lower line to those from Quadrant South. The curve of $\Delta\delta$ for BRADLEY is too irregular to admit of accurate interpolation from a table given for intervals of 5° only. A table for BRADLEY's declinations will be found in *A.J.* 545. $\Delta\delta$ is applicable to the declinations formed (without the correction [illegible]) from [illegible] distances; [illegible] applies to declinations from the Catalogue.

NOTE 9. The corrections for Dorpat 30 [illegible] are applicable to the catalogue declinations [illegible] *Corrections* [illegible] p. 357, F. M.

NOTE 10. Madras 35 refers to DOWNING's new edition of TAYLOR's Catalogue for 1835.

NOTE 11. Radcliffe 1845. The corrections in Tables IV and V correspond to the catalogue declinations corrected for the quantities given in the table, p. viii, of the introduction to the Catalogue.

NOTE 12. The corrections in Tables IV and V for Cape 50 are very uncertain, owing to the very small number of observations of the principal stars contained in the Catalogue.

NOTE 13. As might naturally be expected from the long period embraced in YARNALL's Catalogue there is great uncertainty in $\Delta\delta_s$. South of $-20°$ it is assumed to be zero.

NOTE 14. The values of $\Delta\delta_s$ for Cape 80, adopted in Table IV, are determined from comparison with the Standard Catalogue in zones. The second line applies to the zone $-25°$ to $-35°$; the third line to $-35°$ to $-45°$; and the fourth line, to all declinations south of $-45°$.

NOTE 15. "W.-Ott. 97" indicates GROSSMAN's Catalogue of declinations observed at Von Kuffner's Observatory in Wien-Ottakring, and published in *Abh., Kön. Sächs. Ges. der Wiss.*, Band XXVII, Leipzig.

NOTE 16. To the declinations of the Williamstown observations (Melb. 60) are first applied the corrections on account of latitude, graduation error, and flexure, contained in the table at p. xxi of the introduction to the Catalogue (*Melb. Obs.*, Vol. I). Table V refers to the declinations thus corrected.

NOTE 17. The process by which $\Delta\delta_s$ for Madras 1875 was obtained assumes that a large part of the error of that Catalogue is due to the faulty application of division-correction. Shortly after the publication of that Catalogue, I compared it with the system B_s (*Am. Ephem.* 1881–1899) and its southward extension, B_s (*A.J.* 448–50). This comparison not only indicated recurrence of systematic errors at intervals of 60°, but also that these errors would have been far less striking if the correction for error of graduation had been applied in the reductions with the opposite sign. The following exhibit contains, in the first column, the effect of division correction, D, as adopted in the reductions for the Catalogue; in the second column, the division correction, D_1, as it results from comparison of the Catalogue declinations with the Standard Catalogue; and in the third column that part of adopted $\Delta\delta_s$ which is found to recur at intervals of 60°.

ANALYSIS OF MADRAS DECLINATIONS (1875).

δ	D	D_1	D_r	δ	D	D_1	D_r
°	"	"	"	°	"	"	"
+60	−.35	+.32	+.68	+30	−.22	+.36	+.57
57	−.16	+.34	+.47	27	−.40	+.19	+.61
54	+.20	−.13	−.25	24	−.63	+.15	+.76
51	+.27	+.01	−.27	21	−.51	+.31	+.81
48	+.13	−.02	−.08	18	−.22	+.23	+.49
45	+.01	−.02	−.18	15	−.07	+.23	+.28
42	+.03	−.56	−.39	12	+.08	+.04	−.04
39	+.27	+.04	−.27	9	+.31	.28	−.59
36	+.20	−.10	−.28	6	+.11	−.56	−.64
33	.05	+.25	+.30	3	−.29	−.18	+.06

D represents the smoothed discrepancies which actually exist between the declinations of the Standard Catalogues, B_s and B and those of Madras 75; $D_1 + 2D$ represents the discrepancies which would have existed *if* the correction for graduation error had been applied in the reductions with the contrary sign. Taking the discrepancies without regard to sign we have:

$$\Sigma\, D_1 = 8''.02 \quad ; \quad \Sigma\,[D_1 + 2D] = 4''.04$$

Therefore the agreement with the Standard Catalogue would have been twice as good if the correction for error of graduation had been applied with the opposite sign. The new system of declinations, B, differs so little in 1875 from B and B_s that I have not thought it worth while to repeat this investigation.

The general curve of $\Delta\delta_s$ for Madras 75 has been slightly modified in places to conform better with the system, B.

ADOPTED VALUES OF $\Delta\delta$ FOR MADRAS 75.

δ	Δδ	δ	Δδ	δ	$\Delta\delta_s$	δ	$\Delta\delta_s$
°	"	°	"	°	"	°	"
+80	+.26	+42	−.45	+4	−.34	−34	+0.91
79	+.17	41	.49	3	.00	35	+ .95
78	+.07	40	−.26	2	+.36	36	+1.09
77	−.01	39	−.27	+1	+.46	37	+1.24
76	−.07	38	.38	0	+.66	38	+1.23
75	−.04	37	−.40	−1	+.69	39	+1.16
74	−.06	36	−.36	2	+.66	40	+1.11
73	−.14	35	−.17	3	+.48	41	+ .99
72	−.31	34	+.09	4	+.24	42	+ .89
71	−.47	33	+.29	5	−.05	43	+ .80
70	−.64	32	+.42	6	−.24	44	+ .73
69	−.81	31	+.50	7	−.35	45	+ .75
68	−.97	30	+.54	8	−.29	46	+ .72
67	−.93	29	+.56	9	−.19	47	+ .64
66	−.84	28	+.58	10	−.09	48	+ .46
65	−.70	27	+.60	11	+.01	49	+ .30
64	−.45	26	+.60	12	+.07	50	+ .13
63	−.09	25	+.62	13	+.09	51	− .04
62	+.27	24	+.76	14	+.05	52	− .18
61	+.49	23	+.92	15	+.01	53	− .12
60	+.60	22	+.92	16	−.05	54	.00
59	+.63	21	+.85	17	−.11	55	+ .16
58	+.59	20	+.79	18	−.22	56	+ .44
57	+.41	19	+.67	19	−.25	57	+ .82
56	+.16	18	+.55	20	−.02	58	+1.21
55	−.13	17	+.44	21	−.02	59	+1.45
54	−.34	16	+.34	22	−.12	60	+1.60
53	−.45	15	+.33	23	−.15	61	+1.7
52	−.39	14	+.28	24	−.10	62	+1.7
51	−.31	13	+.16	25	+.10	63	+1.6
50	−.23	12	−.02	26	+.37	64	+1.4
49	−.15	11	−.22	27	+.59	65	+1.2
48	−.10	10	.41	28	+.71	66	+1.1
47	−.11	9	−.62	29	+.81	67	+1.2
46	−.16	8	−.79	30	+.86	68	+1.4
45	−.22	7	.78	31	+.89	69	+1.7
44	−.27	6	.70	32	+.91	−70	+2.0
+43	−.35	+5	−.58	−33	+.93		

NOTE 18. As the result of an analysis of GOULD's General Catalogue made some years ago, but not published, combined with the studies for the present Standard Catalogue, we have the following table of corrections, which are intended to serve as the means for reducing the declinations obtained in each position of the circle to the mean of eight positions, 1872 to 1880. Having reduced the declinations of separate years to the mean, the corrections given in Tables IV and V are still to be applied. As in the case of Madras 35, Cape 50, Cape 80 and the Santiago catalogues, the determination of accurate tables of $\Delta\delta_s$ and $\Delta\delta_s$ for the separate years of GOULD's Catalogue is a problem for the future, to be solved by the use of a great number of additional secondary standards.

CORDOBA 75. $\Delta\delta_s$, TO REDUCE SEPARATE YEARS TO CATALOGUE MEAN.

Year / δ	72	73	74.0 to 75.67	75.67 to 77.0	77	78	79	80	δ
°	"	"	"	"	"	"	"	"	°
0	+.72	—.09	—.10	+.33	—.29	—.54	—.63	+.59	0
— 5	+.78	—.10	.17	+.26	.32	.38	.62	+.54	— 5
10	+.83	.11	—.20	+.18	.38	.22	.59	+.48	10
15	+.87	.14	—.13	+.10	.44	.15	—.53	+.42	15
20	+.89	.17	.06	+.02	.49	—.11	.43	+.34	20
25	+.88	—.19	.00	.02	.52	.12	.28	+.28	25
30	+.72	—.19	+.01	+.06	.51	.14	—.25	+.26	30
35	+.54	—.17	+.01	+.19	.40	.18	.23	+.23	35
40	+.47	.17	+.01	+.27	.30	.24	.20	+.19	40
45	+.41	—.17	+.06	+.25	.25	.32	—.13	+.15	45
50	+.35	—.15	+.13	+.22	—.18	.38	—.04	+.12	50
55	+.28	—.15	+.16	+.20	.13	.41	.00	+.07	55
60	+.23	—.13	+.15	+.18	.07	—.41	.00	+.05	60
65	+.18	.11	+.12	+.16	.04	—.36	.00	+.03	65
70	+.13	—.09	+.09	+.12	.00	—.29	.00	.00	70
75	+.10	—.06	+.07	+.09	.00	—.22	.00	.00	75
80	+.06	—.04	+.04	+.06	.00	—.15	.00	.00	80
85	+.03	—.02	+.02	+.03	.00	—.07	.00	.00	85
—90	.00	.00	.00	.00	.00	.00	.00	.00	—90

$\Delta\alpha_\delta$, TO REDUCE SEPARATE YEARS TO CATALOGUE MEAN.

R.A.									R.A.
h	"	"	"	"	"	"	"	"	h
0	—.02	+.09	+.14	—.16	—.08	—.19	—.01	+.37	12
1	+.01	+.05	+.20	.12	.09	.24	.02	+.30	13
2	+.04	+.01	+.24	—.08	—.09	—.28	.02	+.21	14
3	+.06	—.05	+.26	—.03	.10	—.30	—.03	+.09	15
4	+.08	—.09	+.27	+.02	—.09	—.30	.04	—.02	16
5	+.10	—.12	+.26	+.08	—.08	—.27	.03	—.13	17
6	+.11	—.15	+.23	+.13	.06	—.23	—.03	—.23	18
7	+.11	—.17	+.19	+.16	—.04	—.17	—.03	—.32	19
8	+.10	—.18	+.12	+.18	.02	—.11	.02	.40	20
9	+.09	—.17	+.06	+.20	+.01	—.03	.01	—.43	21
10	+.07	—.16	—.01	+.20	+.03	+.05	.00	—.44	22
11	+.05	—.13	—.08	+.19	+.06	+.13	+.01	.42	23
12	+.02	—.09	—.14	+.16	+.08	+.19	+.01	.37	24

When the argument for $\Delta\delta_s$ is on the right, employ the opposite sign.

NOTE 19. The weight in right-ascension of BRADLEY is computed on the supposition that the equatorial probable error is $\pm 0^s.13 \sec\delta$; and the circumpolar probable error, $\pm 0^s.087 \sec\delta$ (corresponding to 60° of declination); and that the minimum possible probable error is nominally $\pm 0^s.0087$. But since there are elements of systematic error which it would be difficult to determine (especially as to $\Delta\delta_m$) it is thought best not to assign any weight exceeding 0.4. Between +30° and +60° the weights may be interpolated. For equatorial stars we have:

Obs.	p_a
1	0.025
2	0.05
3	0.07

NOTE 20. In greater detail we have for the weight of PIAZZI's right-ascensions:

Obs.	p_a	Obs.	p_a
5–7	0.03	20–26	0.07
8–9	0.04	27–34	0.08
10–14	0.05	35+	0.1
15–19	0.06		

NOTE 21. For Madras 35, DOWNING's new edition of TAYLOR's Catalogue, it has been our practice to assign half weight to the catalogue positions north of +65° and south of —30°. There appears to be a relatively large number of anomalous residuals.

NOTE 22. The weights for the right-ascensions at Pulkowa, determined by means of the great transit, appear to be larger for high declinations in all the series; and in a greater ratio, of course, for Pulkowa 1845. In Table VI, the assumption is that for the right-ascensions of all stars north of the Pulkowa zenith the weights shall be taken from the line marked "N" for the Pulkowa Catalogues; between +20° and +60° the weights should be interpolated between "S," as for +20°, and "N," as for +60°. The differential weights for Pulkowa 92 appear to be somewhat larger than those assigned in the table, but it seemed best to await more evidence before accepting this larger weight.

NOTE 23. The weights in right-ascension for Cordoba 75 (GOULD's General Catalogue) correspond to a single clamp.

NOTE 24. Dr. AUWERS has pointed out that the weights for the stars used as standard by KÜSTNER are greater than for the others (A.N., Bd. 151, s. 227). No distinction has been made in the present discussion; though it is undoubtedly real. The lower grade of weights has been adopted.

NOTE 25. The weights for Albany in right-ascension are for a single clamp.

NOTE 26. In substantial conformity with Table VII the weight in declination for BRADLEY can be computed from the table, p. 21, of the introduction to the catalogue of AUWERS, through multiplication of the numbers in that table by 0.075; but this factor should be very decidedly decreased for zenith distances greater than 75°. For stars observed in two positions of the quadrant, or both below and above pole, the adopted weight may be the sum of the weights for the separate positions.

NOTE 27. For the Pulkowa vertical circle the weights between declinations, +55° and +65°, where the method of reversal is not followed, should be reduced to two-thirds of the values given in Table VII. Since the probable error of a single observation is very small the diminution of weight with zenith distance is more rapid. The following schedule of factors has been adopted:

δ	Factor
+10	0.9
0	0.6
—10	0.4
15	0.3
—20	0.1

NOTE 28. The number of observations most frequently occurring in Pulkowa 1855 is four. The weight in declination corresponding to this is 0.8.

NOTE 29. The weight in declination for Cape 65 within 10° or 15° from the pole seems to be decidedly greater than for zenithal stars.

NOTE 30. Weight, 4, has been assigned to declinations taken from the *Leiden Annals*, Volume II, and weight, 5, to the declinations of 57 fundamental stars, A.N., Bd 80, s. 93.

NOTE 31. The weights for the Cordoba declinations, as given in Table VII, refer to the result for a single position of the circle. The weight assignable to the declination deduced from a combination of the several positions is the sum of the separate weights.

NOTE 32. The weights in declination assigned to Albany [illegible] are due to one of the four positions: Circle AE, AW, BE, BW. The adopted weight is the sum of these separate weights.

ON THE APPARENT EXTENT OF THE ILLUMINATION SURROUNDING A NEW STAR ON THE HYPOTHESIS THAT IT IS REFLECTED LIGHT.

BY SIMON NEWCOMB.

Mr. OTTO C. LEUTHES, of Baltimore, has called my attention to a lack of rigor in the method heretofore adopted of estimating the apparent magnitude of the illuminated nebula surrounding a new star, on the supposition that it shines by reflecting the light of the star. The method as hitherto applied rests on the supposition that the apparent radius of illumination at any moment is determined by a tangent drawn from the earth to the surface of the sphere of illumination. The fault consists in leaving out of consideration the fact that the light sent out by the star in directions near that of the earth will reach us at an earlier moment than it will if sent out at right-angles to that direction. The rigorous method of treatment is this: Let S be the position of the star, E that of the earth, and P that of a particle in the neighborhood of the star. Put

ρ, the distance SP of the nebulous particle from the star.
θ, the angle ESP made by the direction of the ray with that of the earth.
τ, the interval between the time at which outburst of star is seen from the earth, and time of observation.
c, the speed of light.
π, the parallax of the star.
σ, the angular radius of illumination, as seen from earth.

We shall proceed on the supposition that the outburst was a momentary one, which immediately subsided. Then, at the time τ after the outburst is seen, the reflected light visible from the earth will be that from all the particles which fulfil the condition:

$$EP + PS - ES = c\tau \tag{1}$$

Owing to the minuteness of the ratio PS to EP and ES we may treat the lines EP and ES as parallel, so that

$$ES - EP = \rho\cos\theta$$

The equation (1) therefore gives us $\rho(1-\cos\theta) = c\tau$.

$$\text{or}\qquad \rho = \frac{c\tau}{2\sin^2\frac{1}{2}\theta} \tag{2}$$

which is the equation of the required surface.

The apparent radius of the surface, as projected upon the sphere, will be

$$\sigma = \rho\sin\theta\sin\pi = \frac{c\tau\sin\pi}{\tan\frac{1}{2}\theta}$$

or, if we express σ and π in seconds of arc,

$$\sigma = \frac{c\tau\pi}{\tan\frac{1}{2}\theta}$$

This equation implies that we take the earth's mean distance from the sun as the unit of length. Taking also the day as the unit of time we shall have $c = 173$, and the radius of the apparent illumination will become

$$\sigma = \frac{173\,\tau\pi}{\tan\frac{1}{2}\theta} \tag{3}$$

This value increases indefinitely with smaller values of θ. There is, therefore, no well-defined limit to the apparent radius of illumination. The practical limit will depend on the distance to which the nebula extends in the direction of the earth, and upon its density. If we put a for this distance, or for the maximum value of ρ, the minimum value of θ will be given by the equation (2) in the form

$$2\sin^2\tfrac{1}{2}\theta = \frac{c\tau}{a} \tag{4}$$

By substituting this expression in (3) we have for the limitized value of σ

$$\theta_1 = \sqrt{2a\,c\tau}\ .\ \pi\ \cos\tfrac{1}{2}\theta \tag{5}$$

It will be seen that the visible extent of the illumination at any moment is in form a function of two completely unknown quantities, a and π. In addition to this we have another unknown element in the distance at which the light would cease to affect the photographic plate, owing to the faintness of the reflection. I conceive that it would be unprofitable to make hypotheses as to the magnitudes of these uncertain quantities.

The tacit assumption on which KAPTEYN based his estimate is that of $\theta = 90°$. Then, σ being determined from the Lick photographs, the value of π was derived. His result was $0''.02$. I regard so large a parallax as this as altogether without the bounds of reasonable probability, believing that π is more likely to be less than one-tenth of this quantity than greater. Assuming it as great as one-tenth, the illumination would have had to expand with ten times the speed of light in order to make its apparent speed that seen on the Lick photographs. The enigma involved in this conclusion seems to be solved by the considerations just set forth. The principal difficulty which is still left is the faintness of the reflection at the great distances from the star at which the reflecting particles must have been found. The more likely hypothesis seems to be that we have to do with corpuscles thrown out from the star with a speed approximating to that of light.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed Nov. 24.*
Entered February 2, 1903, at Boston, Mass., as second-class matter under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 551.

VOL. XXIII. BOSTON, 1903 DECEMBER 10. NO. 23

ON THE LIGHT-VARIATIONS OF 320 *U CEPHEI*.

BY PAUL S. YENDELL.

The announcement of the variability of *U Cephei*, by CERASKI, in 1880 (*A.N.*, Vol. 97, s. 319), attracted much attention among the observers of variable stars, it being the first of its type, of which at that time but five were known, that had been found since WINNECKE's discovery of *U Coronae* in 1869.

Many observers at once turned their attention to the star. GLASENAPP, WILSING, KNOTT, SCHMIDT, PICKERING, the BAXENDELLS, father and son, published numerous observations during the early years.

Although the main peculiarity of its light-curve was at once noticed, the work of these observers was for the most part directed to the investigation of the star's elements of variation, rather than to the course and character of its light-changes. KNOTT's numerous observations, extending from the time of the star's discovery until 1889, are mostly confined to the three hours on either side of the minimum, and very few of them were made during the time of the star's normal brightness.

The earliest mean light-curve which is known to me is by PICKERING, and was formed from about three hundred photometric observations made at the Harvard Observatory. It was published in 1881, in the Proceedings of the American Academy of Arts and Sciences, Vol. XVI. A discussion of the star variations, including a mean light-curve, was published by WILSING in 1884 (*A.N.*, 2596). In 1889, CHANDLER (*A.J.*, Vol. IX, p. 19), published a discussion of the star's elements of variation, with "Spring" and "Autumn" light-curves, showing the course of the light-changes, with a comparison of the same with the above mentioned curves of WILSING and PICKERING. Since the date of CHANDLER's paper, the only mean light-curve of *U Cephei* that has come to my knowledge is one by BOHLIN, of Upsala, from observations made in 1896, and published in the *A.N.*, 3762.

I began work on *U Cephei* in 1888, and since that time no year has passed without my securing more or less observations of it, although there have been several years during which I have observed no minimum.

Early in the nineties I began to observe the star with a view to the accumulation of material for a mean light-curve of a more or less definitive character, setting my minimum number at a thousand observations, and keeping in view the purpose of securing as nearly as possible as many of the Spring as of the Autumn curve. The latter aim, however, has not been accomplished, mostly from conditions dependent on the weather, and when in 1902 the desired number of observations had been secured, the Autumn observations were so far in the majority that when the list was closed only about one-fourth of the whole number of observations represented the Spring curve.

In the Spring of 1902 I began to collect material from other observers, with the idea of forming a general mean curve from as many different series of observations as I could get together. I already had the published work of KNOTT. BAXENDELL kindly sent me in manuscript his own and his father's observations. CHANDLER, PLASSMAN, SYKORA and SCHWAB all transmitted theirs, so that, including my own material, I had nearly three thousand observations available for my purpose.

Upon further consideration of this plan, the probability suggested itself that the personal differences in the work of the various observers would be likely so far to affect the general result as to detract largely from its value; and after carefully weighing the matter, I abandoned the idea, and each observer's work has been treated by itself. Each series, including my own, was divided into convenient groups in the order of time, and separate mean curves made from the various groups, giving a sequence of curves, representing as many mean epochs, from 1880 till 1903. It was thought that in this way, any progressive change in the course of the light-variations might be brought out.

At the outset, I intended to make use of the Harvard Photometry scale of magnitudes, as being the only one available for stars of all the magnitudes included in the light-range of the variable. Magnitudes for the comparison-stars used were kindly furnished for the purpose by Prof.

PICKERING. But the values for the stars b and d were discordant with their relative values, according to both my own and KNOTT's light-scales, and after reducing a number of observations, I found that the form of the light-curve near the minimum was seriously distorted by this discrepancy. Light-scales were then formed from the other series of observations, and with one exception the same discordance was found to exist in each case. The use of the Harvard magnitudes was thereupon abandoned.

It so happened, however, that very shortly after the reductions had been discontinued for this reason, and when I had almost decided to make use of the provisional magnitude scale formed from my own step-scale, which I had previously used, Dr. MÜLLER, of the Potsdam Astrophysical Observatory, most kindly offered to make photometric measures of these comparison-stars for the purpose of this work, which offer I very thankfully accepted. His results, coming to hand in August, 1902, proved to be accordant in relative values with all the light-scales excepting the one already alluded to, so that simple relations between them were readily established, and the work was resumed. The subjoined table includes the comparison-stars used in all the series under consideration. The first column gives the letters by which they are designated; the second their DM. numbers; the third, headed HP., the Harvard Photometry magnitudes; and the fourth, headed P, the Potsdam measurements.

	DM.	HP	P
k	81 13	6.40	6.58
e	18	7.54	7.43
f	30	7.89	8.04
p	80 34	7.72	7.76
m	81 34	8.56	8.52
g	81 27	8.46	8.53
h	29	8.58	8.57
a	80 21	8.82	8.93
b	22	9.42	9.17
d	81 22	9.00	9.29
c	80 83	–	9.44

The elements on which the present curves were based were suggested by CHANDLER. They satisfy the whole of the observations at my disposal, at least as well as any yet proposed. The elements of 1897 had ceased to represent the star's variations; the departure from them began about 1894, and has gone on increasing, until in the autumn of 1902 the minima were nearly three hours late by them. (See also HARTWIG's VJS Ephemerides, for 1902, p. 269, and 1903, p. 285).

It was therefore necessary to find elements which would fairly represent the observed dates, and the following suggested as above, satisfy the list of minima hereinafter to be given with an average departure of about eight minutes, the algebraic mean of the O−C being $-0^m.48$, corresponding to Epoch 1533.

CHANDLER'S ELEMENTS.

1880 June $23^d\ 7^h\ 13^m.5$ G.M.T. $+\ 2^d\ 11^h\ 49^m\ 44^s.7$ E

In forming the curves, the first question which presented itself, was the division of the observations for the Spring and Autumn curves, as done by WILSING and CHANDLER, and which is necessary in any discussion of observations of this star made by the ARGELANDER method.

As the difference between the two curves is undoubtedly the result of subjective causes, due to the varying presentation of the group formed by the star and its comparison-stars at different hour-angles, it appears that the dividing line should be drawn at the angle at which these disturbances disappear. The principal comparison-stars are assembled in two definite groups, the brighter ones north preceding the star, and the fainter ones south following it, and a line drawn through the approximate centers of these groups, and passing very near the variable, becomes horizontal and parallel to the normal position of the axis of the eyes at hour-angles $2^h\ 24^m$ west and $9^h\ 36^m$ east. The line joining these hour-angles was accordingly taken as the critical line, and all observations taken at angles east of it were used in forming the East or Autumn curve, and all taken west of it for the West or Spring curve.

The observations of each observer having been divided, as mentioned above, into convenient groups in the order of time, mean curves were formed from the several groups, the normals for these curves being generally formed from five observations each, excepting in the more sparsely observed times at the beginning and end of the period of change; in my own group for 1898–1902, the observations being numerous, the normals were formed from ten observations each.

The observations of the late Mr. KNOTT were published in book form in 1899, under the editing of Prof. TURNER of Oxford. I am indebted to the kindness of Mrs. KNOTT for a copy of the volume. KNOTT's observations of *U Cephei* occupy thirty-three pages of the book, twenty-six of which are filled by the observations themselves, which extend from 1880 to 1897, with a few observations on a single date in 1889. There are in all about 850 observations, of which about 660 are available for my purpose, being made by the ARGELANDER method, so that they can be reduced homogeneously with the work of the other observers. The remaining observations are noted as "gauged," the gauging having been done as a check, by the method of limiting apertures. The observations were grouped as follows:

	Spring	Autumn
1880–81	116	136
1882–83	208	61
1884–87	367	20 (no curve)

The observations in 1889 were only ten in number, and have been grouped with those of 1884–87.

The readings from these curves are given in Table I.

The observations of the BAXENDELLS, senior and junior, were kindly forwarded to me in manuscript by the latter. The resulting magnitudes only are given, but as the comparison-stars employed were the same as those used by KNOTT, and as the latter gives BAXENDELL's estimated magnitudes for the purpose of comparison, they were easily reduced to the Potsdam scale by graphic process.

The observations of BAXENDELL, Sr., are 174 in number, extending from 1880 to 1887, and are distributed as follows: Autumn 89, Spring 85.

The readings from the mean curves are in Table II.

The younger BAXENDELL's observations cover a space from 1884 to 1887, and number 117, all except 25 belonging to the Spring curve. The Autumn curve being a single curve, is omitted.

The readings are in Table II.

SPERRA's observations were transmitted in manuscript. There are about 180 of them, of which about one-third are of the star at its normal light, the rest divided between the Spring and Autumn curves. His mean value for the normal light is $6^M.94$.

The readings from the mean curves are contained in Table III.

CHANDLER's observations, to the number of 215, were handed to me also in manuscript. They were made in 1887 and 1888. There are 56 which belong to the Spring curve, and 153 to the Autumn; the remainder are at the normal light, and give a mean value of $7^M.21$.

The readings are given in Table IV.

The observations of PLASSMANN have been sent to me partly in pamphlet form, and partly in manuscript, by himself. They are about three hundred in number, and form two groups, one in 1891, and the other in 1901–02. Those of 1894 all belong to the Spring curve, while those of 1901–02 are divided between the two.

The readings are given in Table V.

SCHWAB's observations were also forwarded in manuscript. There are in all about 250 of them, of which about 50 are of the star at its normal light, and the rest divided pretty evenly between the Spring and Autumn curves, of neither of which, however, are both branches represented. The observations cover a period from December, 1900, to November, 1902. His mean value for the normal light is $6^M.96$.

The readings are given in Table VI.

My own observations of the star were begun 1888 May 16, and when on 1902 Oct. 21, the series for that year came to a close, I found the total number since the beginning to be 1175. Twenty-three of these, depending on comparisons with the stars m and p, whose values on my light-scale were discordant with the photometric magnitudes, were rejected. Of the 1152 remaining available, 866 belonged to the East or Autumn curve, and 286 to the West or Spring curve. Of these, 103 were observations of the star at its normal brightness, made for the purpose of ascertaining whether the light at this phase were constant. Eighty-five were in East hour-angle, belonging to the Autumn curve, and 18 were of the West or Spring group; it was found, however, that there was no sensible difference between the normals formed from the two groups.

The comparison-stars and light-scale used were as follows: the first column gives the notation used; the second, headed DM., their *Durchmusterung* numbers; the third, P, their Potsdam magnitudes; and the fourth, Lt., my own step-scale, formed from all my observations to 1900 July 4, which was retained in the reductions, as in my judgement the later observations would not have sensibly changed it.

	DM	P	Lt.
k	81 13	6.58	31.9
e	18	7.13	24.1
f	30	8.04	19.3
g	27	8.53	14.8
h	29	8.57	14.0
a	80 21	8.93	9.2
b	22	9.17	4.9
d	81 22	9.29	1.9
c	80 23	9.41	0.0

By this light-scale, the value of a step is $0^M.114$ from k to g, and $0^M.062$ from g to c, being nearly twice as great among the brighter stars as among the fainter ones, but pretty constant in each group.

The observations were divided into four groups in the order of time, and each group subdivided into the West (Spring) and the East (Autumn) groups, as follows:

	Spring	Autumn
1888–1890	15 obs.	98 obs.
1891–1894	177	339
1895–1898	30	189
1899–1902	122	400

The observations of the Spring curve for the groups 1888–1890 and 1895–1898 were so few, that no use was made of them excepting in the general mean curves.

The observations were assembled in groups of five, excepting in the Autumn group of 1899–1902, where, being numerous, they were grouped in tens.

The readings for these curves are given in Table VII.

In forming the general mean curve, 781 observations were found available for the Autumn curve, and for the Spring, 268. At the normal light, there were, as mentioned above, 85 observations in East hour-angle, and 18 in West, in all 103; 23 normals were formed from these.

In forming the normals for the Autumn curve as far as practicable 20 observations were used for each normal, so as to give nearly equal weights, but at the beginning and end of the period of change, the observations were less numerous, and the normals are therefore formed from smaller groups. The corresponding normals in the Spring table were formed from 10 observations each.

Table VIII contains these normals. The column $T-t$ gives the interval from the computed time of minimum; M the magnitude; Obs. the number of observations which make up each normal; and e the departure of each normal from the curve as drawn.

The last 15 normals in the Autumn table, and the last 8 in the Spring one fall in the time of the star's normal brightness. They give no indication of any real fluctuation in brightness during that part of the period, the average departure from a mean of $7^{M}.09$ being $0^{M}.03$, and the probable error of a single normal $\pm 0^{M}.02$, the residuals being pretty impartially distributed over the whole forty-eight hours of this portion of the star's period.

The mean minimum light shown is $9^{M}.18$ for the Autumn curve, and $9^{M}.06$ for the Spring curve. Neither curve shows any correction to the time of minimum.

The readings from the general mean curves are given in Table IX.

The duration of the light-changes shown by these curves is longer than it has hitherto been stated. It is from $-5^{h}\,40^{m}$ to $+5^{h}\,40^{m}$, occupying therefore 11 hours and 20 minutes. These limits are well-defined in the Autumn curve, but the beginning of the Spring curve is less satisfactory, being much distorted to about $-2^{h}\,30^{m}$. With this exception, the difference between the two curves are: the general comparative flatness of the Spring curve, its brighter minimum, and its greater asymmetry, as compared with the Autumn curve.

The latter is so far the more fully observed and better made out curve, made from observations taken at far the more favorable season of the year, and the precautions taken to avoid subjective errors have been so unremitting, that it seems to me to be probably a very good approximation to the star's real light-curve. Its departures from actual symmetry are very slight up to $7^{M}.5$, and at $9^{M}.00$, $8^{M}.65$, and $7^{M}.73$ they disappear.

Assuming the curve to be symmetrized by averaging the values of each pair of readings, the probable error of one of these readings is $\pm 0^{M}.025$, while their mean departure from the symmetrized curve is $0^{M}.027$.

Assuming again that the minimum light is constant from -1^{h} to $+1^{h}$, the probable error of one of the nine normals is $\pm 0^{M}.018$, while their mean departure from their mean value is, as in the other part of the curve examined, $0^{M}.027$.

The impression remaining on my mind after fifteen years' constant and careful study of the star, is that the course of the light-changes is really that which would result from an annular eclipse; a symmetrical curve, with inflection increasing with its proximity to the minimum, and an interval of constant minimum light, central at the moment of minimum.

In these series of observations there is no evidence of any progressive change in the star's light-curve. My own series is the only one of those examined in which there is any approach to sufficient observation of the beginning and end of the period occupied by the star's light-changes to furnish any evidence as to possible change in its length, and in this series these points are not made out with enough precision to give any valuable indications in either direction. Examination and comparison of the times at which $8^{M}.00$, at which point the more rapid change is well under way, is passed on the decrease and increase show very considerable differences in the curves of different observers, but no evidence of progressive change.

The difference between the Spring and Autumn values of the observed minimum light varies all the way from almost nothing to fully four-tenths of a magnitude. And whereas in all but one of the other series the Spring curves show the fainter minima, in my own and SCHWAB's the reverse is the case, pointing strongly to the subjective nature of the difference. This curious discrepancy is possibly due to the use of different comparison-stars at the minimum phase, and perhaps also to the fact that, especially of later years, I have been very solicitous to eliminate the hour-angle disturbance from my observations as far as possible. To this practice also I ascribe the fact that in my last four years' group the difference in the two curves at the minimum is very slight, though the Spring curve is much the flatter on both sides of the minimum. In the earliest of my own curves the similarity to SCHOENFELD's curve of *S Cancri*, in the rise from an hour before the minimum point to an hour after is very marked. This entirely disappears in the next group (1891–1894), and does not reappear later. I suspect it to be due to the influence of a knowledge of CHANDLER's 1889 curve to which reference has already been made, which has this peculiarity strongly marked, and whose authority could hardly fail to influence an observer new to the work, when in doubt between two estimates. This may or may not have been the case, but with the gain in experience and confidence the phenomenon vanishes from my results.

A comparison of the minimum brightness found by the various observers gives some apparent sign of change. Of these series, KNOTT's indicates a slow decrease of the minimum light; the others, to SPERRA's in 1894–1897, show a small increase; PLASSMANN's and SCHWAB's are discordant; my own from 1891 to 1902 show an increase of $0^{M}.20$ in the Autumn curves, which is offset by the constancy of the Spring ones, the mean of both being about $0^{M}.09$. The means of KNOTT's and my own series indicate an increase of about a tenth of a magnitude, and this is also the general drift of all the groups.

This change, if real, is very slow, and can only be verified by long observation.

Table X shows the ninety-six minima deduced from the several series by the method of equal brightnesses. In the last column, headed Obs., the letters signify: K, KNOTT; B, BAXENDELL, SR.; b, BAXENDELL, JR.; C, CHANDLER; Y, YENDELL; S, SPERRA; P, PLASSMANN; and Sch., SCHWAB.

The weights in the table are on a scale of five.

TABLE I. KNOTT.

Time from Minimum	1880–1881 Spring	1880–1881 Aut'n	1882–1883 Spring	1882–1883 Aut'n	1884–1887 Spring
h m	M	M	M	M	M
−3 40	. .	. .	7.44	. .	7.59
20	. .	. .	7.56	. .	7.69
3 0	7.55	. .	7.68	. .	7.79
2 40	7.70	. .	7.81	. .	7.92
20	7.90	8.10	7.95	. .	8.08
2 0	8.13	8.37	8.09	8.26	8.28
1 40	8.40	8.67	8.34	8.50	8.54
20	8.69	8.86	8.74	8.74	8.98
1 0	9.08	8.95	9.29	8.99	9.20
0 40	9.31	9.00	9.35	9.06	9.28
−0 20	9.33	9.03	9.34	9.08	9.31
0 0	9.33	9.06	9.34	9.08	9.34
+0 20	9.32	9.04	9.34	9.08	9.35
40	9.30	9.00	9.34	9.05	9.35
1 0	9.30	8.95	9.33	8.95	9.33
20	9.20	8.86	9.31	8.73	9.26
40	8.65	8.62	8.70	8.50	8.78
2 0	8.33	8.29	8.20	8.28	8.14
20	8.10	8.04	. .	8.09	7.89
+2 40	7.92	7.90	. .	7.94	. .

TABLE II.

Time from Minimum	Baxendell, Sr. 1880–1887 Spring	Baxendell, Sr. 1880–1887 Aut'n	Baxendell, Jr. 1884–1887 Spring
h m	M	M	M
−2 40	. .	7.87	7.97
20	. .	8.09	8.13
2 0	8.28	8.31	8.31
1 40	8.50	8.55	8.53
20	8.80	8.82	8.90
1 0	9.13	9.02	9.19
0 40	9.16	9.12	9.20
20	9.18	9.14	9.20
−0 0	9.19	9.12	9.19
+0 20	9.16	9.10	9.18
40	9.14	9.07	9.17
1 0	9.11	9.04	9.13
20	8.94	8.99	9.00
40	8.87	8.63	8.58
2 0	. .	8.35	. .
20	. .	8.16	. .
+2 40	. .	8.00	. .

TABLE III. SPERRA.

Time from Minimum	1894–1897 Spring	1894–1897 Aut'n	Time from Minimum	1894–1897 Spring	1894–1897 Aut'n
h m	M	M	h m	M	M
−4 40	. .	7.14	0 0	9.10	9.06
20	. .	7.26	+0 20	9.08	9.03
4 0	. .	7.38	40	9.03	9.00
3 40	. .	7.50	1 0	8.90	8.95
20	. .	7.63	20	8.62	8.87
3 0	. .	7.78	40	8.35	8.76
2 40	7.91	7.96	2 0	8.10	8.61
20	8.24	8.18	20	7.89	8.47
2 0	8.55	8.50	40	7.71	8.33
1 40	8.83	8.89	3 0	7.54	8.14
20	8.99	9.08	20	7.36	7.94
1 0	9.07	9.10	40	7.19	7.74
0 40	9.10	9.10	4 0	. .	7.53
−0 20	9.11	9.08	+4 20	. .	7.30

TABLE III. (Cont.)

Normal Light.

h m	M		
+12 8	6.92	+41 3	6.92
12 28	6.86	42 16	6.87
18 31	6.95	43 31	6.90
19 11	6.89	44 20	6.89
27 7	6.88	47 26	6.90
27 18	6.92	49 32	6.96
33 46	6.96	51 22	6.99
39 7	6.87	+51 12	7.12
+40 8	6.89	. .	. .

TABLE IV. CHANDLER.

Time from Minimum	1887–1888 Spring	1887–1888 Aut'n	Time from Minimum	1887–1888 Spring	1887–1888 Aut'n
h m	M	M	h m	M	M
−4 20	. .	7.54	−0 20	9.09	9.03
4 0	. .	7.57	0 0	9.07	9.02
3 40	7.64	7.61	+0 20	9.06	9.01
20	7.73	7.67	40	9.05	9.00
3 0	7.84	7.77	1 0	9.01	8.96
2 40	7.97	7.91	1 20	8.89	8.70
20	8.15	8.06	40	8.63	8.42
2 0	8.39	8.23	2 0	8.38	8.20
1 40	8.71	8.44	2 20	8.15	8.02
20	8.98	8.75	40	7.96	7.84
1 0	9.07	8.99	3 0	7.81	. .
−0 40	9.09	9.04	+3 20	7.69	. .

TABLE V. PLASSMANN.

Time from Minimum	1894 Spring	1901–1902 Spring	1901–1902 Aut'n
h m	M	M	M
5 40	. .	. .	7.09
20	. .	. .	7.09
5 0	. .	. .	7.10
4 40	. .	. .	7.11
20	. .	. .	7.14
4 0	. .	7.17	7.18
3 40	. .	7.20	7.23
20	. .	7.24	7.30
3 0	7.37	7.29	7.44
2 40	7.66	7.40	7.72
20	7.95	7.66	8.03
2 0	8.26	8.16	8.31
1 40	8.59	8.61	8.56
20	8.94	8.87	8.80
1 0	9.03	9.02	8.94
0 40	9.03	9.00	8.91
0 20	8.97	9.00	8.90
0 0	8.92	9.00	8.90
+0 20	8.93	9.00	8.90
40	8.98	9.00	8.90
1 0	8.96	8.97	8.86
1 20	8.80	8.73	8.53
40	8.50	8.34	.
2 0	8.45	7.96	
+2 20	7.86	7.58	.

TABLE VI. SCHWAB.

Time from Minimum	1900 Spring	1902 Aut'n	Time from Minimum	1900 Spring	1902 Aut'n	Time from Minimum	Normal Light	Time from Minimum	Normal Light
h m	M	M	h m	M	M	h m	M	h m	M
5 0	. .	7.02	−0 40	9.12	9.20	+ 7 10	7.11	+37 38	6.91
4 40	. .	7.04	−0 20	9.15	9.20	8 31	7.01	40 48	6.86
20	. .	7.07	0 0	9.17	9.20	9 5	6.95	43 43	6.96
4 0	. .	7.12	+0 20	9.18	9.20	11 44	6.92	46 7	6.99
3 40	. .	7.20	40	9.16	. .	13 24	6.91	48 33	6.88
20	. .	7.33	1 0	9.11	. .	19 26	6.93	49 26	7.01
3 0	. .	7.50	1 20	8.90	. .	22 30	6.94	+51 10	6.92
2 40	. .	7.80	40	8.50	. .	22 57	6.98	. .	. .
20	. .	8.13	2 0	8.10	. .	23 41	6.94	. .	. .
2 0	. .	8.50	2 20	7.78	. .	24 20	6.96	. .	. .
1 40	. .	8.72	40	7.52	. .	25 12	6.93	. .	. .
20	8.92	9.03	3 0	7.34	. .	34 19	6.94	. .	. .
1 0	9.06	9.18	+3 20	7.23	. .	+35 16	6.92	. .	. .

TABLE VII. READINGS FROM MEAN CURVES.

	1888–1890		1891–1894				1895–1898		1899–1902			
	Autumn		Spring		Autumn		Autumn		Spring		Autumn	
$T-t$	Before	After	Before	After	Before	After	Before	After	Before	After	Before	After
h m	M	M	M	M	M	M	M	M	M	M	M	M
5 0	. .	. .	. .	. .	. .	. .	. .	. .	. .	7.17	7.16	7.18
4 40	. .	. .	. .	. .	. .	. .	7.15	7.12	. .	7.24	7.22	7.26
4 20	. .	. .	. .	. .	. .	. .	7.23	7.18	7.76	7.31	7.27	7.31
4 0	. .	. .	. .	. .	7.02	. .	7.30	7.24	7.86	7.37	7.34	7.42
3 40	. .	. .	. .	. .	7.10	. .	7.37	7.32	7.97	7.48	7.43	7.49
3 20	. .	. .	. .	. .	7.21	. .	7.44	7.40	8.09	7.60	7.53	7.58
3 0	. .	. .	. .	. .	7.31	. .	7.52	7.58	8.21	7.73	7.64	7.67
2 40	7.74	. .	. .	. .	7.60	7.66	7.67	7.68	8.35	7.88	7.77	7.75
2 20	7.84	. .	8.06	8.21	7.93	7.72	7.98	7.89	8.48	8.05	7.95	7.86
2 0	7.98	8.19	8.47	8.45	8.26	7.93	8.35	7.97	8.64	8.30	8.18	8.03
1 40	8.16	8.62	8.72	8.56	8.57	8.57	8.66	8.50	8.78	8.53	8.48	8.29
1 20	8.52	9.00	8.91	8.73	8.88	9.08	8.94	8.89	8.92	8.77	8.86	8.68
1 0	8.89	9.11	9.02	8.86	9.26	9.26	9.08	9.07	9.00	8.94	9.05	9.02
0 40	9.10	9.17	9.06	8.96	9.30	9.29	9.15	9.13	9.05	9.04	9.10	9.06
0 20	9.24	9.21	9.07	9.02	9.30	9.29	9.17	9.16	9.07	9.08	9.10	9.08
0 0	9.24	. .	9.05	. .	9.29	. .	9.17	. .	9.08	. .	9.09	. .

TABLE VIII. NORMALS FOR MEAN LIGHT-CURVE, 1888–1902.

East (Autumn)				East (Autumn)				East (Autumn)				West (Spring)			
$T-t$	Mag.	Obs.	v	$T-t$	Mag.	Obs.	v	$T-t$	Mag.	Obs.	v	$T-t$	Mag.	Obs.	v
h m				h m				h m				h m			
−4 55.8	7.14	10	+0.01	+ 0 8.1	9.16	20	−0.01	+21 35.6	7.09	9	0.00	−0 57.0	9.03	10	+0.02
4 31.4	7.38	10	+0.20	0 15.9	9.18	20	+0.01	25 11.7	7.12	10	+0.03	0 49.4	8.99	10	−0.04
4 13.0	7.12	8	−0.11	0 34.6	9.18	20	+0.03	27 44.5	7.14	6	+0.05	0 43.7	9.06	10	+0.02
3 41.8	7.37	15	+0.05	0 48.4	9.10	20	−0.02	29 26.1	7.10	6	+0.01	0 26.1	9.07	10	+0.02
3 31.7	7.41	14	+0.04	0 56.6	9.15	20	+0.03	32 34.8	7.06	4	−0.03	0 9.8	9.09	10	+0.03
3 17.3	7.40	15	−0.05	1 6.0	9.05	20	+0.02	42 18.3	7.07	6	−0.02	−0 1.6	9.05	10	−0.01
3 4.1	7.46	20	−0.08	1 13.8	8.96	20	+0.02	46 6.1	7.10	4	+0.01	+0 16.1	9.03	10	−0.01
2 52.0	7.67	20	+0.04	1 20.9	8.75	20	−0.10	49 17.8	7.08	11	−0.01	0 36.1	9.04	10	+0.02
2 40.1	7.82	20	+0.09	1 26.3	8.83	20	+0.10	52 27	7.03	3	−0.06	1 11.8	8.81	10	−0.10
2 22.8	7.86	20	−0.06	1 33.9	8.56	20	−0.02	+54 11.3	7.05	3	−0.04	1 28.0	8.65	10	−0.05
2 10.5	8.12	20	+0.06	1 39.7	8.39	20	−0.01	West (Spring)				1 49.7	8.48	10	+0.05
1 59.8	8.11	20	−0.08	1 47.9	8.33	20	+0.09	− 4 52.1	7.62	10	0.00	2 8.4	8.29	10	+0.07
1 51.6	8.43	20	+0.12	1 56.6	8.08	20	−0.02	3 45.3	7.77	10	+0.01	2 27.7	7.96	10	−0.05
1 44.2	8.41	20	0.00	2 4.6	7.97	20	−0.02	3 18.5	8.24	10	+0.38	2 49.8	7.82	7	+0.01
1 37.5	8.55	20	+0.02	2 21.2	7.81	20	−0.03	2 36.0	8.09	10	−0.03	+1 34.3	7.28	2	0.00
1 30.7	8.56	20	−0.08	2 48.8	7.69	20	0.00	2 30.6	8.10	10	−0.10				
1 21.8	8.84	20	+0.02	3 31.8	7.48	19	0.00	2 20.2	8.35	10	+0.04	+ 8 3	7.06	2	−0.03
1 16.2	8.87	20	0.07	4 56.3	7.13	4	−0.03	1 59.7	8.56	10	0.00	19 13	7.11	2	+0.02
1 8.3	9.06	20	+0.04	5 44.5	7.11	6	+0.02	1 47.4	8.64	10	+0.08	20 31	7.11	2	+0.02
1 1.2	9.08	20	−0.01	7 9	7.09	3	0.00	1 42.1	8.73	10	−0.03	31 13	7.07	3	−0.02
0 52.9	9.14	20	+0.01	9 18.5	7.10	4	+0.01	1 31.7	8.81	10	−0.05	33 15	7.08	3	−0.01
0 41.0	9.16	20	0.00	13 58.1	7.09	6	0.00	1 23.9	8.84	10	−0.04	44 23.5	7.06	2	−0.03
0 28.1	9.16	20	−0.01	16 26.8	7.05	5	−0.04	1 14.9	8.96	10	+0.01	46 27	7.11	2	+0.02
−0 16.9	9.23	20	+0.06	+20 16.2	7.09	5	0.00	− 1 5.3	8.95	10	−0.04	+47 26	6.96	1	−0.13

TABLE IX.

Time from Minimum	East (Autumn) Before	East (Autumn) After	West (Spring) Before	West (Spring) After	Time from Minimum	East (Autumn) Before	East (Autumn) After	West (Spring) Before	West (Spring) After	Time from Minimum	East (Autumn) Before	East (Autumn) After	West (Spring) Before	West (Spring) After
h m	M	M	M	M	h m	M	M	M	M	h m	M	M	M	M
5 40	7.09	7.09	. .	. .	3 30	7.39	7.49	7.82	7.55	1 40	8.49	8.40	8.78	8.56
20	7.10	7.11	. .	. .	20	7.44	7.51	7.86	7.61	30	8.65	8.65	8.87	8.68
5 0	7.12	7.15	7.60	. .	10	7.50	7.58	7.91	7.67	20	8.84	8.85	8.93	8.80
4 50	7.14	7.17	7.62	. .	3 0	7.57	7.64	7.97	7.74	10	9.00	9.00	8.97	8.92
40	7.16	7.21	7.64	. .	2 50	7.64	7.68	8.03	7.81	1 0	9.10	9.07	9.00	8.97
30	7.18	7.25	7.65	7.26	40	7.73	7.73	8.10	7.90	50	9.14	9.11	9.03	9.00
20	7.21	7.29	7.67	7.30	30	7.84	7.78	8.19	7.99	40	9.17	9.14	9.04	9.02
10	7.24	7.33	7.69	7.34	20	7.95	7.85	8.31	8.10	30	9.17	9.16	9.05	9.03
4 0	7.27	7.37	7.72	7.39	10	8.06	7.94	8.43	8.20	20	9.17	9.17	9.06	9.04
3 50	7.30	7.41	7.75	7.44	2 0	8.19	8.05	8.56	8.32	10	9.17	9.18	9.06	9.05
3 40	7.34	7.45	7.78	7.49	1 50	8.34	8.20	8.68	8.43	0 0	9.18	. .	9.06	. .

TABLE X.

E		Gr. M.T.	⊙	O−C	W.	Obs.	E		Gr. M.T.	⊙	O−C	W.	Obs.
		h m							h m				
63	1880 Nov. 27	8 56.9	+3.7	+ 3.2	5	B	1299	1889 May 5	13 57.6	−3.3	+12.0	1	Y
65	Dec. 2	8 45.3	+3.7	+12.1	4	K	1378	Nov. 18	12 32.2	+3.5	+23.5	4	Y
112	1881 Mar. 29	12 47.0	−1.9	+10.2	2	K	1382	28	11 52.4	+3.7	+21.9	3	Y
114	Apr. 3	12 13.3	−2.1	− 3.2	5	K	1512	1890 Oct. 18	13 20.3	+2.6	+ 1.9	5	Y
118	13	11 40.8	−2.6	+ 4.8	4	K	1516	28	12 55.1	+3.0	+21.4	2	Y
124	28	10 31.5	−3.1	− 3.5	2	K	1910	1893 July 6	17 24.9	−2.9	+11.2	4	Y
126	May 3	10 19.1	−3.3	+ 4.4	5	K	1914	16	16 37.1	−2.4	0.8	5	Y
128	8	10 1.5	−3.4	+ 7.2	3	K	1916	21	16 21.6	−2.0	+ 7.6	5	Y
187	Oct. 2	11 49.8	+1.7	+ 5.7	4	K	1922	Aug. 5	15 36.9	−1.2	+21.9	3	Y
193	17	10 40.3	+2.5	− 1.5	4	K	1924	10	14 55.7	−1.0	+ 2.0	5	Y
219	Dec. 21	6 27.5	+3.4	+13.2	1	K	1926	15	14 36.6	−0.7	+ 3.1	5	Y
234	1882 Mar. 18	12 22.0	−1.2	+ 2.2	3	K	1930	25	13 43.6	−0.3	+ 8.2	4	Y
262	Apr. 7	11 2.6	−2.3	+ 3.6	5	K	2111	1894 Nov. 19	18 48.5	+3.6	1.2	1	Y
262	7	11 7.0	−2.3	+ 8.0	5	B	2111	19	18 41.6	+3.6	10.1	5	S
268	22	10 8.0	−3.0	+ 9.9	1	K	2117	Dec. 4	17 39.1	+3.7	11.0	2	Y
270	27	9 42.0	−3.0	+ 1.1	3	K	2117	4	17 36.6	+3.7	13.5	5	S
345	Oct. 31	8 40.6	+3.1	− 1.8	3	K	2123	19	16 39.0	+3.4	9.8	5	S
353	Nov. 30	6 41.0	+3.7	+ 2.2	1	K	2137	1895 Jan. 23	14 20.3	+2.2	6.1	3	Y
394	1883 Mar. 2	12 30.2	−0.3	+ 6.9	4	K	2141	Feb. 2	13 34.3	+1.7	11.6	3	S
398	12	11 47.6	−0.8	+ 4.8	1	K	2181	May 20	18 15.8	−3.6	14.5	2	S
402	22	11 5.3	−1.1	+ 2.9	4	K	2191	June 14	16 25.7	−3.3	21.8	5	Y
406	Apr. 1	10 25.2	−2.9	+ 2.3	1	K	2279	1896 Jan. 12	14 4.3	+2.6	15.6	2	S
487	Oct. 20	8 30.5	+2.7	+ 3.9	4	K	2328	May 13	17 43.6	−3.5	10.0	5	Y
540	1884 Feb. 29	11 31.6	0.0	+ 5.8	1	K	2511	1897 Oct. 26	17 23.0	+2.9	+ 0.2	3	S
548	Mar. 20	10 15.3	−1.4	+10.1	3	K	2813	1899 Sept. 4	18 52.1	+0.1	1.1	4	Y
690	1885 Mar. 9	9 49.2	−0.6	+ 1.0	3	B	2815	9	18 36.9	+0.4	1.5	1	Y
690	9	9 50.6	−0.6	+ 2.1	3	b	2817	14	18 16.1	+0.7	1.5	1	Y
692	14	9 33.9	−1.1	+ 5.2	3	K	2821	24	17 36.6	+1.3	+ 3.7	4	Y
694	19	9 9.8	1.1	+ 0.9	5	K	2825	Oct. 4	16 44.0	+1.9	10.3	4	Y
753	Aug. 3	11 51.2	1.1	+ 9.3	2	B	2829	14	16 12.4	+2.4	+ 2.6	4	Y
753	3	11 50.7	−1.4	+ 5.8	3	b	2983	1900 Nov. 2	13 53.7	+3.1	+ 2.8	5	Y
832	1886 Feb. 26	9 34.3	+0.2	+ 3.2	3	B	3001	Dec. 17	10 34.5	+3.5	10.3	2	Sch.
832	26	9 37.3	+0.2	+ 6.2	5	b	3011	1901 Jan. 11	9 0.2	+2.9	2.7	5	Sch.
966	1887 Jan. 26	10 31.0	+2.1	− 3.1	3	K	3011	11	8 46.9	+2.9	16.9	1	P
970	Feb. 5	9 51.5	+1.5	2.2	4	B	3013	16	8 40.3	+2.5	2.7	2	P
970	5	9 53.6	+1.5	0.1	5	b	3013	16	8 35.8	+2.5	6.9	5	Sch
978	25	8 32.0	+0.3	+ 9.4	3	K	3015	21	8 16.8	+2.4	5.5	5	Sch.
1078	Nov. 1	15 32.1	+3.0	+ 4.6	3	C	3062	May 18	12 16.0	3.6	10.3	2	P
1080	6	15 17.0	+3.3	+12.3	5	C	3064	23	11 54.3	3.7	11.6	3	P
1084	16	14 28.6	+3.5	+ 3.2	2	C	3149	Oct. 7	14 47.1	+2.5	+10.9	4	Y
1086	21	14 4.0	+3.6	+ 1.1	5	C	3135	Nov. 16	11 42.6	+3.5	8.4	5	Sch
1090	Dec. 1	13 35.9	+3.7	+14.2	5	C	3141	Dec. 1	10 38.9	+3.7	10.2	5	Sch
1092	6	13 8.7	+3.7	+ 7.5	1	C	3239	1902 Aug. 2	18 0.4	1.5	+ 9.0	5	Y
1096	16	12 30.2	+3.6	+ 9.9	4	C	3255	Sept 14	15 16.7	+0.8	6.0	5	Y
1133	1888 Mar. 17	18 6.6	1.1	19.3	1	C	3269	Oct. 16	12 38.9	+2.5	18.3	3	Y
1141	Apr. 6	16 35.0	−2.2	9.8	5	C	3275	31	11 53.6	+3.4	1.6	5	Sch
1145	16	16 7.7	−2.7	+ 3.4	5	C	3277	Nov. 5	11 28.6	+3.3	5.9	5	Sch.
1216	Oct. 10	16 2.8	+2.2	+10.5	3	Y	3283	20	10 29.8	+3.6	2.8	5	Sch.

NOMENCLATURE OF NEWLY DISCOVERED VARIABLE STARS.*

Provis. Notation A. N.	Name	Position of 1900.0 R.A.	Position of 1900.0 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-Place R.A.	Chart-Place Decl.	Magnitude Max.	Magnitude Min.	
		h m s	° ′	s	′	h m s	° ′			
11.1903	*RU Andromedae*	1 32 47	+38 9.5	+3.49	+0.31	1 30 41	+37 55.6	9	13	*ph*
15.1903	*Z Cephei*	2 12 48	+81 13	+7.81	+0.28	2 7 6	+81 0	9.10	<13	*ph*
56.1903	*RR Cephei*	2 29 23	+80 42.3	+8.03	+0.27	2 24 15	+80 30.2	9	<13	*ph*
14.1902	*Z Persei*	2 33 40	+41 46.1	+3.81	+0.26	2 30 50	+41 34.3	9	12	*v*
22.1903	*X Camelopardalis*	4 32 36	+74 56	+7.68	+0.12	4 26 48	+74 50	9	13	*ph*
5.1903	*RS Tauri*	5 46 3	+15 51.3	+3.45	+0.02	5 43 28	+15 50.3	8.9	10.11	*v*
1.1903	*Z Aurigae*	5 53 39	+53 18.0	+4.86	+0.01	5 50 3	+53 16.9	9	11	*v*
20.1903	*W Camelopardalis*	6 12 0	+75 32	+8.25	−0.02	6 5 48	+75 32	10.11	12	*ph*
14.1903	*RS Geminorum*	6 55 11	+30 39.8	+3.81	0.08	6 52 21	+30 43.3	9.10	11.12	*ph*
9.1903	*Z Geminorum*	7 4 36	+22 41.0	+3.61	−0.09	6 58 53	+22 44.9	9.10	<12	*v*
16.1903	*RR Monocerotis*	7 12 27	+ 1 16.6	+3.10	−0.10	7 10 7	+ 1 21.2	9	<13	*ph*
13.1903	*RR Geminorum*	7 15 11	+31 4.2	+3.83	−0.10	7 12 18	+31 9.0	10	11.12	*ph*
21.1903	*Y Camelopardalis*	7 27 39	+76 16.9	+8.15	−0.12	7 21 30	+76 22.3	9.10	<11.12	*ph*
4.1902	*Y Geminorum*	7 35 16	+20 39.6	+3.53	−0.13	7 32 37	+20 45.3	8.9		*ph*
2.1903	*Y Draconis*	9 31 5	+78 18.2	+6.98	−0.27	9 25 17	+78 30.1	9	13	*ph*
3.1903	*W Ursae Maj.*	9 36 44	+56 24.6	+4.25	−0.27	9 33 32	+56 36.7	8	9	*v*
4.1903	*Z Draconis*	11 39 19	+72 49.0	+3.15	−0.33	11 37 12	+73 4.0	9.10	12.13	*ph*
57.1903	*T Ursae min.*	13 32 38	+73 56.4	+1.25	−0.31	13 31 42	+74 10.2	9	<13	*ph*
29.1903	*ST Herculis*	15 47 47	+48 47.1	+1.79	−0.18	15 46 27	+48 55.4	7.8	8.9	*v*
18.1902	*W Coronae*	16 11 50	+38 2.7	+2.14	0.15	16 10 14	+38 9.6	7.8	13	*v*
31.1903	*SU Herculis*	17 44 12	+22 34	+2.52	−0.02	17 42 48	+22 35	10	<12	*ph*
76.1901	*RT Ophiuchi*	17 51 51	+11 10.9	+2.81	−0.01	17 49 15	+11 11.5	9	<10	*v*
19.1903	*RZ Lyrae*	18 39 54	+32 41.7	+2.23	+0.06	18 38 14	+32 39.1	10	11.12	*ph*
17.1903	*RY Lyrae*	18 41 15	+34 34.0	+2.17	+0.06	18 39 38	+34 31.4	10	12	*ph*
17.1902	*RW Lyrae*	18 42 7	+43 31.9	+1.82	+0.06	18 40 45	+43 29.2	9	<12	*ph*
10.1903	*RX Lyrae*	18 50 27	+32 42.3	+2.23	+0.07	18 48 46	+32 39.0	11	<15	*ph*
55.1903	*UW Cygni*	20 11 21	+34 11.8	+2.31	+0.18	20 9 37	+34 3.7	9.10	11.12	*ph*
21.1902	*V Sagittae*	20 15 46	+20 47.3	+2.65	+0.19	20 13 47	+20 39.0	9.10	13	*ph*
16.1902	*Z Delphini*	20 28 3	+17 6.2	+2.74	+0.20	20 26 0	+16 57.2	9	<11	*ph*
15.1902	*Y Delphini*	20 36 52	+11 30.9	+2.86	+0.21	20 34 43	+11 21.5	9.10	<13	*v*
58.1903	*UX Cygni*	20 53 34	+39 47.5	+2.26	+0.23	20 51 52	+39 37.2	9	9.10	*ph*
20.1902	*UV Cygni*	21 2 20	+45 22.6	+2.12	+0.24	21 0 45	+45 11.9	11	<12	*ph*
19.1902	*RT Pegasi*	21 59 49	+34 38.2	+2.61	+0.29	21 57 51	+34 25.3	9.10	13.14	*v*
. .	*RS Andromedae*	23 50 19	+48 4.9	+3.01	+0.33	23 48 4	+47 49.5	7.8	8.9	*ph*
12.1903	*Nova Geminorum*	6 37 49	+30 2.6	+3.83	−0.05	6 34 56	+30 5.0	5	–	–

* *From Supplement to Nos. 549–550.*

The Committee for the A.G. Catalogue of Variable Stars:
DUNÉR, HARTWIG, MÜLLER, OUDEMANS.

ERROR IN THE PLACE OF (15) *EUNOMIA* IN THE *JAHRBUCH* FOR 1905.

BY J. C. HAMMOND AND W. W. DINWIDDIE.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent U.S. Naval Observatory.]

On August 20, a plate was exposed on the 6-inch camera by Mr. DINWIDDIE for the purpose of finding the minor planet (41) *Daphne*. A trail was found on the plate which appeared much too bright for that asteroid. The direction of the trail also showed that the motion was entirely different from that of *Daphne*.

After observing it on the 12-inch equatorial on August 21 and 23, Mr. HAMMOND computed a circular orbit from his observations and found that the elements agreed very closely with those of (15) *Eunomia*, which had been searched for photographically by Mr. PETERS in the place given in the *Jahrbuch* for 1905, but without success.

The position of *Eunomia* was then computed by Mr. HAMMOND from the elements and tables of SCHUBERT and was found to agree closely with the observed position. The differences O−C are -48^s in α and $-23'.5$ in δ. The position for August 23 is, $\alpha = 21^h\,52^m\,54^s$; $\delta = +0°\,18'.9$. The daily motion is 1^m in α and $+0.3$ in δ. The time of opposition is August 20 instead of July 29 as given in the *Jahrbuch*.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed Dec. 8.*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 552.

VOL. XXIII. BOSTON, 1903 DECEMBER 31. NO. 24

THE INSTRUMENTAL CONSTANTS IN EQUATORIAL WORK.

BY C. W. FREDERICK.

[Communicated by Rear-Admiral C. M. CHESTER, Superintendent U.S. Naval Observatory.]

In order to eliminate the effect of the constants in equatorial work it is necessary to determine the parallel of the micrometer at the place of each observation. But with a knowledge of the instrumental constants, and a sufficient theory as to their influence in varying the orientation of the micrometer, we could adopt a fixed setting for the parallel from careful determinations made in favorable parts of the sky. Then, at a given place of observation, corrections due to the error of this parallel would be computed for the quantities measured. In the following is given a method for obtaining the constants, with formulas for the deviation of the parallel produced by them, and the construction of tables to facilitate the computations.

The constants may be determined from observations of circumpolar and equatorial stars.

Since λ *Ursae Minoris* and *Polaris* are about six hours apart in right-ascension, one is near culmination when the other is near elongation. The azimuth of the instrumental pole may be obtained from the star at culmination, and the altitude from the star at elongation.

Before making the observations we should determine the reading of the micrometer scale when the movable wire is in the optical axis of the telescope, or better, place the fixed wire in the optical axis, so that coincidence is the required reading. This adjustment may be accomplished by starting the driving clock and reversing the micrometer a few times on a suitable star. To observe for azimuth we clamp the telescope at one side of the pier so the declination axis will lie in the plane of the six-hour circle. Also clamp the micrometer in a horizontal position. Then point the telescope at the star near culmination. As the star moves slowly across the field set the movable thread of the micrometer immediately ahead of it, and note the time of transit, also note the reading of the scale. Two or more such transits should be taken to serve as a check on each other. Then reverse the telescope to the opposite side of the pier, and repeat the same operation. This completes the observation for azimuth. For the altitude the star near elongation is observed in a similar manner, with the telescope above the pier, and clamped so the declination axis will lie in the plane of the meridian. The micrometer should remain clamped in position angle, and not be disturbed during the observations.

The formulas for reducing these observations are easily obtained, and it is not necessary to give their derivation. The quantities observed may be designated as follows:

R_0 = the reading of the micrometer scale when the movable wire is in coincidence with the optical axis of the telescope.

R = the mean of the settings of the movable wire in any given position of the telescope.

θ = the sidereal time corresponding to R, mean of the transits.

The position of the telescope may be shown by indices E, W, or U (east, west, up); and the position of the micrometer by subscripts E, W, U, or D. These may refer to the positive direction of the micrometer scale, or to the micrometer head if it is opposite the zero of the scale. Thus R^{E}_{W} is the mean of the scale readings taken when the telescope is east of the pier, and the micrometer clamped so the scale increases westward, or head west. Put

α_c, ρ_c = the apparent right-ascension and polar distance of the star at culmination;

α_e, ρ_e = the same for the star at elongation;

r = the refraction of the atmosphere at the altitude of the pole;

φ = the latitude of the observatory;

$\tau = \theta - \alpha$.

Let the constants be as follows:

η = the distance of the instrumental pole westward from the true pole measured along the six-hour circle; termed azimuth above.

ξ = the distance of the instrumental pole above the true pole measured along the meridian, termed altitude above.

$i+90°$ = the angle between the polar axis produced northward and the declination axis produced through the telescope tube. Flexure is included in this quantity.

c = the collimation. The angle between the optical axis of the telescope produced through the objective and the declination axis produced through the tube is $90°+c$.

ϵ = the flexure of the declination axis when in a horizontal position, positive when the end joining the tube bends downward.

e = the flexure of the telescope tube when horizontal, positive when the objective end of the tube bends most.

For the reduction of the above observations we have the following approximate formulas. When the micrometer is clamped head outward from the pier,

$$
(1a)\qquad
\begin{aligned}
\eta &= \rho\ \sin\tfrac{1}{2}(\tau_{\rm W}^{\rm W}+\tau_{\rm E}^{\rm E})+\tfrac{1}{2}(R_{\rm W}^{\rm W}-R_{\rm E}^{\rm E})\\
i_1-c &= \rho_c \sin\tfrac{1}{2}(\tau_{\rm W}^{\rm W}-\tau_{\rm E}^{\rm E})+\tfrac{1}{2}(R_{\rm W}^{\rm W}+R_{\rm E}^{\rm E})-R_0\\
\xi &= \rho_e \cos\tau_{\rm U}^{\rm U}+R_{\rm U}^{\rm U}-R_0-(i_1-c)+\epsilon\cos q+r
\end{aligned}
$$

When the micrometer is clamped head inward next the pier,

$$
(1b)\qquad
\begin{aligned}
\eta &= \rho_c \sin\tfrac{1}{2}(\tau_{\rm E}^{\rm W}+\tau_{\rm W}^{\rm E})+\tfrac{1}{2}(R_{\rm W}^{\rm E}-R_{\rm E}^{\rm W})\\
i_1-c &= \rho_c \sin\tfrac{1}{2}(\tau_{\rm E}^{\rm W}-\tau_{\rm W}^{\rm E})-\tfrac{1}{2}(R_{\rm E}^{\rm W}+R_{\rm W}^{\rm E})+R_0\\
\xi &= \rho_c \cos\tau_{\rm D}^{\rm U}+R_0-R_{\rm D}^{\rm U}-(i_1-c)+\epsilon\cos q+r
\end{aligned}
$$

If we wish to eliminate R_0 by reversing the micrometer during each observation we have for the reduction,

(1c)

$$
\begin{aligned}
\eta &= \rho_c \sin\tfrac{1}{4}(\tau_{\rm W}^{\rm W}+\tau_{\rm W}^{\rm E}+\tau_{\rm E}^{\rm E}+\tau_{\rm E}^{\rm W})+\tfrac{1}{4}(R_{\rm W}^{\rm W}+R_{\rm W}^{\rm E}-R_{\rm E}^{\rm E}-R_{\rm E}^{\rm W})\\
i_1-c &= \rho_c \sin\tfrac{1}{4}(\tau_{\rm W}^{\rm W}-\tau_{\rm W}^{\rm E}-\tau_{\rm E}^{\rm E}+\tau_{\rm E}^{\rm W})+\tfrac{1}{4}(R_{\rm W}^{\rm W}-R_{\rm W}^{\rm E}+R_{\rm E}^{\rm E}-R_{\rm E}^{\rm W})\\
\xi &= \rho_c \cos\tfrac{1}{2}(\tau_{\rm U}^{\rm U}+\tau_{\rm D}^{\rm U})+\tfrac{1}{2}(R_{\rm U}^{\rm U}-R_{\rm D}^{\rm U})-(i_1-c)+\epsilon\cos q+r
\end{aligned}
$$

The quantity i_1-c is the distance from the instrumental pole at which the optical axis of the telescope passes when the instrument is revolved in declination. The term $\rho_c \sin\frac{1}{2}(\tau^{\rm W}-\tau^{\rm E})$ in equations (1a) and (1b), and the corresponding term in (1c), becomes negative for lower culmination. The term is really $\rho_c\frac{1}{2}(\sin\tau^{\rm W}-\sin\tau^{\rm E})$, so when the hour-angle is in the second or third quadrant a positive increment of τ will produce a negative increment of $\sin\tau$.

The collimation, flexure of the declination axis, and flexure of the tube, may be determined from observations of equatorial stars. For the collimation we take several transits of an equatorial star near the meridian, half with the telescope east of the pier, and half with the telescope west, reading both verniers of the hour-circle after each transit. The thread over which the transits are taken should be carefully placed in the optical axis of the telescope. If the flexure of the tube is considerable, or for safety we wish to eliminate its differential effect on c, the transits may be taken, two with the telescope on one side of the pier, four on the opposite side, then two more in the first position. For the reduction we have,

$$
(2)\qquad c_m = \tfrac{1}{2}(\theta_m^{\rm E}-\theta_m^{\rm W})-\tfrac{1}{2}(\tau_m^{\rm E}-\tau_m^{\rm W})-0.0003\,\tfrac{1}{2}(\theta_m^{\rm E}-\theta_m^{\rm W})
$$

where $\theta_m^{\rm E}$, $\theta_m^{\rm W}$, are the means of the sidereal times of transits taken telescope east, and telescope west, respectively, and $\tau_m^{\rm E}$, $\tau_m^{\rm W}$, are the means of the corresponding readings of the hour-circle. The term $-0.0003\,\frac{1}{2}(\theta_m^{\rm E}-\theta_m^{\rm W})$ is a correction for differential refraction. Its coefficient -0.0003 is practically the same for any observatory.

The collimation thus determined is affected by a component of the flexure of the declination axis, so that $c=c_m+\epsilon\cos q\cos\tau_m$. This component will be much diminished by observing for collimation at a large hour-angle, say between four and five hours. By taking an observation in both west hour-angle τ_w and east hour-angle τ_e the flexure of the tube may also be deduced. The observations are made in the same manner as for the meridian, except it is not necessary to reverse the telescope more than once for each observation, as the differential flexure of the tube may be sufficiently well eliminated by making the observation at west hour-angle in the reverse order from that at east hour angle. Thus, if at τ_w we observe in the order, telescope east, telescope west, at τ_e we should observe in the order, telescope west, telescope east. This is the natural procedure in manipulating the instrument. For the reductions we have

$$
\begin{aligned}
c_w &= \tfrac{1}{2}(\theta_w^{\rm E}-\theta_w^{\rm W})-\tfrac{1}{2}(\tau_w^{\rm E}-\tau_w^{\rm W})-M\tfrac{1}{2}(\theta_w^{\rm E}-\theta_w^{\rm W})\\
c_e &= \tfrac{1}{2}(\theta_e^{\rm E}-\theta_e^{\rm W})-\tfrac{1}{2}(\tau_e^{\rm E}-\tau_e^{\rm W})-M\tfrac{1}{2}(\theta_e^{\rm E}-\theta_e^{\rm W})
\end{aligned}
\qquad(3)
$$

Subscripts here indicate the hour-angle. M may be computed by placing $p=90°$ in equation (349), page 458, *Chauvenet*, Vol. II.

From these observed collimations we have

$$
\begin{aligned}
c-\cos\tfrac{1}{2}(\tau_m^{\rm E}+\tau_m^{\rm W})\,\epsilon\cos q &= c_m\\
c-\cos\tfrac{1}{2}(\tau_w^{\rm E}+\tau_w^{\rm W})\,\epsilon\cos q &= c_w\\
c-\cos\tfrac{1}{2}(\tau_e^{\rm E}+\tau_e^{\rm W})\,\epsilon\cos q &= c_e
\end{aligned}
\qquad(4)
$$

with which to determine the true collimation c, and incidentally ϵ.

For the 26-inch equatorial at the Naval Observatory the quantity $\epsilon\cos q$ is about $90''=6^{\rm s}$.* This is the maximum component of the flexure of the declination axis in the plane of the equator. Therefore when observing the pole stars, for the one at culmination the hour-circle is set $6^{\rm s}$ past $0^{\rm h}$ for telescope east, and $6^{\rm s}$ less than $0^{\rm h}$ for telescope west, in order to bring the declination axis properly into the plane of the six-hour circle.

* At this point attention may be called to a misleading omission in certain equations of CHAUVENET. On page 383, Vol. II, a statement is made to the effect that the term $\varepsilon\cos\varphi\cos\tau$ is small, and can be omitted. This is not likely to be the case. Equations (262), page 384, *Chauvenet*, Vol. II, should be written.

$$
\begin{aligned}
\varepsilon\cos\varphi+c\ \sec\delta-i_1\ \tan\delta &= \tfrac{1}{2}[(t_2-t_1)-(T_2-T_1)]\\
\varepsilon\cos\varphi+c\ \sec\delta'-i_1\ \tan\delta' &= \tfrac{1}{2}[(t'_2-t'_1)-(T'_2-T'_1)]
\end{aligned}
$$

See Pulkowa Observatory Publications, Vol. for 1889. Description of the 30-inch Refractor, page 58.

For the flexure of the tube, put

$$\theta_w = \tfrac{1}{2}(\theta_w^{W} + \theta_w^{E}),\ \theta_e = \tfrac{1}{2}(\theta_e^{W} + \theta_e^{E}),$$
$$\tau_w = \tfrac{1}{2}(\tau_w^{W} + \tau_w^{E}),\ \tau_e = \tfrac{1}{2}(\tau_e^{W} + \tau_e^{E})$$

then we have

$$(5) \qquad \rho \cos q = \frac{\theta_w - \theta_e + \tau_e - \tau_w + \alpha_e - \alpha_w - r_w - r_e}{\sin \tau_w - \sin \tau_e}$$

α_e, α_w, are the apparent right-ascensions of the stars observed at east, and west hour-angles respectively. Stars of the seventh or eighth magnitude within fifteen minutes of the equator may be picked up as required, and their right-ascensions obtained from the Nicolajew A.G. Catalogue; r_e, r_w, are the refractions in right-ascension for τ_e and τ_w. They are assumed both positive. At a given observatory the values of r, and also of M above, may be computed for the equator as in the following table for Washington.

τ_w	M	r	τ_e
h m		s	h m
4 0	0.0011	6.69	20 0
4 10	0.0013	7.41	19 50
4 20	0.0016	8.25	19 40
4 30	0.0019	9.27	19 30
4 40	0.0023	10.52	19 20
4 50	0.0029	12.09	19 10
5 0	0.0041	14.13	19 0

Computed for thermometer = 50° Fah., barometer = 30 inches. To correct for other temperatures and barometers,

$$r_0 = r + r\,\frac{50^\circ - F^\circ}{500^\circ} + r\,\frac{B^{in} - 30}{30}$$

By the above methods a full set of constants may be obtained with little labor. But in carrying on a series of observations for the constants it will not be necessary to take a full set each time. The flexural terms will show no seasonal changes in the nature of the case, and a few careful determinations will answer for them. So ordinarily it will only be necessary to observe the pole stars, and one equatorial star, a very brief operation indeed.

In addition to the constants enumerated above there is another influence at work to disturb the parallel of the micrometer. This is the torsion of the telescope tube. It is probably caused by the distortion of the central casting of the tube under the weight of the telescope, its circular section becoming gibbous. The effect would be as if the declination axis should bend more perpendicular to the tube than parallel to it. But for practical reasons let us assume the torsion to be produced by the weight of a finder attached to the eye-end of the tube opposite the declination axis, also as a test for symmetry, suppose a second finder attached 90° in position angle from the declination axis. Put

μ = the maximum torsion produced by the first finder,
μ' = the maximum torsion produced by the second finder.

The values of μ and μ' can be determined by means of levels placed on the micrometer box. Two small spirit levels may be attached to a tee-shaped piece of metal, and secured to the micrometer box in such a way that their respective axes will be parallel and perpendicular to the axis of the telescope tube. Then when the level bubbles are in the center of their scales, let

p_e^{D} = the reading of the position-circle when the telescope is beneath the pier and object glass east;
p_w^{U} = the reading when the telescope is above the pier and object glass west;
p_e^{U} = the reading for telescope above the pier and object glass east;
p_w^{D} = the reading for telescope beneath the pier and object glass west.

The telescope is set each time with the declination axis as nearly as possible in the instrumental meridian, and the tube as nearly as possible in the instrumental equator. In these positions let

f = the component of the torsion due to the first finder, $f = \mu \sin q$;
f' = the component of the torsion due to the second finder, $f' = \mu' \cos q$;
q' = the altitude of the instrumental pole.

Then we have

$$\left.\begin{aligned} f &= \tfrac{1}{4}(p_w^{D} + p_e^{U} - p_e^{D} - p_w^{U}) - i_1 \\ f' &= \tfrac{1}{4}(p_e^{U} + p_w^{U} - p_e^{D} - p_w^{D}) \\ q' &= 90^\circ - \tfrac{1}{4}(p_w^{U} + p_w^{D} - p_e^{U} - p_e^{D}) \\ \xi &= q' - q \end{aligned}\right\} \qquad (6)$$

With large telescopes the torsion of the tube is likely to have more influence in varying the parallel of the micrometer than all the other constants, except of course in the zenith and northern regions of the sky. The quantity f' will probably be small. It will usually be represented by the errors of the observations only.

We pass now to the effect of the constants on micrometer measurements. A series of observations should be made to determine the stability of the constants with regard to seasonal or other changes. And from this series mean values may be adopted for use throughout the year. Or summer values and winter values may be adopted if there is reason for doing so. Put

λ = the deviation of the micrometer parallel due to the constants, positive when the vertical wire strikes to the eastward of the true pole of the heavens.

The formulas for λ may easily be obtained by considering each constant separately. Let s be a star at which the telescope is pointed, and p the pole of the heavens. Then for ξ and η we take the component of each perpendicular to the arc sp and find the angle subtended at s.

$$\lambda = \xi \sec \delta \sin \tau - \eta \sec \delta \cos \tau$$

The deviation produced by the flexure of the tube is the difference of the parallactic angles at the extremities of the arc through which the tube bends.

$$\lambda = -\rho \cos q \tan \delta \sin \tau$$

For the inclination of the axes we construct a right triangle upon sp as an hypothenuse, and side i_1 adjacent the pole, measured eastward when the telescope is east of the pier. The deviation is the angle at s.

$$\lambda = +i_1 \sec\delta$$

This term becomes negative when the telescope is west of the pier.

For the collimation we construct a right triangle upon sp as an hypothenuse, and side c adjacent s, measured eastward when the telescope is east of the pier. The deviation is the complement of the angle at s,

$$\lambda = -c \tan\delta$$

This term becomes positive when the telescope is west of the pier.

The flexure of the declination axis need not be considered, as its component in right-ascension does not affect the parallel, and its component in declination is included in i_1.

The torsion of the tube we assume proportional to the component of gravity taken perpendicular to the plane of the tube and declination axis. When the telescope is east of the pier we have

$$\lambda = +\mu(\sin\varphi\cos\delta - \cos\varphi\sin\delta\cos\tau)$$

Substituting the observed quantity f for $\mu\sin\varphi$,

$$\lambda = +f\cos\delta - f\cot\varphi\sin\delta\cos\tau$$

The signs are changed in this expression when the telescope is west of the pier. If f' should be appreciable we should have, $\lambda = \pm f'\sin\tau$.

Now assume a fundamental parallel, or setting of the position-circle, p_m, the error of which we are to compute for any position of the telescope. It is most natural to take the parallel determined by the trail of an equatorial star along the fixed short wire of the micrometer when the telescope is east of the pier, and in the meridian. Then λ for this position should be zero. But taking the sum of the above equations when δ and τ are zero,

$$\lambda = i_1 - \eta + f$$

Therefore subtracting this expression from the sum of the above equations we have the deviation for the setting p_m.

Taking the sum of the terms which are independent of the position of the telescope,

(7a)

$$\lambda_1 = \eta - i_1 - f - \eta\sec\delta\cos\tau + (\xi\sec\delta - e\cos\varphi\tan\delta)\sin\tau$$

Taking the sum of the terms which change sign when the telescope is reversed in position,

$$(7b)\quad \lambda_2 = i_1\sec\delta - c\tan\delta + f\cos\delta - f\cot\varphi\sin\delta\cos\tau$$

Then we have,

$$(7)\qquad \begin{aligned} \lambda^{E} &= \lambda_1 + \lambda_2 \quad . \quad \text{for Telescope East of Pier} \\ \lambda^{W} &= \lambda_1 - \lambda_2 \quad . \quad \text{for Telescope West of Pier} \end{aligned}$$

These designations for the position of the telescope are not sufficiently general. But if we call the direction of increasing right-ascensions, east, for any given part of the sky at which the telescope is pointed, the designation becomes general.

In order to avoid laborious computing in the reduction of observations it is necessary to tabulate the values of λ^E and λ^W. It will be sufficient to compute them for every five degrees in declination, and every thirty minutes in hour-angle.

The value of p_m may now be easily deduced from observations made in any part of the sky, and either position of the telescope by adding λ from the table, and a correction for refraction, to the observed parallel. By taking many observations in different regions of the sky, in different positions of the instrument, and by different methods we should arrive at an accurate mean value of p_m, and incidentally test the theory of the constants. The stability of p_m would also appear with continued observing.

Adopting a value of p_m we are ready for the reduction of observations. For position angles we have

$$p_0 = p + \lambda - p_m \tag{8}$$

in which p_0 is the true, and p the observed angle. In satellite work where many observations are made in one neighborhood, p_m should be derived principally from parallel determinations made in the same neighborhood. In this case the parallel will be determined by the trail of a star on the long wire of the micrometer, and p_m will be reading of the position-circle less ninety degrees.

When observing by rectangular coordinates the position-circle is set at p_m, and $90° + p_m$. Then we have

$$\begin{aligned} \Delta\alpha'_0 &= \Delta\alpha' + \Delta\delta\ \sin\lambda \\ \Delta\delta_0 &= \Delta\delta - \Delta\alpha'\ \sin\lambda \end{aligned} \tag{9}$$

in which $\Delta\alpha'_0$, $\Delta\alpha'$, are the true and observed micrometer measurements in right-ascension, not reduced to the equator; $\Delta\delta_0$, $\Delta\delta$, are the true and observed measurements in declination. For this reduction the values of the natural sine of λ in units of the fifth decimal place should be tabulated for every five degrees in declination, and thirty minutes in hour-angle.

When $\Delta\alpha$ is determined by transits the formulas become

$$\begin{aligned} \Delta\alpha_0 &= \Delta\alpha + \tfrac{1}{15}\,\Delta\delta\sin\lambda\sec\delta \\ \Delta\delta_0 &= \Delta\delta \end{aligned} \tag{10}$$

The values of $\frac{1}{15}\sin\lambda\sec\delta$ should be tabulated for this reduction.

These instrumental corrections will be found of about the same magnitude as the corrections for differential refraction. But there is some degree of uncertainty about them, as there are mechanical imperfections of the instrument, and temperature disturbances, the effect of which cannot be computed. However, the action of the known constants is positive, and observations are improved with as much certainty by applying the above corrections as they are in the case of refraction.

The stability of the constants for the 26-inch equatorial may be seen from the following results.

Date	Temp. Fah.	η	i_1-c	ξ	c from stars	c from coll's	$z\cos\delta$	$\epsilon\cos\delta$ from stars	ϵ from coll's
1902	°	″	″	″	″	″	″	″	″
July 13	69	+116	−54	−71					
27	76	+113	−55	−73					
Aug. 3	80	+115	−51	−73					
11	70	+115	−55	−75					
28	67	+121	−55	−74					
Sept. 23	70	+115	−52	−73	. .	+113	. .	.	. .
Oct. 22	51	+116	−52	−73	+114	. .	+94	. .	. .
27	64	. .	. .	. .	. .	+113	.	. .	+10
Nov. 5	56	. .	. .	. .	. .	. .	.	.	+ 4
28	35	+116	−52	−68	+107	+108	.	.	. .
1903									
Jan. 9	27	. .	.	. .	+113	+119	+96	+ 2	− 2
9	20	+114	−57	−67	. .	. .	. .	. .	.
Feb. 17	14	+111	−56	−63	+115	. .	+95	+ 4	. .
Mar. 13	55	+114	−50	−70	. .	.	. .	. .	. .
Apr. 21	48	+113	−55	−65	+113	. .	+98	+ 6	.
June 21	70	+113	−53	−74	+114	. .	+96	+ 5	. .

Adopted Constants

$\eta = +115''$
$\xi = -72$
$i_1-c = -54$
$c = +113$
$i_1 = +59$
$\epsilon\cos\varphi = +5$

The collimators used in the above determinations of c and ϵ were attached to the dome in a vertical position, and sighted into each other by means of mirrors. They were very unstable, the images of the wires moving as much as ten or fifteen seconds vertically, and half as much horizontally during an observation.

TORSION OF THE TUBE FROM LEVELS PLACED ON THE MICROMETER BOX.

Date	Temp.	f	f'	ξ
1903	°	°	°	″
May 5	. .	+0.021	+0.001	−97
26	64	+0.023	−0.001	−73
June 25	70	+0.017	−0.004	−81
26	68	+0.016	0.000	−80

Date	Temp.	f	f'	ξ
1903	°	°	°	″
July 13	74	+0.022	+0.004	−94
13	75	+0.013	+0.002	−69
24	72	+0.014	0.000	−78
24	72	+0.017	+0.002	−79

Adopt $f = +0°.018$. For these determinations the levels were fastened to the micrometer box with lead wire. The position-circle was read by careful estimation to half-hundredths of a degree, the verniers reading only to fiftieths. The quality of the observations may be judged, aside from the agreement of the results in ξ, by the knowledge that f' should be zero, and the value of ξ about $-72''$.

ON THE SPIRAL CHARACTER OF THE NEBULOSITIES SURROUNDING γ *CASSIOPEAE*.

BY J. M. SCHAEBERLE.

Half-hour exposure photographs of the region surrounding γ *Cassiopeae*, taken with the 13-inch reflector, show that in various position-angles and at various distances there are rows of stars connected by nebulous streams all of which seem to originate in γ *Cassiopeae*, and many of these streams plainly appear to return to the star, so that in their course other streams are crossed at various angles, producing appearances strongly suggestive of similar structures to be found in the Great Cluster in *Hercules*. Especially is this the case near the beginning of the 3d quadrant where, up to distances of more than 15′ from the central star, these intersections are so numerous that much confusion of detail exists in this region. There are isolated patches of nebulosity in various position-angles. The two known objects* lie on a heavy stream which seems to leave the central star near the middle of the 3d quadrant.

With γ *Cassiopeae* on the optical axis all these patches are so far from the center of the plate that considerable uncertainty still exists for the regions which have not yet been photographed near the optical axis. With this axis half-way between the two known condensations, a broad nebulous spiral-like band can be traced from near the beginning of the second quadrant (where it is something more than a quarter of a degree from γ *Cas.*) through the brighter parts of the two known objects, on towards and nearly up to the naked-eye star in the 4th quadrant. Within 5′ of the optical axis this band appears to be made up largely of star-like condensations, each surrounded by a nebulosity which has a tendency to form outlines similar to the two neighboring nebulas; the width of this band varies from 1′ to 5′ or more. A second fainter band from 5′ to 7′ inside of and nearly parallel with the one just described can be traced from the middle of the 1st quadrant well into the 4th quadrant; its outline is somewhat

* First photographed by BARNARD and WOLF.

irregular, the average width being about 1′, and the distance from *Gamma* 15′ ±.

A composite drawing, made with the aid of many negatives having different pointings, covering the whole region within half a degree of γ *Cassiopeae* will be required before definite conclusions can be drawn from photographs having such a limited field of view. During the past month but one night was suitable for work with this instrument, and owing to local conditions observations can only be made near the meridian.

Ann Arbor, December 16, 1903.

ON THE PHYSICAL STRUCTURE OF THE GREAT CLUSTER IN *HERCULES*,

BY J. M. SCHAEBERLE.

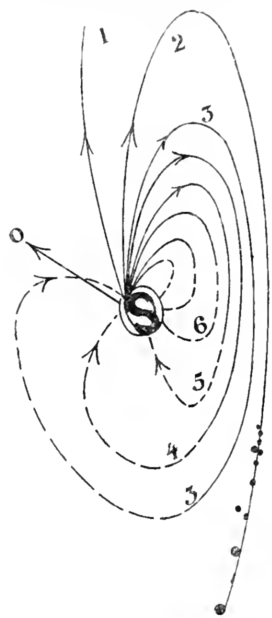
Several months ago a few photographs of the cluster in *Hercules* were taken with the 13-inch reflector; these negatives were found to give unmistakable evidence of a spiral structure in this object. Nebulous streams joining certain stars in curved lines could be traced up to the very center of the cluster. The puzzling feature, however, was that there seemed to be two spirals, the more pronounced one being clock-wise, the other counter clock-wise. As certain other observations were demanding attention at that time, the matter was laid aside with the intention of making this cluster a special study when it again got into good position for observation. The discovery, however, that a precisely similar structure on a much larger scale exists in the stars and nebulosity surrounding γ *Cassiopeae*,* led me again to examine the above mentioned negatives with the result that the physical structure of this cluster seems to be very simple.

The clock-wise spiral is formed by the inner streams of *outgoing* matter; the seeming counter clock-wise spiral is formed by that part of each stream which contains *returning* matter. The plane of the spiral is not normal to the line of sight.

My interpretations of these negatives can, perhaps, best be followed with the aid of the accompanying figure roughly deduced from the photographs. To avoid confusion of detail, streams from only one branch are represented; the broken lines refer to the seeming counter clock-wise spiral. For purposes of approximate orientation a few conspicuous star-images are inserted.

Streams with the greatest velocity (and consequently least density) have the initial direction 0. All outgoing streams between 0 and 2 have velocities still too great to form closed curves. Individual masses in any of the streams 3 to 8 describe orbits in which the minor axes are nearly zero. The periodic time for masses in stream 3, for instance, is about equal to three fourths of the time required for the central mass to make one complete rotation on its axis.

The slower moving masses in streams with large initial inclinations to the normal will describe orbits of less eccentricity: a number of these are seen to encircle the two branches (in the shape of the letter S) near the origin; and as the lower branch is less dense than the upper, these encircling streams give a horse-shoe-like outline to this rather conspicuous central nebulosity; they also form the outer boundary of the *lanes* which Lord ROSSE first observed, the inner boundary being formed by the upper part of the central figure S. The third, nearly radial lane, is included between streams 1 and 2, returning streams also play an important part in the formations at this place. A similar, less conspicuous structure exists on the other branch.



The star-like masses in this cluster are probably of various shapes and have various velocities of axial rotation; this seems to be indicated by the variations in brightness which some of these objects are known to undergo.

If two *dot* tracings of the figure are made (the distances between the dots (star images) increasing with the distance from the origin, measured on the curve) and one of these placed upside down against the other — thus giving the

* In the opinion of the writer a majority of the stars—both bright and faint—within half a degree of γ *Cassiopeae* belong to a single physical system.

combination for both branches — a resemblance between the transparency and the original cluster will be recognized.

It is hoped that a negative suitable for enlargement, showing the essential features, will be obtained when this cluster is again in a favorable position for observation. It is quite probable that a long exposure taken with a suitable instrument will show that the whole cluster is but the central condensation, made up of the heavier masses, of a great spiral nebula.

Ann Arbor, November 20, 1903.

OBSERVATIONS OF COMETS AND MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By THEO I. KING.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

Washington M.T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ	Red. to App. Pl.
			COMET *b* 1902 III (PERRINE).					
1902 d h m s			m s	′ ″	h m s	° ′ ″		s ″
Sept. 4 14 12 13.7	1–2	12 , 6	−0 38.20	+ 0 2.0	3 14 2.01	+36 13 18.2	n9.4773 0.0000	+4.07 + 1.2
5 13 47 32.9	3	12 , 6	−0 30.71	+ 9 13.8	3 12 15.71	+36 40 56.2	n9.5311 0.0514	+4.11 + 1.4
6 11 38 11.1	1	8 , 4	−0 47.41	− 5 4.6	3 11 27.73	+37 6 53.8	n9.7255 0.4523	+4.17 + 1.4
7 12 43 37.7	5	13 , 7	+0 1.86	+ 2 6.1	3 9 17.17	+37 38 30.8	n9.6168 0.2233	+4.21 + 1.7
			COMET *a* 1903 I (GIACOBINI).					
1903								
Feb. 26 7 7 36.6	6	19 , 4	+4 21.33	+ 2 11.3	23 55 28.25	+11 20 15.0	9.6711 0.7110	−0.20 + 0.6
Mar. 3 7 18 57.8	7	28 , 6	+2 32.75	+ 3 35.1	0 5 41.51	+16 5 32.8	9.6773 0.7198	−0.17 0.0
4 7 19 44.2	8	33 , 9	−0 25.82	+ 0 36.9	0 7 13.82	+16 23 12.7	9.6780 0.7210	−0.16 − 0.1
12 7 4 31.0	9	21 , 7	+0 1.66	− 6 59.6	0 21 58.81	+17 15 1.0	9.6800 0.7205	−0.14 − 0.8
13* 6 45 38.6	10	5 , 1	−1 25.59	− 3 0.2	0 23 11.56	+17 1 26.8	9.6785 0.7191	0.14 − 1.0
			(42) *Isis.*					
1902								
July 2 13 2 50.5	11	6 , 8	+1 36.01	− 1 10.9	20 16 9.10	−27 30 11.7	n8.8552 0.9037	+3.75 −19.8
			(6) *Hebe.*					
1903								
Apr. 17 12 14 31.8	12	28 , 6	+0 5.84	+13 11.2	13 40 25.58	+11 5 7.7	8.1756 0.6107	+2.29 − 9.9
21 11 14 23.3	13	30 , 6	−0 49.23	−13 31.7	13 37 0.21	+11 28 21.2	n8.7292 0.6057	+2.30 − 9.6
27 12 1 29.1	14	30 , 6	−1 7.01	− 0 12.3	13 31 51.99	+11 57 2.9	9.0012 0.6012	+2.31 − 8.9
28 10 1 5.8	14	39 , 6	−1 51.83	+ 3 23.9	13 31 10.20	+12 0 39.2	n9.1253 0.6003	+2.31 − 8.8

Mean Places of Comparison-Stars for the beginning of the year.

✱	α	δ	Authority	✱	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 14 36.17	+36 13 15.0	B.D. +36 680 [illegible]	8	0 8 9.80	+16 23 5.9	Berlin, A.G., A. [illegible]
2	3 11 7.52	+36 20 1.9	Lund, A.G. 1719	9	0 21 57.32	+17 22 1.1	Berlin, A.G., A. 100
3	3 13 12.31	+36 31 11.0	Lund, A.G. 1712	10	0 24 10.29	+17 4 28.0	Berlin, A.G., A. 119
4	3 12 11.00	+37 11 57.0	Lund, A.G. 1704	11	20 14 29.34	−27 29 20.6	Gould Gen. Cat., 27849
5	3 9 11.37	+37 36 31.7	Lund, A.G. 1680	12	13 40 17.45	+10 51 36.4	Leipzig, A.G., I. 4923
6	23 51 7.12	+11 18 3.1	Leipzig, A.G., I. 9490	13	13 37 47.14	+11 42 2.5	Leipzig, A.G., I. 4914
7	0 3 8.96	+16 1 57.4	Berlin, A.G., A. 11	14	13 32 59.72	+11 57 24.1	Leipzig, A.G., I. 4893

* This observation was made by Mr. J. C. HAMMOND, and could not be completed on account of clouds.

′ Micrometrical comparison with ✱2.

PERIOD OF 320 *U CEPHEI*,

BY S. C. CHANDLER.

The appearance of Mr. YENDELL's paper on the light-curve of *U Cephei*, in the last number of this journal, makes timely the following remarks on its period. A new investigation of this has been made within the past year. For this calculation I reduced anew all the series of observations whose details were available in such form as to enable me to determine the times of minima on a homogeneous plan, independent as possible of the peculiarities in the form of the light-curve. This object seemed to be best secured by adopting as the point of reference, or in-

stant of normal minimum, the mean of the times, before and after minimum, when the star is near the middle of its variation, and fluctuating most rapidly. The points selected for this use were the magnitudes 8.3 and 8.6. The series so treated embraced the observations of KNOTT, WILSING, YENDELL, SPERRA, and my own. The results were grouped to form normals, as follows:

No. Min.	Mean E	Wt.	O—C	No. Min.	Mean E	Wt.	O—C
			m				m
3	55	2	+0.6	3	1139	5	− 0.8
5	122	6	+1.3	1	1216	2	+ 8.9
4	199	6	+2.8	2	1514	1	+ 7.0
6	258	9	−1.2	7	1920	7	+ 4.4
4	337	4½	−2.4	5	2117	9	− 2.6
5	400	8	−3.5	3	2154	4	− 3.5
5	471	9½	−5.0	1	2328	1	−11.3
4	516	5	−6.1	5	2821	5	+ 3.0
2	692	3	−0.8	1	2983	1	+ 7.3
2	972	4	−7.4	1	3246	3½	− 4.8
7	1086	10½	+9.8				

The column O−C shows the representation of the normal observed times by the elements, *

1880 June 23 $7^h\ 46^m.0$ (Gr.) + $2^d\ 11^h\ 49^m\ 44^s.55$ E

It may be concluded from this comparison that during the twenty-four years' interval since 1880 there has been no sensible deviation from a uniform period. On the contrary, these elements give, for the minimum indicated by SCHWERD's observations in 1828 (Epoch −7636), a deviation, O−C = +917^m. Unless, therefore, we are prepared to reject this indication, the period cannot be constant. I have discussed the circumstances relating to SCHWERD's data in *A.J.* IX, p. 50. For the present we must give over the attempt to ascertain the nature of the inequality of the period, and rest on the value of the above elements.

* These values differ very slightly from those used by YENDELL in *A.J.* 551 (p. 214), (1880 June 23 $7^h\ 43^m.5$ + $2^d\ 11^h\ 49^m\ 44^s.7$ E), which were furnished him before I had incorporated some of the recent observations.

EPHEMERIS OF WINNECKE'S COMET FOR THE APPEARANCE OF 1903–1904.

[BY C. HILLEBRAND, FROM *A.N.* 3916.]

For Berlin Midnight.

1904	App. α	App. δ	log r	log Δ
	h m s	° ′ ″		
Jan. 0	17 30 51.40	−17 46 46.7	9.988836	0.272241
1	35 47.19	18 0 34.6		
2	40 44.73	18 13 56.4		
3	45 43.71	18 26 51.5		
4	50 44.16	18 39 19.1	9.981012	0.270012
5	17 55 46.06	18 51 17.6		
6	18 0 49.31	19 2 49.2		
7	5 53.80	19 13 50.2		
8	10 59.47	19 24 20.8	9.974601	0.268606
9	16 6.23	19 34 19.9		
10	21 14.02	19 43 47.6		
11	26 22.67	19 52 43.1		
12	31 32.15	20 1 5.7	9.969782	0.268027
13	36 42.33	20 8 54.7		
14	41 53.11	20 16 10.0		
15	47 4.38	20 22 51.3		
16	52 16.03	20 28 58.1	9.966685	0.268248
17	18 56 27.91	20 34 29.7		
18	19 2 40.09	20 39 26.2		
19	7 52.27	20 43 47.7		
20	13 4.39	20 47 33.7	9.965417	0.269255
21	19 18 16.37	−20 50 43.9		
Jan. 22	19 23 28.07	−20 53 18.6		
23	28 39.41	20 55 18.0		
24	33 50.27	20 56 41.8	9.966004	0.271006
25	39 0.52	20 57 29.8		
26	44 10.08	20 57 42.6		
27	49 18.85	20 57 20.6		
28	54 26.72	20 56 24.0	9.968430	0.273468
29	19 59 33.56	20 54 52.7		
30	20 4 39.30	20 52 47.4		
31	9 43.88	20 50 8.8		
Feb. 1	14 47.17	20 46 57.1	9.972633	0.276601
2	19 49.09	20 43 12.5		
3	24 49.57	20 38 55.9		
4	29 48.53	20 34 8.0		
5	34 45.89	20 28 49.3	9.978474	0.280344
6	39 41.55	20 23 0.2		
7	44 35.45	20 16 41.5		
8	49 27.55	20 9 54.2		
9	54 17.77	20 2 38.9	9.985794	0.284645
10	20 59 6.02	19 54 56.3		
11	21 3 52.26	19 46 47.2		
12	21 8 36.47	−19 38 12.6		

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, LYNN, MASS. *Closed Dec. 30.*
Entered February 2, 1903 at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

INDEX

TO THE TWENTY-THIRD VOLUME

K.

L.

M.

N.

O.

P.

Q.

R.

W.

Y.

PLATES.

THE

ASTRONOMICAL JOURNAL

FOUNDED BY B. A GOULD

EDITED BY
SETH C. CHANDLER

ASSOCIATE EDITORS
ASAPH HALL AND LEWIS BOSS

VOLUME XXIV
JANUARY 1904 TO SEPTEMBER 1905
NUMBERS 553-576

BOSTON
1905

CONTENTS.

NUMBER 559.

NUMBER 560.

NUMBER 561.

NUMBER 562.

NUMBER 563.

NUMBER 564.

NUMBER 565.

NUMBER 566.

NUMBER 567.

NUMBER 568.

NUMBER 569.

NUMBER 570.

NUMBER 571.

NUMBER 572.

NUMBER 573.

NUMBER 574.

NUMBER 575.

NUMBER 576.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 553.

VOL. XXIV. BOSTON, 1904 JANUARY 8. NO. 1

REVISION OF ELEMENTS OF THIRD CATALOGUE OF VARIABLE STARS.

By S. C. CHANDLER.

The following table contains a revision of the elements of the stars contained in the Third Catalogue of Variable Stars (*A.J.* 379), utilizing all the suitable material for that purpose that has been published since that Catalogue was issued. Little explanation is needed. The headings of the columns and their signification are the same as in the Catalogue itself. At the end of the column "Elements of Maximum" an asterisk denotes that the original elements have been retained unchanged, the recent observations indicating no certain correction to them. In the same place an initial letter indicates that the elements rest on some other authority, as detailed in the notes at the end of the table. The column "Basis of Elements" gives the number of maxima and minima, with the limiting dates, of the data used in the determination.

No.	Star	$M-m$	Elements of Maximum, Greenwich M.T. Epoch	Period		Inequalities		Basis of Elements M	m	Dates	
22	V Sculpt.		241 1642	+295.5	E		R	5+		[illegible]	1899
62	S Sculpt.	161 ?	239 5578	+366.0	E			9+		[illegible]	1896
103	T Androm.	110	239 8848	+281.0	E			12	5	1855	1901
107	T Cassiop.	247	240 4540	+443.5	E			20	13	1852	1902
110	S Tucanae	75:	1 1344	+241	E			5.		1880	1899
112	R Androm.	120:	0 0141	+410.3	E	+30 sin (12° E + [illegible])		27	8	1827	1903
114	S Ceti	145	0 5159	+320.6	E			12	[illegible]	1872	1902
146	T Sculpt.	85	1 1343	+202	E			5	1	1872	1898
213	U Cassiop.	100	1 0253	+276.0	E			9	3	[illegible]	1896
294	W Cassiop.	208	1 0559	+404	E			5	5	1887	[illegible]
320	U Cephei		Min. 1880 June 23 7h 45m.0 + 2d 11h 49m 44s.55 E						79	1880	1902
401	U Sculpt.		241 0515	+328.0	E		R	5		[illegible]	[illegible]
419	U Androm.	–	5192	+348	E			4		[illegible]	[illegible]
432	S Cassiop.	280	0 1603	+610.5	E	+27 sin (15° E + 50)		19	7	[illegible]	[illegible]
434	S Piscium	163:	0 2608	+404.3	E	+18 sin (10° E + 352)		13	1	[illegible]	[illegible]
466	U Piscium	83	0 7721	+172.9	E			20	[illegible]	[illegible]	[illegible]
513	R Piscium	143	0 2927	+344.3	E			23	2	[illegible]	[illegible]
659	X Cassiop.	–	[illegible]	480 – 420				7	2	[illegible]	[illegible]
678	U Persei	165	241 1353	+318	E			13	8	[illegible]	[illegible]
715	S Arietis	165:	0 1867	+292.2	E	Periodic inequality		14	[illegible]	[illegible]	[illegible]
782	R Arietis	91.5	0 2849.0	+186.55	E	+7 sin (5° E + 2.5)	*	53	24	[illegible]	[illegible]
806	o Ceti	125:	1 5575.0	+331.693	E	+17.5 sin (4° 56' E + 307.2) + [illegible] (1)		102		[illegible]	[illegible]
845	R Ceti	70	0 3028	+167.0	E	Periodic inequality	*	28	5	[illegible]	[illegible]
893	U Ceti	122:	0 9522	+235.8	E		*	14	2	1877	[illegible]
906	R Triang.	123	1 1631	+268.0	E			10	6	1881	[illegible]

(1) +11.5 sin (3°.85 E + 124°.5) + 12[illegible] sin (9°.12 E + 71°.8) + [illegible]

No.	Star	M − m	Elements of Maximum, Greenwich M.T. Epoch	Period	Inequalities		Basis of Elements M	m	Dates incl.
976	T Arietis	127	240 5249d	+313d E		*	13	10	1871–1903
1018	R Horologii	151	1 1179	+405 E		R			1893–1899
1090	β Persei		Min. 1888 Jan. 3d 8h 11m.2	+2d 20h 48m 55s.6 E + (*)				565	1782–1901
1113	U Arietis		211 2406	+370 E			10		1892–1903
1166	X Ceti	76	1 1401	+182 E			11	2	1890–1903
1222	R Persei	96	0 1039	+210.3 E	+15 sin (8° E+120°)		35	7	1833–1900
1357	U Eridani	121 :	1 0199	+239 E			5	1	1886–1897
1386	T Eridani	110 :	1 1323	+251 E			5	1	1889–1898
1414	λ Tauri	–	Min. 1887 Dec. 6d 11h 57m	+3d 22h 52m.2 E	Per. Ineq.	*	–	60	1796–1902
1577	R Tauri	140 :	0 1262	+325 E		*	28	3	1798–1903
1582	S Tauri	70 :	0 0155	+380.0 E	−0.15 E²		21	2	1855–1903
1623	T Camelop.	–	1 2091	+370 E		*	8	2	1891–1903
1635	R Reticuli	–	240 1907	+279.5 E			5 +		1861–1899
1651	R Doradus	100	0 9960	+315 E	Irregular	R	–	–	1886–1899
1662	R Caeli		1 5258	+398 E		R	–		1893–1899
1701	R Pictoris	–	1 5080	+160 E	Irregular	R			1896–1899
1717	V Tauri	92	0 5060	+170.1 E			27	9	1826–1903
1761	R Orionis	168	239 8676	+378.5 E	Periodic inequality		15	1	1816–1901
1771	R Leporis	212	240 1936.7	+436.1 E	Periodic inequality	*	20	14	1855–1897
1800	W Orionis	16	1 4708.0	+ 32.32 E		R	7	8	1899–1902
1803	T Leporis	–	1 1343	+360 E	Irregular	*	7		1889–1898
1805	V Orionis	125	1 1778	+267 E			8	2	1856–1902
1850	S Pictoris	–	1 3162.5	+428.5 E			8 +	–	1873–1899
1855	R Aurigae	235	0 1486	+458.6 E	+19 sin (12° E+228°)		24	14	1862–1902
1894	T Columbae	105	1 1272	+225 E		R	7 +	1	1873–1899
1944	S Orionis	194	0 4095	+413 E	Large irregularity	*	15	6	1863–1897
1981	S Camelop.	162 :	1 2285	+328 E ?	Large irregularity		7	1	1892–1902
2013	U Aurigae	–	1 1753	+405.5 E			8		1891–1903
2059	S Columbae	–	1 0574	+327 E			2 +	–	1887–1899
2080	R Columbae	–	. . . .	320–330 ?			–	–	– –
2100	U Orionis	148	0 9877	+375 E		*	18	9 :	1885–1903
2141	R Octantis	–	1 5170	+330 E		R	–	–	1895–1899
2213	η Geminor.	–	Min. 2402546	+231.4 E	Period increasing ?		–	16	1844–1898
2258	V Aurigae	–	. . . .	320d–340d			6	5	1886–1897
2266	V Monoc.	160 :	0 8853	+332.0 E		*	9	3	1853–1903
2279	T Monoc.	7.93	0 9633.63	+ 27.0122 E		Y *	–	–	1884–1902
2415	W Monoc.	–	1 0617	+262.5 E		H *	4	–	1887–1897
2478	R Lyncis	186	0 5796	+379.2 E	+14 sin (15° E+270°)		20	13	1874–1902
2509	ζ Geminor.	5.015	1 0640.60	+10.15382 E		*	–	–	– –
2528	R Geminor.	121	0 3370.0	+370.2 E	+35 sin (6° E+78°)	*	22	10	1796–1899
2530	V Can. min.	–	1 1266	+364 E		P *	3	–	1889–1895
2539	R Can. min.	130 :	0 0089	+337.7 E		*	21	2	1796–1897
2583	L₂ Puppis	59	0 4876	+140.2 E			9 +	3 +	1872–1899
2610	R Can. Maj.	–	Min. 1887 Mar. 26d 15h 18m	+1d 3h 15m 46s.0 E		*	–	37	1887–1902
2625	V Geminor.	132	0 7754	+276 E		*	16	3	1857–1901
2676	U Monoc.	18.0	0 5275	+ 46.10 E	Periodic inequality	*	52	45	1873–1902
2684	S Can. min.	164	0 1629	+330.3 E	+20 sin (12° E+30°)	*	18	7	1856–1901
2691	T Can. min.	–	0 4138	+322.7 E	Periodic inequality ?	*	11	–	1854–1899
2735	U Can. min.	175	240 7760	+410 E	Large irregularity	*	8	10	1880–1901
2742	S Geminor.	120 :	239 7546	+293.8 E			24	1	1848–1901

(*) : +147m sin (0°.024 E+226°) + 22m sin (° E+216°)

No.	Star	M — m	Elements of Maximum, Greenwich M.T. Epoch	Period	Inequalities		Basis of Elements M	m	Dates incl.
			d	d					
2776	W Puppis	54	241 3628.4	+120.8	E	R	3+	1+	1896–1899
2780	T Geminor.	–	239 6369.5	+288.1	E	•	20		1848–1903
2815	U Geminor.	–	241 5029	+ 86.3	E		–		1901
2852	V Puppis		Min. 1900 Jan. 1ᵈ 5ʰ 5ᵐ	+1ᵈ 10ʰ 51ᵐ 26ˢ.7 E		R	–		1891–1899
2857	U Puppis		240 8148	+315	E	•	6	–	1881–1899
2946	R Cancri	125:	239 7553	+362	E +60 sin(6° E+141°)		24	1	1830–1901
2976	V Cancri	116	40 4568	+272.1	E		15	6	1855–1901
3010	V Carinae	2.16	1 5026.78	+ 6.6951	E	R			1892–1899
3055	X Carinae	–	Min. 1900 Jan. 1ᵈ 2ʰ 44ᵐ	+12ʰ 59ᵐ 29ˢ.9 E		R	–		1893–1899
3060	U Cancri		239 7962	+305	E	•	19	–	1853–1900
3087	T Velorum	1.10	41 5022.78	+ 4.6392	E	R	–		1892–1899
3109	S Cancri		Min. 1867 Aug. 31ᵈ 14ʰ 2ᵐ.89	+9ᵈ 11ʰ 37ᵐ 45ˢ E		S •	–		1892
3128	R Pyxidis	–	241 1350	+355	E ?	•	9	–	1851–1895
3170	S Hydrae	100	239 9379	+256	E Periodic inequality		24	3	1852–1901
3184	T Hydrae		39 9739	+288.8	E	•	22		1851–1900
3186	T Cancri	–	Min. 239 9706	+482	E	•	–	10	1858–1905
3264	W Cancri		241 1694	+383	E		8		1890–1901
3355	W Carinae	0.97	1 5021.64	+ 4.3709	E	R	–		1892–1899
3407	S Antliae		Min. 1888 Apr. 13ᵈ 12ʰ 33ᵐ.0	+0ᵈ 7ʰ 46ᵐ 48ˢ.18 E			–	72	1888–1902
3416	S Velorum	–	Min. 1900 Jan. 1ᵈ 3ʰ 44ᵐ	+5ᵈ 22ʰ 24ᵐ 21ˢ.1 E		R	–		1894–1899
3417	U Velorum	30	241 5078	+ 62	E Irregular	R	–		1894–1899
3418	R Carinae	136	0 1653	+309.5	E +25 sin(9° E+279°)	•	20	14	1772–1897
3425	X Hydrae	–	1 2180	+296	E	P •	2	1	1895–1896
3477	R Leo. min.	165	0 2308	+370.5	E +20 sin(10° E+300°)	•	19	7	1796–1897
3493	R Leonis	144	236 2907	+312.8	E Periodic inequality	•	54	19	1757–1901
3495	l Carinae	13	240 4637.4	+ 35.520	E		37	7	1871–1903
3567	V Leonis		0 8515	+273.1	E		12		1855–1903
3569	RR Carinae		. . . .	365	or irregular	R	–		
3637	S Carinae	86	0 4922	+148.7	E	•	14	3+	1874–1899
3662	Z Carinae		1 5276	+394.0	E	R	–		1895–1899
3777	Y Carinae	1.07	1 5021.40	+ 3.6401	E	R	–	–	1893–1899
3825	R Urs. Maj.	110	239 7949	+302.1	E +11 sin(8° E+258°)		54	29	1843–1902
3922	U Carinae	5.5	241 5034.0	+38.7397	E	R			1891–1899
3994	S Leonis	93:	0 0752	+189.5	E Periodic inequality		27	1	1859–1901
4225	X Centauri	140	241 1150	+314.0	E	•	6+	2	1889–1899
4260	W Centauri	–	241 1134	+203.0	E		5+		1889–1899
4315	R Comae	120:	239 9304	+361.8	E		16	1	1831–1903
4364	S Muscae	3.45	241 5029.18	+ 9.657	E	R	–		1891–1899
4377	T Virginis	153:	0 0891	+339.5	E		12	2	1861–1901
4407	R Corvi		0 3476	+318.5	E Periodic inequality	•	16		1796–1901
4415	T Crucis	2.07	1 5028.32	+ 6.7322	E	R			1895–1899
4429	R Crucis	1.40	1 5027.39	+ 5.82485	E	R			1891–1899
4488	U Centauri	106?	1 5046	+216.8	E	R			1895–1899
4492	Y Virginis	85:	0 8880	+218.8	E	•	8	2	1883–1899
4514	T Urs. Maj.	107.5	0 0705.8	+257.2	E +20 sin(9° E+90°)	•	49	28	1843–1903
4521	R Virginis	68.5	238 1934.8	+145.47	E +20 sin(° E+216°)+(¹)	•	84	30	1809–1901
4536	R Muscae	0.26	240 4656.60	+ 0.88217	E				1871–1899
4557	S Urs. Maj.	108	0 0571	+226.5	E +35 sin(5°.4 E+194°)		71	19	1790–1903
4596	U Virginis	88	0 2781	+206.92	E −0.006 E²		27	8	1813–1902
4611	S Crucis	1.49	1 5026.92	+ 1.68989	E	R			1891–1899

(¹) +4.8 sin(° E+343°)

No.	Star	M − m	Elements of Maximum, Greenwich M.T. Epoch	Period	Inequalities		Basis of Elements M	m	Dates incl.
			d	d					
1805	W Virginis	8.20	240 2708.27	+ 17.2711 E		*	51	51	1866 1895
1816	V Virginis		0 0456.5	+250.5 E		*	19		1857 1900
1826	R Hydrae	190	1 1951.0	+425.15 E	−0.36 E^2 + 15 sin (7°.5 E + 202°)	*	27	6	1781 1898
1847	S Virginis	157 :	239 7507	+376.9 E	Periodic inequality		21	2	1795 1900
1896	T Centauri	46	241 2985	+ 91.5 E			5 +	3 +	1891–1899
4940	W Hydrae		1 1061	+384 E		*	8		1875 1897
4918	R Can. Ven.		1 0742	+333 E			8		1888 1899
5037	RR Virginis		0 7483	+217 E		*	11		1873 1899
5070	Z Virginis		0 7851	+307.5 E			10	1	1855 1900
5095	R Centauri	60 :	0 1573	+160.5 E		*	8	3	1871 1892
5156	X Bootis		. . . .	121.5 ?		*	–	–	–
5157	S Bootis	132	0 1606	+270.0 E	+60 sin (3°.6 E + 358°)		45	22	1790 1902
5171	RS Virginis		1 1510	+355 E			9	–	1890 1902
5190	R Camelop.	142	0 3987	+269.5 E	+65 sin (1° E + 218°)		16	22	1862 1903
5192	V Centauri	1 47	1 5025.52	+ 5.19394 E		R	–		1891–1899
5194	V Bootis	102	0 9419	+256 E		D *	17	9	1880 1902
5237	R Bootis	101.5	239 9812	+223.3 E	+9 sin (9° E + 117°)		39	20	1858 1901
5249	V Librae	119 :	240 8579	+255.2 E			7	2	1880 1900
5321	S Lupi	–	1 1951	+345 E			5 +	–	1891 1899
5338	U Bootis	95	0 7778	+177.5 E	Large irregularity		20	12	1857 1902
5374	δ Librae		Min. 1867 Oct. 25d 9h 17m.5	+ 2d 7h 51m 22s.8 E		*	–	99	1797–1902
5396	S Apodis		241 5196	+298.0 E		R	–	–	1898–1899
5402	T Tri. aus.		. . . .	0.98			–	–	–
5405	RT Librae		241 3035	+252 E			7	–	1853–1903
5430	T Librae	105	0 7105	+238 E		*	8	3	1878–1900
5438	Y Librae		0 0919	+272 E		*	8	–	1861–1899
5465	R Tri. aus.	1.01	0 4623.71	+ 3.38922 E			–	–	1871–1899
5484	U Coronae	–	Min. 1870 Mar. 25d 9h 30m	+ 3d 10h 51m 11s.7 E + (1)			–	56	1858–1903
5494	S Librae	93	240 5692	+192.1 E		*	12	2	1853–1903
5501	S Serpentis	–	238 8724	+368.5 E	+116 sin (1° E + 62°)		39		1791–1903
5504	S Coronae	120	240 0647	+361.2 E	+8 sin (12° E + 327°)		38	11	1860–1902
5511	RS Librae	130 :	1 1190	+221 E			9	2	1889–1903
5566	RU Librae		. . . .	320 ?			6	–	1888–1901
5583	X Librae	80	0 7183	+163.6 E		*	17	1	1878–1898
5593	W Librae		0 7126	+205.5 E			11	–	1878–1897
5601	S Urs. min.	156	1 1623	+325 E			9	5	1890 1900
5617	U Librae		0 5363	+226.2 E	Periodic inequality	*	12	–	1849–1896
5644	Z Librae		0 7109	+295 E		*	7	–	1878–1898
5675	V Coronae	171	0 7279	+356.5 E		*	17	7	1857–1901
5677	R Serpentis	151	238 8491	+357.2 E	+35 sin (4° E + 48°)		39	7	1783 1902
5682	R Lupi	117 ?	241 5024	+231.5 E		R	–	–	1896 1899
5688	R Librae		239 9800	+212.1 E			8	–	1858–1903
5704	RR Librae	–	240 9703	+276.7 E			9	–	1851–1903
5713	S Tri. aus.	2.10	1 5023.41	+ 6.3231 E		R	–	–	1892–1899
5750	U Tri. aus.	0.63	1 5022.02	+ 2.5683 E		R	–	–	1892–1899
5758	X Herculis	60	1 1541	+ 93.5 E	Periodic inequality		14	11	1890–1899
5761	Z Scorpii		0 5292	+370 E		*	11	–	1854–1895
5768	RR Herculis	–	1 3119	+238 E ?			4	–	1891–1897
5770	R Herculis		0 2440	+317.7 E	+17 sin (10° E + 322°)		29	–	1825–1902
5776	X Scorpii		0 6364	+199.0 E		*	11	–	1876–1897

(1) + 80m sin (0°.06 E + 78°)

No.	Star	$M-m$	Epoch	Period		Inequalities		Basis of Elements M	n	Dates
5795	W Scorpii	130:	2406101ᵈ	+221ᵈ.5	E			15	2	1876 1900
5823	S Normae	1.1	1 5029.15	+ 9.7525	E		R			1892 1899
5830	R Scorpii		0 1591	+224.1	E			27		1839 1900
5831	S Scorpii	–	2392162.1	+176.7	E		·	31		1837 1900
5856	W Ophiuchi	–	2408276	+329.8	E	Period decreasing ?		6		1823 1900
5887	V Ophiuchi	176:	0 5660	+302.5	E	Periodic inequality		13	2	1874 1901
5889	U Herculis	171	0 0708	+411.4	E	+.0225 E² — .0035 E³		20	8	1825 1902
5903	Y Scorpii	–	0 6138	+365	E			5		1876 1900
5928	T Ophiuchi	–	2400507	+361	E ?			5		1860 1883
5931	S Ophiuchi	–	2399495	+233.8	E		·	11		1857 1903
5949	R Arae	–	Min. 1900 Jan. 5ᵈ 7ʰ 35ᵐ	+4ᵈ 10ʰ 12ᵐ 7ˢ.9 E			R	–		1891 1899
5950	W Herculis	128	2407537	+280.2	E	+26 sin (13° E + 354°)		19	7	1857 1902
5955	R Draconis	108	0 6715.8	+245.6	E			32	20	1797 1902
6005	S Draconis		. . . . . .	. . . .		Irregular period		6	3	1892 1897
6041	S Herculis	152	2399197	+308.3	E	+35 sin (9° E + 86°)		31	11	1840 1902
6050	RS Scorpii	–	. . . . . .	300–350		Irregular period		–		1873 1899
6062	RR Scorpii	130:	2410146	+282	E			7+	1+	1846 1899
6071	RV Scorpii	1.41	2115026.04	+ 6.0622	E		R	–		1834 1899
6132	R Ophiuchi	–	2399507	+302.2	E			13		1842 1900
6170	RW Scorpii	–	2411112	+387	E		·	5+		1890 1899
6189	U Ophiuchi		Min. 1881 July 17ᵈ 15ʰ 32ᵐ + 20ʰ 7ᵐ.6903 E — 3ᵐ.0t + 0ᵐ.3t² ($t - \frac{E}{1000}$					–	138	1863 1902
6207	Z Ophiuchi	168	2112590	+348	E			3	6	1893 1899
6225	RS Herculis		1 3771	+223	E			9	2	1896 1902
6275	S Octantis	107 ?	1 5091	+265	E		R			1898 1899
6331	RU Scorpii	–	1 1229	+378	E			5+		1889 1899
6368	X Sagittarii	2.896	0 4291.78	+ 7.01185	E		·	401	381	1866 1896
6401	Y Ophiuchi	6.22	0 8691.25	+ 17.1207	E		·	55	53	1882 1899
6442	Z Herculis		Min. 1894 July 28ᵈ 11ʰ 8ᵐ.2	+3ᵈ 23ʰ 49ᵐ.515 E (¹)			H ·			
6449	T Draconis	182	1 3173	+426	E			7	7	1894 1902
6472	W Sagittarii	3.00	0 2849.15	+ 7.5916	E		·	394	372	1866 1899
6500	R Pavonis	109 ?	1 5106	+229	E		R			1895 1899
6512	T Herculis	79	0 3399	+165.0	E	+10 sin (5° E + 110°)		51	31	1856 1902
6546	RS Sagittarii		Min. 1871 Sept. 5ᵈ 16ʰ 0ᵐ	+2ᵈ 9ʰ 58ᵐ 36ˢ.0 E			·			1871 1899
6573	V Sagittarii	2.10	2110175.10	+ 5.7731	E			96	96	1886 1899
6608	RV Sagittarii		1 1211	+320	E			6		1889 1899
6613	d Serpentis		. . . . . .	8.72			V ·			
6624	T Serpentis		0 0909	+341.8	E			19		1861 1901
6630	U Sagittarii	2.97	0 1215.0	+ 6.7446	E					1866 1899
6682	X Ophiuchi	208	1 0061	+335	E			9	5	1851 1900
6733	R Scuti		. . . . . .	71.1		Large irregularity	·			
6758	β Lyrae		Min. 1855 Jan. 6ᵈ.00[1] + 12ᵈ.908009 E + 3ˢ.855 $e^{...}$ – .0147 $e^{...}$				·			
6760	κ Pavonis	4.0	2104765.0	+ 9.102	E		·			1871 1899
6791	R Lyrae	22.7	1 0559.3	+ 46.4	E	Large irregularity	·	22	21	1887 1894
6811	R Cor. aus.	40 ?	1 5050	+ 89.2	E	Large irregularity	R			1896 1899
6849	R Aquilae	138	2399163	+355.0	E	0.63 E + 0.005 E²		22	9	1854 1902
6854	Y Aquilae		. . . . . .	. . . .		Not variable				
6871	V Lyrae	139:	2112705	+375	E			8	2	1893 1901
6892	RX Sagittarii		1 3102	+332	E			5		1894 1902
6894	S Lyrae	115:	1 2654	+218	E			8	1	1897 1900
6900	W Aquilae		1 2656	+480	E			4		1893 1898

(¹) Sec. min. 45ʰ later. (²) Where $t = \frac{E}{1000}$. PANNEKOEK's elements. Secondary minimum midway.

No.	Star	M – m	Elements of Maximum, Greenwich M.T. Epoch	Period	Inequalities		Basis of Elements M	m	Dates incl.
6903	T Sagittarii	–	240 2869^d	+375.8^d E	+0.50 E^2		11		1865–1898
6905	R Sagittarii	138	0 2796	+269.0 E	+18 sin (10° E+320°)		20	3	1849–1901
6921	S Sagittarii	102 :	0 2865	+230.7 E	+15 sin (10° E+110°)		16	1	1863–1900
6923	Z Sagittarii	226 :	1 0865	+452 E		*	5	1	1888–1899
6943	T Sagittae	–	. . . . . .	165 ?					–
6981	U Aquilae	2.18	1 0170.15	+ 7.0240 E		*	121	99	1886–1899
7045	R Cygni	157	239 8501	+425.9 E	+0.01 E^2		33	14	1817–1902
7077	T Pavonis		241 1126	+244 E			5	–	1893–1899
7085	RT Cygni	88	1 0511	+190.5 E			19	15	1877–1900
7106	S Vulpec.	26.5	0 2239	+ 67.5 E	Periodic inequality	*	51	47	1836–1895
7118	X Aquilae	–	1 2690	+348 E		*	6	–	1854–1897
7120	χ Cygni	171.5	236 5136.5	+406.02 E	+.0075 E^2+25 sin (5° E+272°)	*	96	16	1687–1902
7122	S Pavonis		. . . . . .	389 ?			–	–	– –
7124	η Aquilae	2.25	239 6168.625	+ 7.176381 E		S *	–	–	– –
7139	RR Sagittarii	140 :	241 2002	+338 E ?		*	3	4	1879–1892
7149	S Sagittae	3.40	0 6602.60	+ 8.38320 E		*	87	84	1876–1899
7151	RU Sagittarii	91	1 5220	+239 E		R		–	1896–1899
7155	RR Aquilae		1 3327	+395 E			9	–	1888–1902
7162	RS Aquilae	–	0 3775	+406 E			6	1	1888–1901
7192	Z Cygni	125 :	1 0312	+265 E		*	11	4	1887–1897
7220	S Cygni	163	0 2119	+323 E	+0.015 E^2 Large irregularity		13	5	1841–1902
7234	R Capric.	–	0 0391	+344 E	Systematic irregularity		15		1859–1900
7242	S Aquilae	72	0 2553	+146.7 E	Secondary phases	*	29	29	1855–1897
7245	R Telescop.	–	1 5331	+372 E		R	–	–	1896–1899
7252	W Capric.	87 :	0 4985	+208.0 E			13	1	1872–1898
7257	R Sagittae	17.0	0 0358.5	+ 70.56 E	+6.5 sin (2°.25+47°) (1)		66	169	1859–1897
7260	Z Aquilae	65	1 3131	+127.2 E			10	3	1876–1899
7261	R Delphini	130 :	0 2190	+284.1 E	+26 sin (9° E+200°)		17	2	1851–1902
7266	RT Sagittarii	130 ?	1 5182	+301 E		R	–	–	1896–1899
7299	U Cygni	229	0 4596	+461.3 E	Systematic irregularity		21	18	1871–1902
7399	W Delphini	–	Min. 1896 Jan. 5^d 13^h.7	+4^d 19^h 21^m.2 E		P *	–	–	– –
7404	R Microsc.	64	241 3510.6	+138.8 E		P *	4	–	1889–1897
7428	V Cygni	220	240 8244	+418 E		*	10	2	1857–1896
7431	S Delphini	118	240 2621	+277.5 E		*	21	13	1863–1902
7437	X Cygni	6.8	1 0190.90	+ 16.3855 E		*	120	117	1886–1901
7444	T Delphini	–	0 2133	+331.2 E		*	19	0	1855–1901
7448	W Aquarii	–	. . . . . .	. . . . . .			–	–	– –
7450	V Aquarii	–	. . . . . .	. . . . . .			–	–	– –
7455	U Capric.	–	239 9573.5	+202.5 E	+20 sin (5° E+285°)		15	–	1852–1899
7456	RR Cygni	–	. . . . . .	. . . . . .			–	–	– –
7458	V Delphini	–	241 1722	+540 E		*	6	–	1890–1899
7468	T Aquarii	88	0 1096	+203.3 E	+8 sin (7°.5 E+255°)	*	25	6	1794–1898
7482	U Pavonis	–	1 1924	+288 E			5	–	1891–1899
7483	T Vulpec.	1.41	0 9819.01	+ 4.43578 E		L	–	–	1885–1899
7488	Y Cygni	–	Min. { Even 1886 Dec. 9^d 11^h 31^m.0 +1^d 11^h 57^m 26^s.1 E } { Odd 1886 Dec. 9^d 9^h 24^m.3 +1^d 11^h 57^m 18^s.0 E }			D	–	–	– –
7492	RZ Cygni	137	1 2662	+280 E			6	3	1893–1899
7495	S Indi	–	1 5100	+405.7 E		R	–	–	1896–1899
7502	X Delphini	120 :	1 3450	+277 E			6	2	1895–1902
7514	T Octantis	55	1 5021	+205 E		R	–	–	1896–1899
7560	R Vulpec.	62.0	0 2500	+136.8 E	+18 sin (4°.5 E+61°)		42	11	1807–1902

(1) Secondary max. and min. follow principal max. and min. 35^d and 33^d, respectively.

No.	Star	$M-m$	Elements of Maximum, Greenwich M.T. Epoch	Period		Inequalities	Basis of Elements	M	m	Dates
			d	d						
7571	V Capric.	–	240 3212	+ 156.7	E	Periodic inequality		13		1867 1897
7577	X Capric.	117 :	0.3196	+ 218.1	E	+ 20 sin (10° E + 50°)	·	12	1	1867 1896
7590	Z Capric.	–	1.3525	+ 356	E			7		1876 1899
7609	T Cephei	208	0.5359	+ 387	E			24	19	1789 1902
7659	T Capric.	149	239.8878	+ 269.2	E		·	16	2	1850 1897
7733	Y Capric.	–	240.9790	+ 206	E		·	6		1871 1895
7751	W Cygni	70.0	0.9506	+ 131.5			·	28	26	1885 1895
7779	S Cephei	267	0.2389	+ 485.8	E	+ 0.05 E²		19	17	1789 1902
7783	RU Cygni	–	. . . . . .	396 ?		Irregular period				1890 1897
7795	RV Cygni	–	. . . . . .	425 ?		Irregular period				1862 1895
7813	R Gruis	–	1.2397	+ 331	E			4 –		1892 1899
7896	V Pegasi	115 :	1.3353	+ 303	E			4	1	1895 1902
7907	U Aquarii	–	0.6105	+ 258	E ?		·	11		1875 1898
7909	S Pisc. aus.	–	1.1620	+ 272	E		H ·	6		1890 1898
7911	T Pegasi	–	0.2155	+ 373.8	E	Periodic inequality		19		1822 1900
7994	R Pisc. aus.		0.5086	+ 292	E		·	5 –		1872 1899
7999	X Aquarii		1.3365	+ 315	E			5		1889 1899
8039	T Gruis	64	1.5038	+ 141	E		R			1898 1899
8040	S Gruis	–	1.5330	+ 410	E		R			1898 1899
8068	S Lacertae	113	241.1930	+ 234	E			7	4	1889 1897
8073	δ Cephei	38.6	1840 Sept. 26d 10h 50m	+ 5d 8h 47m 39.3 E		− 0.0008 E² (1)	·	300	300	1785 1899
8153	R Lacertae		240.8857	+ 299.8	E		·	12	0	1856 1899
8230	S Aquarii		0.0395	+ 279.7	E	Periodic inequality	·	19	0	1798 1899
8290	R Pegasi	172 :	239.7158	+ 377.5	E	+ 60 sin (7°.5 E + 225°)		25	7	1841 1902
8324	V Cassiop.	107	241.2789	+ 231.5	E			11	9	1857 1899
8369	W Pegasi		1.3185	+ 341	E			4		1895 1899
8373	S Pegasi	138	0.2210.5	+ 317.5	E			13	4	1864 1903
8512	R Aquarii		238.2847.6	+ 387.16	E	+ 35 sin (10° E + 235°)	·	30	1	1811 1903
8588	R Phoenic.	137 ?	241.5099	+ 270	E		R			1895 1899
8591	V Cephei	220	0.8886	+ 360	E		·	6	8	1882 1895
8594	R Tucanae		1.5135	+ 275	E		R			1896 1899
8597	V Ceti	–	0.7590	+ 261	E		·	10		1879 1898
8598	U Pegasi	131.9	1894 Sept. 22d 18h 13m.2	+ 1h 29m 50s.67 E				35	60	1894 1899
8600	R Cassiop.	182	239.8371	+ 431.6	E	+ 32 sin (9° E + 60°)		35	14	1850 1902
8604	S Phoenic.	66	241.5058	+ 151.2	E		R			1895 1899
8622	W Ceti	–	241.3565	+ 366	E			6	4	1896 1903

(1) — 0.000 000 15 E

NOTES.

320. *U Cephei.* While there is an undoubted inequality, its exact nature is not yet clear. The elements given, provisionally, represent the best uniform period that will satisfy observations since 1880, disregarding SCHWERD's (1828).

980. *V Persei.* BOHLIN's period of 318 days not consistent with some of the data.

1018. *R Horologii.* ROBERTS's elements adopted in preference to those of the Third Catalogue, although it is not easy to conciliate them with some of the data.

2815. *U Geminorum.* Provisional new epoch formed from 5 max. in 1900–01; old period retained.

3493. *R Leonis.* Definitive investigation not yet made. Present O — C, —20d.

5484. *U Coronae.* Elements provisional, awaiting new obsns., which are much needed. They give O — C = +12m from average of seven minima by YENDELL in 1900–02–03; and satisfy well all the anterior data.

5677. *R Serpentis.* Existence of periodical inequality with cycle of about 90 periods is certain, but complicated with other terms.

6760. *κ Pavonis.* The exact period is in doubt, for the present, owing either to uncertainty in enumerating periods between widely separated groups of obsns., or possibly to secular variation of period. Thus ROBERTS's elements (*A.J.* 491–492) [illegible] with some data inaccessible to him, while the elements of [illegible] give a difference for his epoch. [illegible] 2.98 [illegible] (9.09) or mine (9.1062) is correct must be left in abeyance for the present, until the ambiguity can be cleared up.

7122. *S Pavonis.* ROBERTS's elements do not satisfy [illegible] observations.

8073. *δ Cephei.* Elements of Third Catalogue retained. [illegible] (*A.N.* 3855) has a larger coefficient for term in E², but this is not borne out by later observations.

8598. *U Pegasi.* Difference of 0m.15 in brightness at alternate minima requires verification.

In the column of Elements the initial R denotes ROBERTS; G, GUTHNICK; L, LUIZET; Y, YENDELL; H, HARTWIG; P, PICKERING or PAUL; D, DUNÉR; S, SCHÖNFELD or SCHUR.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A 4¼-INCH REFRACTOR, BY A. W. QUIMBY.

1903		Time	New Grs	Total Grs.	Total Spots	Fac Grs	Def.
July	1	8	1	2	6	1	fair
	2	7	..	2	6	1	fair
	3	4	..	2	5	1	fair
	4	8	..	2	3	3	fair
	5	8	1	2	3	1	poor
	6	4	..	2	10	1	fair
	7	8	..	2	7	..	poor
	8	8	1	3	41	..	fair
	9	7	..	3	36	..	fair
	10	4	..	3	21	1	poor
	11	8	..	3	10	1	fair
	12	8	..	3	16	2	fair
	14	8	1	3	4	2	fair
	15	8	..	3	8	2	fair
	16	5	1	3	16	2	good
	17	8	..	3	16	1	fair
	19	8	..	2	8	1	fair
	20	7	..	2	7	1	fair
	21	8	..	2	3	1	poor
	22	4	..	1	1	2	fair
	23	5	..	1	2	1	fair
	24	8	1	2	12	1	good
	25	5	..	1	10	1	v. good
	26	9	..	1	18	1	fair
	27	8	1	2	16	2	poor
	28	7	..	2	20	2	poor
	29	7	..	1	5	1	poor
	30	10	1	2	5	2	fair
	31	9	..	..	..	2	fair
Aug.	1	8	..	..	..	..	fair
	2	8	1	1	5	..	fair
	3	8	..	1	1	..	poor
	4	7	..	..	..	..	poor
	5	10	1	1	1	2	fair
	6	11	1	..	..	..	poor
	7	5	2	2	13	3	fair
	8	8	1	3	13	3	fair
	9	8	..	3	32	3	good
	10	7	..	3	28	2	good
	*11	11	..	3	13	2	poor
	*12	8	..	3	12	2	poor
	*13	7	2	1	14	3	poor
	*14	4	.	1	12	2	poor
	15	8	..	3	10	2	good
	16	1	.	3	14	2	fair
	17	8	..	2	12	2	fair
	18	8	.	1	6	2	fair
	19	9	..	1	3	1	poor
	20	1	..	1	1	2	poor
	21	8	1	1	1	2	poor
	22	9	..	1	2	2	poor
	23	9	..	1	2	1	poor
	24	8	..	1	1	1	fair
	25	7	..	1	3	..	fair
	*26	7	..	1	1	..	fair
	*27	7	1	1	4	..	poor
	*30	2	..	..	..	..	poor
Aug.	*31	2	..	..	..	..	poor
Sept.	*1	10	..	..	..	..	poor
	2	7	..	..	..	..	fair
	3	9	..	..	..	..	fair
	4	8	..	..	..	..	fair
	5	8	..	..	..	..	fair
	6	9	..	..	..	..	fair
	7	8	..	..	..	..	fair
	10	2	1	1	5	1	good
	11	8	..	1	1	1	good
	12	5	..	1	2	1	fair
	13	8	..	1	3	1	fair
	14	8	..	1	4	1	fair
	15	1	2	3	18	1	fair
	16	1	..	2	10	2	poor
	17	2	..	2	3	2	poor
	18	3	..	1	2	2	fair
	19	8	..	..	..	3	fair
	20	9	..	..	..	2	fair
	21	7	..	..	..	1	good
	22	8	..	..	..	..	fair
	23	9	1	1	6	..	fair
	*24	11	..	1	12	..	fair
	*25	8	..	1	10	..	fair
	26	8	..	1	16	1	good
	27	2	..	1	7	1	fair
	28	9	1	2	1	2	fair
	29	9	..	1	1	1	fair
	30	9	..	..	..	..	fair
Oct.	1	8	..	..	..	..	fair
	2	7	1	1	1	1	fair
	3	9	..	1	11	1	fair
	4	8	..	1	12	1	fair
	5	8	1	2	20	2	fair
	6	10	..	2	40	1	fair
	7	11	1	3	83	1	fair
	8	8	..	2	40	..	v. poor
	12	11	..	2	74	1	fair
	13	8	1	3	78	2	good
	14	7	..	3	76	3	good
	15	8	..	2	63	2	fair
	16	8	..	2	11	1	fair
	18	7	..	1	1	1	fair
	19	7	..	1	1	..	fair
	*20	8	..	1	1	..	fair
	*21	8	..	1	1	..	fair
	*22	7	..	1	1	..	poor
	*23	10	..	1	1	..	poor
	24	8	..	1	1	..	fair
	25	1	1	2	2	1	fair
	26	8	..	1	8	1	fair
	27	10	..	1	12	1	poor
	28	10	..	1	22	1	fair
	29	7	..	1	41	1	fair
	30	7	1	2	43	1	poor
	31	9	..	2	66	1	fair
Nov.	1	8	..	2	80	2	good
	2	1	1	3	22	1	poor
	3	7	..	3	21	1	poor
	4	1	1	1	32	1	fair
	5	9	..	1	131	1	good
	6	8	..	3	64	1	fair
	7	8	..	3	77	1	poor
	8	9	..	3	98	1	fair
	9	8	..	3	112	1	fair
	10	9	1	3	76	1	fair
	11	8	..	2	72	2	fair
	12	7	..	1	10	1	poor
	13	8	..	1	10	1	fair
	14	11	..	1	38	1	fair
	15	9	..	1	22	1	poor
	16	10	1	2	11	2	poor
	17	9	..	1	1	1	poor
	18	10	..	1	1	1	fair
	19	8	..	1	1	..	fair
	20	7	..	1	1	..	poor
	21	10	..	1	2	..	fair
	22	4	..	1	3	..	fair
	23	9	..	1	1	..	poor
	24	10	1	2	3	1	fair
	25	7	..	2	2	1	poor
	26	9	..	2	5	3	good
	27	9	1	3	13	3	good
	28	8	..	2	6	..	poor
	29	9	0	2	6	1	poor
	30	10	1	3	13	2	fair
Dec.	1	7	..	3	6	1	poor
	3	10	2	5	13	1	poor
	4	3	3	5	21	1	fair
	5	2	..	1	30	3	fair
	6	9	..	1	22	1	fair
	7	10	1	5	31	2	fair
	8	8	1	6	25	3	poor
	10	8	..	1	18	1	poor
	11	9	1	4	18	2	fair
	12	9	..	4	11	2	fair
	13	1	..	1	13	2	fair
	14	9	..	1	13	2	fair
	15	3	1	4	15	2	fair
	16	10	1	5	10	3	poor
	17	9	..	5	13	2	poor
	18	1	..	4	8	2	poor
	19	8	..	3	9	2	fair
	21	9	..	3	15	1	fair
	22	9	..	2	11	3	fair
	23	4	..	2	5	1	fair
	26	8	..	..	..	..	v. poor
	27	1	..	..	..	1	poor
	28	9	2	2	5	1	fair
	29	2	1	3	12	1	fair
	30	12	..	3	10	1	fair
	31	11	..	2	9	1	fair

* 2¼-inch refractor.

CONTENTS

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME PRESS OF THOS P. NICHOLS, LYNN, MASS. *Closed Jan. 7.*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 554.

VOL. XXIV. BOSTON, 1904 JANUARY 21. NO. 2

EXAMPLES OF PERIPLEGMATIC ORBITS.

By G. W. HILL.

In the motion of material points it is well known that the determination of the orbits may be considered quite apart from the question what positions upon the orbits the points have at a given time. When the first portion of the problem has been completely investigated, the second is reduced in general to a mere matter of quadratures. GYLDÉN's later investigations in this line have rendered this division of procedure familiar. Our investigation will be confined to the motion of two points in the same plane.

In this plane, having adopted [illegible] longitude and [illegible] The line of departure [illegible] which [illegible] chosen arbitrarily [illegible] periodic functions of [illegible] integral number of circumferences [illegible] the latter, which must be permitted to extend from $-\infty$ to $+\infty$. Then if μ and μ' are two constants, and we put

$$\frac{\mu}{r} - 1 = \rho\,, \qquad \frac{\mu'}{r'} - 1 = \rho'\,,$$

the differential equations

$$\frac{d^2\rho}{dv^2} + \rho = 0\,, \qquad \frac{d^2\rho'}{dv^2} + \rho' = 0\,,$$

are, as is well known, those of two [illegible] at the pole. If, more generally [illegible] are such that they can be written [illegible] form

$$\frac{d^2\rho}{dv^2} = \frac{dV}{d\rho}\,, \qquad \frac{d^2\rho'}{dv^2} = \frac{dV}{d\rho'}\,,$$

V may be called the orbital potential. The present discussion will be limited to the case where V does not explicitly involve v. In the foregoing simple case we have

$$V = -\tfrac{1}{2}(\rho^2 + \rho'^2)\,.$$

A more general form for this function would be

$$V = f(\rho) + f'(\rho')$$

and the [illegible] other, and [illegible] of [illegible] But if the differen[illegible] not thus simply [illegible] be given at th[illegible] tion of variables [illegible] may be said [illegible] being impossible to determine one of [illegible] virtue, at least, of elimination of the [illegible] latter [illegible] L[illegible]'s [illegible]

[illegible]

[illegible] be made [illegible] there would [illegible] employment of L[illegible]'s series [illegible] be proportional [illegible] arise.

Let us suppose [illegible]

$$V = [illegible]$$

Then the [illegible]

[illegible]

It [illegible] and of by multiplying [illegible] placing [illegible] and [illegible] radii by the equations

$$r = \frac{\mu}{\mu + \rho}\,, \qquad r' = \frac{\mu'}{\mu' + \rho'}\,,$$

ρ and ρ' will be determined by the equations

$$\frac{d^2\rho}{dv^2} + \rho + \rho\rho' + \tfrac{1}{2}\rho'^2 = 0$$
$$\frac{d^2\rho'}{dv^2} + \rho' + \rho\rho' + \tfrac{1}{2}\rho^2 = 0$$

which differ from the former only in that μ is replaced by unity.

These equations have the integral

$$\frac{d\rho^2}{dv^2} + \frac{d\rho'^2}{dv^2} + \rho^2 + \rho'^2 + \rho\rho'(\rho+\rho') = C^2$$

we write C^2 instead of C in order to avoid a radical sign in some of the following relations. When ρ and ρ' are interchanged, the equations remain the same; thus the relation $\rho = \rho'$ constitutes a particular integral of the system of differential equations.

Adopt for exhibiting graphically the simultaneous values of ρ and ρ' (simultaneous with reference to the independent variable v) a system of rectangular coordinates, x exhibiting the value of ρ, and y the value of ρ'. Then the representative point P must lie on the negative side of the curve whose equation is

$$x^2 + y^2 + xy(x+y) - C^2 = 0$$

in order that $\frac{d\rho}{dv}$ and $\frac{d\rho'}{dv}$ may be real. This cubic will have a closed branch surrounding the origin if C^2 falls below a certain limit. It crosses the axes of x and y on both sides of the origin at distances therefrom, equal in all four cases, to C. Its intersections with the right line whose equation is $x+y=0$, and which bisects two of the angles made by the axes, are also at a distance C from the origin. On the other hand, its intersections with the line bisecting the remaining angles, whose equation is $x-y=0$, are given by the roots of the equation

$$2x^2 + 2x^3 - C^2 = 0$$

But this cubic cannot have more than one real root unless C^2 does not exceed $\frac{8}{27}$. This is the condition necessary and sufficient that the original cubic should have a closed branch including the origin. As we wish to confine our attention to the case where the radii are restricted to finite limits, we suppose that C fulfils the mentioned condition, and that the representative point P is always within the closed branch.

When x is at a maximum or minimum in the original cubic, the equation

$$2(1+x)y + x^2 = 0$$

must be satisfied. Multiply this by $\frac{1}{2}y$ and subtract the product from the cubic; the result is

$$x^2 + \tfrac{1}{2}x^2 y - C^2 = 0$$

But the previous equation yields

$$y = -\frac{1}{2}\,\frac{x^2}{1+x}$$

Hence the quartic

$$x^2 - \frac{1}{4}\,\frac{x^4}{1+x} = C^2$$

by its roots, which immediately embrace 0 between them, furnishes the limits of both the variables ρ and ρ'. However, we are not under the necessity of solving the quartic for the purpose of obtaining these limits; evidently, for C we may substitute a function of another constant rendering the solution easy.

The quartic, in a developed form, is

$$x^4 - 4x^3 - 4x^2 + 4C^2x + 4C^2 = 0$$

To remove the second term from this put $x = z + 1$, and we have

$$z^4 - 10z^2 - 4(4-C^2)z - 7 + 8C^2 = 0$$

We can adopt indeterminates q, q', R, such that the roots of this quartic are

$$\begin{aligned} z_1 &= \sqrt{R} + \sqrt{q+q'\sqrt{R}} \\ z_2 &= -\sqrt{R} + \sqrt{q-q'\sqrt{R}} \\ z_3 &= \sqrt{R} - \sqrt{q+q'\sqrt{R}} \\ z_4 &= -\sqrt{R} - \sqrt{q-q'\sqrt{R}} \end{aligned}$$

Then q, q', R are determined by the equations

$$q + R = 5 \quad . \quad q'R = 4 - C^2$$
$$R^3 - 5R^2 + 2(4-C^2)R - \left(\frac{4-C^2}{2}\right)^2 = 0$$

Put, for simplicity, $4 - C^2 = m$, then

$$\begin{aligned} z_1 &= \sqrt{R} + \sqrt{5-R+mR^{-\frac{1}{2}}} \\ z_2 &= -\sqrt{R} + \sqrt{5-R-mR^{-\frac{1}{2}}} \\ z_3 &= \sqrt{R} - \sqrt{5-R+mR^{-\frac{1}{2}}} \\ z_4 &= -\sqrt{R} - \sqrt{5-R-mR^{-\frac{1}{2}}} \end{aligned}$$
$$R^3 - 5R^2 + 2mR - \tfrac{1}{4}m^2 = 0$$

The solution of the last equation, regarding m as the unknown, is

$$m = 4R \pm 2R\sqrt{R-1}$$

whence it follows that

$$C^2 = 4(1-R) \mp 2R\sqrt{R-1}$$

In order that C may be real R should exceed unity, and the cubic in R has always at least one root greater than 1; for, if we make $R = 1$, the left member becomes $-\frac{1}{4}C^4$, while, for $R = +\infty$, the result is $+\infty$.

If we make $\sqrt{R-1} = c$, we have

$$C^2 = 2c(1-c)^2$$

If we adopt the right member of this as a substitute for C^2, it is plain that the roots of the quartic will be expressible without the intervention of cubic radicals. While C^2 goes from 0 to $\frac{8}{27}$, c goes from 0 to $\frac{1}{3}$. In terms of c we have

$$x_1 = 1 + \sqrt{1+e^2} + \sqrt{4-e^2+(4-2e)\sqrt{1+e^2}}$$
$$x_2 = 1 - \sqrt{1+e^2} + \sqrt{4-e^2-(4-2e)\sqrt{1+e^2}}$$
$$x_3 = 1 + \sqrt{1+e^2} - \sqrt{4-e^2+(4-2e)\sqrt{1+e^2}}$$
$$x_4 = 1 - \sqrt{1+e^2} - \sqrt{4-e^2-(4-2e)\sqrt{1+e^2}}$$

Then x_4 is evidently the lower limit of the values of ρ and ρ' and x_2 the upper limit of the same. The values of these limits are tabulated below for every 0.01 in e.

LIMITING VALUES OF ρ AND ρ' AS FUNCTIONS OF e.

e	Lower	Upper	e	Lower	Upper	e	Lower	Upper
0.00	0.0000	0.0000	0.12	−0.4529	+0.4385	0.24	−0.5962	+0.5391
0.01	−0.1404	+0.1403	0.13	0.4684	0.4516	0.25	0.6050	0.5435
0.02	0.1972	0.1968	0.14	0.4832	0.4637	0.26	0.6135	0.5470
0.03	0.2399	0.2390	0.15	0.4971	0.4747	0.27	0.6216	0.5500
0.04	0.2751	0.2735	0.16	0.5103	0.4849	0.28	0.6295	0.5526
0.05	0.3056	0.3031	0.17	0.5229	0.4942	0.29	0.6370	0.5546
0.06	0.3326	0.3290	0.18	0.5348	0.5027	0.30	0.6443	0.5562
0.07	0.3569	0.3520	0.19	0.5462	0.5104	0.31	0.6513	0.5574
0.08	0.3791	0.3727	0.20	0.5571	0.5175	0.32	0.6580	0.5581
0.09	0.3996	0.3915	0.21	0.5675	0.5239	0.33	0.6615	0.5585
0.10	0.4186	0.4086	0.22	0.5775	0.5297	[illegible]	[illegible]	[illegible]
0.11	−0.4363	+0.4242	0.23	−0.5871	+0.5348			

To illustrate the matter let us take a particular case, the radii being represented by the formulas

$$r = \frac{\mu\rho}{\mu+\rho} \quad , \quad r' = \frac{\mu\rho'}{\mu+\rho'}$$

suppose that the values of the four constants involved are

$$\rho = 1 \quad , \quad \rho' = 2 \quad , \quad \mu = 2 \quad , \quad e = 0.2$$

The limiting values of r are

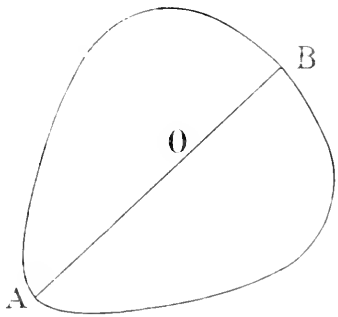
$$r = \frac{2}{2+0.5175} = 0.794 \quad , \quad r = \frac{2}{2-0.5571} = 1.386$$

and those of r' double these

$$r' = 1.589 \quad , \quad r' = 2.772$$

Here the upper limit of r is less than the lower limit of r'; hence the orbits have no point in common, and do not interfere with each other. We shall call this the quality of noninterference. It will be seen at once that the values of ρ, ρ', μ, e can be varied through a considerable range without the failure of this quality. But here is evidently an opportunity to apply LINDSTEDT's series in integrating the differential equations determining ρ and ρ'. Thus the applicability of these series does not imply dynamical instability in the motions which can take place upon the two orbits.

The form of the cubic circumscribing the values of ρ and ρ' for the special case noted above, where $e = 0.256$, is shown in the adjacent figure (the scale is two inches to the unit). O is the origin, and the right line AOB, passing through that point and bisecting the angle between the axes of coordinates, is the path of the representative point P for the case where $\rho' = \rho$, and the solution of the differential equations is a periodic one. It may be noted that this point in general never attains the closed branch of the cubic curve, as this cannot happen unless the values $\frac{d\rho}{dv} = 0$, $\frac{d\rho'}{dv} = 0$ are simultaneous.*



It is interesting to know whether the orbits are periplegmatic in the sense of GYLDÉN. With his notation we should have

$$\frac{d^2\frac{\rho}{\mu}}{dv^2} + \frac{\rho}{\mu} = \frac{1}{\mu}(\rho\rho' + \ldots) = P$$

$$\frac{d^2\frac{\rho'}{\mu}}{dv^2} + \frac{\rho'}{\mu} = \frac{1}{\mu}(\rho\rho' + \tfrac{1}{2}\rho \ldots) = P'$$

* The infinite branch is not given in the diagram, as it is useless for our purposes. The curve is species 67, and is shown in Fig. 71 of NEWTON's *Enumeratio linearum tertii ordinis*, printed at the end of Dr. SAMUEL CLARKE's Latin translation of NEWTON's Optics.

For the quality in question P and P' must not fall below -1. As the greatest value of $\rho\rho'+\frac{1}{2}\rho'^2$ or $\rho\rho'+\frac{1}{2}\rho^2$ is $\frac{2}{3}$, if μ exceeds this, the orbits will be periplegmatic.

The treatment of the differential equations is, in general, easier if we make the linear transformation

$$u = \tfrac{1}{2}(\rho+\rho') \quad , \quad s = \tfrac{1}{2}(\rho-\rho')$$

They then take the form

$$\frac{d^2u}{dv^2} + u + \tfrac{3}{2}u^2 - \tfrac{1}{2}s^2 = 0$$

$$\frac{d^2s}{dv^2} + s - us = 0$$

The radii of the orbits are represented by the equations

$$r = \frac{\mu\rho}{\mu+u+s} \quad , \quad r' = \frac{\mu\rho'}{\mu+u-s}$$

The integral, in terms of the new variables, is

$$\frac{du^2}{dv^2}+\frac{ds^2}{dv^2}+u^2+s^2+u^3-us^2 = \tfrac{1}{2}C^2$$

The adoption of the solution $s=0$, satisfying the equations, leads directly to a periodic solution of them. In this case we have the single differential equation

$$\frac{du^2}{dv^2} = \tfrac{1}{2}C^2 - u^2 - u^3$$

to be integrated. Make the substitution

$$u = g + g'\cos 2\psi$$

g and g' being constants; then

$$4g'^2(1-\cos^2 2\psi)\frac{d\psi^2}{dv^2} = \tfrac{1}{2}C^2-(g+g'\cos 2\psi)^2-(g+g'\cos 2\psi)^3$$

Let g and g' be so chosen that the right member of this, equated to zero, may have the two roots $\cos 2\psi = \pm 1$. Then g and g' are determined by the equations

$$\tfrac{1}{2}C^2 - (g+g')^2 - (g+g')^3 = 0$$
$$\tfrac{1}{2}C^2 - (g-g')^2 - (g-g')^3 = 0$$

or by

$$\tfrac{1}{2}C^2 - g^2 - g'^2 - g^3 - 3gg'^2 = 0$$
$$2g + 3g^2 + g'^2 = 0$$

If we divide both members of the last differential equation by $1-\cos^2 2\psi$ the result is

$$4g'^2\frac{d\psi^2}{dv^2} = \tfrac{1}{2}C^2 - g^2 - g^3 + g'^3\cos 2\psi$$

But, eliminating C^2, this becomes

$$4\frac{d\psi^2}{dv^2} = 1+3g+g'\cos 2\psi = 1+3g+g'-2g'\sin^2\psi$$

If we put $\sqrt{3}\,g' = \sin\theta$, then will $3g = \cos\theta - 1$, and

$$\frac{d\psi^2}{dv^2} = \frac{1}{2\sqrt{3}}\left[\sin(\theta+60^\circ)-\sin\theta\sin^2\psi\right]$$

If next

$$k^2 = \frac{\sin\theta}{\sin(\theta+60^\circ)}$$

we have

$$\frac{d\psi^2}{dv^2} = \tfrac{1}{4}\frac{1}{\sqrt{1-k^2+k^4}}(1-k^2\sin^2\psi)$$

and to u may be given the form

$$u = \tfrac{1}{3}\left(\frac{1+k^2}{\sqrt{1-k^2+k^4}}-1\right)-\frac{k^2}{\sqrt{1-k^2+k^4}}\sin^2\psi$$

It will be seen that k takes the place of the arbitrary constant C^2 which is attached to the integral. In the Gudermannian notation for elliptic functions, putting m for $\dfrac{1}{2\sqrt[4]{1-k^2+k^4}}$, and c being an arbitrary constant,

$$\sin\psi = sn\,(mv+c) = sn\,x$$

and

$$u = \frac{1}{\sqrt{1-k^2+k^4}}\left(\frac{1+k^2}{3}-\tfrac{1}{3}\sqrt{1-k^2+k^4}-k^2\,sn^2x\right)$$

The value of C is of interest; we have

$$\tfrac{1}{2}C^2 = g^2+g^3+g'^2(1+3g)$$
$$= \tfrac{1}{27}(3-6\cos\theta+3\cos^2\theta-1+3\cos\theta-3\cos^2\theta+\cos^3\theta)$$
$$+\tfrac{1}{3}(1-\cos^2\theta)\cos\theta$$
$$= \tfrac{2}{27}(1+3\cos\theta-4\cos^3\theta) = \tfrac{2}{27}(1-\cos 3\theta)$$
$$C^2 = \tfrac{8}{27}\sin^2\tfrac{3}{2}\theta$$

If C^2 is wanted in terms of k we have

$$C^2 = \tfrac{4}{27}\left(1-\frac{1-\frac{3}{2}k^2-\frac{3}{2}k^4+k^6}{(1-k^2+k^4)^{\frac{3}{2}}}\right)$$

The argument on which u depends is

$$\frac{\pi}{2K}\frac{1}{\sqrt[4]{1-k^2+k^4}}v+c$$

where K, as usual, denotes the period of the elliptic integral; or, it is

$$\frac{1}{\sqrt[4]{1-k^2+k^4}}\,\frac{1}{1+(\frac{1}{2})^2k^2+(\frac{1\cdot 3}{2\cdot 4})^2k^4+(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6})^2k^6+\ldots}v+c$$
$$= (1-\tfrac{15}{64}k^4-\tfrac{15}{64}k^6-\tfrac{5621}{16384}k^8+\tfrac{393}{8192}k^{10}+\ldots)v+c$$

It is to be noted that the square of k is absent from the latter expression, hence this parameter must become quite a large fraction before a marked difference results in the period.

An expression in terms of the nome q may be preferred. The period has the equivalent

$$\frac{2K\sqrt{k'}}{\pi}\sqrt[4]{1+\frac{k^4}{k'^2}}$$

where $k' = \sqrt{1-k^2}$. But the first factor has the value

$$\frac{2K\sqrt{k'}}{\pi} = 1+4\left[-\frac{q^2}{1+q^2}+\frac{q^6}{1+q^4}-\frac{q^{12}}{1+q^6}+\frac{q^{20}}{1+q^8}-\ldots\right]$$

and the second can be derived from

$$\frac{k}{\sqrt{k'}} = 4\sqrt{q}\,\frac{[1+q^2+q^6+q^{12}+\ldots]^2}{[1+2q^4+2q^{16}+\ldots]^2-[2q+2q^9+2q^{25}+\ldots]^2}$$

The series for the period or its reciprocal in powers of q is tardily convergent, and it seems better to retain the foregoing expressions where the law of progression is obvious.

If we put $k = \sin \eta$, q may be derived by tentation from the equation

$$\frac{\sin^2 \frac{1}{2}\eta}{1+\sqrt{\cos\eta}} = \frac{q+q^9+q^{25}+\ldots}{1+2q^4+2q^{16}+\ldots}$$

When $k = 1$, the numerator and denominator of the second member become divergent series, but the proper value of q, in this case, is unity. K may be derived from

$$\sqrt{\frac{2K}{\pi}} = 1+2q+2q^4+2q^9+2q^{16}+\ldots$$

To have u expressed as a periodic function of its argument substitute for the transcendental function $\mathrm{cn}^2 x$ its equivalent

$$\frac{2\pi^2}{k^2K^2}\Bigg[\frac{q}{1-q^2}-\frac{2q^2}{1-q^4}-\frac{3q^3}{1-q^6}-\ldots$$
$$+\frac{q}{1-q^2}\cos\left(\frac{\pi}{K}x\right)+\frac{2q^2}{1-q^4}\cos\left(2\frac{\pi}{K}x\right)$$
$$+\frac{3q^3}{1-q^6}\cos\left(3\frac{\pi}{K}x\right)+\ldots\Bigg].^*$$

There is still another linear transformation of the differential equations worthy of notice. In order to remove from the potential the terms of three dimensions which are products, let us put

$$\rho = u+hu' \quad , \quad \rho' = u'+hu$$

where h is either of the complex cube roots of unity, or such that

$$h^2+h+1 = 0$$

Then

$$\frac{1}{2}\frac{d\rho^2+d\rho'^2}{dv^2} = -\frac{1}{2}h\frac{du^2-4du\,du'+du'^2}{dv^2}$$
$$V = \frac{1}{2}h(u^2-4uu'+u'^2)+\frac{1}{2}(u^3+u'^3)$$

Hence it is seen that the differential equations take the form

$$\left[\frac{d^2}{dv^2}+1\right](2u'-u) = \frac{3}{2}hu^2$$
$$\left[\frac{d^2}{dv^2}+1\right](2u-u') = \frac{3}{2}hu'^2$$

or, if, as a symbol of operation, we put

$$\mathsf{D} = \frac{2}{3}h\left[\frac{d^2}{dv^2}+1\right]$$

the simple form

$$\mathsf{D}[2u'-u] = u^2 \quad , \quad \mathsf{D}[2u-u'] = u'^2$$

* For these formulas in elliptic functions consult Briot, *Traité Élémentaire des Fonctions Elliptiques*, p. 207, Eq. (124); pp. 210–211, Eqs. (5) and (6); p. 210, Eq. (3); p. 172, Eq. (17).

Thus, if from the [illegible] one of the [illegible] we subtract the other, and on the [illegible] operate with D, the result is the same as if we [illegible] the [illegible] able. Simple as are these equations, no completely satisfactory general expressions of the unknowns for [illegible] range in longitude have been found.

In applying Lindstedt's series to the integration of these equations we should assume

$$u = \Sigma\, A \ldots \quad [illegible] \quad = \Sigma\, A' \ldots$$

where the A and A' are constants as well as [illegible] and [illegible] and i and i' are integers reaching from $-\infty$ to $+\infty$. The substitution of these values in the equations shows that A, A', k, k' must satisfy, for each combination i, i', the conditions

$$[(ik+i'k')^2+1]\,(2A'\ldots-A\ldots) = \ldots h\,\Sigma\,A\ldots A\ldots$$
$$[(ik+i'k')^2+1]\,(2A\ldots-A'\ldots) = \ldots h\,\Sigma\,A'\ldots A'\ldots$$

These equations should suffice for determining the A and A' as well as k and k' in terms of the four arbitrary constants introduced by the integration. But two of these constants are involved in the expressions only through addition to the two elementary arguments [illegible] and [illegible]; thus the mentioned quantities involve only two arbitrary parameters. Since u and u' as periodic functions of v involve only cosines, we have the conditions

$$A_{\ldots} = A_{\ldots} \quad , \quad A'_{\ldots} = A'_{\ldots}$$

If, besides k and k', either of the two groups of coefficients A and A' is known, the other is deducible.

The differential equations may be reduced to a system in which all are of the first order; employing for this purpose those in terms of the variables u and s, the closed curve enveloping the area in which the differential coefficients are real has the equation

$$s^2 = \frac{[illegible]}{1 [illegible]}$$

The maximum value of [illegible] is then [illegible], and the maximum of [illegible] corresponds to the value of [illegible] given by the smaller positive root of

$$[illegible] = 0$$

Thus, if we put

$$[illegible] = 1 [illegible]$$

it will be found that

$$s^2 = [illegible]$$

And if [illegible] we shall have approximately [illegible]

In place of the two variables [illegible] and s we employ the [illegible] such that

$$\frac{du}{dv} = -[illegible] \quad , \quad s = [illegible]\cos[illegible] \quad , \quad [illegible] = -[illegible]\sin[illegible]$$

The integral equation will then be expressed in the form

$$y^2 + u^2 + u^3 + e'^2(1 - u\cos^2 l') = \tfrac{1}{2}\,C^2$$

which gives

$$\tfrac{1}{2}\,e'^2 = \tfrac{1}{2}\,\frac{\tfrac{1}{2}\,C^2 - y^2 - u^2 - u^3}{1 - u\cos^2 l'} = W$$

And the differential equations are

$$\frac{du}{dv} = -y$$

$$\frac{dy}{dv} = u + \tfrac{3}{2}\,u^2 - \tfrac{1}{2}\,e'^2\cos^2 l'$$

$$\frac{d\,(e'\cos l')}{dv} = -e'\sin l'$$

$$\frac{d\,(e'\sin l')}{dv} = (1-u)\,e'\cos l'$$

The third and fourth are equivalent to

$$\frac{d\,.\log e'}{dv} = -\tfrac{1}{2}\,u\sin 2l' \quad , \quad \frac{dl'}{dv} = 1 - u\cos^2 l'$$

From the latter it is plain that l' and v advance together; thus l' will serve equally well as v for the independent variable. By division and elimination of e'^2, the first and second equations become

$$\frac{du}{dl'} = -\frac{y}{1 - u\cos^2 l'}$$

$$\frac{dy}{dl'} = \frac{u + \tfrac{3}{2}\,u^2}{1 - u\cos^2 l'} - \tfrac{1}{2}\,\frac{\tfrac{1}{2}\,C^2 - u^2 - u^3}{(1 - u\cos^2 l')^2}\cos^2 l'$$

or, as they may be written

$$\frac{du}{dl'} = \frac{\partial W}{\partial y} \quad , \quad \frac{dy}{dl'} = -\frac{\partial W}{\partial u}$$

These equations may be still further varied by putting

$$u = e\cos l \quad , \quad y = e\sin l$$

Then if

$$W = \tfrac{1}{2}\,\frac{\tfrac{1}{2}\,C^2 - e^2 - e^3\cos^3 l}{1 - e\cos l\cos^2 l'}$$

we have

$$\frac{d\,.\,\tfrac{1}{2}\,e^2}{dl'} = \frac{\partial W}{\partial l} \quad , \quad \frac{dl}{dl'} = -\frac{\partial W}{\partial\,.\,\tfrac{1}{2}\,e^2}$$

After u and y or e and l have been determined in terms of l' through the integration of these equations, v can be found by a quadrature from

$$\frac{dv}{dl'} = \frac{1}{1 - u\cos^2 l'}$$

and thence, by inversion, l' in terms of v, and thus the problem completely solved.

W can be developed in an infinite series of the form

$$\Sigma_{i,\,i'}\,A_{i,\,i'}\cos\,[il + 2i'l']$$

For putting

$$\beta = \frac{2 - u - 2\sqrt{1-u}}{u}$$

we have

$$W = \frac{\tfrac{1}{2}\,C^2 - y^2 - u^2 - u^3}{\sqrt{1-u}}\left[\tfrac{1}{2} + \beta\cos 2l' + \beta^2\cos 4l' + \beta^3\cos 6l' + \ldots\right]$$

From this u and y may be eliminated by substituting their values in terms of e and l.

The integrals of the two differential equations may then be approximated to by a series of DELAUNAY transformations, as the function W is quite similar to DELAUNAY'S R in the lunar theory. The only noteworthy differences being that here there are only two unknowns in place of DELAUNAY'S six, and only one constant parameter C instead of DELAUNAY'S three n', e', a'.

We may give here DELAUNAY'S rule for making a transformation. If we have integrated the differential equations (L is put for $\tfrac{1}{2}\,e^2$)

$$\frac{dL}{dl'} = \frac{\partial W}{\partial l} \quad , \quad \frac{dl}{dl'} = -\frac{\partial W}{\partial L}$$

when W is limited to the terms involving one argument $il + i'l'$ (the constant term is included) and have found in this manner (θ designating the argument)

$$\theta = \theta_0(l'+c) + \theta_1\sin\theta_0(l'+c) + \theta_2\sin 2\theta_0(l'+c) + \theta_3\sin 3\theta_0(l'+c) + \ldots$$

$$L = L_0 + L_1\cos\theta_0(l'+c) + L_2\cos 2\theta_0(l'+c) + L_3\cos 3\theta_0(l'+c) + \ldots$$

c being a constant, and $\theta_0,\ \theta_1,\ \theta_2,\ \ldots\ L_0,\ L_1,\ L_2,\ \ldots$ being known functions of another constant (e_0 for instance), we can replace

$$L \text{ by } L_0 + L_1\cos(il + i'l') + L_2\cos 2\,(il + i'l') + \ldots$$

$$l \text{ by } l + \frac{\theta_1}{i}\sin(il + i'l') + \frac{\theta_2}{i}\sin 2\,(il + i'l') + \ldots$$

and we shall have, for determining the new variables e_0, l, precisely the same equations

$$\frac{dL}{dl'} = \frac{\partial W}{\partial l} \quad , \quad \frac{dl}{dl'} = -\frac{\partial W}{\partial L}$$

provided, first, that we put for W the function obtained by making the preceding substitutions in the old function W (complete) augmented by the quantity

$$-\frac{i'}{i}\,(L - L_0) + \frac{i'}{2i}\,(\theta_1 L_1 + 2\theta_2 L_2 + 3\theta_3 L_3 + \ldots)$$

second, that we regard the new variables L as connected with e_0 by the relation

$$L = L_0 + \tfrac{1}{2}\,(\theta_1 L_1 + 2\theta_2 L_2 + 3\theta_3 L_3 + \ldots)$$

DOUBLE-STAR MEASURES,

By JOHN A. MILLER and W. A. COGSHALL.

The stars in the list that follows are selected from those noted as double by the Albany observers while making the observations for the catalog of the *Astronomische Gesellschaft*, Zone 1°–5°. The list, kindly furnished us by Professor S. W. Burnham, contains only such stars as have not hitherto been measured, or at least whose measures have not been published. The measures were made with the 12-inch refractor of the Kirkwood Observatory, the method of work being the same as that followed while measuring the list selected from the Berlin A.G Catalog. *A.J.*, No. 546.

The letter C. follows the measures made by Mr. Cogshall, the letter M. those made by myself. The positions given are for 1900.0.

John A. Miller.

DM. 3°490. A.G. 1020. 9M.0 ; 9M.5.
$\alpha = 3^h\,28^m\,0^s.11$; $\delta = +3°\,48'\,28''.4$

t	θ_0	ρ_0	
	°	″	
1903.039	354.0	5.99	
.140	354.6	6.35	
.164	352.6	6.06	
1903.114	353.7	6.13	C.

DM. 2°635. A.G. 1165. 8M.7.
$\alpha = 3^h\,54^m\,21^s.20$: $\delta = +2°\,23'\,52''.4$
Not double. M.

DM. 4°817. A.G. 1540. 8M.9 ; 9M.3.
$\alpha = 4^h\,57^m\,7^s.77$; $\delta = +4°\,10'\,43''.1$

1903.039	175.6	8.69	
.170	177.8	9.14	
.227	177.5	9.13	
1903.145	177.0	8.99	M.

DM. 4°818. A.G. 1544. 8M.8 ; 10M.2.
$\alpha = 4^h\,57^m\,24^s.2$; $\delta = +4°\,27'\,22''.8$

1903.039	281.6	30.29	
.118	280.2	30.36	
.227	281.1	31.11	
1903.128	280.9	30.59	M.

DM. 2°859. A.G. 1570. 9M.0 ; 9M.1
$\alpha = 5^h\,1^m\,5^s.13$; $\delta = +2°\,19'\,6''.5$

1903.023	358.0	1.56	
.161	356.8	2.21	
.206	356.3	1.86	
1903.131	357.0	1.88	C.

DM. 2°996. A.G. 1796. 9M.0 ; 10M.5.
$\alpha = 5^h\,27^m\,51^s.93$: $\delta = +2°\,23'\,52''.5$

1903.023	232.6	4.87	
.161	234.9	4.78	
.118	234.8	4.43	
1903.105	234.1	4.69	M.

DM. 2°1328. A.G. 2325. 8M.5 ; 9M.5.
$\alpha = 6^h\,33^m\,54^s.34$; $\delta = +2°\,25'\,29''.8$

1903.023	307.8	31.80	
.068	306.9	34.81	
.164	307.1	31.80	
1903.085	307.3	31.80	C.

DM. 3°1382. A.G. 2381. 8M.0.
$\alpha = 6^h\,39^m\,33^s.75$: $\delta = +3°\,46'\,55''.2$
Not double.

DM. 5°1396. A.G. 2390. 9M.1 ; 10M.2.
$\alpha = 6^h\,40^m\,19^s.75$: $\delta = +4°\,59'\,35''.2$

t	θ	ρ	
	°	″	
1903.039	21.2	25.13	
.068	21.1	24.51	
.118	21.3	24.56	
1903.075	21.2	24.73	M.

DM. 2°1427. A.G. 2419. 8M.8 ; 10M.2.
$\alpha = 6^h\,45^m\,49^s.77$: $\delta = +1°\,58'\,24''.5$

1903.039	263.5	2.41	
.068	264.6	2.22	
.164	262.6	2.67	
1903.090	263.6	2.43	C.

DM. 2°1457. A.G. 2495. 8M.6 ; 10M.3.
$\alpha = 6^h\,49^m\,32^s.46$: $\delta = +3°\,41'\,4''.4$

1903.068	209.4	6.94	
.118	205.5	6.98	
.227	207.0	7.02	
1903.137	207.3	6.98	M.

DM. 2°1463. A.G. 2500. 8M.9 ; 10M.3.
$\alpha = 6^h\,50^m\,14^s.03$: $\delta = +2°\,42'\,6''.9$

1903.039	251.9	14.23	
.161	250.8	11.65	
.227	250.1	11.56	
1903.113	251.0	11.18	C.

DM. 3°1469. A.G. 2508. 8M.5.
$\alpha = 6^h\,51^m\,8^s.19$: $\delta = +3°\,19'\,36''.0$
Not double.

DM. 2°1503. A.G. 2558. 9M.0 ; 9M.3.
$\alpha = 6^h\,55^m\,18^s.19$: $\delta = +2°\,17'\,46''.8$

1903.039	92.9	3.74	
.120	92.4	3.77	
.161	90.1	3.63	
1903.108	91.8	3.71	M.

DM. 3°1508. A.G. 2566. 8M.3 ; 10M.8.
$\alpha = 6^h\,55^m\,52^s.30$; $\delta = +3°\,11'\,56''.9$

1903.039	269.6	6.44	
.120	269.7	6.92	
.164	268.8	6.66	
1903.108	269.4	6.67	C.

DM. 2°1514. A.G. 2570. 8M.9 ; 10M.7
$\alpha = 6^h\,56^m\,21^s.07$: $\delta = +2°\,41'\,21''.3$

t	θ	ρ	
1903.039	204.6	20.50	
.164	204.3	20.80	
.216	204.4	20.33	
1903.149	204.4	20.55	M.

DM. 2°1645. A.G. 2754. 9M.0 ; 9M.5.
$\alpha = 7^h\,14^m\,56^s.25$; $\delta = +1°\,56'\,19''.7$

1903.164	320.2	21.86	
.216	321.0	23.80	
1903.205	320.6	24.33	C.

DM. 1°1877. A.G. 2963. 8M.7 ; 9M.7.
$\alpha = 7^h\,36^m\,0^s.63$; $\delta = +1°\,22'\,14''.7$

1903.161	97.3	5.36	
.216	98.0	5.71	
1903.205	97.6	5.53	M.

DM. 4°1922. A.G. 3218. 9M.0 ; 9M.5.
$\alpha = 8^h\,4^m\,52^s.27$; $\delta = +4°\,20'\,57''.6$

1903.161	28.5	1.77	
.216	28.0	1.69	
1903.205	28.3	1.73	M.

DM. 1°2302. A.G. 3784. 9M.0 ; 11M.0.
$\alpha = 9^h\,21^m\,37^s.41$; $\delta = +1°\,30'\,56''.3$

1902.320	66.5	3.52	
1902.370	...	3.10	
1903.161	69.7	3.26	
1902.618	68.1	3.29	C.

DM. 3°2445. A.G. 4181. 9M.1 ; 9M.3.
$\alpha = 10^h\,55^m\,23^s.49$; $\delta = +3°$ [illegible]

1902.359	127.0	1.71	
1902.370	126.3	1.70	
1903.241	126.4	1.66	
1902.657	126.5	1.69	M.

DM. 2°2449. A.G. 4308. 8M.7 ; 10M.0.
$\alpha = 11^h\,29^m\,37^s.43$; $\delta = +2°\,2'\,57''.0$

1902.320	6.9	2.09	
.370	6.1	1.88	
.414	4.4	1.34	
1902.368	5.8	1.77	C.

DM. 3°2610. A.G. 1179. 9m.0 : 10m.0.
$\alpha = 12^h 19^m 35^s.88$; $\delta = +2° 58' 32''.9$

t	θ	ρ	
1902.320	216.9	6.50	
.370	217.9	6.25	
.392	215.1	6.31	
1902.361	216.6	6.35	M.

DM. 2°2550. A.G. 1501. 8m.3 : 9m.0.
$\alpha = 12^h 25^m 56^s.55$; $\delta = +2° 39' 43''.17$

1902.323	288.6	1.16	
1902.418	290.1	1.23	
1903.288	286.7	1.32	
1902.674	288.1	1.24	C.

DM. 2°2651. A.G. 1677. 9m.0 : 10m.1.
$\alpha = 13^h 11^m 18^s.11$; $\delta = +2° 42' 51''.0$

1902.923	307.9	3.80	
1903.119	307.7	3.12	
1903.147	305.1	3.22	
1903.063	306.9	3.38	M.

DM. 3°2846. A.G. 1860. 9m.0 : 10m.0.
$\alpha = 14^h 5^m 36^s.72$; $\delta = +3° 11' 32''.4$

1902.659	186.9	1.82	
1902.468	184.9	1.94	
1903.119	188.0	1.70	
1902.748	186.8	1.82	C.

DM. 2°2831. A.G. 1980. 9m.1 : 9m.3.
$\alpha = 14^h 26^m 36^s.71$; $\delta = +2° 16' 53''.7$

1902.419	158.2	1.72	
1902.468	163.7	1.20	
1903.288	158.5	1.95	
1902.725	160.1	1.62	M.

DM. 1°3055. A.G. 5276. 9m.0 : 10m.5.
$\alpha = 15^h 40^m 21^s.99$; $\delta = +4° 50' 47''.1$

t	θ	ρ	
1902.419	118.7	1.98	
.468	115.7	1.89	
.501	117.9	2.08	
1902.463	117.4	1.98	M.

DM. 1°3270. A.G. 5503. 9m.0 : 10m.2.
$\alpha = 16^h 33^m 0^s.51$; $\delta = +1° 50' 48''.6$

1902.419	191.3	2.43	
.468	193.0	2.36	
.502	191.0	3.00	
1902.463	191.8	2.60	M.

DM. 1°3387. A.G. 5662. 9m.0 : 9m.8.
$\alpha = 17^h 3^m 8^s.44$; $\delta = +1° 51' 18''.8$

1903.419	243.1	26.95	
.473	241.9	27.12	
1903.446	242.5	27.18	C.

DM. 3°3596. A.G. 6092. 9m.0 : 9m.1.
$\alpha = 18^h 2^m 43^s.25$; $\delta = +3° 16' 29''.5$

1902.419	88.1	1.87	
.468	88.9	1.83	
.504	88.1	1.96	
1902.463	88.4	1.89	M.

DM. 3°3571. A.G. 6188. 9m.0 : 9m.5.
$\alpha = 18^h 11^m 27^s.40$; $\delta = +3° 17' 19''.4$

1902.807	280.0	2.61	
1903.447	277.4	2.36	
1903.687	281.5	2.84	
1903.313	279.7	2.60	M.

DM. 2°3562. A.G. 6189. 9m.1 : 10m.2.
$\alpha = 18^h 14^m 38^s.70$; $\delta = +2° 4' 27''.8$

1902.419	36.9	7.14	
1902.793	34.8	7.62	
1903.714	35.9	7.17	
1902.975	35.9	7.31	M.

DM. 3°3769. A.G. 6321. 8m.2 : 9m.2.
$\alpha = 18^h 35^m 5^s.32$; $\delta = +3° 15' 56''.7$

t	θ	ρ	
1902.419	318.5	22.03	
1903.447	318.7	21.73	
1902.933	318.6	21.88	C.

DM. 1°4233. A.G. 7036. 8m.5 : 10m.
$\alpha = 20^h 8^m 18^s.82$; $\delta = +1° 10' 14''.6$

1902.419	358.2	3.51	
.785	358.0	2.54	
1902.602	358.1	3.01	C.

DM. 4°4170. A.G. 7147. 8m.5 : 8m.5.
$\alpha = 20^h 26^m 13^s.11$; $\delta = +4° 52' 23''.8$

1902.419	235.4	1.60	C.

DM. 2°4232. A.G. 7239. 9m.2 : 9m.3.
$\alpha = 20^h 38^m 3^s.56$; $\delta = +2° 31' 30''.4$

1902.793	275.1	4.35	
1903.701	273.6	4.99	
1903.714	272.5	4.89	
1903.403	273.7	4.74	M.

DM. 3°4428. A.G. 7272. 9m.0 : 9m.2.
$\alpha = 20^h 43^m 22^s.9$; $\delta = +4° 3' 55''.1$

1902.775	260.1	5.02	
1902.848	261.0	5.04	
1903.701	263.2	5.40	
1903.108	261.4	5.15	M.

DM. 4°4586. A.G. 7348. 8m.4 : 10m.0.
$\alpha = 20^h 53^m 11^s.53$; $\delta = +4° 24' 4''.8$

1902.775	287.9	11.35	
1902.848	289.4	10.01	
1903.701	287.0	12.30	
1903.108	288.1	11.23	

OBSERVATIONS AND CIRCULAR ELEMENTS OF PLANET [1898 DW].

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY.

By W. WALTER DINWIDDIE. [Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.

A new asteroid was found on a photographic plate exposed by Mr. PETERS, Dec. 11, 1903. From its position and motion it could not be identified with any whose place is given in the *Jahrbuch*. I have computed a circular orbit from my micrometer obsns. of Dec. 15, and Dec. 18, and find the elements agree with the elements of [1898 DW] in every respect except the position of the planet in the orbit, and this may be accounted for by eccentricity.

1903 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
Dec. 15 11 27 41	1	10 , 10	$-0^m 9^s.94$	$-9' 18''.2$	$5^h 42^m 1^s.99$	$+9° 20' 30''.3$	9.0028n	0.6364	$+4^s.57$	$-7''.4$
Dec. 18 13 52 16	2	33 , 7	−2 15.15	+0 22.0	5 38 57.05	+9 9 8.9	9.3616	0.6516	+4.60	−7.2

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	$5^h 41^m 47^s.48$	$+9° 29' 55''.9$	A.G.Leipzig II 2399	2	$5^h 36^m 7^s.00$	$+9° 8' 5''.41$	A.G.Leipzig II 2337

Epoch 1903, Dec. 18.5 $\omega = 216° 1' 26''$, $\Omega = 230° 11' 49''$, $i = 15° 16' 52''$, $\mu = 849''.85$, $\log a = 0.413803$.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed Jan. 20*,
Entered February 2, 1903 at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 555.

VOL. XXIV. BOSTON, 1904 FEBRUARY 1. NO. 3

DEFINITIVE ORBIT OF COMET 1845 III.

By HENRY A. PECK.

The discovery of the third comet of 1845 is credited to COLLA of Parma, although the claim of priority rests on the narrow margin of a few hours as against that of BOND of Harvard. The first observations for position were made June 2, at Cambridge, U.S.A. It was not until three days after that observations began to be made in Europe. The comet was very brilliant at discovery, being visible to the naked eye, and having a tail a degree long. Quite a large number of observers, on both sides of the Atlantic, reported it as an independent discovery. Four days after its first announcement it passed near *Capella* about midnight and was estimated by RICHTER of Berlin as of the third magnitude. For nearly two weeks it could be observed at lower culmination with meridian instruments. The last observation of this character was made at Hamburg June 15.

From the first, the similarity of its orbit and physical appearance to TYCHO BRAHE's bright comet of July 1596 was recognized. The observations of this latter comet were again gone over by VALZ and J. R. HIND. The short period of time embraced by the observations, as well as their somewhat uncertain character, lead to varying results, according to the hypothesis adopted for their reduction. Our present knowledge of the orbit of 1845 III rests upon the work of D'ARREST. From the meridian observations, together with the best from the remaining series at his command, he deduced a parabola, a hyperbola and an ellipse with a period of 249 years. Either of these three satisfied the observations used by him with sufficient accuracy. It has long been recognized that a thorough revision of all this work was desirable. The present article contents itself with the single question as to the most probable orbit of 1845 III during the time of visibility, reserving all other phases of the subject for another time and place.

The entire series of observations is embraced in a period of twenty-nine days, beginning at Cambridge, U.S.A., June 2, and closing at Washington with a single observation on July 1. This last observation is of the greatest importance. It fortunately enables us to answer without hesitation the question as to the form of the orbit, in so far as this form relates to the possibility of the identity of the comet with that of 1596.

For the purpose of the reductions from the equinox and ecliptic of 1845.0 to that of the date of observation, the following values of the independent star numbers were computed from the data in BAUSCHINGER's *Tafeln zur Theoretischen Astronomie*. The dates are with reference to Paris Mean Noon.

COMPUTATION OF THE EPHEMERIS.

	f	$\log g \sin G$	$\log g \cos G$	G	$\log$ [illegible]	H	$\log$ [illegible]	
	″			° ′		° ′		
June 2	+31.76	+0.757	+1.110	22 29	1.174	196 50	1.308	−2.55
3	.90	.758	.112	27	.176	195 57	.308	42
4	32.05	.759	.114	25	.178	195 4	.308	29
5	.20	.760	.116	23	.180	194 10	.309	16
6	.35	.762	.118	21	.182	193 17	.309	03
7	.49	.763	.150	18	.184	192 24	.309	1.90
8	.65	.764	.152	16	.186	191 31	.309	76
9	.80	.765	.154	13	.187	190 38	.310	63
10	.95	.766	.156	11	.189	189 45	.310	50
11	33.11	.767	.158	8	.191	188 52	.310	36
12	.26	.768	.160	5	.193	187 59	.310	23
13	+33.41	+0.769	+1.162	22 2	1.195	187 7	1.310	1.09

	f	log g sin G	log g cos G	G	log g	H	log h	i
	″			° ′		° ′		″
June 14	+33.57	+0.770	+1.164	21 59	1.197	186 14	1.310	−0.96
15	.72	.771	.166	56	.199	185 21	.311	.82
16	.87	.772	.168	53	.200	184 29	.311	.69
17	34.02	.772	.170	49	.202	183 36	.311	.55
18	.18	.773	.172	45	.204	182 43	.311	.42
19	.33	.774	.174	42	.206	181 51	.311	.28
20	.49	.774	.176	38	.208	180 58	.311	.15
21	.64	.775	.178	34	.209	180 6	.311	−0.01
22	.80	.775	.180	30	.211	179 13	.311	+0.12
23	.95	.776	.182	27	.213	178 21	.311	.26
24	35.11	.776	.184	23	.214	177 28	.311	.40
25	.26	.777	.185	19	.216	176 36	.311	.53
26	.41	.777	.187	11	.218	175 43	.311	.67
27	.57	.777	.189	9	.219	174 51	.311	.80
28	.72	.777	.191	5	.221	173 58	.311	.94
29	.87	.777	.193	1	.223	173 6	.310	1.07
30	36.03	.777	.195	20 56	.224	172 13	.310	.21
July 1	.18	.777	.197	51	.226	171 20	.310	.34
2	.33	.778	.198	47	.228	170 28	.310	.47
3	+36.48	+0.778	+1.200	20 43	1.229	169 35	1.310	+1.61

The following positions of the sun were found with the assistance of NEWCOMB's *Tables of the Sun*. As before, the dates are with reference to Paris Mean Noon.

	App. Long. of ☉	log R	Lat. of ☉	Sidereal Time
	° ′ ″		″	h m s
June 2	71 44 12.7	0.0063393	+0.17	4 43 14.01
3	72 41 39.6	63990	+0.05	47 10.57
4	73 39 5.6	64562	−0.07	51 7.13
5	74 36 30.8	65108	.19	55 3.69
6	75 33 55.1	65630	.27	59 0.25
7	76 31 18.5	66127	.38	5 2 56.82
8	77 28 40.9	66599	.45	6 53.37
9	78 26 2.3	67049	.50	10 49.93
10	79 23 22.7	67476	.52	14 46.49
11	80 20 42.0	67881	.51	18 43.04
12	81 18 0.5	68267	.48	22 39.60
13	82 15 17.9	68634	.39	26 36.15
14	83 12 34.6	68982	.31	30 32.70
15	84 9 50.4	69312	.18	34 29.26
16	85 7 5.5	69626	−0.04	38 25.81
17	86 4 19.9	69924	+0.09	42 22.36
18	87 1 33.8	70206	.25	46 18.93
19	87 58 47.1	70473	.37	50 15.48
20	88 56 0.0	70726	.49	54 12.05
21	89 53 12.4	70965	.59	58 8.61
22	90 50 24.6	71190	.66	6 2 5.17
23	91 47 36.5	71401	.68	6 1.72
June 24	92 44 48.2	0.0071595	+0.67	6 9 58.28
June 25	93 42 59.9	0.0071773	+0.65	6 13 54.83
26	94 39 11.7	71934	.60	17 51.39
27	95 36 23.6	72075	.52	21 47.94
28	96 33 35.7	72196	.44	25 44.50
29	97 30 47.9	72295	.31	29 41.05
30	98 28 0.4	72372	.19	33 37.61
July 1	99 25 13.1	72423	+0.07	37 34.17
2	100 22 26.0	72448	−0.05	41 30.73
3	101 19 39.0	0.0072447	−0.16	6 45 27.29

APPARENT OBLIQUITY OF ECLIPTIC.

	° ′ ″
June 2	23 27 28.1
4	28.0
16	27.9
18	27.8
23	27.9
29	27.8
July 1	27.7
6	27.8

In using this table the value remains the same until changed by a succeeding value.

From these data the following coordinates of the sun were computed. The right-ascension contains the aberration, and the rectangular coordinates are referred to the true equinox and equator. The equation of time is to be added to apparent time.

	α apparent of ☉	Equation of Time	X	Y	Z
	h m s	m s			
June 2	4 40 51.49	+2 22.52	+0.317895	+0.883982	+0.383593
3	44 57.65	12.92	.301791	.888854	.385706
4	49 4.20	2.93	.285599	.893473	.387710
5	53 11.10	1 52.59	.269325	.897837	.389604
6	57 18.32	41.93	.252975	.901945	.391385
7	5 1 25.85	30.97	.236554	.905798	.393056
8	5 33.66	19.71	.220067	.909392	.394615
9	5 9 41.72	+1 8.21	+0.203519	+0.912729	+0.396063

	α apparent of ☉	Equation of time	X	Y	Z
	h m s	m s			
June 10	5 13 50.00	+0 56.49	+0.186915	+0.915805	+0.397398
11	17 58.50	44.54	.170259	.918622	.398621
12	22 7.19	32.41	.153557	.921181	.399731
13	26 16.04	20.11	.136814	.923477	.400728
14	30 25.04	+0 7.66	.120032	.925515	.401612
15	34 34.16	−0 4.90	.103218	.927291	.402384
16	38 43.39	17.58	.086377	.928806	.403042
17	42 52.70	30.34	.069512	.930057	.403587
18	47 2.08	43.15	.052628	.931050	.404018
19	51 11.50	56.02	.035730	.931782	.404336
20	55 20.94	1 8.89	.018822	.932255	.404541
21	59 30.37	21.76	+0.001910	.932464	.404632
22	6 3 39.80	34.63	−0.015004	.932413	.404610
23	7 49.18	47.46	.031917	.932101	.404475
24	11 58.50	2 0.22	.048817	.931526	.404226
25	16 7.75	12.92	.065707	.930689	.403863
26	20 16.89	25.50	.082579	.929592	.403387
27	24 25.92	37.98	.099431	.928233	.402796
28	28 34.81	50.31	.116256	.926613	.402092
29	32 43.53	3 2.48	.133049	.924732	.401275
30	36 52.08	14.47	.149808	.922590	.400345
July 1	41 0.42	26.25	.166526	.920186	.399301
2	45 8.51	37.78	.183198	.917522	.398145
3	6 49 16.34	+3 49.05	−0.199819	+0.914597	+0.396875

As a basis for the ephemeris the parabolic elements of d'Arrest were used. These elements are found in *A.N.* XXIII, p. 351, and are as follows:

$$T = \text{June } 5.68951 \text{ Paris M.T.}$$

$$\left.\begin{aligned} \omega &= 75^\circ\ 48'\ 16'' \\ \Omega &= 337\ \ 48\ \ 49 \\ i &= 131\ \ \ 4\ \ 52 \end{aligned}\right\} 1845.0$$

$$\log q = 9.6032278$$

and to these correspond the equatorial coordinates

$$\begin{aligned} x &= [9.981653]\, r \sin[180^\circ\ 48'\ 23.2'' + v] \\ y &= [9.966390]\, r \sin[277\ \ 47\ \ \ 3.3 + v] \\ z &= [9.675527]\, r \sin[\ 57\ \ 18\ \ \ 8.8 + v] \end{aligned}$$

Ephemeris for Paris Mean Noon.

	α apparent	δ apparent	log r	log Δ	Ab. time —(in days)
June	h m s	° ′ ″			
2	3 14 59.61	+36 32 51.3	9.616313	9.9261	0.00487
3	29 52.92	38 32 55.6	.610296	.9186	479
4	46 33.51	40 24 31.4	.606049	.9125	472
5	4 4 57.04	42 3 31.9	.603701	.9085	467
6	24 50.33	43 25 58.3	.603324	.9065	465
7	45 50.67	44 28 34.1	.604950	.9066	465
8	5 7 27.04	45 9 13.1	.608470	.9094	468
9	29 3.90	45 27 20.0	.613834	.9137	473
10	50 6.33	45 23 56.0	.620868	.9204	480
11	6 10 4.89	45 1 19.0	.629384	.9282	489
12	28 38.43	44 22 34.2	.639172	.9379	500
13	45 35.51	43 31 10.5	.650022	.9486	513
14	7 0 52.61	42 30 28.8	.661725	.9601	526
15	14 32.20	41 23 26.9	.674090	.9721	541
16	7 26 40.67	+40 12 34.8	9.686914	9.9849	0.00557
17	7 37 26.50	+38 59 50.7	9.700129	9.9978	0.00574
18	46 58.68	37 46 42.2	.713522	0.0109	592
19	55 26.11	36 34 16.8	.727013	.0239	610
20	8 2 57.12	35 23 17.3	.740512	.0367	628
21	9 39.24	34 14 15.3	.753947	.0494	647
22	15 39.01	33 7 30.0	.767261	.0621	666
23	21 2.18	32 3 11.5	.780411	.0742	685
24	25 53.68	31 1 23.4	.793363	.0862	704
25	30 17.75	30 2 6.9	.806090	.0978	723
26	34 18.02	29 5 20.8	.818577	.1091	742
27	37 57.53	28 10 59.0	.830844	.1202	761
28	41 18.87	27 18 55.3	.842783	.1309	780
29	44 24.33	26 29 3.4	.854491	.1415	799
30	47 15.92	25 41 17.2	.865933	.1516	818
July 1	49 55.26	24 55 30.2	.877111	.1614	837
2	52 23.69	24 11 34.7	.888027	.1710	855
3	8 54 42.54	+23 29 24.1	9.898686	0.1803	0.00874

The Comparison Stars

The basis of the star positions is to be found in the various zones of the A.G., re-enforced whenever possible with material from other star catalogues. Among these are American Ephemeris, B.B. VI, Bessel's Zones, Fundamental Catalog of the A.G., the various Greenwich and Pulkowa Catalogues, Harvard, Paris, Yarnall and the second Washington Catalogue. Proper motions have been applied where possible. In this part of the work I have been greatly assisted by Professor Boss and Mr. Roy of the Dudley Observatory, who furnished series of positions from their card catalogue of the brighter stars. The adopted positions are:

No.	α 1845.0	δ 1845.0	No.	α 1845.0	δ 1845.0
	h m s	° ′ ″		h m s	° ′ ″
1	3 21 59.77	+38 3 39.5	35	7 19 29.77	+41 9 34.4
2	3 31 11.02	42 1 59.8	36	7 20 19.21	40 46 14.5
3	3 39 18.20	41 29 18.3	37	7 20 22.77	40 40 28.0
4	4 4 56.24	41 42 13.2	38	7 21 35.53	40 50 10.9
5	4 22 31.40	42 43 34.6	39	7 30 41.74	39 33 47.9
6	4 50 57.87	44 17 25.8	40	7 32 38.71	39 39 46.8
7	4 54 9.47	44 50 2.8	41	7 34 23.11	39 56 51.3
8	4 55 22.89	49 30 15.7	42	7 36 18.06	37 53 12.5
9	5 18 43.45	45 41 47.7	43	7 39 21.58	38 24 8.4
10	5 37 19.64	45 10 27.1	44	7 40 58.01	37 28 6.6
11	5 48 9.66	44 55 27.6	45	7 42 10.44	38 19 50.6
12	5 56 54.48	45 33 51.8	46	7 43 28.07	38 55 29.9
13	5 59 50.10	44 58 21.6	47	7 44 33.73	38 14 42.6
14	6 0 53.16	45 4 22.8	48	7 59 34.90	36 9 1.5
15	6 14 49.18	44 48 34.1	49	8 0 5.46	35 54 49.2
16	6 15 24.07	44 53 30.6	50	8 11 41.54	34 25 17.6
17	6 17 49.45	44 58 57.5	51	8 19 27.92	33 12 3.1
18	6 28 43.00	44 8 31.5	52	8 22 56.23	31 21 11.1
19	6 28 55.77	44 27 49.2	53	8 24 24.32	31 24 17.1
20	6 31 47.79	44 39 58.7	54	8 26 29.25	30 32 51.3
21	6 34 11.54	44 23 28.5	55	8 28 39.84	30 33 18.8
22	6 35 33.72	43 23 28.9	56	8 29 4.49	31 14 58.1
23	6 37 37.21	43 59 47.6	57	8 31 58.74	29 25 29.9
24	6 38 10.34	43 55 29.8	58	8 32 0.60	29 23 52.7
25	6 49 9.20	43 11 18.5	59	8 32 9.61	30 32 27.6
26	6 49 29.09	43 10 22.9	60	8 32 33.38	29 26 14.5
27	6 55 56.73	43 5 49.2	61	8 35 50.18	31 15 15.6
28	6 56 37.37	42 58 19.8	62	8 37 18.35	29 19 20.3
29	7 0 59.16	39 33 59.7	63	8 37 46.07	28 43 32.6
30	7 7 37.71	42 8 35.0	64	8 13 8.75	28 50 11.1
31	7 10 5.55	42 56 12.1	65	8 13 21.22	28 55 6.4
32	7 11 46.93	42 56 59.1	66	8 46 21.95	28 30 53.3
33	7 13 23.84	40 58 50.2	67	8 19 41.57	+24 33 40.8
34	7 16 49.13	+39 37 13.6			

The Observations.

So far as known, the following is a complete list of the observations of this comet:

Altona. Observations found in *A.N.* XXIII. The observers were Peterson and Schumacher. No particulars are given as to instrument used. It is noted that the observation of June 10 is made with a ring micrometer and not fully reduced. The first observations of June 15 and 17 depend on stars whose position is not given. None of these are used.

Ashurst. In *Monthly Notices*, Vol. VI, p. 254, are mentioned observations made by R. Snow, Esq., June 9, 10, 11 and 12. No data are given by which accurate positions can be deduced. The observations are therefore useless.

Berlin. Observations reduced from the *Berliner Beobachtungen*, Bd. III. They consist of a series of meridian observations made at lower culmination by Galle, and also of filar micrometer observations made upon the equatorial. Some of the observations are also found in *A.N.* XXIII.

Bonn. Observations by Argelander with the five-foot telescope and published in *A.N.* XXIII.

Breslau. Two observations by Schubert and published without particulars in *A.N.* XXIII. The times of observations are not given but the coordinates are reduced to $11^h\ 0^m$ Breslau M.T.

Brussels. Meridian observations made at lower culmination and found in *C.R.* XX, *A.N.* XXIII, and also in *Ann. de l' Obs. de Bruxelles*, XII. The right-ascensions were observed on the meridian transit by Bouvy, Liagre and Quetelet; the declinations on the mural circle by Houzeau.

Cambridge, Eng. Meridian observations at lower culmination. One is reported by Hind in *A.N.* XXIII. The others are found in the *Cambridge Observations*, XVI. After searching in several observatory libraries for this apparently somewhat rare volume, the observations were furnished me by Professor Graham, the present director at Cambridge. The right-ascensions were observed by Morgan on the transit instrument and the declinations by Challis on the mural circle. Professor Graham also says: "There are a few extra meridian observations in manuscript, made on the same nights and on June 16; but the discussion of these is so confusing, and in the case of the last so decidedly in error, that I fear they could not be relied on in the determination of the orbit."

Cambridge, Mass. Observations contained in the *Proceedings of the American Academy of Arts and Sciences*, Vol. I. In response to a note of enquiry, Professor Pickering states that "it is not certainly known what instrument was employed in the observations you mention, but probably it was the five-foot telescope mentioned on page XVIII of Vol. I of the Annals of this observatory." The following notes accompanying the observations have a bearing on their use for the present purpose:

June 2. The observations of this morning are made with the spider-line micrometer, and under favorable circumstances.

June 4. The differences of A.R. were obtained this day from the hour circle of the equatorial, which reads to single seconds of time.

June 6. The observations are made as on the 4th. On the 9th and 10th the observations are made with the spider-line and annular micrometers.

June 25. Observed with the spider-line and annular micrometers, the comet being still sufficiently bright to bear illumination.

The observations of June 14 and June 24 depend upon stars that can not with certainty be identified. They have therefore been disregarded.

Geneva. A single observation by Plantamour published in *A.N.* XXIII.

Göttingen. A single meridian observation by Gauss published in *A.N.* XXIII.

Greenwich. A series of observations contained in the *Greenwich Observations* for 1845, and made upon the equa-

torial by MAIN, ROGERSON and WILLIAM RICHARDSON. Both the right-ascensions and declinations depend entirely on circle readings. The grade of the entire series is so poor that it has been disregarded in computing the orbit.

Hamburg. These observations consist of a series made by RÜMKER on the meridian circle in both coordinates, a second series of right-ascensions by FUNK on the transit instrument and a single filar-micrometer observation by RÜMKER. All are published in *A.N.* XXIII, and the first series is also in *C.R.* XX.

Königsberg. Two series of observations. The first was made by BUSCH in the meridian at lower culmination, the other was made upon the heliometer by WICHMAN. They are to be found in *A.N.* XXIII, XXIX, and also in the *Königsberger Beobb.*

Kremsmünster. Observations by RESLHUBER published in *A.N.* XXIII. The series, which is quite extended, is of such poor quality that it has been entirely disregarded.

London. Observations made by HIND at Mr. BISHOP's private observatory. They are published both in *A.N.* XXIII and in the volume known as *Bishop's Observations.* The observations are made in part on the equatorial and in part with an altitude and azimuth instrument mounted as a transit circle. The equatorial had a seven-inch lense of about eleven-foot focus and was furnished with a wire micrometer. The following notes have a bearing on the orbit:

June 10. Only an approximate instrumental place was obtained.

June 12. Good differential and meridian observations.

June 18. Only instrumental places taken.

Modena. A long series from June 6 to June 27. They are to be found in *A.N.* XXIII and *C.R.* XX. The observers were BIANCHI and GOBBI. From the period they cover, they would be of the greatest value if reliable. They are however worthless and present a fine example of misdirected zeal. They have of necessity been entirely disregarded in the following discussion.

Padua. Another series of m[illegible] found in *A.N.* XXIII. These wo[illegible] been of great value as they cover the later portion of [illegible]. No account has been taken of them, however, in the following discussion.

Paris. The observations of MAUVAIS, LAUGIER, EUGENE BOUVARD, FAYE and GOUJON, are given in Vol. XIX of the *Annales de l'Observatoire de Paris.* I have omitted the observation of the 6th, the first observation of the 7th and that of the 19th. The first two are referred to comparison stars from one to two degrees distant in declination, and for the last the comparison star could not with certainty be identified.

Vienna. From the *Annalen der Sternwarte in Wien* (S. II) XIII. The observers were JELINEK, HORNSTEIN and SCHAUB. All the observations were made with the ring micrometer, except that on the 12th the filar micrometer was used. An observation for the 7th is given in *A.N.* XXIII but has not been retained here. On the 18th the observer professes to have made differential filar micrometer observations, using two stars seven and fourteen degrees distant. On this account the observation has been disregarded.

Washington. These observations appear in the first volume of the *Astronomical Journal* and also in the second volume of the *Washington Observations.* Those used have been reduced from the data furnished by the latter volume. Several observations have been reluctantly abandoned because the comparison stars could not be absolutely identified. The observers were COFFIN and HUBBARD. The telescope was an equatorial of 9.65 inches aperture and 14 feet 4.3 inches focal length. It was furnished with a filar micrometer. On account of the length of time covered, this series is the most important of all.

COMPARISON OF OBSERVATIONS WITH THE EPHEMERIS.

In comparing the observations with the ephemeris, the time of observation has been corrected for aberration and then reduced to the meridian of Paris.

Date	Place	α apparent	π	O − C Δα cos δ	δ apparent	π	O − C Δδ	✱
June		h m s	s	"	° ' "	"	"	
2.85143	Cambr., U.S.A.	3 27 34.19	−0.61	+ 3.4	+38 45 26.4	+ 6.9	+ 1.4	1
4.83925	Cambr., U.S.A.	4 1 56.93	.61	(+35.1)	41 48 34.8	7.8	1.6	2
4.84846	Washington	2 4.21	.66	− 0.7	49 33.3	7.8	10.8	4
5.57585	Paris	16 14.92	.42	0.2	42 53 7.5	10.0	1.9	5
5.60181	Paris	15.49	.50	− 7.8	55 18.2	9.4	+ 2.5	5
6.83732	Cambr., U.S.A.	42 39.71	0.51	(+3' 1".5)	44 19 38.8	7.1	1.7	3
7.38457	Berlin	51 6.56	+0.28	+ 1.7	16 16.5	10.3	+ 6.1	7
7.43128	Paris	55 5.71	.24	−15.1	18 45.8	10.7	+ 4.9	8
7.43757	Berlin	15.09	.08	− 3.0	54.3	10.7	− 2.8	6.7
7.45831	Berlin	12.14	+0.00	3.2	49 16.5	10.8	2.1	Mer.
7.48780	Königsberg	56 20.90	−0.18	+ 8.3	51 59.9	10.5	2.1	7
8.40232	Vienna	5 16 10.16	+0.27	− 3.5	+45 19 43.0	+10.1	(−18.3)	9

Date	Place	α apparent	τ	O — C Δα cos δ	δ apparent	π	O — C Δδ	*
June		h m s	s	"	° ' "	"	"	
8.45080	Königsberg	5 17 13.63	+0.00	— 1.5	+45 20 1.9	+10.6	+ 3.8	Mer.
9.45663	Berlin	38 46.16	.11	3.2	28 11.6	10.5	+ 0.1	10
9.45941	Konigsberg	49.49	.01	— 2.3	9.4	10.4	— 1.8	10
9.46303	Konigsberg	54.79	.00	+ 1.3	11.3	10.5	+ 5.9	Mer.
9.48299	Berlin	39 19.50	.00	— 4.5	6.7	10.6	1.9	Mer.
9.49264	Hamburg	31.80	.00	3.8	10.1	10.6	7.0	Mer.
9.50567	London	48.53	.06	0.3	3.4	10.5	0.8	10
9.52149	London	40 7.70	+0.00	9.5	6.6	10.6	+ 9.9	Mer.
9.53859	Berlin	30.02	—0.20	— 3.8	27 54.3	10.6	— 0.8	10
9.56878	Cambr.,U.S.A.	41 8.27	+0.60	+ 6.5	42.4	8.3	— 4.7	11
10.39523	Geneva	58 7.64	.32	—10.7	. . . .	. .	. .	. .
10.39609	Geneva	. . . .	. .	. .	17 9.0	9.9	+13.2	. .
10.41266	Breslau	27.70	.27	(21.9)	16 22.3	13.6	— 9.8	. .
10.42102	Bonn	39.17	.34	— 3.3	24.1	9.7	— 1.5	12
10.43107	London	52.10	.37	+ 5.5	15 45.0	9.5	(+32.2)	. .
10.44515	Bonn	59 8.13	.25	— 3.0	52.0	10.0	— 2.2	13
10.44716	Konigsberg	11.55	.10	+ 2.9	52.1	10.3	+ 1.0	13
10.46146	Berlin	32.14	.11	— 1.8	30.2	10.3	+ 2.0	14
10.47464	Konigsberg	44.87	.00	+ 2.1	19.1	10.4	4.6	Mer.
10.49150	Berlin	6 0 4.69	+0.01	— 1.2	. . . .	. .	. .	14
10.49306	Königsberg	6.65	—0.07	1.8	14 50.1	10.3	0.1	13
10.49459	Berlin	8.29	.00	3.5	19.1	10.5	1.7	Mer.
10.50419	Hamburg	20.10	.00	— 0.1	42.1	10.5	+ 7.8	Mer.
10.50419	Hamburg	20.22	.00	+ 0.8	. . . .	. .	. .	Mer.
10.51958	Berlin	38.45	.09	— 1.3	12.7	10.3	— 0.9	14
10.51992	Brussels	38.27	.00	— 7.7	15.0	10.5	+ 2.1	Mer.
10.53167	Cambr., Eng.	53.73	.00	+ 6.9	13 55.0	10.5	— 1.9	Mer.
10.53167	Cambr., Eng.	53.49	—0.00	+ 4.1	55.3	10.5	1.6	Mer.
10.61682	Cambr.,U.S.A.	2 33.49	+0.48	— 7.6	11 49.7	9.1	— 5.6	11
11.42081	Bonn	6 18 3.12	+0.36	— 9.0	+41 46 35.7	+ 9.4	— 1.5	15
11.42722	Bonn	10.82	.34	6.6	30.2	9.4	+ 7.9	17
11.43127	Vienna	15.79	.38	— 2.1	8.3	9.7	— 5.3	15.20
11.43585	Konigsberg	21.32	.17	+ 0.5	2.2	9.9	0.2	17
11.45724	Berlin	45.10	.17	— 0.9	45 3.7	9.9	—10.4	15
11.48534	Königsberg	19 16.50	.00	3.1	44 13.2	10.2	+ 4.7	Mer.
11.49692	Altona	29.14	.06	5.3	. . . .	. .	. .	15,16,18,19,20
11.50014	Altona	. . . .	. .	. .	43 31.5	10.1	— 3.3	15,18,19,20
11.50526	Berlin	38.89	.00	1.4	22.6	10.3	+ 0.3	Mer.
11.51485	Hamburg	49.45	.00	2.8	5.0	10.3	5.2	Mer.
11.51485	Hamburg	49.20	.00	5.5	. . . .	. .	. .	Mer.
11.52988	Brussels	20 6.10	.00	— 0.0	42 25.0	10.3	+ 0.5	Mer.
11.54231	Cambr., Eng.	20.44	.00	+ 2.3	41 52.4	10.3	— 2.8	Mer.
11.54370	London	21.08	.00	— 7.3	42 2.8	10.3	+10.9	Mer.
11.59529	Cambr.,U.S.A.	21 18.61	.56	+ 2.9	39 53.2	7.1	1.7	11
12.37910	Vienna	35 15.08	.53	— 1.0	4 15.6	8.3	0.6	20
12.41142	Altona	48.36	.36	— 9.2	2 38.5	8.9	+ 1.8	22
12.41245	Breslau	36 7.34	.33	(+3' 14".8)	20.2	14.1	— 8.4	. .
12.41325	Bonn	35 50.21	.40	— 1.9	27.7	8.8	— 3.6	22
12.43623	Bonn	36 13.19	.33	— 8.8	29.6	9.3	+ 8.7	21
12.43921	Altona	17.28	.28	+ 1.7	. . . .	. .	. .	23
12.46086	Bonn	38.70	.25	— 6.2	0 6.3	9.5	+ 1.0	23
12.47317	London	50.78	.27	11.3	43 59 26.2	9.4	— 1.2	22
12.47529	Königsberg	54.04	.07	1.5	23.9	9.8	+ 3.2	22
12.48359	Altona	37 2.25	.14	3.5	58 53.2	9.8	— 2.6	23
12.51483	Berlin	34.20	.00	— 3.2	57 19.8	10.1	+ 0.7	Mer.
12.52441	Hamburg	44.49	.00	+ 2.7	56 54.0	10.1	+ 5.3	Mer.
12.52441	Hamburg	44.19	.00	— 0.5	. . . .	. .	. .	Mer.
12.52472	Gottingen	44.36	.00	2.0	53.6	10.1	6.3	Mer.
12.54045	Brussels	59.16	.00	14.5	11.0	10.1	0.4	Mer.
12.55182	Cambr., Eng.	6 38 11.64	+ 0.00	— 3.6	+43 55 29.9	+10.1	+ 6.7	Mer.

Date	Place	α apparent	π	O − C Δα cos δ	δ apparent	π	O − C Δδ	*
June		h m s	s	s	° ′ ″	″	″	
12.55323	London	6 38 13.12	+0.00	− 3.3	+43 55 15.8	+10.1	3.0	Mer.
12.59985	Washington	59.72	.62	1.5	53 52.9	7.1	− 3.9	24
13.44405	Altona	52 34.14	.28	6.8	5 2.8	9.1	− 1.0	27
13.44622	Berlin	36.79	.25	− 1.7	4 51.0	9.2	0.2	25, 26
13.52320	Berlin	53 48.01	.00	− 2.7	0 15.0	9.9	− 1.9	Mer.
13.53276	Hamburg	56.45	.00	− 0.5	42 59 11.5	9.8	+ 6.4	Mer.
13.53276	Hamburg	56.61	.00	− 1.2	. . . . .	. .	. .	Mer.
13.56014	Cambr., Eng.	54 21.57	.00	+ 0.7	57 54.1	9.9	− 3.6	Mer.
13.57778	Washington	35.65	.65	−13.2	56 56.1	6.1	+ 0.6	28
13.59111	Cambr., U.S.A.	16.84	.57	(24.5)	55 56.7	7.2	− 9.6	31, 32
14.44026	Königsberg	7 7 1.70	.21	− 2.7	1 27.9	9.1	− 1.7	30
15.42308	Bonn	7 19 50.27	+0.38	1.6	+10 53 44.7	+ 8.0	− 1.3	35
15.44508	Bonn	20 6.97	.33	− 1.2	52 5.2	8.1	1.4	33
15.45595	Altona	14.16	.26	8.3	51 35.4	8.6	+15.0	38
15.46342	Bonn	19.01	.28	−14.8	50 17.9	8.6	− 0.7	38
15.47555	Königsberg	29.38	.12	− 1.1	49 57.0	9.1	+ 0.6	35
15.54599	Hamburg	21 20.97	.00	3.1	44 56.5	9.2	0.1	Mer.
15.54599	Hamburg	20.77	.00	0.8	. . . . .	. .	. .	Mer.
15.59127	Washington	53.05	.60	3.1	11 48.1	5.9	1.7	36
15.59881	Washington	58.55	.59	− 3.7	15.2	6.1	1.2	37
16.41594	Berlin	31 18.30	.31	− 1.7	39 12 22.3	8.0	2.1	39
16.41857	Paris	19.22	.13	10.4	18.0	7.6	9.4	34
16.42582	Bonn	24.65	.37	2.8	11 37.4	7.8	0.7	11
16.44951	Paris	39.89	.37	1.8	10 5.5	8.1	+12.5	29
16.45871	Berlin	45.68	.24	+ 3.1	. . . . .	. .	. .	39, 40
16.47969	Königsberg	32 0.17	.12	+ 3.2	37 40.1	8.8	− 0.5	11
17.41125	Vienna	41 29.58	.40	2.7	38 29 38.1	7.5	− 0.9	46
17.45074	Königsberg	52.86	.19	+ 0.8	26 17.0	8.3	+ 4.0	43
17.45770	Altona	57.71	.25	+11.7	11.5	8.1	− 0.9	42
17.46629	Berlin	42 1.53	.21	− 1.5	25 21.0	8.2	10.9	43
17.58564	Washington	43 9.11	.56	3.0	16 56.3	5.3	+ 2.0	45
17.60280	Cambr., U.S.A.	18.25	.49	10.4	15 31.8	6.3	2.5	42
17.60715	Washington	21.13	.53	7.0	28.0	5.8	+11.1	47
18.42835	Paris	50 43.41	.39	+ 4.3	37 15 32.3	7.2	7.5	44
18.43608	London	46.00	.36	−12.7	11.0	7.1	(19.2)	. .
19.41495	Königsberg	58 39.45	.26	+ 0.6	36 4 30.1	7.5	+ 1.1	48
19.58836	Cambr., U.S.A.	59 57.56	.47	3.2	35 52 25.7	5.6	+13.3	49
20.42112	Hamburg	8 5 52.03	+0.30	+ 3.9	+34 53 1.1	+ 7.0	(−19.5)	. .
20.44176	Altona	6 1.31	.27	(14.6)	52 12.7	7.2	−11.8	50
21.41611	Königsberg	12 13.77	.21	+ 3.2	33 46 7.2	7.1	+ 3.4	50
23.40733	Paris	23 3.16	.35	−12.3	31 37 39.6	5.9	2.5	56
23.40733	Paris	3.53	.35	− 7.6	39.5	5.9	2.4	61
23.41981	Paris	8.10	.34	+ 3.8	36 4.3	6.1	11.0	64
23.59432	Washington	58.10	.13	− 5.1	26 9.4	4.6	+ 4.6	53
23.60570	Washington	24 1.85	.12	− 0.5	25 19.9	4.8	− 2.8	52
24.58988	Washington	28 31.73	.12	− 4.6	30 26 27.8	4.4	(+21.9)	55
24.59030	Washington	31.83	.12	4.7	7.5	4.4	+ 6.4	59
24.60182	Washington	35.02	.11	− 2.7	17.5	4.6	− 2.8	54
25.37600	Vienna	31 51.70	.38	(+17.7)	29 40 44.6	5.0	(+20.7)	62
25.59192	Cambr., U.S.A.	32 11.92	.37	− 1.7	28 28.1	4.8	(+19.9)	62
25.59842	Washington	32 14.23	.39	− 4.4	. . . . .	. .	. .	57
25.59842	Washington	13.45	.39	+ 3.2	. . . . .	. .	. .	58
25.59842	Washington	14.01	.39	− 3.0	27 57.4	4.6	+ 5.4	60
25.60717	Washington	16.18	.40	− 1.2	22.4	4.7	5.7	62
26.58085	Cambr., U.S.A.	36 27.89	.37	+ 5.1	28 33 26.8	4.6	+ 2.2	63, 64, 65, 66
July								
1.58192	Washington	8 51 22.29	+0.34	− 3.2	+21 29 36.3	+ 3.8	3.3	67

In forming the normal places the following weights were given the various series of observations:

Berlin,	2.0	Hamburg,	1.5
Königsberg,	2.0	Vienna,	1.5
Washington,	1.5		

All others are given the weight unity, except when enclosed in ( ). These latter are excluded.

Normal Places and Equations of Condition.

	$\Delta\alpha\cos\delta$	Wt.	$\Delta\delta$	Wt.
	$''$		$''$	
June 6.0	−0.97	13.5	+1.61	15.5
10.0	−1.55	43.0	+1.48	38.0
12.0	−2.71	52.5	+0.51	49.5
16.0	−1.08	36.5	+2.10	31.5
24.0	−1.91	22.0	+2.23	16.0
July 1.5	−3.2	1.5	−3.3	1.5

Before proceeding to the solution of the equations of condition an ephemeris was computed from the elliptic elements of d'Arrest. As will be recalled, these are founded on the hypothesis of possible identity with Tycho Brahe's comet of 1596. In this manner the coefficients were shown to be of sufficient accuracy to meet any demands made upon them. It was also proven that the apparent validity of these elliptic elements arises from the short arc used. During the period covered by the meridian observations, there is substantial agreement between the places computed from the parabolic and elliptic elements, but immediately afterwards a sharp divergence in the right-ascensions begins. The differences for the normal dates were

	$E-P$		$E-P$
	s		s
June 6.0	+0.11	June 16.0	−0.03
10.0	−0.03	24.0	−3.74
12.0	+0.05	July 1.5	−9.10

six-place logarithms being used.

The Schönfeld equations and their solution are

$\partial\kappa$	$\kappa\sqrt{2}\,\partial T$	∂q	$\partial\lambda$	$\partial\nu$	$\frac{\partial e}{2}$		
+9.6694	−0.2622	−9.6610	+8.7342	+6.8234	+7.8367	+9.9868	= 0
+9.6534	−0.2466	−9.3992	+9.0930	+8.7212	+8.9763	+0.1903	
+9.5572	−0.1777	−8.9819	+9.1441	+8.9524	+9.1093	+0.4330	
+9.1513	−9.9640	+9.3605	+9.1212	+9.1898	+9.2409	+0.0334	
−9.2250	−9.3758	+9.8171	+8.8280	+9.3451	+9.3823	+0.2810	
−9.4949	−7.9816	+9.9141	+8.0949	+9.3843	+9.4826	+0.5051	
+9.2348	−9.8460	+0.0308	−9.2052	−7.2944	+7.4383	−0.2068	
−9.1059	+9.1051	+0.0730	−9.1967	−8.8249	+8.0981	−0.1703	
−9.3891	+9.6361	+0.0887	−9.2189	−9.0272	+7.6595	−9.7076	
−9.5581	+9.8298	+0.0879	−9.2248	−9.2934	−7.9611	−0.3222	
−9.5666	+9.7403	+0.0120	−9.0068	−9.5239	+7.4702	−0.3483	
−9.5236	+9.5871	+9.9299	−8.3106	−9.6000	+8.4953	+0.5185	

the coefficients being given by their logarithms.

The normal equations are

$\partial\kappa$	$\kappa\sqrt{2}\,\partial T$	∂q	$\partial\lambda$	$\partial\nu$	$\frac{\partial e}{2}$		
+1.4842	−1.9865	−1.6934	+1.0282	+0.9349	+0.6172	+2.1105	= 0
−1.9865	+2.5648	+1.9171	−1.5261	−1.4376	−1.3919	−2.6675	
−1.6934	+1.9174	+2.3601	−1.4508	−1.2859	+0.6259	−2.3684	
+1.0282	−1.5261	−1.4508	+0.7927	+0.6890	+0.4109	+1.8359	
+0.9349	−1.4376	−1.2859	+0.6890	+0.8165	+0.4847	+1.7937	
+0.6172	−1.3919	+0.6259	+0.4109	+0.4847	+0.5788	+1.6327	

From these are formed the elimination-equations.

$$\partial\kappa - 0.5023\,\kappa\sqrt{2}\,\partial T - 0.2092\,\partial q + 9.5440\,\partial\lambda + 9.4507\,\partial\nu + 9.1330\,\frac{\partial e}{2} + 0.6263 = 0$$

$$\kappa\sqrt{2}\,\partial T - 0.1001\,\partial q + 7.7651\,\partial\lambda - 6.6665\,\partial\nu - 9.2894\,\frac{\partial e}{2} - 9.9696$$

$$\partial q - 9.2767\,\partial\lambda - 8.9879\,\partial\nu - 8.8024\,\frac{\partial e}{2} - 0.2275$$

$$\partial\lambda + 0.2517\,\partial\nu + 0.0425\,\frac{\partial e}{2} + 1.0964$$

$$\partial\nu + 9.2321\,\frac{\partial e}{2} + 0.4605$$

$$\frac{\partial e}{2} + 0.9517$$

From these the values of the unknown quantities are

$$\delta\kappa = +0''.25 \qquad \delta\lambda = -0''.18$$
$$\kappa\sqrt{2}\,\delta T = +0.10 \qquad \delta\nu = -1.36$$
$$\delta q = +0.96 \qquad \frac{\delta e}{2} = -8.95$$

The corrections to the parabolic elements are

$$\delta T = +0.00008 \qquad \delta i = -0''.51$$
$$\delta\omega = -0''.86 \qquad \delta q = +0.0000017$$
$$\delta\Omega = -1.69 \qquad \delta e = -0.0000868$$

and therefore the elliptic elements.

$$T = \text{June } 5.68959 \text{ Paris M.T.}$$
$$\left.\begin{aligned}\omega &= 75^\circ\ 48'\ 15''.4\\ \Omega &= 337\ 48\ 47.3\\ i &= 131\ \ 4\ 51.5\end{aligned}\right\}1845.0$$
$$\log q = 9.603233$$
$$\log e = 9.999962$$

A substitution in the equations of condition yields as residuals

$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$
$''$	$''$	$''$	$''$
+0.2	+0.8	+0.8	+0.1
+ .2	+0.3	+ .1	+ .7
−0.8	−0.9	−0.9	−1.5

and places computed from the elements are in agreement with the limits of error of a six-place table.

The most superficial examination of these figures shows that the value of the eccentricity is not reliable, as its probable error is in considerable excess of δe. In order to examine the limits within which it can vary, the other variables were expressed as functions of the last with the result that

$$\delta\kappa = +2''.42 \quad +0.2442\,\frac{\delta e}{2}$$
$$\kappa\sqrt{2}\,\delta T = +1.00 \quad +0.0695\,\frac{\delta e}{2}$$
$$\delta q = +0.02 \quad -0.1041\,\frac{\delta e}{2}$$
$$\delta\lambda = -7.33 \quad -0.7981\,\frac{\delta e}{2}$$
$$\delta\nu = -2.89 \quad -0.1706\,\frac{\delta e}{2}$$

Substituting in the weighted equations of condition the values of $\Sigma\,pvv$ are

δe	$\Sigma\,pvv$	δe	$\Sigma\,pvv$
+60	222	−20	160
+40	194	−40	163
+20	174	−60	180
0	163	−80	202

and for a period of 249 years $\Sigma\,pvv = 45000''\pm$, showing that the orbit is not with certainty to be distinguished from a parabola.

Adopting $\delta e = 0$ as the only hypothesis that can be considered as consistent with all the facts

$$\delta T = +0.00020 \qquad \delta i = -7''.81$$
$$\delta\omega = +1''.54 \qquad \delta\log q = 0$$
$$\delta\Omega = -1.34$$

and we may adopt as the definitive elements the system

$$T = \text{June } 5.68971 \pm 0.00038 \text{ Paris M.T.}$$
$$\left.\begin{aligned}\omega &= 75^\circ\ 48'\ 17''.5 \pm 9''.43\\ \Omega &= 337\ 48\ 47.7 \pm 5.90\\ i &= 131\ \ 4\ 44.2 \pm 7.46\end{aligned}\right\}1845.0$$
$$\log q = 9.603228 \pm 33 \text{ (unit being sixth place)}$$

Since these corrections are apparently the accidents of computation, we may conclude that the complete examination of the observations substantiates the parabolic orbit of D'ARREST, while it shows that his ellipse must be abandoned as being contrary to fact.

Syracuse University, 1903 December 9.

OBSERVED MAGNITUDES OF 62.1903 *ANDROMEDAE* (DM. +43°462).

By J. A. PARKHURST.

This star (11s following 787 *W. Andromedae*) is considered variable by Father HAGEN (*A.N.* 3917). I have the following observations, made when using it as a comparison star for the variable. The magnitude scale is nearly that of the DM., the neighboring stars +43°467 and +43°460 being $8^M.9$ and $9^M.0$ in the DM., and $9^M.06$ and and $8^M.91$ by my measures.

Visual				Photometric			
Date		Aper	Mag.	Date		Aper	Mag.
1899 Feb	6	6	9.5	1902 Feb	4	10	9.22
Oct.	18	6	9.5	Mar	4	12	9.05
	23	6	9.4		27	12	9.28
	28	6	9.3	Oct	29	12	9.10
Nov	4	6	9.2	1903 Nov	17	12	9.23
1900 Feb	16	6	9.6		18	12	9.28
					19	12	9.24
				Dec	6	6	9.33
					21	6	9.33

The fluctuations noted seem scarcely greater than would be expected in case of a reddish-yellow star.

Yerkes Observatory, 1904 Jan. 3.

OPPOSITION-TIME OF (15) *EUNOMIA*.

Prof. BAUSCHINGER writes to note that the error of the *Ephemeris* for 1905 in the opposition-time [illegible] spoken of in *A.J.* 551, had already been corrected by FÖRSTERLING last August, in *A.N.* 3892. [illegible]

OBSERVATIONS OF THE SATELLITES OF *URANUS*,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By W. W. DINWIDDIE.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1903 Washington M.T.	p	Wash. M.T.	s	1903 Washington M.T.	p	Wash. M.T.	s
			Titania.				
	h m s	°	h m s	"	h m s	°	h m s	"
Apr. 28 15 33 16	181.78	15 34 6	25.50	June 21 11 52 22	246.73	11 52 57	33.07
29 15 46 6	219.86*	15 46 51	30.99	30 10 58 28	256.29	10 58 24	32.70
May 28 14 4 2	337.32	14 2 53	32.33	July 19 9 43 20	320.37	9 43 3	31.83
June 3 13 0 13	228.58	13 2 18	31.80	21 9 13 59	43.64	9 14 13	32.45
20 12 7 0	204.72	12 7 27	32.78	24 9 2 49	166.74	9 3 19	31.59
			Titania–Oberon.				
Apr. 28 15 54 24	345.97	15 54 47	73.36	June 21 12 8 15	13.81	12 8 8	55.12
29 15 57 17	17.58	16 3 43	70.89	30 11 15 8	168.84	11 15 23	28.59
May 28 14 30 31	96.67	14 30 54	49.07	July 19 9 59 34	51.72	9 59 34	29.66
June 3 13 23 56	192.88	13 24 30	11.41	21 9 29 53	86.47	9 30 6	14.22
20 12 22 57	341.35	12 22 58	60.70	21 9 18 11	89.03	9 19 10	22.17
			Oberon.				
June 3 13 46 38	217.01	13 48 12	45.31	June 30 11 34 43	215.34	11 36 55	43.52
20 12 40 26	308.19	12 40 56	43.22	July 21 9 43 52	57.93	9 44 42	42.76
21 12 24 58	337.26	12 25 29	43.05	24 9 32 32	136.11	9 33 6	43.87

* Only five measures of position angle.

The above measures were all made by double distances, four comparisons on each side of coincidence; and eight position angles. In the measures of *Titania* and *Uranus*, and *Oberon* and *Uranus*, the disk of *Uranus* was bisected. A magnifying power of six hundred diameters was used. The measures have been corrected for refraction. *Oberon* and *Titania* have been almost, if not quite, as difficult at this opposition, as the Satellite of *Neptune* was at the last opposition of *Neptune*, and on account of the low declination of *Uranus*, there was much less chance for observing. The comparison of *Oberon* with the planet was always the last measure, which accounts for the fewer number of observations on *Oberon*, as the seeing would not continue good enough to observe the satellites for any length of time. *Oberon* was not quite so difficult to see as *Titania*. The seeing was good on Apr. 29, and June 30; fair on June 21, and July 19; and poor for all other measures.

Washington, D.C., 1903 *October* 12.

MISSING STAR DM. +19°2773,

By ZACCHEUS DANIEL.

On 1900 April 3, while looking for *T Bootis* with the 10-inch telescope at the Bucknell Observatory, I found that DM. +19°2773 [(1855) $14^h\,7^m\,53^s.1$, +19° 48'.8; $9^M.5$], was missing from the assigned place. The region was examined on 1900 April 3, June 23, July 21, 23, 27, Aug. 1, 22, 31, Oct. 17; 1901 July 10, Sept. 5; 1902 May 5, Aug. 25; 1903 May 15, 21, June 13, 21. No star brighter than 12^M was seen at or near the place. A very faint star, however, was seen not far off, probably a little south following.

At my request Prof. EDWARD C. PICKERING has had some of the Harvard photographs of this region examined. With his kind permission, I quote his report in full as follows:

"An examination has been made of plates taken on May 23, 1891; March 7, 1894; May 10, 1895; June 4, 1896; June 22, 1897, February 12, 1898; May 26, 1899; March 16, 1900, and March 31, 1903, all of which contain the region of +19°.2773. No star appears on them in, or near, the position given for this object. It is probable, therefore, that if it exists it was fainter than 11^M on all of these dates."

On each of the seventeen dates given above, I also looked for 5097 *T Bootis*, but no star was ever seen in its place.

The Observatory, Princeton, New Jersey, 1903 *July* 11.

VARIABILITY IN THE BRIGHTNESS OF *IRIS*,

Prof. E. C. PICKERING telegraphs, Jan. 26: "WENDELL finds *Iris* variable. Six hours. Range quarter of a magnitude."

CONTENTS

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed Jan*
Entered February 2, 1903 at Boston, Mass. as second-class matter under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 556.

VOL. XXIV. BOSTON, 1904 FEBRUARY 17. NO. 4

THE THEOREMS OF LAGRANGE AND POISSON ON THE INVARIABILITY OF THE GREATER AXES IN AN ORDINARY PLANETARY SYSTEM.

By G. W. HILL.

The remarks of this article follow a line very similar to that of the article in No. 527 of this *Journal*, and, to avoid the tedium of restating the explanation of the fundamental notation employed, I take the liberty of referring the reader to that place. However, we cannot here use the three theorems there stated, as it is proposed to take account of terms of three dimensions with respect to planetary masses. But we employ the notion there expounded of hypothetical planets, arranging the planets in such an order that the one for which $i = 1$ is that whose greater axis we especially wish to consider; the first actual planet and its hypothetical are then identical. We adopt the distinction there defined between terms periodic and terms secular.

Let there be ν planets in the system. The canonical elements $L_i, G_i, H_i, l_i, g_i, h_i$ and the function F have the significations of the mentioned article, and the differential equations to be satisfied are the same. Let P denote the general periodic argument, and S the general secular argument. Let K denote the general (that is, without any hint of individuality), coefficient usually multiplying the cosine or sine of any argument. We write above each K the number indicating its order of magnitude with respect to planetary masses; thus $\overset{2}{K}$ signifies a coefficient factored in the lowest dimension by squares and products of planetary masses. K, in general, is a function of all the linear elements. Σ will be used to indicate a sum of terms in number either finite or infinite.

The first term of

$$F \text{ or } m\sum\frac{\mu}{2a}$$

is of the dimension 1 with respect to planetary masses; we denote it by $\overset{1}{F}$. Developing F in a periodic series, we can write it as the sum of three terms, as follows:

$$F = \overset{1}{F} + \Sigma_s \overset{2}{K}\cos S + \Sigma_p \overset{1}{K}\cos P$$

Here it must be understood that S can receive the value 0, but P not. Also $\overset{2}{K}$ does not mean that the order of the coefficient is 2 in every case, but only that it is never less than 2.

Let us now assume a pure function of the variables $L_1, L_2, \ldots \ldots, L_\nu$, which we may write

$$f(L_1, L_2, \ldots \ldots, L_\nu)$$

or of the variables $a_1, a_2, \ldots \ldots, a_\nu$, to be written

$$f(a_1, a_2, \ldots \ldots, a_\nu).$$

f is to be finite, continuous, and of the zero order with respect to planetary masses.

We now propose to apply the principle of the DELAUNAY transformation to the establishment of the theorems with which we are engaged. DELAUNAY makes his transformations in the three polar coordinates of the moon; we have to make ours in the one function $f(L_1, L_2, \ldots \ldots, L_\nu)$. From the infinite number of periodic arguments P we select one θ, in which the positive or negative integers multiplying the angular variables are prime to each other, and take the part of F, which may be regarded as dependent on the sole argument θ. We call this F_θ, and write, similarly to DELAUNAY,

$$-F_\theta = -B + A_1\cos\theta + A_2\cos 2\theta + A_3\cos 3\theta + \ldots$$

$-B$ is the absolute term of F_θ, so that if K_0 denotes the coefficient of the second part of F when $S = 0$, we have

$$B = F + K_0$$

Let us now make the DELAUNAY transformation necessary to remove from F the terms factored by $\cos\theta$, $\cos 2\theta$, $\cos 3\theta$, ... The formulas for this purpose are, if L denotes any linear variable,

Replace $\overset{1}{L}$ by $L + \overset{2}{M_0} + M_1\cos\theta + M_2\cos 2\theta + M_3\cos 3\theta + \ldots$

and, if l denotes any angular variable,

Replace l by $l + N_1\sin\theta + N_2\sin 2\theta + N_3\sin 3\theta + \ldots$

where the M and N are functions of the new set of linear variables. By the addition of $\overset{3}{M}_0$ to the first formula we secure the advantage that the new linear variables are the conjugates of the new angular variables. M_0 is necessarily two orders higher with respect to planetary masses than the term which precedes it. The fact of the rest of the M and N being of the orders indicated above them is due to the circumstance that the motion of the argument θ is of the zero order.

We have now to inquire what changes, if any, are produced in the qualities of the coefficients of the three terms of F by this transformation. It is evident that $[F]$ is reduced by it to a function of the linear variables only. Hence, when the substitution is made in the term $\overset{1}{F}$, the new terms, which arise and are of the second order, precisely cancel the old terms $-A_1 \cos\theta - A_2 \cos 2\theta - \ldots\ldots$, and the remainder are of the form

$$\Sigma\,.\,\overset{3}{K}\cos S + \Sigma\,.\,\overset{3}{K}\cos P$$

Moreover, when the substitution is made in the second and third terms of F, the new terms arising are also of the same form. Hence the new form of F can be written

$$F = \overset{1}{F} + \Sigma\,.\,\overset{2}{K}\cos S + \Sigma\,.\,\overset{2}{K}\cos P$$

where $\overset{1}{F}$ has precisely the same expression as before. Hence the quality of F is unchanged by the execution of the transformation. We need only bear in mind that the coefficients of $\cos\theta$, $\cos 2\theta$, ..., are now not of the form $\overset{2}{K}$, but of the form $\overset{3}{K}$.

We may suppose a second Delaunay transformation to be made with the object of removing the terms of F having as periodic arguments the multiples of another θ of the same quality as before. The result will be that after the transformation F will have the same quality as before. Thus it is possible to conceive that an infinity of Delaunay transformations may be made in such a way that the third term of F wholly disappears, and F takes the form

$$F = \overset{1}{F} + \Sigma\,.\,\overset{2}{K}\cos S$$

Let us next inquire what happens when these transformations are made in the expression.

$$f(L_1, L_2, \ldots\ldots L_n)$$

It is quite plain that, after the first transformation depending on the periodic argument θ, we shall have the equation

$$f(L_1, L_2, \ldots\ldots L_n) = f(L_1, L_2, \ldots\ldots L_n) + \Sigma\,.\,\overset{2}{K}\cos S + \Sigma\,.\,\overset{1}{K}\cos P$$

where it must be understood that the L appearing under the functional sign f in the left member have their original signification, but, in the right member, their signification as modified by the transformation. That the periodic portion should have coefficients of the form $\overset{1}{K}$ is so obvious that it needs no formal demonstration; but that the secular portion has coefficients of the form $\overset{2}{K}$ results from the fact that its terms can arise only from the multiplication of two periodic terms $\overset{1}{K}\cos P$ and $\overset{1}{K}\cos P'$, where $P+P'$ or $P-P'$ is an S.

Now make the second Delaunay transformation: it is obvious that we have still the same equation

$$f(L_1, L_2, \ldots\ldots L_n) = f(L_1, L_2, \ldots\ldots L_n) + \Sigma\,.\,\overset{2}{K}\cos S + \Sigma\,.\,\overset{1}{K}\cos P$$

where it is necessary to note only that the L appearing in the second member have the signification as twice modified. Next, suppose that the infinite number of Delaunay transformations conceived to be made for the purpose of removing all periodic terms from the periodic development of F, have also been made here. The result will still be the equation

$$f(L_1, L_2, \ldots\ldots L_n) = f(L_1, L_2, \ldots\ldots L_n) + \Sigma\,.\,\overset{2}{K}\cos S + \Sigma\,.\,\overset{1}{K}\cos P$$

where the L in the second member have the signification as last modified.

Consider now the variability of this last group of the L. Since the angular variables $l_1, l_2, \ldots\ldots l_n$ conjugate to them have altogether disappeared from the last modified expression for F, we have generally

$$\frac{dL_i}{dt} = \frac{\partial F}{\partial l_i} = 0$$

Consequently the last group of the modified L forms a series of constants. Thus, if we please, we may write the foregoing equation

$$f(L_1, L_2, \ldots\ldots L_n) = \text{a constant} + \Sigma.\overset{2}{K}\cos S + \Sigma.\overset{1}{K}\cos P$$

But, for our purpose, it is necessary that the right member of this should appear as an explicit function of the time. Hence a new class of Delaunay transformations must be made, having for object the removal from of all the terms (the absolute excepted) constituting the second portion $\Sigma\,.\,\overset{2}{K}\cos S$. These transformations would then turn upon the secular arguments of the group S, and, after the requisite infinity of them had been performed, F would be reduced to the absolute term which is a function of the linear variables: thus

$$F = \overset{1}{F} + \overset{2}{K}_0$$

All the linear variables are now constant, for, in addition to what has been stated in reference to the L, we have generally

$$\frac{dG_i}{dt} = \frac{\partial F}{\partial g_i} = 0 \quad , \quad \frac{dH_i}{dt} = \frac{\partial F}{\partial h_i} = 0$$

Suppose that all these transformations are made in the term $\Sigma.\overset{1}{K}\cos P$ of $f(L_1, L_2, \ldots\ldots, L_n)$. It is evident,

from the general principles underlying the representation of the integrals of our differential equations by LINDSTEDT's series, that this term will undergo a change expressed by the apparently tautological equation

$$\Sigma . \overset{1}{K} \cos P = \Sigma . \overset{1}{K} \cos P$$

in the second member of which, however, $\overset{1}{K}$ is absolutely constant, and P is a linear function of the time expressed by $\theta_0 (t + c)$, where θ_0 and c are constants, the first being of the zero order with respect to planetary masses. Thus, throughout this second class of transformations, no terms cross over from the portion $\Sigma . \overset{1}{K} \cos P$ to the portion $\Sigma . K \cos S$. Then, if we are concerned only about the secular inequalities of $f(L_1, L_2, \ldots \ldots L_n)$, we can omit its last term and write

$$f(L_1, L_2, \ldots \ldots L_n) = \text{a constant} + \Sigma . \overset{2}{K} \cos S$$

in which it is understood that $\Sigma . K \cos S$ has not undergone any of the latter class of transformations.

At this stage we give up the process of approximating to the integrals of our differential equations through DELAUNAY transformations, and adopt the process of elaborating them in positive integral powers of the time, making use of the generalized theorem of MACLAURIN. The equations in terms of the variables last used in the earlier class of transformations have expressions of which the type is

$$\frac{dG_i}{dt} = \frac{\partial F}{\partial g_i}, \qquad \frac{dH}{dt} = \frac{\partial F}{\partial h_i}$$

$$\frac{dg_i}{dt} = -\frac{\partial F}{\partial G_i}, \qquad \frac{dh}{dt} = \frac{\partial F}{\partial H}$$

where F has the expression

$$F = \overset{1}{F} + \Sigma . \overset{2}{K} \cos S$$

The integrals in series of powers of the time are

$$G_i = K + \overset{2}{K}t + Kt^2 + \ldots \qquad H = K + \overset{2}{K}t + Kt^2 + \ldots$$

$$g_i = K + \overset{1}{K}t + \overset{2}{K}t^2 + \ldots \qquad h = K + \overset{1}{K}t + \overset{2}{K}t^2 + \ldots$$

the K being all constant and of the order with respect to planetary masses indicated above them. If we substitute these values in the portion $\Sigma . K \cos S$ of the [illegible] $f(L_1, L_2, \ldots \ldots L_n)$ there arises a constant term, equivalent to the value of $\Sigma . \overset{2}{K} \cos S$ at the origin of time, which coalesces with the preceding constant. This is followed by terms involving t, t^2, t^3, etc. Then it is almost immediately apparent that

$$f(L_1, L_2, \ldots \ldots L_n) = K + \overset{3}{K}t + \overset{4}{K}t^2 + \overset{5}{K}t^3 + \ldots$$

It is evident that this relation may also be written

$$f(a_1, a_2, \ldots \ldots a_n) = K + \overset{3}{K}t + \overset{4}{K}t^2 + \overset{5}{K}t^3 + \ldots$$

We may therefore state the theorem of POISSON in a more general form than has been customary, as follows:

The secular variation of any finite and continuous function of the zero order with respect to planetary masses, of the instantaneous greater axes of the orbits described by the hypothetical planets of a planetary system, when developed in powers of the time, is, at least, of three dimensions in reference to the same masses.

The qualification "at least" is necessary, for the function f may be such that all terms of the third order in the coefficient of t identically vanish, and then K must be written $\overset{4}{K}$. In the second place, since the function f probably has an infinite number of maxima and minima, if it so happen that the origin whence t is counted coincides with one of these, the coefficient of t must vanish, not identically, but on account of the special values received by the parameters at that epoch.

It will be perceived that the method here followed for the demonstration of the theorem has marked advantages over those employed heretofore. The truth of the theorem is now so obvious that a formal proof seems scarcely necessary. The DELAUNAY transformation is to be credited for this advance. An objection may be raised against the method that it is valid only in the case where F never *potentially* becomes infinite.* But, in the opposite case, there seems no valid distinction between secular and periodic inequalities, and LINDSTEDT's series ought to be rejected as being a possible mode of expressing the coordinates.

* See *Transactions of the American Mathematical Society*, [illegible] p. 270.

NOTES ON VARIABLE STARS. — No. 39.

BY HENRY M. PARKHURST.

4315 *R Comae Ber.* Reducing the weight of the maxima as given in *A.J.* 487 and 513, in consequence of the especially unfavorable conditions of the observations, the following elements well represent the original maximum in 1831 and my maxima obtained from 1894 to 1903,

$$2389865.5 + 361.6 \text{ E} + 15 (10 \text{ E} + 80)$$

The interference of the twilight has become less intrusive, yet at the observation of Aug. 1, *R* could not be seen when it sank behind the new building.

5465 *RT* [illegible] New [illegible] 63 days [illegible] to the maximum [illegible] prevented by the cloudy sky from [illegible] and the additional interval of 25 days [illegible] only the observations made by Mr. PERRY, I [illegible] the following elements, which satisfy [illegible] of 7 different years

$$2411.3796 + 252 \text{ E}$$

Results of Observations.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	W.	Mag.	Factors		d	Remarks
				1903								
2735	*U Canis min.*	Min.	6195	Mar. 21	21	–	E	–				Wave near min. *A.J.* 470.
2780	*T Geminorum*	Max.	6199	Mar. 25	69	–49	3	8.8				Little change near max.
3493	*R Leonis*	Min.	6199	Mar. 25	171	52	9	10.2				
3567	*V Leonis*	Max.	6177	Mar. 3	28	–24	5	9.27			–	
4315	*R Comae Bere.*	Max.	6317.2	July 21	73	–0.9	9	8.90	4.81	6.34	8	Elements above.
4492	*Y Virginis*	Max.	6319	July 23	34	–	E	10.0				
4573	*RU Virginis*	Max.	6481	Jan. 1	6	–	E			–		
4596	*U Virginis*	Max.	6233	Apr. 28	65		E	–				
4665	*RT Virginis*	Max.	6268 :	June 2		–	1					379d approx. period.
5249	*V Librae*	Max.	6246	May 14	30	–	E					Elements. *A.J.* 444.
5338	*U Bootis*	Max.	6267	June 1	48		E					Period not confirmed.
5405	*RT Librae*	Max.	6320	July 24	10	+ 4	9	9.02	2.50	1.04	33	Revised elements above.
5494	*S Librae*	Max.	6273	June 7	55	+ 5	6	–	–	–	–	
5501	*S Serpentis*	Max.	6343	Aug. 16	75	+68	3	8.1	–	–	–	Corr. regularly increasing.
5511	*RS Librae*	Max.	6281	June 15	27	+ 8	2		–	–	–	Max. following first obs.
5677	*R Serpentis*	Max.	6352	Aug. 25	78	+ 7	5		–	–	–	
5688	*R Librae*	Max.	6273	June 7	64	–10	3		–	–	–	
5704	*RR Librae*	Max.	6326	July 30	24	–32	6		–	–	–	
5796a	*RU Herculis*	Max.	6347	Aug. 20		–	3		–	–	–	4362+495 E ?
5887	*V Ophiuchi*	Max.	6257	May 22	85	–	E	–	–	–	–	Observations indecisive.
5931	*S Ophiuchi*	Max.	6345	Aug. 18	72	+17	7			–	–	Prob. minor max. earlier.
6044	*S Herculis*	Min.	6304	July 8	56	– 1	1	12.7	1.07	1.07	73	From two observations only.

Individual Observations.

Including Observations by Arthur C. Perry

2735 *U Canis min.*

(Cont. from 513 Comp Stars 470

Julian	Calendar	Mag.
	1903	
6138	Jan. 23	11.2
6169	Feb. 23	11.2
6175	Mar. 1	9.3
6178	4	10.0
6192	18	11.2]

2780 *T Geminorum.*

Continued from 403

	1903	
6138	Jan. 23	11.9
6169	Feb. 23	8.6
6172	26	9.3
6175	Mar. 1	8.8
6178	4	9.1
6192	18	8.8
6203	29	9.3
6217	Apr. 12	8.8
6229	24	8.7
6237	May 2	9.8]

3493 *R Leonis.*

Continued from 487

	1903	
6191	Mar. 17	9.28_2
6192	18	10.12_2
6194	20	10.71_2
6200	26	9.76_2
6201	27	10.17_2
6205	31	10.72_2
6263	May 28	9.5
6266	31	9.5

3567 *V Leonis.*

(Cont. from 441. Comp Stars 441

Julian	Calendar	Mag.
	1903	
6171.6	Feb. 25	9.7
6172.6	„ 26	8.94_2
6177.6	Mar. 3	9.52_2
6180.6	6	9.37_2
6189.6	15	9.77
6201.6	27	9.57_2
6205.6	31	10.55_2

4315 *R Comae Ber.*

(Cont. from 513. Comp Stars 415)

	1903	
6287.6	June 21	11.35
6292.6	26	11.24
6293.6	27	10.11 :
6297.6	July 1	10.25_1
6298.6	2	9.69_{10}
6300.6	4	10.17_{10}
6302.6	6	10.04_2
6303.6	7	9.95_{11}
6304.6	8	9.85_{10}
6305.6	9	9.79_{10}
6306.6	10	9.62_{10}
6311.6	15	9.19_{10}
6312.6	16	8.92_{10}
6313.6	17	9.03_{11}
6315.6	19	8.84_{10}
6317.6	21	8.88_{10}
6319.6	23	8.89_{10}
6320.6	24	8.97_{10}
6321.6	25	8.98_{10}
6323.6	27	8.92_{10}
6328.6	Aug. 1	9.20]

4492 *Y Virginis.*

Cont. from 470 Comp. Stars 415

Julian	Calendar	Mag.
	1903	
6268.6	June 2	11.9]
6311.6	July 15	10.0

4573 *RU Virginis.*

Continued from 470.

	1903	
6263.6	May 28	11.0
6266.6	31	11.2
6311.6	July 15	10.9

4596 *U Virginis.*

Continued from 487.

	1903	
6263.6	May 28	9.2
6266.6	31	9.2
6268.6	June 2	9.4
6311.6	July 15	11]

4665 *RT Virginis.*

Continued from 513

	1903	
6263.6	May 28	8.85
6266.6	31	8.8
6268.6	June 2	8.85
6297.6	July 1	8.7
6311.6	15	8.8

5249 *V Librae.*

(Cont. from 487 Comp Stars 388

	1903	
6258	May 23	11.2
6260	25	10.8
6268.6	June 2	11.2
6311.6	July 15	12]

5338 *U Bootis.*

Continued from 487.

Julian	Calendar	Mag.
	1901	
5510.5	May 5	10.87_2
	1903	
6263.6	May 28	11.2
6266.6	31	11.3
6268.6	June 2	11.2

5405 *RT Librae.*

Continued from 513.

	1903	
6256	May 21 to	
6311	July 15 :] 6 dates	
6311.6	July 15	8.8P
6315.6	19	9.0P
6320.6	24	8.8P
6336.6	Aug. 9	9.2 :
6337.6	10	9.1
6341.5	14	9.11
6342.5	15	9.11

5494 *S Librae.*

(Continued from 513.

	1903	
6263.6	May 28	9.1
6266.6	31	9.2
6268.6	June 2	9.0
6269.6	3	8.94_2
6311.6	July 15	9.9

5501 *S Serpentis.*

Cont. from 513. Comp Stars 388

	1901	
5553.6	June 17	11.0P
	1903	
6263.6	May 28	12

5501 *S Serpentis.*—Cont.

Julian	Calendar	Mag.
	1903	
6342.6	Aug. 15	8.1
6360.5	Sept. 2	8.1
6364.6	6	8.08_2
6374.5	16	9.06

5511 *RS Librae.*

Cont from 490 (Comp Stars 388)

	1901	
5553.6	June 17	9.2P
	1903	
6266.6	May 31	9.7
6311.6	July 15	10.9
6311.6	15	10.7P
6315.6	19	10.7P
6320.6	24	10.7P

5677 *R Serpentis.*

(Cont from 499. Comp Stars 476)

	1901	
5554.6	June 18	8.9P
	1903	
6263.6	May 28	12
6342.6	Aug. 15	7.9
6360.5	Sept. 2	8.2
6364.6	6	8.59_2

5688 *R Librae.*

Cont. from 476. Comp. Stars 476)

	1901	
5511.6	May 6	9.7]
5512.6	7	10.9]
5521.6	16	11]
	1903	
6268.6	June 2	9.8
6287.6	21	10.2
6293.6	27	10.6

5704 RR Librae. (Continued from 513.)

Julian	Calendar	Mag.
	1903	
6311.6	July 15	9.5 r
6315.6	19	9.1 r
6320.6	24	8.7 r
6344.5	Aug. 17	9.3
6348.5	21	9.10_2
6350.5	23	9.07_2

5796a RU Herculis. (Continued from 513.)

Julian	Calendar	Mag.
	1901	
5551.6	June 18	11.1 p
	1903	
6345.6	Aug. 18	9.7
6347.5	20	8.94_2
6348.5	21	9.85_2
6350.5	23	9.57_2
6360.5	Sept. 2	10.2

5887 V Ophiuchi. (Cont. from 513. Comp. Stars 456.)

Julian	Calendar	Mag.
	1901	
5553.6	June 17	8.2 p
	1903	
6269.6	June 3	9.67_2
6313.6	July 17	10.3 p
6315.6	19	10.1 p
6320.6	24	9.6 p
6334.6	Aug. 7	10.10
6344.6	17	9.62_2

5887 V Ophiuchi. Cont.

Julian	Calendar	Mag.
	1903	
6347.6	Aug. 20	10.03
6362.5	Sept. 4	10.0

5931 S Ophiuchi. (Continued from 456.)

Julian	Calendar	Mag.
	1903	
6313.6	July 17	9.9 p
6315.6	19	10.0 p
6320.6	24	9.9 p
6341.6	Aug. 14	10.26

5931 S Ophiuchi. Cont.

Julian	Calendar	Mag.
	1903	
6342.6	Aug. 15	9.89
6344.6	17	9.52
6347.6	20	9.95_2
6369.5	Sept. 11	10.8

6041 S Herculis. (Cont. from 513. Comp. Stars 388.)

Julian	Calendar	Mag.
	1903	
6269.6	June 3	10.8
6342.6	Aug. 15	10.6

COMPARISON STARS, 1893–1903.

1582 S Tauri.

Star	DM.	Mag.	n
1 T	+10°586	8.18	7
W	+10°589	8.98	13
Y	+10 584	9.13	16
1 Y	+ 9°581	10.28	7
Z	+ 9°588	9.82	19
1 Z	+ 9°589	9.88	27
2 Z	+ 9°591	10.05	17
d	6n1p V	10.59	13
e	18p V	11.33	7
f	6s3f 2Z	11.36	6

1941 S Orionis.

Star	DM.	Mag.	n
F	−4 1141	7.65	21
N	−5°1274	8.16	3
1 N	−5°1277	8.96	1
Q	−4°1155	8.72	17
U	−5°1273	9.82	22
1 U	−4°1117	9.95	28
W	−4°1144	11.06	1
X	−4°1148	10.20	21
j	6f W	12.26	3
k	5s2p X	12.51	2

2100 U Orionis.

Star	DM.	Mag.	n
O^2	+20°1162	4.80	1
U	+19 1126	6.02	10
U^2	+19 1110	6.03	13
E	+20°1156	6.72	37
F	+19°1113	8.21	10
K	+20°1171	8.25	28
N	+19°1131	8.72	3
P	+20 1168	8.73	37
Q	+20°1178	9.58	9
W	+20 1172	10.26	4

2401 R Geminorum.

Star	DM.	Mag.	n
H	+30°1332	7.41	11
K	+29 1312	7.59	10
P	+30 1306	8.65	14
S	+30°1320	9.50	9
T	+30°1316	9.20	14
W	+30 1321	9.76	2
Y	+30°1309	9.98	8
Z	+30 1330	10.57	3
a	5s T	9.96	14
d	8p a	11.20	5

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY.

By C. W. FREDERICK.

(Communicated by Rear Admiral C. M. CHESTER, U. S. N., Superintendent.)

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ (α)	log pΔ (δ)	Red. to App. Pl. (α)	Red. to App. Pl. (δ)
				(334) *Chicago.*						
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Jan. 22 13 20 17	1	t19, 4	+2 39.06	+0 51.1	7 7 32.13	+19 58 17.7	9.441	0.511	+2.05	12.2
25 8 51 51	2	d 8, 5	0 25.00	+1 28.5	7 5 13.01	+20 1 2.4	n9.366	0.491	+2.05	12.3
30 11 53 30	3	d10, 10	0 1.28	+4 3.9	7 2 37.21	+20 11 6.5	9.261	0.471	+2.05	12.4
				(102) *Miriam.*						
Feb. 21 9 26 9	4	t28, 6	+0 32.00	0 15.0	9 45 9.89	+20 2 56.0	9.434	0.507	+2.09	15.7
22 9 18 10	5	d10, 10	0 12.21	0 47.9	9 44 19.92	+20 13 7.6	n9.346	0.484	+2.11	15.6
				(381) *Myrrha.*						
Oct. 20 11 6 55	6	d 8, 8	0 29.11	+2 48.5	2 51 35.49	1 54 17.2	n9.328	0.756	+3.69	[illegible]
21 11 43 11	7	t29, 6	+2 9.34	+2 39.4	2 50 52.81	1 59 22.5	n9.438	0.758	+3.71	[illegible]
27 10 17 37	8	t30, 6	+1 12.74	2 32.6	2 46 56.81	2 21 1.1	n9.384	0.759	+3.77	[illegible]
				(420) *Bertholda.*						
Oct. 21 9 51 17	9	t35, 7	1 32.38	+1 11.6	1 48 1.32	+17 6 4.0	9.364	0.543	+4.15	+17.5
22 9 33 30	10	t30, 6	1 4.88	+2 47.9	1 47 18.95	+17 0 57.2	n9.417	0.551	+4.16	+17.6
25 9 21 7	11	t30, 6	1 16.91	5 36.2	1 45 11.05	+16 45 6.7	9.403	0.555	+4.17	+18.0
26 8 11 16	12	t30, 6	+1 20.10	+7 18.6	1 44 29.75	+16 39 53.0	9.490	0.579	+4.16	+18.1
				(371) *Bohemia.*						
Oct. 22 10 16 55	13	t30, 6	2 3.01	1 6.2	2 40 58.55	+26 11 54.1	n9.351	0.355	+4.49	+12.5
26 9 21 10	14	d 8, 8	+0 31.48	9 15.6	2 37 27.91	+25 56 18.5	n9.548	0.446	+4.53	+12.6
28 9 1 10	15	t30, 6	+2 36.57	1 50.6	2 35 38.92	+25 47 31.3	n9.572	0.467	+4.54	+12.8

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(409) Aspasia.					
	d h m s			m s	′ ″	h m s	° ′ ″			s	″
Oct. 29 9 23 54	16	t29 , 6	+3 52.05	8 14.0	2 15 38.33	+22 25 13.8	n9.530	0.502	+4.14	+12.6
Nov. 2 9 7 18	17	t29 , 6	−0 17.95	+ 6 26.0	2 11 59.56	+21 55 38.7	n9.522	0.507	+4.16	+12.8
3 8 28 48	17	t30 , 6	−1 11.73	1 2.4	2 41 5.78	+21 48 10.3	n9.582	0.544	+4.16	+12.8
6 9 12 50	18	t30 , 6	−1 55.51	+ 6 11.2	2 38 18.55	+21 24 25.4	n9.463	0.493	+4.17	+13.2
6 9 51 59	19	t30 , 6	−1 1.15	+11 44.3	2 38 16.99	+21 24 12.6	n9.340	0.159	+4.47	+13.4
					(199) Byblis.					
Nov. 6 11 23 18	20	t27 , 6	−1 7.82	− 5 23.7	2 9 12.32	5 59 31.3	8.161	0.791	+3.77	+14.6
8 9 1 42	21	t30 , 6	+0 15.41	− 9 55.7	2 8 16.12	− 6 0 8.8	n9.356	0.785	+3.77	+14.7
8 9 29 38	22	t30 , 6	+1 43.70	+ 5 31.3	2 8 15.46	− 6 0 8.1	n9.246	0.788	+3.77	+11.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	7 1 51.04	+19 57 38.5	Berlin B, A.G. 2816	12	1 13 5.49	+16 32 16.0	Berlin A, A.G. 513
2	7 6 5.96	+20 2 46.2	Berlin B, A.G. 2824	13	2 42 57.07	+26 12 18.8	Camb. (Eng.), A.G. 1458
3	7 2 39.47	+20 10 14.7	Berlin B, A.G. 2798	14	2 36 18.93	+26 5 21.5	Camb. (Eng.), A.G. 1418
4	9 44 35.80	+20 3 26.7	Berlin B, A.G. 3851	15	2 32 57.81	+25 49 11.6	Camb. (Eng.), A.G. 1393
5	9 41 30.02	+20 11 11.1	Berlin B, A.G. 3853	16	2 41 11.84	+22 33 15.2	Berlin B, A.G. 830
6	2 52 1.24	− 1 57 49.0	Nicolajew, A.G. 619	17	2 42 13.05	+21 48 59.9	Berlin B, A.G. 835
7	2 48 39.79	− 2 2 15.3	Strassburg, A.G. Zones	18	2 40 9.59	+21 17 31.0	Berlin B, A.G. 826
8	2 45 20.30	− 2 21 41.9	Strassburg, A.G. Zones	19	2 39 16.97	+21 12 14.9	Berlin B, A.G. 820
9	1 49 29.55	+17 1 31.9	Berlin, A, A.G. 546	20	2 10 16.37	− 5 54 25.2	Wien, A.G. Zone 295
10	1 18 19.67	+16 57 51.7	Berlin, A, A.G. 541	21	2 7 26.94	− 5 50 27.8	Wien, A.G. Zone 295
11	1 46 23.82	+16 50 24.9	Berlin, A, A.G. 532	22	2 6 27.99	− 6 5 54.2	Wien, A.G. Zones 92, 209

The star places from the Strassburg Zones were furnished through the courtesy of the Director of the Observatory at that place. The second observation of *Chloe* is by W. W. DINWIDDIE. Planets 420, 371, 409, and 199, were found photographically by Mr. G. H. PETERS, and 402, by Mr. W. W. DINWIDDIE.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

BY W. WALTER DINWIDDIE. [Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(236) Honoria.					
	d h m s			m s	′ ″	h m s	° ′ ″			s	″
May 1 11 55 58	1	t15 , 8	+2 9.31	− 1 32.4	14 19 6.31	− 7 13 30.6	8.6910	0.8037	+2.64	− 8.1
5 10 57 51	1	t 9 , 6	+1 24.18	+ 1 33.9	14 18 21.18	− 7 37 24.3	n8.7669	0.8028	+2.64	− 8.1
11 13 8 14	2	t18 , 10	+0 51.89	− 1 21.8	14 13 45.91	− 7 6 41.9	9.3946	0.7904	+2.65	− 8.0
					(335) Roberta.					
May 5 12 8 30	3	t15 , 8	−1 17.45	− 0 14.2	14 13 39.85	6 30 14.1	8.5175	0.7950	+2.65	− 6.3
9 11 10 19	1	t16 , 7	+1 9.98	− 7 39.6	14 40 4.97	− 6 10 17.8	8.1763	0.7925	+2.64	− 5.7
10 10 33 16	5	10 , 10	−0 5.36	− 7 3.8	14 39 13.99	− 6 5 51.6	n9.0381	0.7906	+2.66	− 6.3
					(333) Badenia.					
Sept. 13 11 21 16	6	10 , 10	−0 26.29	− 9 55.0	22 31 9.26	−10 56 59.4	8.6301	0.8238	+3.53	+24.4
14 10 51 14	7	t30 , 6	+1 49.38	+ 1 33.1	22 30 26.62	−10 59 16.3	n8.1796	0.8262	+3.53	+24.4
15 10 18 17	7	t29 , 6	+1 7.34	− 0 41.2	22 29 14.58	−11 1 30.6	n8.8604	0.8256	+3.53	+24.4
18 10 11 31	8	t29 , 6	+2 5.85	+ 2 15.3	22 27 40.28	−11 7 18.2	n8.7648	0.8265	+3.53	+24.3
10 54 38	9	t28 , 6	+1 37.57	− 2 6.5	22 27 39.09	−11 7 51.2	8.4702	0.8270	+3.52	+24.3
20 9 12 0	8	t30 , 6	+0 18.86	− 1 25.7	22 26 23.28	−11 11 29.3	n9.1983	0.8225	+3.52	+24.2
					(362) Harnia.					
Sept. 14 11 40 12	10	t30 , 6	−0 36.26	− 1 43.8	0 51 42.17	− 0 12 18.8	n9.2843	0.7420	+3.47	+20.8
15 11 2 16	10	t30 , 6	−1 22.11	− 4 10.2	0 50 56.33	− 0 11 15.1	n9.3987	0.7427	+3.48	+20.9
20 11 13 25	11	t30 , 6	+0 57.40	+ 8 2.3	0 46 43.21	− 0 27 52.6	n9.1242	0.7444	+3.55	+21.4
21 12 5 53	11	10 , 10	+0 3.39	+ 5 17.8	0 45 49.21	− 0 30 37.0	n8.9036	0.7447	+3.56	+21.5
22 13 44 2	12	10 , 10	−0 9.02	+11 15.9	0 44 51.43	− 0 33 31.2	9.0935	0.7452	+3.57	+21.6

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(393) Lampetia.					
d h m s			m s	′ ″	h m s	° ′ ″				
Sept. 20 10 14 50	13	t30 . 6	−0 51.79	0 51.5	0 51 35.97	+18 1 59.7	n9.1162	0.5363	+3.87	+19.1
21 9 43 26	11	t30 . 6	+2 29.38	+ 1 47.1	0 50 56.75	+17 51 8.7	n9.5463	0.5794	+3.87	+19.5
22 10 12 8	15	10 . 10	+0 5.80	1 1.4	0 50 14.04	+17 12 20.7	n9.4774	0.5576	+3.88	+19.5
23 10 8 14	16	t30 . 6	+1 37.15	− 1 35.0	0 49 31.69	+17 30 35.8	n9.4739	0.5596	+3.89	+19.8
25 10 12 18	17	t30 . 6	+0 39.92	− 2 36.1	0 48 1.27	+17 5 58.3	n9.3445	0.5400	+3.90	+20.2
					(332) Siri.					
Sept. 20 12 35 6	18	t30 . 6	+1 24.76	− 1 58.6	0 22 31.31	− 0 7 21.7	8.2161	0.7444	+3.58	+22.6
21 11 13 23	19	t30 . 6	+3 19.85	− 6 11.9	0 21 17.33	− 0 11 33.3	n9.1319	0.7449	+3.59	+22.7
22 11 10 17	20	t30 . 6	+1 11.54	+ 3 28.7	0 20 57.64	− 0 15 18.8	n9.1218	0.7426	+3.60	+22.7
24 9 57 4	21	10 . 10	+0 25.02	+ 0 21.6	0 19 19.73	− 0 21 13.5	n9.3938	0.7434	+3.61	+22.8
12 10 26	22	t30 . 6	+1 16.12	3 25.0	0 19 14.91	− 0 24 36.1	7.7148	0.7411	+3.61	+22.8
					(163) Erigone.					
Sept. 22 12 35 27	23	t30 . 6	−3 19.98	+ 6 2.6	0 18 38.65	− 1 21 31.4	8.6092	0.7536	+3.58	+22.6
24 10 35 53	24	10 . 10	−0 21.93	− 9 5.2	0 16 57.90	− 1 37 41.0	n9.2127	0.7540	+3.60	+22.8
11 13 24	25	t30 . 6	−2 27.49	+ 8 28.5	0 16 56.65	− 1 37 52.7	n9.0147	0.7548	+3.59	+22.8
25 11 31 40	26	t35 . 6	+1 24.34	+ 1 13.6	0 16 2.62	− 1 46 21.9	n8.7630	0.7563	+3.60	+23.0
11 59 22	25	t30 . 6	−3 22.60	− 0 13.2	0 16 1.55	− 1 46 34.4	n7.1912	0.7565	+3.60	+22.8
					(73) Klytia.					
Sept. 25 13 4 10	27	t33 . 6	+2 15.95	+ 8 18.0	0 59 58.55	+ 4 2 17.6	8.5948	0.6993	+3.62	+20.6
27 10 9 27	27	t30 . 6	+0 28.80	+ 0 18.3	0 58 11.45	+ 3 54 18.0	n9.4319	0.7084	+3.67	+20.7
28 11 32 26	28	t30 . 6	−1 1.31	+ 6 1.7	0 57 10.13	+ 3 49 12.2	n9.0571	0.7027	+3.68	+20.8
29 13 58 41	29	t30 . 6	+2 19.59	+ 3 40.0	0 56 5.06	+ 3 44 55.1	9.2616	0.7062	+3.69	+21.1
14 21 57	30	t30 . 6	+2 11.96	− 1 52.9	0 56 4.01	+ 3 44 49.3	9.3611	0.7075	+3.69	+21.1
					(154) Bertha.					
Sept. 27 11 0 45	31	10 . 10	+0 19.00	+ 2 5.0	1 2 12.66	− 9 10 26.2	n9.2850	0.8077	+3.49	+21.2
11 28 14	32	t30 . 6	+1 2.90	− 0 6.9	1 2 11.70	− 9 10 28.4	n9.1457	0.8108	+3.49	+21.3
Oct. 13 11 14 45	33	t30 . 6	+1 38.70	− 1 11.0	0 48 25.63	− 9 24 36.4	n8.1809	0.8158	+3.59	+21.0
11 32 13	34	t30 . 6	−1 2.19	9 0.5	0 48 25.02	− 9 24 35.8	8.3063	0.8158	+3.59	+20.9
					(343) Ostara.					
Sept. 29 8 54 51	35	t28 . 6	+0 55.71	+ 9 14.0	0 7 35.59	− 3 27 25.4	n9.4737	0.7636	+3.60	+23.4
9 21 8	36	t29 . 6	−2 27.34	+ 6 1.7	0 7 34.72	− 3 27 30.0	n9.4078	0.7658	+3.60	+23.2
Oct. 12 10 47 48	37	t30 . 6	+2 39.26	+ 3 59.0	23 56 25.28	− 1 1 16.3	8.4321	0.7756	+3.59	+23.4
13 13 28 55	37	t30 . 6	+1 49.71	+ 2 21.6	23 55 35.76	1 2 53.7	9.5404	0.7654	+3.59	+23.4
					(301) Bavaria.					
Sept. 29 10 35 3	38	t30 . 6	+4 47.31	− 6 13.1	0 12 18.29	− 1 1 35.2	9.1261	0.7742	+3.60	+23.2
Oct. 13 10 19 51	39	t28 . 6	−0 29.17	+ 1 13.1	0 2 28.40	− 5 19 3.2	n8.5264	0.7859	+3.59	+23.0
					1903 [illegible]					
Sept. 29 11 37 17	40	10 . 10	0 26.51	+ 3 16.7	0 12 27.68	2 19 57.0	7.9182	0.7611	+3.61	+23.2
12 7 35	41	10 . 10	+0 15.36	− 9 5.7	0 12 26.49	2 20 11.6	8.7181	0.7610	+3.62	+23.2
30 11 46 12	42	t28 . 6	−1 50.70	+ 1 33.3	0 11 33.65	2 28 16.7	8.3058	0.7625	+3.62	+23.2
					(419) [illegible]					
Sept. 29 13 11 47	43	t30 . 6	+0 46.36	+ 6 9.1	1 12 6.74	+ 2 26 49.8	8.7899	0.7145	+3.66	+20.1
Oct. 2 14 15 15	44	10 . 10	0 18.67	+ 1 15.9	1 9 31.31	+ 2 20 35.5	9.5212	0.7197	+3.68	+20.3
11 17 21	45	t30 . 6	2 9.01	+ 6 11.7	1 9 30.27	+ 2 20 29.4	9.4224	0.7211	+3.68	+20.2
13 15 1 18	46	t28 . 6	3 24.60	7 16.5	0 59 38.51	+ 1 22 23.2	9.5617	0.7322	+3.74	+20.8
					(379) Huenna.					
Sept. 30 12 53 52	47	10 . 10	+0 3.11	5 18.5	0 28 33.22	+ 2 21 10.7	9.0770	0.7178	+3.68	+22.6
13 27 2	48	t30 . 6	1 29.97	5 6.3	0 28 32.43	+ 2 21 3.6	9.2601	0.7190	+3.68	+22.5
Oct. 13 14 12 4	49	t28 . 6	1 0.93	11 8.2	0 19 28.56	+ 1 13 23.7	9.5431	0.7326	+3.69	+23.0
14 13 56 32	50	t25 . 6	0 31.23	+ 7 5.6	0 18 51.00	+ 1 8 16.2	9.5216	0.7326	+3.68	+23.1

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	11 16 54.36	— 7 38 50.1	Wien, A.G.Z. 53, 251	26	0 11 31.68	1 48 1.5	Nicolajew, A.G. 11
2	11 12 51.40	7 5 12.1	Wien, A.G.Z. 12	27	0 57 38.98	+3 53 39.0	Albany, A.G. 268
3	11 14 54.65	6 29 53.6	Wien, A.G.Z. 14, 321	28	0 58 7.76	+3 43 16.7	Albany, A.G. 273
4	11 35 52.35	- 6 2 32.5	Strassburg, A.G. Zones	29	0 53 41.78	+3 10 51.0	Albany, A.G. 245
5	11 39 16.69	- 5 58 41.5	Yarnall III, 6476	30	0 53 18.36	+3 46 21.1	Albany, A.G. 241
6	22 31 32.02	-10 47 28.8	Camb.(U.S.) A.G.Zones	31	1 1 50.47	9 12 52.1	Wien, A.G. Zones {199A, 297, 200, 285.
7	22 28 33.71	-11 1 13.8	Camb.(U.S.) A.G.Zones	32	1 1 5.31	9 10 12.8	Wien.A.G.Z.199A.200,285
8	22 25 30.90	11 10 27.8	Fund. Catalogue 1483	33	0 16 13.34	9 20 16.1	Wien.A.G.Z.199A,200.297
9	22 25 58.00	-11 6 12.0	Camb.(U.S.) A.G.Zones	34	0 19 23.92	9 15 56.2	Wien,A.G.Z.199A,200.286
10	0 52 14.96	0 10 55.8	Nicolajew, A.G. 177	35	0 6 36.28	-3 37 2.8	Strassburg, A.G. Zones
11	0 45 12.26	- 0 36 16.3	Nicolajew, A.G. 151	36	0 9 58.46	-3 33 57.9	Strassburg, A.G. Zones
12	0 14 56.88	0 45 8.7	Nicolajew, A.G. 148	37	23 53 42.43	4 5 38.7	Fund. Catalogue 1590
13	0 52 23.89	+18 5 32.1	Berlin A., A.G. 266	38	0 7 57.35	3 55 15.1	Strassburg, A.G. Zones
14	0 48 23.50	+17 52 2.1	Berlin A., A.G. 238	39	0 2 53.98	5 23 39.3	Strassburg, A.G. Zones
15	0 50 4.36	+17 13 5.6	Berlin A., A.G. 248	40	0 12 50.58	—2 21 6.9	Strassburg, A.G. Zones
16	0 47 50.35	+17 34 51.0	Berlin A., A.G. 233	41	0 12 7.51	—2 11 32.1	Strassburg, A.G. Zones
17	0 47 20.45	+17 8 14.2	Berlin A., A.G. 230	42	0 13 20.53	—2 33 13.2	Strassburg, A.G. Zones
18	0 21 6.00	- 0 5 18.7	Nicolajew, A.G. 64	43	1 11 16.72	+2 30 20.6	Albany, A.G. 344
19	0 18 23.86	— 0 5 11.1	Nicolajew, A.G. 53	44	1 9 16.30	+2 18 29.3	Albany, A.G. 339
20	0 19 42.50	— 0 19 10.2	Nicolajew, A.G. 60	45	1 11 35.60	+2 13 21.5	Albany, A.G. 346
21	0 18 51.10	- 0 21 57.9	Nicolajew, A.G. 55	46	1 2 59.37	+1 29 18.9	Albany, A.G. 303
22	0 17 55.18	- 0 21 33.9	Nicolajew, A.G. 52	47	0 28 26.43	+2 26 36.6	Albany, A.G. 105
23	0 21 55.05	— 1 27 56.6	Nicolajew, A.G. 67	48	0 29 58.72	+2 25 47.1	Albany, A.G. 116
24	0 17 16.23	— 1 28 58.6	Nicolajew, A.G. 50	49	0 20 25.80	+1 24 8.9	Albany, A.G. 75
25	0 19 20.55	— 1 46 44.0	Nicolajew, A.G. 58	50	0 19 18.55	+1 1 17.5	Nicolajew, A.G. 57

The star places from the Strassburg and Cambridge (U.S.) Zones were furnished through the courtesy of the Directors of the observatories at those places.

Planets 236, 335, 333, 332, 163, 73, 343, 301 and 449, were found photographically by Mr. G. H. PETERS, 362 by myself. The photographic plates were often made use of afterwards for finding the asteroids quickly in the field of the 26-inch equatorial.

* This asteroid was found on a plate exposed by Mr. PETERS on Sept. 28. The following circular elements were computed from the first observation of Sept. 29, and the observation of Sept. 30.

Epoch Sept. 30.5, $u = 203^\circ 4' 35''$, $\Omega = 160^\circ 58' 9''$,
$i = 4^\circ 51' 20''$, $\mu = 1050.72$, $\log a = 0.352183$

The magnitude was estimated, Sept. 29 (11.7), Sept. 30 (11.8). It was clear on Oct. 2 for only a short time, and then cloudy until Oct. 13, when the object could not be found again.

THE ASTRONOMICAL JOURNAL.

Founded by B. A. GOULD in 1849. Six volumes were issued to 1861. The publication was resumed in 1886, with the seventh volume, and has been continued to volume twenty-four.

Each volume consists of twenty-four numbers of at least eight quarto pages each, with table of contents and alphabetical index. Subscription price, \$5.00, payable in advance.

The *Journal* will be sent currently, postage prepaid to any part of the world, upon receipt of a letter or order giving name and address.

Foreign subscriptions can be remitted by postal money order on post office at Cambridge or Boston, Massachusetts; domestic subscriptions either by postal money order or by check on any bank in the United States.

Back volumes may still be obtained as below:

Volume I (a few numbers missing), according to completeness,		\$4.50 to \$7.00
Volumes II to VI, complete, each	. . . .	4.50
" VII to XXIII, " "	. . . .	3.75

Single numbers can still be supplied, to complete imperfect volumes or for other purposes.

Address S. C. CHANDLER, 16 Craigie Street, Cambridge, Massachusetts.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, \$5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, LYNN, MASS. *Closed Feb 1*
Entered February 2, 1903 at Boston, Mass. as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 557.

VOL. XXIV. BOSTON, 1904 MARCH 4. NO. 5

THEORY OF THE MUTUAL PERTURBATIONS OF PLANETS MOVING AT THE SAME MEAN DISTANCE FROM THE SUN, AND ITS BEARING ON THE CONSTITUTION OF *SATURN'S* RINGS AND THE COSMOGONY OF LAPLACE.

By JOHN N. STOCKWELL.

The theories of the mutual perturbations of the planets of the solar system have been undergoing the process of development during a period of more than two hundred years; and they are now so nearly perfect that the places of the planets in the heavens can be very accurately predicted for any past or future date. The amount of labor required for the development of a planetary theory depends very much on the eccentricity and inclination of its orbit: but it also depends to a much greater extent on the ratio of its mean distance from the sun to that of the disturbing planet. For the planets of the solar system this ratio of mean distances varies between 0.0128871 in the case of *Mercury* and *Neptune*, and 0.7233323 in the case of *Venus* and the *Earth*; but in the system of *Saturn's* satellites this ratio rises to 0.8258 in the case of *Titan* and *Hyperion*. For theoretical reasons, however, it is important to investigate the nature of the perturbations which would take place between two planets moving at the same mean distance from the sun; and this is what I propose to do in the present article.

The great difficulty which presents itself in the solution of this problem arises from the circumstance that no simple relation between the distances of two planets from each other has hitherto been discovered by means of which this distance can be determined directly in functions of the time as the independent variable. It is easy to express this distance analytically; and if we suppose that a and a' denote the mean distances of the disturbed and disturbing planets from the sun, respectively, the square of the distance between them will be expressed by the equation,

$$\Delta^2 = a^2 + a'^2 - 2aa' \cos\beta \tag{1}$$

the orbits being supposed circular and in the same plane; while β denotes the difference of the heliocentric longitudes of the two planets, and is therefore a function of the time t. If we put $a \div a' = \alpha$, equation (1) will give

$$\Delta^2 = a'^2 \{1 + \alpha^2 - 2\alpha\cos\beta\} \tag{2}$$

In the planetary theories the quantity $1 \div \Delta^3$ is required, and it must be computed from the equation

$$\frac{1}{\Delta^3} = \frac{1}{a'^3} \{1 + \alpha^2 - 2\alpha\cos\beta\}^{-\frac{3}{2}} = \frac{1}{a'^3}\, u^3 \tag{3}$$

Now if we assume that

$$u^3 = \tfrac{1}{2} b_{\frac{3}{2}}^{(0)} + b_{\frac{3}{2}}^{(1)} \cos\beta + b_{\frac{3}{2}}^{(2)} \cos 2\beta + b_{\frac{3}{2}}^{(3)} \cos 3\beta + \&c. \tag{4}$$

the coefficients $b_{\frac{3}{2}}$ will be functions of α, and are called the LAPLACE coefficients of the perturbative function.

In order to show the different degrees of convergency of the series (4) for the different planets of the solar system, I here give a few examples by way of illustration.

In the case of *Mercury* disturbed by *Neptune* we have $\alpha = 0.0128871$; and we get

$$\frac{a'^3}{\Delta^3} = 1.000374 + 0.0386783 \cos\beta + 0.0006231 \cos 2\beta + 0.00000937 \cos 3\beta;$$

in which three variable terms of the series are sufficient to give the value of the function correct to six decimal places.

In the case of *Mercury* disturbed by *Venus* we have $\alpha = 0.5351604$, and we get

$$\frac{a'^3}{\Delta^3} = 2.407072 + 3.035438 \cos\beta + 1.950491 \cos 2\beta + 1.192288 \cos 3\beta + \&c.$$

and it would be necessary to extend the series to the term $\cos 23\beta$ in order to get the value correct to only five decimals.

In the case of *Venus* disturbed by the *Earth* we have $\alpha = 0.7233323$, which is the largest value of α among

the principal planets of the solar system. This value of α gives

$$\frac{a'^3}{\Delta^3} = 1.996255 + 8.871665\cos\beta + 7.38676\cos 2\beta + 5.95117\cos 3\beta + \&c.$$

Lastly, for the satellites *Titan* and *Hyperion* of the *Saturnian* system we have $\alpha = 0.82578531$; and this gives

$$\frac{a'^3}{\Delta^3} = 11.65816 + 22.16477\cos\beta + 20.3391\cos 2\beta + 18.2937\cos 3\beta + 16.2366\cos 4\beta + 14.2738\cos 5\beta + 12.1577\cos 6\beta + 10.8110\cos 7\beta + \&c.$$

These illustrations are sufficient to show that this method of development may be very easily applied to all cases of planetary perturbation in which α is rather small; but when α equals or exceeds 0.75 the labor required for its application becomes very great. This is evident from the last two series given above, which converge with extreme slowness; thus showing that a very great number of terms must be computed in order that a very moderate degree of approximation may be obtained.

But when α becomes equal to unity, which is the case when the two planets are moving at the same mean distance from the sun, this method of development fails completely; for all the coefficients $b^{(0)}$, $b^{(1)}$, &c., become infinite and equal to each other; and mathematicians cannot make use of numbers which are infinitely great, in their investigations.

It is fortunate, however, that analysis is able to furnish a solution of our problem by a method of development which is wholly free from the use of infinite series, and at the same time rigorously exact.

We shall now attend to this new development, and for this purpose shall resume the consideration of equation (3). If we put $\alpha = 1$ in equation (3) it will reduce to the following:

$$\frac{a'^3}{\Delta^3} = \left\{2(1-\cos\beta)\right\}^{-\frac{3}{2}} = \frac{1}{8\sin^3\frac{1}{2}\beta} = u^3 \tag{5}$$

Equation (5) is entirely rigorous, and is also wholly free from infinite quantities except for the particular case in which $\beta = 0$; and in that case, in circular orbits the two bodies would unite and form a single body.

If we now denote the mass of the disturbing planet by m', and the potential function by R, the forces acting on the disturbed planet in the direction of, and perpendicular to, the radius-vector of the disturbed planet will be given by the equations

$$\left(\frac{dR}{dr}\right) = \frac{m'}{a'^2}\cos\beta + \frac{m'a}{a'^3}u^3 - \frac{m'}{a'^2}u^3\cos\beta \tag{6}$$

$$\left(\frac{dR}{dv}\right) = -\frac{m'a}{a'^2} + \frac{m'a}{a'^2}u^3\sin\beta \tag{7}$$

And if we substitute in these the above value of u^3 they will become

$$\left(\frac{dR}{dr}\right) = \frac{m'}{a^2}\left\{\cos\beta + \frac{1}{4\sin\frac{1}{2}\beta}\right\} \tag{8}$$

$$\left(\frac{dR}{dv}\right) = \frac{m'}{a}\left\{\frac{1}{8\sin^3\frac{1}{2}\beta} - 1\right\}\sin\beta \tag{9}$$

These values are evidently constant in the case of two planets moving at the same mean distance from the sun.

Now the perturbation of the radius-vector is given by the differential equation

$$\left.\begin{aligned}\frac{d\delta r}{dt} = a^3\frac{n}{\mu}\left\{\cos v\int dt\cos v\left(\frac{dR}{dr}\right)\right\} - a^3\frac{n}{\mu}\left\{\sin v\int dt\sin v\left(\frac{dR}{dr}\right)\right\} \\ + 2a^2\frac{n}{\mu}\left\{\cos v\int dt\sin v\left(\frac{dR}{dv}\right)\right\} - 2a^2\frac{n}{\mu}\left\{\sin v\int dt\cos v\left(\frac{dR}{dv}\right)\right\}\end{aligned}\right\} \tag{10}$$

in which v denotes the longitude of the disturbed planet.

Performing the operations indicated on the terms multiplied by $\left(\frac{dR}{dr}\right)$ they become

$$a^3\frac{n}{\mu}\left\{\sin v\cos v - \sin v\cos v\right\}\left(\frac{dR}{dr}\right) = 0 \tag{11}$$

and therefore terms depending on $\left(\frac{dR}{dr}\right)$ produce no perturbation of the radius-vector.

If we in like manner perform the operations indicated on the terms multiplied by $\left(\frac{dR}{dv}\right)$ they become

$$-2a^2\frac{n}{\mu}\left\{\sin^2 v + \cos^2 v\right\}\left(\frac{dR}{dv}\right) = -2a^2\frac{n}{\mu}\left(\frac{dR}{dv}\right) \tag{12}$$

and equation (10) becomes

$$\frac{d\delta r}{dt} = -2a^2\frac{n}{\mu}\left(\frac{dR}{dv}\right) = 2a\frac{m'}{\mu}n\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta \tag{13}$$

since $a = a'$.

Equation (13) gives by integration

$$\delta r = 2a\frac{m'}{\mu}nt\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta \tag{14}$$

an equation which gives the perturbation of the radius-vector in terms of the time t. We shall now discuss this equation.

Since β denotes the angular distance between the two planets, at the sun's center, we shall evidently have $\delta r = 0$, when $\beta = 180°$; whence it follows that there would be no perturbation of the radius-vector when the two bodies were situated at the opposite extremities of a diameter of the orbit. If β is less than 180° and greater than 60° the co-

efficient of t is positive, and the radius-vector would increase with time.

If $\beta = 60°$ the factor $1 - \frac{1}{8\sin^3\frac{1}{2}\beta} = 0$; and consequently there would be no perturbation of the radius-vector when the two planets formed an equilateral triangle with the sun. When β is less than 60° the coefficient of t becomes negative, and the radius-vector would diminish with the time.

The function $\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta$ is a maximum and equal to 0.7265688 when $\beta = 108° 21' 30''$; whence it follows that the radius-vector increases most rapidly when $\beta = 108° 21' 30''$.

We shall now consider the perturbations of the motion in longitude. The *direct* action on the longitude is given by the equation

(15)

$$\frac{d\delta_0 v}{dt} = -\frac{1}{n^2}\int\left(\frac{dR}{dr}\right)dt = \frac{m'}{\mu}n^2 t\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta$$

and the *indirect* action through the radius-vector is determined by the following equation:

$$(16) \qquad \frac{d\delta_1 v}{dt} = -2\frac{n}{a}\delta r = -4\frac{m'}{\mu}n^2 t\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta$$

Equations (15) and (16) give by integration

$$(17) \qquad \delta_0 v = \frac{1}{2}\frac{m'}{\mu}n^2 t^2\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta$$

$$(18) \qquad \delta_1 v = -2\frac{m'}{\mu}n^2 t^2\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta$$

and the sum of these two equations gives the whole perturbation in longitude. Therefore we get

$$(19) \qquad \delta v = -\frac{3}{2}\frac{m'}{\mu}n^2 t^2\left\{1 - \frac{1}{8\sin^3\frac{1}{2}\beta}\right\}\sin\beta$$

Equations (5)–(19) are entirely rigorous, and we shall now proceed to discuss them.

We first observe, however, that the angle β denotes the longitude of the disturbed planet diminished by that of the disturbing planet; or in symbols, if nt and $n't$ denote the mean longitudes of the disturbed and disturbing planets, we shall always have $\beta = nt - n't$. This explanation is important; for without it we should be unable to determine in what direction the perturbation took place.

We have already observed that the coefficient of t in the value of δr, is positive when β is greater than 60°, and less than 180°. It therefore follows that the coefficient of t^2 in the value of δv is negative for these same values of β, and consequently the value of δv would be negative, and the planet would recede more and more from the point of no disturbance; but the reciprocal action of the disturbed on the disturbing planet would cause it to advance until the distance between them was reduced to 60°, when the perturbations would cease entirely. We may therefore state this general theorem of celestial mechanics, namely:

If two planets are moving in the same plane, and at the same mean distance from the sun, and are situated at an angular distance greater than 60° and less than 180° from each other, as viewed from the sun, their mutual perturbations will cause them to approach each other until their distance apart becomes equal to 60°; and their approach will be most rapid when their angular distance apart is equal to 108° 21′ 30″.

We have also seen that the coefficient of t in the value of δr is negative when β is less than 60°. The coefficient of t^2 is therefore positive in the value of δv; it therefore follows that the disturbed body would be thrown forward, and the disturbing body would be thrown backward by their mutual perturbations when their angular distance apart was less than 60°.

We may therefore state this general proposition, namely:

If two planets are moving in the same plane, and at the same mean distance from the sun, and are situated at an angular distance of less than 60° apart as seen from the sun, their mutual perturbations will cause them to recede from each other until their distance apart becomes equal to 60°; and they will always remain in a condition of stable equilibrium at that distance apart, and will revolve around the sun forever free from mutual disturbance.

On the other hand, if they were placed at a distance of 180° apart, they would move free from *mutual* disturbance; but a slight derangement from that position by any cause would at once be continued by their mutual attraction until their distance apart had reached the angle of 60°, in which position it would remain forever. The angular distance of 180° apart is therefore one of unstable equilibrium; and no two planets could permanently remain in that position.

We shall now consider the problem of the constitution of *Saturn's Rings*, and the cosmogony of Laplace.

According to equations (14) and (19) the radius-vector and longitude are free from perturbation when the three bodies, the sun and two planets form an equilateral triangle. But six equilateral triangles fill the circle. There might, therefore, be six equal planets revolving in the same orbit at angular distances of 60° apart, and all be free from perturbation, since each would be symmetrically situated with respect to all the others. Moreover, there might be another set of six equal planets introduced midway between the first six, and as each would be symmetrically situated with respect to all the others they would all be free from perturbation. And, finally, the number of planets might be increased indefinitely so as to form a complete ring like that of *Saturn*, and as each planet would be symmetrically situated with respect to the others its motion would be free from the effects of

perturbation; or in other words, it would be in a position of stable equilibrium with respect to the forces acting upon it, and would remain relatively at rest forever.

According to equations (8) and (9) the forces

$$\left(\frac{dR}{dr}\right) \text{ and } \left(\frac{dR'}{dr'}\right)$$

become infinite when $\beta = 0$. But in that case the two bodies would coincide. It therefore becomes an interesting problem to determine their mutual perturbations when they approach that condition, since the forces must become relatively quite strong.

We shall therefore suppose that a disturbing body in the ring of *Saturn* has a mass equal to a sphere of one mile in diameter of the same mean density of *Saturn*; and if we further suppose this sphere to be condensed to a point, the radius of its sphere of activity with respect to *Saturn* would be only 520 feet; or in other words, if a particle of matter were brought within a less distance than 520 feet of the disturbing body it would describe an orbit around that body; while if it were at a greater distance than 520 feet it would revolve around *Saturn* the same as any other part of the ring.

If the disturbing body in the ring had a mass equal to that of a sphere one hundred miles in diameter, of the same mean density as *Saturn*, it would have only 25 miles for the radius of its sphere of activity with respect to *Saturn*, if reduced to a point by condensation. Now, a distance of 25 miles on *Saturn's* ring would subtend an angle of only 64″ if viewed from the center of *Saturn*, supposing it to be 80,000 miles distant; while if viewed from the earth it would subtend an angle of only 0″.0054, and would hardly be visible in the most powerful telescopes.

A homogeneous ring of satellites would therefore be in a condition of stable equilibrium; but if any finite portion of the ring should become weighted by a mass of matter comparable to that of one of the satellites, the ring would be destroyed, not by precipitation on the planet, but by the formation of three satellites moving at the same distance from *Saturn*, the one preceding and the other following the disturbing satellite by 60°. LAPLACE demonstrated that a symmetrical homogeneous solid ring would be in a condition of unstable equilibrium, and would move forward in its own plane until the ring came in contact with *Saturn*, when it would be broken into fragments and precipitated upon the planet. He therefore imagined that such a catastrophe would be averted if the ring was made up of irregular solids; but he gave no demonstration that such a constitution of the ring would afford any sustaining power, or produce a condition of stable equilibrium. *Saturn's* ring has now been carefully observed during two centuries and a half; and as no indications of irregularities of structure nor perturbations of motion have been detected we may logically conclude that it is in a condition of stable equilibrium with all the acting forces; and is therefore composed of millions of satellites so small that they do not disturb each other's motion any more than the grains of sand in a sand-blast on the surface of the earth.

It is now somewhat more than one hundred years since LAPLACE published his cosmogony or theory of the genesis of the solar system, under the name of "*Nebular Hypothesis*." The general principles of this theory are so well known and understood that it is unnecessary to enter into any details at this time concerning them. They are so plausible that they have very generally been accepted by scientific men as a general and satisfactory solution of the great problem of *world making*. It is evident, however, that LAPLACE forgot his usual caution, and gave to the world an ingenious speculation without submitting any of its details to the rigid scrutiny of mathematical computation; — trusting more to the celebrity of the author's name for its acceptance than to any inherent merits of the theory itself. For it is a curious fact, that among all the elements of the theory there is only one that seems susceptible of demonstration.

It is not my purpose at this time to enter into a general discussion of the merits of the nebular hypothesis, but only to show that some of its *assumptions* are incompatible with the operation of gravitating forces; leaving for another occasion a more complete consideration of the subject.

Assuming for the present, however, that a ring of matter has been left behind by the contraction of a rotating nebula, LAPLACE has *assumed* that if one portion of the ring was much heavier and larger than other portions of the ring, it would break up into several pieces, and the largest pieces would gradually attract all the remaining parts of the ring to itself; and the matter thus collected together would further condense into a planet and its satellites, having motions of rotation and translation in the same direction as the primitive nebula. But we have already seen that the effect of the mutual attraction of two parts of a ring would be to separate them still further apart provided their primitive distance was less than 60°; while if their primitive distance apart was greater than 60°, and less than 180°, the two parts would approach each other until they were 60° apart, when they would have attained a position of equilibrium with all the acting forces, and a nearer approach would be impossible. The assumption by LAPLACE that the matter of which the ring was composed would concentrate by the mutual attraction of its different parts, into a single planet or satellite is therefore not sustained by a rigorous calculation.

In this connection it may be proper to mention the result of a computation made by the writer soon after the discovery of the satellites of *Mars*, in regard to the formation of rings by a rotating nebula which is undergoing

the process of condensation by cooling. According to the principle of the conservation of areas, when a rotating nebula shrinks in volume its rotation on its axis must become more rapid. Now the force of gravity at the surface of a sphere varies inversely as the square of the radius, and the centrifugal force varies inversely as the cube of the radius. The centrifugal force, therefore, varies much more rapidly than the force of gravity; and if at a given time the force of gravity exceeds the centrifugal force, as the process of contraction goes on the time will come when the centrifugal force will just equal the force of gravity. When that condition is reached, the particles of matter subjected to these two forces will cease to fall towards the equator of the shrinking nebula, but will constitute the outer convex surface of a ring in process of formation. As time goes on the centrifugal force becomes greater than the force of gravity, and the particles of matter at the equator of the nebula, instead of being left at that distance from the center of the nebula would be thrown off by the greater centrifugal force, in the same manner as water is thrown off from the circumference of a rapidly rotating grindstone.

Now, in order that a ring of matter may be *left behind*, it would be necessary to stop the rotation of the nebula, either wholly or in part, and allow it to assume a spherical form by means of the force of gravity alone. In this way a ring of matter might be left behind the shrinking nebula, but the theory as explained by its author fails to provide or suggest any method by which this object could be attained. A correct theory of creation must be able to explain not only the general plan, but all the subordinate details of its development; and since the nebular hypothesis wholly fails to satisfy that requirement it evidently rests on no logical foundation.

Cleveland, 1904 *Jan.* 20.

AN ORBIT OF *ENCELADUS*,

BY HERBERT R. MORGAN.

The observations used in the following orbit were made by Prof. T. J. J. SEE, on the 26-inch equatorial of the U.S. Naval Observatory, in 1901, and are published in the *Astronomische Nachrichten*, Nr. 3806. On June 9 the position-angle and distance should be:

$$p = 278°.95 \quad , \quad s = 59''.70$$

June 18 was out 1″, and Aug. 17 was incomplete; these two were not used. On nine nights, when the seeing was bad, and the residuals were over 0″.8, less weight was given.

The mass of *Saturn*, $\mu = 3495.3$, was considered known, and the distances, Δ, of the satellites derived therefrom. The peri-Saturnium of *Enceladus* is known from theory; $\pi = 2l_{Di} - l_{En}$. The solution gives only the eccentricity. All formulas used are given by H. STRUVE. The longitudes are reckoned on *Saturn's* equator, as the fundamental plane, from its node, (N_1), on the *Earth's* equator.

ASSUMED ELEMENTS.

1901 July 8.0 Greenwich M.T.

	Enceladus	*Tethys*
ϵ	176°.195	36°.697
e	. . . .	0.0
π	257°.28	. . . .
$I\pi$	+123°.13 t	. . . .
γ	0.0	61°.36
θ	. . . .	177°.0
$I\theta$	−152°.7 t	72°.5 t
n	262°.73199	190°.69795
Δ	31″.116	12″.605

$$\left.\begin{matrix} \textit{Saturn's}\text{ equator} \\ 1901.518 \\ \text{Mass} = \mu = 3495.3 \end{matrix}\right\{ \begin{matrix} N_1 = 126°\ 30'.0 \\ J_1 = \ \ 6°\ 55'.6 \\ \omega_1 = \ 42°\ 17'.2 \end{matrix}$$

CORRECTIONS.

	Enceladus	*Tethys*
$d\epsilon =$	−0°.100 ± 0°.079	−0°.116 ± 0°.058
$e =$	0.0032 ± 0.0006	
$\theta =$	208°.6	
$\gamma =$	25°.5	

$[nn] = 12''.41$, $[vv] = 9''.68$, Prob. error of equation ±0″.213

There were one hundred and two equations of condition, with an average residual of ±0″.35; the solution only reducing this to ±0″.34.

The longitude and eccentricity of *Enceladus* agree with STRUVE'S values, while the longitude of *Tethys* is [illegible] greater than that found by him in 1901. As the *Earth* was way above the plane of *Saturn's* equator, the solution, while giving a good eccentricity, would not give the inclination and node closely, and the inclination is [illegible] too large. A solution in 1900 gave 11°.4, while 1898 gave 9°.0.

It is hoped that when the earth again passes through the ring plane in the next few years, a series of observations and reductions of the nature of STRUVE'S at the last such passage, may be undertaken. [illegible] val of fifteen years for determining [illegible].

Naval Observatory.

OBSERVATIONS OF 1072 ρ *PERSEI*,

MADE AT THE STUDENTS' OBSERVATORY, UNIVERSITY OF VIRGINIA,

BY CHARLES P. OLIVIER.

During the latter part of 1901, my attention was called to the variable ρ *Persei*, and ever since that time observations have been taken of its brightness. The results follow:

Date		Mag.
1900 Oct.	9	3.8
	17	3.63
	26	3.8
	29	3.4
1901 Oct.	5	3.73
	11	3.70
Nov.	8	3.51
	9	3.61
	30	3.68
Dec.	7	3.90
	8	3.90
	10	4.47
	11	4.20
	12	4.12
	15	4.12
	17	4.11
	18	4.00
	21	3.90
	24	3.70
	30	3.98
	31	4.00
1902 Jan.	1	3.91
	2	4.00
	3	3.91
	4	3.81
	5	3.81 +
	6	3.81
	9	3.88
	11	3.88
	12	3.88
	13	3.72
	14	3.53
	16	3.58
	17	3.64
Feb.	3	3.97
	3	4.05
	4	4.13
	6	3.96
	7	4.03
	8	3.90
	9	4.04
	10	3.90
	13	3.90
1902 Feb.	18	3.81
	19	3.55
	23	3.44
	26	3.45
Mar.	3	3.44
	6	3.49
	7	3.70
	9	3.40
	10	3.46 ±
	11	3.50
	12	3.59
	13	3.59
	19	3.59
	22	3.49
	23	3.49
	25	3.60
	29	3.72
	30	3.90
	31	3.81
Apr.	5	3.85
	6	3.58
	9	3.81
July	1	3.90
	14	3.98
	28	4.00
Aug.	7	4.12
	10	3.68
	17	3.70
	22	3.49
	24	3.58
	26	3.65
Sept.	7	3.86
	11	3.90
	13	3.68
	22	3.90
	23	4.00
	29	3.68
Oct.	1	3.58
	2	3.70
	7	3.55
	8	3.49
	9	3.58
	21	3.58
1902 Oct.	22	3.68
	24	3.58
	25	3.70
	29	3.87
	31	3.77
Nov.	1	3.68
	21	3.90
	28	3.63
Dec.	3	3.58
	5	3.58
	6	3.61
	8	3.54
	9	3.59
	10	3.59
	16	3.59
	17	3.75
	18	3.75
	19	3.75
	31	3.96
1903 Jan.	1	4.08
	7	3.73
	8	3.70
	9	3.50
	11	3.68
	15	3.68
	23	3.75
	30	3.77
	31	3.90
Feb.	5	3.64
	9	3.59
	11	3.58
	12	3.59
	13	3.60
	17	3.73
	19	3.64
	20	3.64
	24	3.64
Mar.	13	3.65
	17	3.56
	20	3.56
	23	3.56
	25	3.41
	26	3.49
1903 Mar.	31	3.56
Apr.	1	3.65
	8	3.57
June	29	3.81
July	1	3.79
	13	3.61
	15	3.50
	20	3.64
	24	3.64
	24	3.75
Aug.	11	3.90
	20	4.08
	25	3.91
Sept.	12	3.50
	15	3.61
	17	3.71
	18	3.33?
	21	3.49
	21	3.60
	22	3.91
	23	4.07
	24	3.91
	26	3.90
	28	3.90
	29	3.69
	30	3.77
Oct.	1	3.49
	8	3.54
	12	3.60
	13	3.79
	14	3.78
	17	3.90
	18	3.75
	19	3.84
	26	3.90
	27	3.69
	28	3.65
Nov.	6	3.39
	7	3.39
	10	3.35
	11	3.29
	12	3.29
	18	3.40
1903 Nov.	19	3.35
	20	3.35
	24	3.39
	27	3.44
	30	3.47
Dec.	3	3.49
	10	3.90
	11	3.91
	14	4.07
	15	4.02
	16	4.02
	17	3.90
	17	3.80
	18	3.76
	21	3.49
	22	3.47
	23	3.49
	25	3.44
	26	3.44
	28	3.50
	31	3.50
1904 Jan.	4	3.64
	5	3.46
	7	3.37
	8	3.38
	9	3.44
	13	3.80
	15	3.77
	16	3.81
	18	3.77
	19	3.77
	20	3.70
	25	3.70
	26	3.81
	27	3.81
	30	3.73
Feb.	1	3.68
	3	3.59
	4	3.61
	5	3.69
	6	3.69
	8	3.71 ±
	11	3.80
	12	3.80

The light-curve obtained from the above results shows that the star has had no regularity in its variations during the period covered. By a comparison of the Harvard, Oxford and Potsdam measures, the following magnitudes for the comparison stars were obtained:

	M		M
γ *Persei*	3.02	δ *Persei*	3.09
ε *Cassiopeae*	3.44	α *Trianguli*	3.49
ν *Persei*	3.90	ι *Persei*	3.91
τ *Persei*	4.01	θ *Persei*	4.18
ι *Persei*	4.22		

The variable was compared with from 1 to 9 stars at each observation.

OBSERVATIONS OF SUNSPOTS,

MADE AT AMHERST COLLEGE OBSERVATORY,

BY ROBERT H. BAKER.

1903		h	New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots
May	17	1	1	3			1	3	1	3
	18	1	–	–					1	2
	19	1		–					1	2
	20	1	1	9			1	5	2	11
	21	1	1	4			1	1	2	10
	22	1	–						2	6
	22	23		1	–				1	2
	24	1		2					1	1
	25	0	2	6					3	10
	26	12		1					2	11
	28	21			1	1				
	30	1	1	2					1	2
	31	0			1	2				
June	1	2								
	2	0	–	–						
	3	0	1	3			1	3	1	3
	3	21								
	5	11								
	6	4								
	10	2					–			
	13	2	2	10			1	6	2	10
	16	2	1	5	1	4	2	5	2	5
	17	6							2	5
	18	0	1	9		–	1	6	3	14
	21	12		1					3	9
Oct.	13	2							2	15
	14	2	1	7	1	1	1	1	2	21
	15	2							2	8
	16	3				5			2	3
	18	2		–	1	2			1	1
	20	3	1	2					2	3
	21	3	–			–			2	3
	22	2	–						2	3
	23	2			–				2	3
	25	2	1	2		–	1	2	2	3
	27	4		3	1	1		3	1	5
	28	2		1					1	6
	29	2		9					1	15
	30	1	1	3			1	3	2	18

1903		h	New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots
Oct.	31	2		22					2	40
Nov.	1	0		2					2	32
	2	2	2	15			1	2	4	47
	3	2		7					1	51
	4	3	1	5			1	4	1	58
	7	3		9	1	8			3	37
	9	2		65					3	102
	10	2							3	80
	11	2							3	65
	12	2			1	3			2	58
	13	2	1	1			1	1	2	35
	14	2		3					2	38
	15	3				–			1	17
	18	2	1	1	1	17	1	1	1	1
	19	2		1					1	2
	20	2		3					1	5
	21	2							1	3
	24	2							1	2
	24	22	1	3			1	3	1	3
	27	0	1	2			1	2	2	5
	27	22							2	5
	30	2	1	11			1	1	3	16
Dec.	1	2	1	2					1	16
	4	2	1	13	1	1	1	13	3	16
	6	2	1	11	1	1			3	28
	10	22	2	8	2	6	2	8	3	23
	12	2							3	20
	13	2		3					3	14
	14	2	1	1			1	1	3	12
	15	2	1	1	1	4	1	1	3	9
	15	22		–					3	7
	17	2		1					3	8
	17	22		1					3	7
	21	11	2	6					2	6
	22	2		1					2	7
	25	2							2	1
	28	2	1	2	2	4	1	2	1	2
	30	0	1	1			1	4	2	6
	31	2							2	6

Observations in May and June made with 7½-inch refractor. Faculae seen except on June 10.

Observations in October, November and December made with 6-inch reflector, except with 2-inch refractor, from Oct. 1[illegible]–29 and Dec. 21–31. Faculae seen except Oct. 20–25, Dec. 21, 22, 25, 31.

OBSERVATIONS OF VARIABLE STARS,— No. 9.

BY WM. F. SPERRA.

103 *U Andromedae.*

Eight observations of this star, between 1901 October 29 and 1902 January 7, give as the date of maximum 1901 November 22, at $8^M.1$.

The first observation was at $9^M.5$, and the last at $10^M.8$.

320 *U Cephei.*

1900 April 4, 18 observations, from $13^h\,28^m$ to $18^h\,57^m$.

Greenw. Time of minimum by single curve, $16^h\,15^m$
" " " mean curve, $16\ 26.5$ w: 10
" " " equal lights, $16\ 21.5$

	Before	After	Mean
M	h m	h m	h m
8.4	14 35	18 11	16 23.0
8.6	14 17	18 1	16 21.0
8.8	14 58	17 52	16 23.5
9.0	15 0	17 42	16 21.0
9.2	15 6	17 26	16 16.0
	Mean (Greenw. M.T.)		16 21.5

Lowest observed light $9^M.4$.

As in my results (*A.J.* 397), CHANDLER's mean light-curve was used. By using YENDELL's mean light-curve (*A.J.* 551, p. 219), the resulting time of minimum is $16^h\ 37^m.8$. Using CHANDLER's elements (*A.J.* 551, p. 214): viz., 1880 June $23^d\ 7^h\ 43^m.5$ G.M.T., $+2^d\ 11^h\ 19^m\ 14^s.7$ E, the computed time of minimum is $16^h\ 24^m.5$; and from his definitive elements in the "Revision, &c." (*A.J.* 553) it is $16^h\ 19^m.8$; thus, correcting for light-equation, giving $O-C = -5^m.1$, and $-0^m.8$, respectively.

2815 *U Geminorum.*

	M		M
1900 April 4.57	9.50	1901 March 31.56	10.46
5.54	9.42		
8.56	9.95		

7792 *SS Cygni.*

Continuation of the series published in *A.J.* 476, to which refer for magnitudes and comparison stars.

	M		M
1900 June 30.62	8.37	1900 Oct. 27.60	11.36
July 14.61	9.88	Dec. 23.49	8.73
Sept. 2.54	11.36		

Barberton, Ohio, 1904 Jan. 27.

1901 June 16 to July 5, seven observations at normal light.

	M		M
1901 July 5.66	11.17	1901 July 11.66	8.85
6.68	10.77	12.61	9.10
8.65	8.60	14.65	10.42
9.65	8.41	19.63	11.36
10.64	8.65		

These observations yield as the time of maximum 1901 July 9.35, at $8^M.45$.

1901 July 21 to Aug. 12, six observations at normal light.

	M		M
1901 Aug. 18.58	10.52	1901 Aug. 25.58	8.47
21.63	8.50	Sept. 3.63	10.92
22.63	8.51		

These indicate a maximum for Aug. 23±, at $8^M.5$.

1901 Sept. 12 to Oct. 7, eight normal observations.

	M		M
1901 Oct. 14.60	9.03	1901 Oct. 21.58	9.44
15.12	8.61	22.63	10.32

1901 Nov. 1 to Dec. 1, five normal observations.

	M		M
1901 Dec. 11.52	8.34	1901 Dec. 12.51	8.58

	M		M
1903 July 22.65	10.18	1903 July 28.63	8.32
23.64	8.94	31.58	8.41
24.63	8.36	Aug. 5.63	9.68
27.67	8.33		

These indicate as the time of maximum 1903 July 27.50, at $8^M.32$.

1903 Sept. 21.58 $10^M.75$

VARIABILITY OF (135) *HERTHA*.

A despatch from Kiel, February 20, states that PALISA's planet *Hertha* is variable; range half a magnitude; short period.

CORRIGENDA.

	N	Col.				
A.J. 504, p. 188:	182	$nt_1 - \Omega$	*for*	362°.6159	*put*	352°.6159
A.J. 504, p. 188:	305	$nt_1 = n't$	*for*	108°.0624	*put*	188°.0624

A.J. 556, p. 28, col. 2, line 14 from bottom, *read* "removal from F of all"

CONTENTS

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed March 3*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 558.

VOL. XXIV. BOSTON, 1904 MARCH 22. NO. 6

PROVISIONAL RESULTS OF AN EXAMINATION OF THE PROPER MOTIONS OF CERTAIN FAINT STARS,

BY GEORGE C. COMSTOCK

In Vol. X of the *Observations de Poulkova* OTTO STRUVE has published a series of proper motions of faint stars derived from micrometric comparisons of these stars with neighboring bright ones of known proper motion. Some of these stars are as faint as the twelfth magnitude, and so far as I am aware this list represents the first attempt ever made at determining the motions of stars of this class, or indeed of any class fainter than the ninth magnitude. But small heed seems ever to have been paid to these determinations, and they cannot be considered to have produced any effect upon current views of the structure of the stellar system, although they furnish precisely the kind of data most needed for this purpose. Indeed, it is not feasible from an inspection of the published data to determine whether the resultant proper motions are real quantities, or are merely residual errors of observation and of the assumed reduction elements. For the most part they depend upon micrometric observations at only two epochs, separated by an interval of about a quarter of a century, and the assumed proper motions of the bright comparison stars are far from corresponding to the best data now attainable.

The method, however, has seemed to me sufficiently promising to warrant further attempts at its application, and as the elapsed time interval separating the present epoch from that of the earlier observations of these stars by the STRUVES amounts on the average to something more than half a century, I have repeated with the 40 cm. equatorial of the Washburn Observatory the micrometric measures of all these stars fainter than the ninth magnitude for which suitable early observations are available. The presentation of these observations and their detailed discussion is reserved for another place, and I here confine myself to a preliminary exposition of the methods and results of that discussion.

For the older data required in the determination of proper motions I have relied exclusively upon the observations of W. STRUVE and O. STRUVE, made in the twenty years following 1835. For corresponding recent data I have been compelled to rely mainly upon my own observations, although these are supplemented, in some measure, by other data, most frequently that of the Lick observers. I have combined with the foregoing material a sufficient number of intermediate observations to furnish a control upon the assumed linear character of the relative motion of the stars, but I have made no attempt to utilize all of the extant data, since most of it falls near the mean epoch of the earlier and later observations, and would therefore add very little to the weight of the resulting proper motions. I have therefore limited myself, for the most part, in the selection of intermediate data to the observations of DEMBOWSKI, HALL, BURNHAM and the Pulkowa observers. In every case the mean of the position angles measured by a given observer during a brief period (one to four years) was reduced to the epoch 1850.0, and was then combined with the corresponding mean of the distances to furnish rectangular coordinates, dA perpendicular, and dD parallel to the hour-circle. The weights of these coordinates were assumed to be independent of the observer, and to be functions solely of the number of individual observations entering into the adopted mean position angle and distance, in accordance with the following scheme:

No. of Observations	Assumed Weight
1	0.5
2 to 10	1.0
10 to ∞	2.0

Cases in which the weight 2.0 was assigned are extremely rare.

Each star furnishes two sets of equations of the type,

$$x_0 + x_1(t - 1850.0) = dA \; , \quad y_0 + y_1(t - 1850.0) = dD$$

which were treated by the method of least-squares and

definitive values obtained for the four unknowns, together with the weights of their determination. If the rectangular components of the proper motion of the bright star be then added to the x' and y' we shall obtain the proper motion of the star under investigation, provided there is no physical connection between the two bodies. I have excluded from consideration in the present article all cases in which this condition (absence of physical connection) is not satisfied, the criterion applied for this purpose being a large proper motion of the bright star not shared by its faint companion. Whenever possible I have taken the assumed proper motion of the comparison stars from NEWCOMB's *Fundamental Catalogue*. For stars not contained in this catalogue I have myself derived, with the assistance of Mr SEBASTIAN ALBRECHT, proper motions referred to NEWCOMB's system, using for this purpose the formulas, precession constants and systematic corrections of the *Fundamental Catalogue*. I have, however, extended my data for proper motions beyond the list of catalogues for which NEWCOMB gives systematic corrections, and have derived the requisite corrections to these catalogues from the current literature of astronomy.

The following example represents the principal steps in the process of deriving the proper motion of one of these stars, viz.: a star of the 10.6 magnitude near *Aldebaran*. It may be regarded as fairly typical of the entire series, although the resulting proper motion is unusually large.

COMPARISON OF *α Tauri* WITH ✱ $10^{M}.6$.

Date	Observer	No. of Obs.	p (1850.0)	s	dA	c	dD	r
			° ′	″	″	″	″	″
1836.06	Σ	2	36 5.0	109.04	+64.22	−0.08	+88.13	−0.09
53.82	OΣ	9	35 19.6	111.61	64.54	− .14	91.05	+ .18
55.36	OΣ	9	35 24.5	111.78	64.75	+ .04	91.10	− .01
56.95	OΣ	6	35 25.3	112.11	64.98	+ .24	91.37	+ .02
84.16	HΣ	3	34 21.1	115.11	65.12	− .20	95.28	− .15
1902.40	Comstock	3	33 50.3	118.22	+65.83	+0.12	+98.19	+0.03

Concluded Values

$$dA = +64''.60 + 0''.0213\,(t-1850.0)$$
$$dD = +90.34 + 0.1495\,(t-1850.0)$$

	Proper Motions *α Tauri*	Proper Motions ✱ $10^{M}.6$	Weight
	″	″	
	+0.0690	+0.0903	2884
	−0.1889	−0.0394	

The weight 2884 given in the preceding exhibit is that of the relative annual motion of the two stars in each coordinate, *i.e.*, that of the variable part of both dA and dD. To determine the probable error corresponding to unit-weight I have discussed the residuals from 914 equations furnished by the observations of 99 stars, without distinction of observer or coordinates, and find thus for the probable error of an equation of unit-weight, $r_1 = \pm 0''.15$. Corresponding to this datum the probable errors of the quantities above derived are each $\pm 0''.0028$. According to NEWCOMB the probable error of the proper motion of *α Tauri* is in each coordinate $\pm 0''.0016$, and the resulting probable error of the annual proper motion of the faint star is $\pm 0''.0032$ in each coordinate.

As a control upon the assumption that equal weights are to be attributed to all observers, I have derived from the entire data at my disposal the value of r_1 for each observer for whom 20 or more residuals are available, and their values are shown in the following table:

Observer	No. of rs.	r_1
		″
Σ	122	±0.14
OΣ	308	.15
HΣ	44	.19
DOUBIAGO	24	.20
Δ	116	.14
HALL	26	.16
β	64	.17
COMSTOCK	198	.11

The variations here shown, while not inconsiderable, are in part fictitious, the large time-coefficient tending to produce a better representation of the observations at the beginning and end of the series, than is the case for intermediate data, and it does not appear that any marked improvement in the results could be obtained by assigning different weights to the different observers. From a comparison of the data furnished by NEWCOMB's *Fundamental Catalogue* with the probable errors of the proper motions of faint stars here obtained, it appears that in general the precision of these proper motions does not differ sensibly from the proper motions of NEWCOMB's stars that are fainter than the fifth magnitude, the average probable error of a proper motion in declination being for NEWCOMB's stars $\pm 0''.0054$, and for the tenth and eleventh magnitude stars here considered $\pm 0''.0052$. The precision in right-ascension is a trifle less. So far, therefore, as internal consistency of the data is concerned the proper motions that form the subject of this paper must be regarded as quite comparable with those of the fainter fundamental stars.

It is possible, however, to apply to these proper motions a further and independent criterion of their intrinsic worth, and to that end I have sought to derive from them a determination of the elements of the sun's motion of translation. Although the amount of data is scanty, the stars are well distributed in right-ascension, and the problem is

not quite so hopeless as it might seem *a priori*. I have used for this purpose AIRY's method, *i.e.*, the determination of the rectangular coordinates of the solar motion, involving in each equation into which these coordinates enter, an assumption with regard to the parallax of the given star. In place of the rather crude assumptions frequently made in this connection I have employed in computing the coefficients of the equations an extrapolation of KAPTEYN's formula, $\pi_{\mu,m} = (0.905)^{m-5.5}\sqrt[1.4]{0.0387\mu}$ (*Publications*... *Groningen*, No. 8, p. 24). This gives for any star in terms of its brightness, m, and proper motion μ a hypothetical parallax that in the mean corresponds well to the average distance of such stars. Although the formula professedly relates only to stars brighter than the ninth magnitude, it will appear later that the extrapolation is justified as far as the twelfth magnitude.

In view of the wide range of utility presented by this equation I reproduce here the following tables, based upon it, that I have employed in computing the parallaxes that enter into the coefficients of my equations. The argument for A is the stellar magnitude, and for B it is the annual proper motion, expressed in seconds of arc of a great circle, and multiplied by such a factor (10^n) as will suffice to bring it within the range of the tabular arguments.

TABLE FOR KAPTEYN'S HYPOTHETICAL PARALLAXES.

$\log \pi = A + B - 0.712\,n$

m	A	$\mu.10^n$	B	n	$0.712\,n$
0	9.24	0	$-\infty$	0	0.00
1	.20	1	0.00	1	0.71
2	.15	2	.21	2	1.42
3	.11	3	.33	3	2.14
4	.07	4	.42		
5	9.02	5	.49		
6	8.98	6	.55		
7	.93	7	.60		
8	.89	8	.64		
9	.85	9	.67		
10	.80	10	.71		
11	.76	11	.74		
12	8.71	12	0.77		

As an example of the application of these tables I select the star above compared with α *Tauri*, and obtain

m	10.6	B	0.70
$100\,\mu$	9.85	$0.712\,n$	1.42
A	8.78	$\log \pi$	8.06

$\pi = 0''.011$

In the table on page 18 of this article there are shown the proper motions of 68 faint stars, together with other data pertinent to their use. Reserving for another publication the individual equations arising from these several stars, I present as sufficient for the present purpose the following normal equations involving the equatorial, rectangular coordinates of the annual solar motion, and their relation to the observed proper motions of these stars, all of which are included between the magnitudes 8.5 and 12.5. No one of these proper motions has been hitherto employed for this purpose.

NORMAL EQUATIONS FOR DETERMINING THE SOLAR MOTION

In ΔA.

$$
\begin{aligned}
+7.23\,X + 0.59\,Y &= +10.56 \\
+0.59\,X + 7.06\,Y &= -31.30
\end{aligned}
$$

In ΔD

$$
\begin{aligned}
+1.65\,X - 0.19\,Y - 1.38\,Z &= +\ 2.21 \\
-0.19\,X + 1.51\,Y + 0.30\,Z &= +\ 0.38 \\
-1.38\,X + 0.30\,Y + 11.23\,Z &= +21.36
\end{aligned}
$$

From the solution of these equations I find $X = +1.956$, $Y = -3.773$, $Z = +2.242$, with the weights 8.70, 8.55, 11.00, respectively. These values furnish as the apex of the solar motion relative to the group of stars under consideration, R.A. $= 297°$, Decl. $= +28°$, which may be compared with NEWCOMB's determination from proper motions of the brighter stars, *A.J.*, No. 457, R.A. $= 277°.5$, Decl. $= +35°$, and with CAMPBELL's spectroscopic result, *A-P.J.* XIII, 83, R.A. $= 277°.5$, Decl. $= +20°$.

In view of the character of the data employed the agreement of these numbers with previous determinations of the apex must be regarded as eminently satisfactory, the deviation from the commonly adopted mean position of the apex being not more than 20°. I regard this agreement as proof *a posteriori* that the proper motions in question are real quantities, and not mere computation results, and I proceed, therefore, to consider some of the conclusions that may be derived from these proper motions, subject, of course, to confirmation by similar but more extensive data to be hereafter obtained.

It is an immediate inference from the preceding results that the bright and faint stars, considered as separate groups, are at rest relative to each other, since they have substantially the same relation to the sun's motion. The supposed rotation of stars fainter than the sixth magnitude relative to those brighter than this limit, if real, does not extend to stars fainter than the ninth magnitude.

From the values of X, Y and Z, above given, I obtain as the sun's linear motion 4.805 radii of the earth's orbit per annum. This number depends upon the magnitude of the coefficients in the several observation-equations, and its approximate agreement with CAMPBELL's spectroscopic result, 4.2 per annum, is the required proof that the values of the parallaxes derived by extrapolation from KAPTEYN's formula are not seriously in error. Multiplying the assumed parallaxes by the constant factor 1.14, required to reduce the resulting solar velocity to agreement with the spectroscopic results, I obtain as the mean parallax of the

stars under investigation. $\pi_m = 0''.00499$, corresponding to the mean magnitude 10.5.

Following the methods of KAPTEYN I have resolved the total proper motion of each star into components respectively parallel and perpendicular to the great circle passing through the assumed apex of the sun's way (R.A. = 275°, Decl. = +30°), and have used these components to determine the average linear velocity of the motion of these stars, adopting as the unit in which to express their velocities the number of radii of the earth's orbit traversed in a year. I find thus as the mean velocity of the 68 stars the number 7.2, which is, in part at least fortuitously, in exact agreement with CAMPBELL's mean spectroscopic result for the brighter stars of magnitude approximately 4.0. It does not, however, correspond to his supposition of an increasing velocity of translation for the fainter stars, the average star here considered being at least six magnitudes fainter than the average of CAMPBELL's list, and showing no appreciable difference of velocity.

I have also employed KAPTEYN's method of determining the mean parallax of the stars from the component of their proper motion parallel to the sun's way, and I thus find as the centennial proper motion of the sun, seen from the mean distance of these stars, the so-called secular parallax, the value 1''.656. Dividing this number by 420, CAMPBELL's value of the centennial motion of the sun expressed in radii of the earth's orbit, I obtain as the average parallax of the stars in question, the value 0''.00394. The difference between this value and the preceding result for π_m is due in large measure to the differently assumed positions of the apex of the solar motion. I adopt as a definitive result for the mean parallax of the stars here considered the mean of the two determinations, $\pi_m = 0''.0045$. KAPTEYN's formula furnishes for this group of stars $\pi_m = 0''.0042 \pm$.

I am inclined to regard the adopted value of π_m as a fair approximation to the mean parallax of stars having the same average brightness as those under consideration, but the number 10.5 adopted as representing the mean magnitude of these stars must be considered as subject to revision, since it is derived solely from the casual estimates of brightness made by the double-star observers above named. As a basis for this revision Mr. J. A. PARKHURST has undertaken to determine at the Yerkes Observatory the photometric brightness of these stars, but this data is still far from being available, and provisionally I am constrained to assume that the average of these ocular estimates of brightness made by eight different astronomers, does not differ widely from the photometric standard, and that the average magnitude of the stars is fairly represented on the photometric scale by the number 10.5.

Associating this number with the value of π_m above derived, I proceed to inquire their relation to a concept of the stellar system represented by the following two hypotheses: *A*. That the fainter stars are of equal intrinsic brilliancy with the brighter ones: *B*. That the illumination furnished by a given star varies inversely as the square of the star's distance. From these hypotheses and the assumed value of the light ratio, $\rho = (100)^{\cdot 2}$, it follows immediately that a diminution of five magnitudes in brightness corresponds to a ten-fold increase of distance. KAPTEYN has determined as one of the chief results of the paper above cited, that the mean parallax of the stars of magnitude 5.5 considered without distinction of spectral type, is $\pi_{5.5} = 0''.0158$, and from this we find by application of the principle just stated that the mean parallax of the stars here under investigation should be $\pi_{10.5} = 0''.0016$, *i.e.*, but little more than a third of the observed value of this quantity. This discordance will be still further increased if we make the common assumption that among the more distant and fainter stars, spectra of the first type are relatively more numerous than among the brighter stars. After making all due allowance for uncertainty in the data employed, the discordance between the observed and the concluded mean parallax appears too great to be attributed either to errors of determination or to a fortuitous selection of non-representative stars. In the absence of some such explanation it seems necessary to modify at least one, and possibly both of the hypotheses *A* and *B*. The necessity for such modification is indeed implicitly recognized in KAPTEYN's formula for the mean parallax of the stars of a given magnitude, *m*, brighter than 9.0, contained in the volume already cited.

$$\pi_m = 0''.0158\,(0.78)^{m-5.5}$$

This formula, although professedly an empirical one, and devoid of theoretical basis, is grossly inconsistent with the two hypotheses in question, but may be reconciled with either of them taken alone.

If the inconsistencies here presented are to be removed by the abandonment of hypothesis *A*, there must be substituted in place of the statement: The fainter stars are of equal intrinsic luminosity with the brighter stars, — its antithesis: The intrinsic luminosity of stars diminishes with their apparent brightness in such a ratio that a star of the tenth magnitude possesses only one-tenth of the luminosity of a star of the fifth magnitude. If such a diminution of luminosity be admitted we must regard the stellar system as a cluster, of probably finite extent, in which the central bodies are of very considerable intrinsic brightness, but with increasing distance from the center this brilliancy diminishes rapidly, and probably tends toward zero at the limits of the system. While this concept of the stellar system is not inconsistent with the data in hand, it is certainly not established by that data, since

our defective theory may equally well be brought into harmony with the observed parallaxes through a modification of the alternative hypothesis *B*.

The ordinary physical law repesented by this hypothesis, *B*, obtains with entire rigor only in case of an infinite transparency of the medium through which the light is transmitted, and there is abundant indication in the known presence of meteoric matter, and in the faint nebulous background covering much of the sky, that the conditions for infinite transparency are not satisfied in the celestial spaces. Even in the absence of such gross matter one might well hesitate at postulating the existence of an ethereal medium that should transmit energy in absolutely undiminished amount over a space many millions of times greater than the radius of the earth's orbit.

I am aware that the so-called absorbing medium in space has been made the subject of several investigations based upon an enumeration of the stars of different orders of magnitude, and that these investigations have uniformly failed to establish its existence. It is perhaps not too much to affirm that they have equally failed to demonstrate its absence, and that at present the question of such a medium is wholly an open one, beyond the probability of solution by the methods hitherto employed. I have therefore considered it legitimate to inquire what modification of hypothesis *B* is required to reconcile the observed parallaxes with hypothesis *A*, *i.e.*, what amount of absorption of starlight per unit of distance traversed will bring $\pi_{5.5}$ into harmony with $\pi_{10.5}$, and with the observed mean parallaxes of stars of other magnitudes.

Assuming as a rough approximation that the absorption is of uniform amount in all directions, and that it follows the ordinary exponential law, it is evident that the fainter stars must have a preponderant influence in determining its amount, since they are far more largely influenced by it than are the brighter and nearer ones. Numerically, the absorption is represented by a constant, ϵ, such that $1-\epsilon$ measures the amount of light transmitted through a stratum of space equal in thickness to the radius of the earth's orbit. Using all of KAPTEYN's data (*Publications, Groningen*, No. 8, p. 12) reduced to CAMPBELL's value of the solar motion, and combining with it my own value of $\pi_{10.5}$, I find from a least-square solution that the value of the constant is $\epsilon = 8{\cdot}10^{-8}$, and corresponding to this value and to the assumption of uniform average luminosity among the stars of different magnitudes, I obtain the following representation of the observed data. The symbol v contained in the last column is the residual obtained by KAPTEYN in the application of his formula to the more limited data employed by him.

The numbers in the last column would have been somewhat smaller had KAPTEYN employed CAMPBELL's value of the sun's velocity.

COMPARISON OF OBSERVED AND COMPUTED MEAN PARALLAXES, FOR $\epsilon = 8{\cdot}10^{-8}$.

m	Obs'd π	Comp. π	O—C	v
	"	"		
2.7	0.0355	0.0318	+0.0037	+0.0067
4.4	.0191	.0196	— 5	— 10
5.4	.0136	.0144	— 8	— 20
6.0	.0120	.0112	+ 8	+ 5
6.9	.0083	.0089	— 6	— 7
8.6	.0059	.0062	— 3	+ 4
10.5	.0045	.0043	+ 2	. . . .

The accuracy with which the data from the second to the eleventh or twelfth magnitude is represented by the hypothesis of an extremely minute extinction of light in its transmission through space, leaves little to be desired, save perhaps in the case of stars of the third magnitude and brighter. The discordance here found agrees in sign with the result that should follow if, as is supposed on other grounds, the sun is a member of a limited group of stars relatively near to it. On the hypothesis of such a progressive absorption of light the relation between parallax, π, and magnitude m is of the form

$$\log \pi_m = C - \frac{m}{5} + \frac{a\epsilon}{\pi_m}$$

where a is a known coefficient, and C and ϵ are constants to be determined. Introducing the values of these constants furnished by the preceding least-square solution, I obtain the following formula from which the values of the computed π, above given, were derived.

$$\log \pi_m = 8.93 - \frac{m}{5} + \frac{0''.00357}{\pi_m}$$

This equation, although transcendental in form, is readily solved by trial, and represents the provisional result of the present investigation in so far as it relates to the mean relation between stellar magnitude and parallax.

If the parallaxes be converted into linear distances, and these distances be plotted as ordinates with the corresponding stellar magnitudes as abscissae, it will be found that the resulting locus is very nearly a straight line, and corresponding to this we have the somewhat striking result that from the second to the twelfth magnitude the relation between magnitude (m) and distance (d) is very approximately represented by the equation $d = 4(m-1)$. The unit of distance here employed is a million times the radius of the earth's orbit.

I have used this relation to express approximately and in simple form the amount of absorption of starlight assigned to the several orders of magnitude by the assumed value of the constant ϵ. Transforming this constant so that it shall relate to a unit a million times greater than that hitherto employed, I obtain $\epsilon = 0.077$, and representing by T the amount of transmitted light expressed as a fraction of the light emitted by any star, we have

$$T = (1-\epsilon)^{4(m-1)}$$

With sufficient precision and greater convenience this relation may be approximately expressed in the form

$$\log T = \frac{1-m}{7}$$

from which the values of T shown in the following table have been computed. The data upon which the formula is based extend only to the twelfth magnitude, and beyond this limit the assigned values are purely extrapolations.

Amount of Transmitted Light for $\epsilon = 8 \cdot 10^{-8}$.

m	2	4	6	8	10	12	14	16	18	20
T	0.72	0.37	0.19	0.10	0.052	0.027	0.014	0.007	0.004	0.002

It appears that despite the very small value obtained for the coefficient of absorption, if the existence of this absorption be admitted, the distances of the fainter stars are such as to produce an almost total extinction of their light.

In at least one respect the above determination of a numerical factor representing the amount of the extinction of the light is open to serious objection. If such absorption is produced by gross matter in space the absorption, at least for the remoter stars, will not be uniform, but will be relatively greater in those regions where sparsely strewn matter is abundant, *i.e.*, in the nebulous regions remote from the galaxy. We might well expect to find stars of a given magnitude that are in high galactic latitudes, appreciably nearer to the earth than are stars of similar appearances but situated near the galaxy; and this difference, if one exists, should become more pronounced with diminishing brightness of the star. I have sought to examine this possibility somewhat further by tabulating with the galactic latitude as argument the total centennial proper motion μ of each star in the appended list of proper motions, and by

Table of Observed Proper Motions of Faint Stars.

No.	Mag.	Comp. Star	R.A.	Decl.	Centennial μ A	Centennial μ D	Wt.	No.	Mag.	Comp. Star.	R.A.	Decl.	Centennial μ A	Centennial μ D	Wt.
			h m	°	″	″					h m	°	″	″	
1	11.2	α Androm.	0 3	29	+0.68	−1.76	0.26	35	10.2	σ Bootis	14 30	30	−1.01	+0.02	0.13
2	11.1	42 Piscium	0 17	13	−1.54	−3.55	.33	36	11.7	"	14 30	30	+0.13	−5.34	.09
3	9.8	α Cassiop.	0 35	56	−0.28	−0.53	.29	37	10.8	γ Serpentis	15 52	16	−1.69	+0.20	.13
4	10.9	μ Cassiop.	1 1	54	−1.84	+2.26	.15	38	9.9	ρ Coronae	15 57	34	+0.69	−2.92	.13
5	10.3	6 h Persei	2 7	51	−0.35	−0.33	.12	39	10.3	49 Serpentis	16 9	14	−2.22	+0.51	.10
6	9.6	o Ceti	2 14	−3	−0.27	−0.95	.14	40	10.3	σ Coronae	16 11	34	−0.80	−1.46	.35
7	10.1	θ Persei	2 37	49	+1.70	−0.20	.13	41	9.8	γ Herculis	16 17	19	−2.12	−2.48	.22
8	9.3	41 Arietis	2 44	27	+2.21	−0.63	.13	42	12.0	ω Herculis	16 20	14	−2.01	−0.92	.05
9	11.5	"	2 44	27	−1.43	−0.05	.11	43	9.8	41 Herculis	16 40	6	+0.21	−2.32	.08
10	11.2	"	2 44	27	+0.07	−1.22	.10	44	11.1	60 Herculis	17 1	13	−1.03	+0.26	.09
11	9.5	A¹ Tauri	3 59	22	+0.93	−2.24	.08	45	9.6	72 Herculis	17 17	33	−0.98	−2.88	.16
12	12.3	40 Eridani	4 11	−8	+3.02	−2.73	.12	46	11.4	54 Ophiuchi	17 30	13	−1.05	−1.74	.34
13	10.6	α Tauri	4 30	16	+9.03	−3.94	.29	47	10.6	ψ Draconis	17 45	72	+0.86	−1.09	.07
14	10.3	1 Orionis	4 44	7	−0.48	−0.12	.13	48	10.1	η Serpentis	18 16	−3	−2.49	−0.37	.23
15	9.5	λ Aurigae	5 12	40	+0.60	−0.57	.23	49	10.4	109 Herculis	18 19	22	−0.22	−2.37	.09
16	9.3	111 Tauri	5 19	17	+0.39	+1.07	.17	50	9.5	α Lyrae	18 34	39	+0.07	−0.86	.42
17	9.7	θ Aurigae	5 53	37	−0.77	−1.29	.17	51	9.6	11 Aquilae	18 54	13	−1.05	−1.75	.29
18	10.9	"	5 53	37	−0.20	−2.28	.10	52	11.3	3 Cygni	19 21	25	−0.50	−2.00	.06
19	8.9	56 Aurigae	6 40	44	+2.95	−0.69	.19	53	10.4	Σ 2532	19 25	3	+0.53	−1.19	.28
20	10.2	15 Lyncis	6 49	59	+0.96	+0.04	.14	54	10.3	σ Draconis	19 33	69	−1.56	−0.66	.08
21	12.0	45 Geminor.	7 3	16	−1.88	−0.61	.35	55	11.5	θ Cygni	19 34	50	−1.11	−1.38	.08
22	9.8	α Geminor.	7 28	32	−1.15	−1.19	.16	56	10.2	α Aquilae	19 46	9	+0.05	−0.67	.25
23	12.5	α Can. Min.	7 34	5	−1.49	−2.95	.17	57	11.0	β Delphini	20 33	14	+0.04	−1.97	.33
24	10.8	β Geminor.	7 39	28	−0.15	−0.65	.27	58	11.9	κ Delphini	20 34	10	+1.26	−1.02	.25
25	11.1	"	7 39	28	−0.65	−2.30	.13	59	12.4	ϵ Cygni	20 42	34	−1.35	−0.61	.11
26	9.6	B.A.C. 2751	8 9	60	−0.61	−1.52	.09	60	11.3	56 Cygni	20 46	44	+1.18	−1.85	.08
27	12.3	δ Cancri	8 39	19	−2.31	−5.37	.03	61	10.4	61 Cygni	21 2	38	+2.48	−0.65	.16
28	10.7	θ Hydrae	9 9	3	−3.32	+2.61	.12	62	10.4	δ Equulii	21 10	10	−1.28	−1.89	.45
29	10.5	δ Leonis	11 9	21	+0.04	−0.66	.13	63	10.9	κ Pegasi	21 40	25	+0.57	+0.02	.30
30	9.7	81 Leonis	11 20	17	−0.18	−1.66	.20	64	9.8	15 Cephei	22 1	59	+6.04	−2.54	.06
31	11.5	61 Urs. Maj.	11 36	35	+2.11	−0.01	.13	65	11.3	ι Pegasi	22 2	25	+0.14	−1.84	.14
32	10.9	β Virginis	11 45	2	−0.84	−2.68	.13	66	10.0	Arg. 528	22 46	13	+8.99	+8.43	.14
33	10.9	42 Comae	13 5	18	−1.69	−1.98	.13	67	10.7	λ Androm.	23 33	46	+5.90	−1.18	.04
34	10.8	43 Comae	13 7	28	−2.04	−2.95	0.18	68	8.8	85 Pegasi	23 57	27	+0.42	−0.56	0.33

The numbers in the last column of the above table relate solely to the determination of the relative motions of bright and faint stars, and do not include the uncertainty of μ arising from the probable errors of the assumed proper motions of the comparison stars

similarly tabulating in each case the component of the total motion, μ_A, that is perpendicular to the sun's way, and is therefore unaffected by the solar motion. Grouping the stars, and taking mean results, I obtain the following table:

RELATION OF PROPER MOTION TO GALACTIC LATITUDE.

Galactic N.P.D.	**	Mean m	μ	μ_A	Area
° °			″	″	
0– 50	14	10.6	2.64	1.68	7400
50– 80	18	10.5	2.21	1.11	9700
80–100	15	10.4	1.75	1.05	7000
100–130	18	10.5	3.07	1.70	9700
130– ..	2	10.4	2.40	2.40	. .

The last column of the table represents, in square degrees, the areas of the several zones within which the stars are grouped.

Both the total motion and its resolved component seem to show distinct evidence of passing through a minimum at the galaxy, and if this relation shall hereafter be established by more abundant data its explanation will doubtless be that in the plane of the galaxy, stars of a given brightness are more remote than in the direction perpendicular to that plane. This conclusion tends to fortify the supposition of an absorption by sparsely distributed gross matter, and implies that the milky way is a stratum in which a closer grouping of the stars is accompanied by a relative paucity of interspersed matter.

It must be conceded that the conclusions above drawn from the proper motions of only 68 stars rest upon an all too slender basis, but the manner in which the stars were selected seems to free them in unusual degree from the imputation of a biased choice that might render them not fairly typical of the great body of stars of similar brightness. I cannot at present comprehend why apparent proximity to a bright star of considerable proper motion should impair the representative character of a faint star, or tend to assign to it a proper motion systematically different from that of other stars of similar brightness. In the absence of such a prejudicial effect the chief objection to the inferences above drawn must rest upon the legitimacy of applying the doctrine of chances to so small a body of data. The approximate determination of the sun's way, given by the proper motions of these stars, may furnish some measure of the extent to which the application is justified in the present case; I hope, however, to obtain in the not distant future a more radical test of the conclusions above presented, through the discussion of a second series of similar observations of other stars now in progress.

Washburn Observatory, 1904 March.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY.

By J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			(324) *Bamberga.*							
h m s			m s	′ ″	h m s	° ′ ″			s	″
Aug. 7 12 50 31	1	25 , 5	+2 17.13	1 13.2	22 26 53.29	12 1 59.3	n8.840	0.832	+3.27	+23.5
7 13 8 45	2	25 , 5	+2 11.59	1 7.7	22 26 52.65	12 1 51.3	n8.508	0.833	+3.27	+23.5
9 11 18 1	3	30 , 6	+1 56.95	0 4.5	22 25 22.63	11 50 54.6	n9.360	0.820	+3.31	+23.6
11 11 31 1	4	30 , 6	1 15.99	−2 9.4	22 23 42.25	11 39 28.1	n9.271	0.823	+3.33	+23.6
11 11 57 40	5	30 , 6	+2 5.13	+3 32.1	22 23 40.95	11 39 20.5	n9.136	0.827	+3.31	+23.6
21 10 42 58	6	30 , 6	−1 14.90	+0 21.2	22 11 6.32	10 42 6.4	n9.268	0.818	+3.46	+24.1
22 10 51 47	7	30 , 6	+2 37.88	0 57.3	22 13 3.08	10 56 10.0	n9.202	0.819	+3.47	+24.1
24 10 7 34	8	30 , 6	+1 1.87	+5 46.1	22 10 58.11	10 24 32.5	n9.348	0.812	+3.49	+24.1
			(15) *Eunomia.*							
Aug. 21 10 7 18	9	30 , 6	−0 13.06	9 56.3	21 54 52.91	+0 18 2.4	n9.325	0.738	+3.47	+23.8
22 10 3 23	10	30 , 6	+1 36.41	+4 55.8	21 53 53.89	+0 18 36.5	n9.323	0.738	+3.47	+23.8
23 9 46 11	11	30 , 6	+0 43.60	−10 5.1	21 52 55.00	+0 18 56.5	n9.364	0.738	+3.48	+23.9
23 10 14 6	9	30 , 6	2 12.22	9 5.3	21 52 53.76	+0 18 53.6	9.260	0.737	+3.48	+24.0
24 9 33 53	10	30 , 6	0 21.25	+5 29.5	21 51 56.24	+0 19 10.4	n9.388	0.738	+3.48	+24.0
Sept. 2 9 59 57	12	30 , 6	+2 39.07	+0 56.1	21 43 15.65	+0 15 22.1	n9.066	0.738	+3.49	+24.8
2 10 22 39	13	30 , 6	−2 23.70	3 7.0	21 43 14.67	+0 15 20.1	n8.857	0.738	+3.50	+24.8
3 11 1 40	14	30 , 6	+1 20.13	3 44.6	21 42 18.72	+0 14 21.4	8.333	0.738	+3.50	+24.8
4 11 0 15	12	30 , 6	+0 48.14	1 9.9	21 41 25.02	+0 13 16.5	8.349	0.738	+3.49	+24.9

1903 Washington M.T.	⋆	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
					(110) Lydia.					
h m s			m s	″	h m s	″			s	″
Sept.10 11 27 8	15	29 , 6	−2 3.94	+ 4 55.1	21 59 54.24	−22 27 54.9	8.968	0.884	+3.60	+22.6
14 10 17 27	16	30 , 6	+2 21.85	+ 4 32.0	21 59 13.62	−22 29 11.9	n8.674	0.886	+3.60	+22.4
12 9 10 26	16	30 , 6	+1 12.43	+ 3 25.2	21 58 33.90	−22 30 18.7	n9.251	0.877	+3.60	+22.4
20 9 19 54	17	30 , 6	+1 38.12	+10 20.3	21 53 44.94	−22 32 51.7	n8.920	0.885	+3.56	+21.6
					(56) Melete.					
Sept. 2 10 59 0	18	30 , 6	+1 21.99	+ 1 18.3	22 21 50.09	0 28 26.8	n8.882	0.745	+3.52	+24.7
3 10 28 18	18	30 , 6	+0 44.67	− 8 18.7	22 21 9.77	0 38 3.7	n9.100	0.746	+3.52	+24.8
4 10 22 23	19	30 , 6	−1 47.13	− 0 0.3	22 20 29.28	0 47 51.6	n9.112	0.747	+3.52	+24.8
10 10 36 2	20	35 , 8	−0 4.53	+ 4 32.9	22 16 40.79	− 1 47 36.1	n8.692	0.756	+3.52	+25.2
11 9 24 21	21	30 , 6	+1 49.36	− 4 1.2	22 16 7.71	− 1 57 1.2	n9.252	0.757	+3.52	+25.1
					(192) Nausikaa.					
Sept. 3 14 33 49	22	30 , 6	+1 19.03	− 4 38.8	23 16 56.33	− 4 28 3.4	n9.036	0.778	+3.48	+24.1
4 14 26 31	22	30 , 6	+0 23.19	− 0 13.7	23 15 10.50	− 4 26 38.2	n9.050	0.778	+3.49	+24.2
10 12 0 48	23	30 , 6	+1 8.95	+ 1 23.2	23 9 54.69	− 4 18 44.0	8.154	0.778	+3.53	+24.4
11 10 47 13	23	30 , 6	+0 13.32	+ 2 34.8	23 8 56.07	− 4 17 32.4	n9.087	0.776	+3.54	+24.4
12 9 51 0	24	30 , 6	+0 13.60	+11 9.7	23 7 59.78	− 4 16 19.9	n9.337	0.773	+3.54	+24.5
					(306) Unitas.					
Sept.11 11 47 36	25	30 , 6	+2 18.75	+ 5 57.3	23 7 39.07	−11 53 58.1	6.022	0.832	+3.52	+24.3
12 10 43 42	25	30 , 6	+1 34.15	− 2 57.9	23 6 54.47	−12 2 53.3	n9.078	0.830	+3.52	+24.3
13 9 37 51	26	30 , 6	+2 50.24	− 4 37.7	23 6 10.13	−12 11 38.6	n9.371	0.821	+3.52	+24.3
14 9 10 15	27	30 , 6	+3 1.72	− 0 58.5	23 5 25.01	−12 20 27.4	n9.439	0.817	+3.53	+24.3
15 8 33 8	28	30 , 6	+0 2.76	+ 4 55.6	23 4 40.72	−12 29 1.9	n9.513	0.808	+3.53	+24.3
					(27) Euterpe.					
Sept.12 11 32 16	29	30 , 6	+1 31.98	− 4 4.3	23 36 7.56	− 5 32 50.7	n8.900	0.787	+3.52	+24.0
13 10 22 26	30	30 , 6	+2 33.60	− 0 38.1	23 35 14.15	− 5 38 54.3	n9.308	0.784	+3.52	+24.1
14 10 1 46	30	30 , 6	+1 38.04	− 6 54.5	23 34 18.60	− 5 45 10.7	n9.364	0.783	+3.53	+24.1
15 9 10 6	31	30 , 6	−1 4.01	+ 8 9.2	23 33 23.95	− 5 51 16.3	n9.488	0.777	+3.53	+24.1
18 10 1 54	32	30 , 6	−0 21.43	− 0 11.6	23 30 30.77	− 6 10 22.8	n9.293	0.788	+3.55	+24.3
					(67) Asia.					
Sept.13 11 43 46	33	30 , 6	−2 56.52	− 6 36.0	23 38 50.49	+ 3 55 23.0	n8.733	0.701	+3.60	+23.6
15 9 53 53	34	30 , 6	+0 3.99	− 1 51.1	23 37 19.92	+ 3 37 46.9	n9.385	0.709	+3.61	+23.9
15 10 14 22	35	30 , 6	+2 31.11	− 0 33.3	23 37 19.17	+ 3 37 38.5	n9.316	0.708	+3.61	+24.0
18 11 0 42	36	30 , 6	+1 23.18	− 3 27.6	23 34 56.01	+ 3 9 15.7	n8.967	0.709	+3.61	+24.3
18 11 18 27	37	30 , 6	+0 11.43	+ 5 31.1	23 34 55.20	+ 3 9 9.3	n8.760	0.709	+3.61	+24.3
					(196) Philomela.					
Sept.14 11 42 57	38	30 , 6	+0 25.61	− 7 18.6	23 50 57.42	−12 36 34.4	n8.866	0.835	+3.47	+23.8
15 11 0 38	39	25 , 5	+2 21.54	− 7 58.3	23 50 13.41	−12 40 57.0	n9.171	0.832	+3.48	+23.9
19 11 32 9	40	20 , 4	−3 56.17	+ 3 1.6	23 47 9.50	−12 58 10.3	n8.680	0.838	+3.50	+23.7
24 11 32 46	41	40 , 8	+1 44.11	+ 0 3.3	23 43 21.67	−13 17 11.4	7.173	0.840	+3.53	+23.5
24 11 45 12	42	30 , 6	−2 37.88	+ 4 1.0	23 43 21.30	−13 17 14.8	8.432	0.840	+3.52	+23.5
					(51) Nemausa.					
Sept.15 11 45 44	43	20 , 10	+0 17.57	+ 5 16.7	23 56 32.78	− 0 31 10.8	n8.842	0.745	+3.56	+23.4
18 12 5 4	44	30 , 6	−0 13.24	+ 1 3.6	23 54 3.18	− 1 0 54.9	n7.396	0.750	+3.58	+23.6
19 12 11 30	45	30 , 6	+0 56.23	+ 0 6.6	23 53 12.81	− 1 10 49.5	8.299	0.751	+3.58	+23.7
19 12 27 26	46	30 , 6	−1 3.08	− 3 58.4	23 53 12.44	− 1 10 56.9	8.714	0.751	+3.58	+23.7
21 10 34 18	47	29 , 6	+0 2.78	− 2 3.4	23 51 35.93	− 1 30 0.4	n9.184	0.753	+3.59	+23.8
					(63) Ausonia.					
Sept.20 11 0 9	48	30 , 6	−0 50.11	+ 1 48.3	0 16 18.19	+ 6 15 46.1	n9.200	0.678	+3.66	+22.5
20 11 17 47	49	30 , 6	+0 25.62	+ 3 14.1	0 16 17.49	+ 6 15 43.7	n9.096	0.676	+3.66	+22.5
21 10 54 46	49	25 , 6	−0 33.24	− 0 10.6	0 15 18.64	+ 6 12 19.1	n9.203	0.679	+3.67	+22.6
21 11 39 9	50	30 , 6	+2 27.44	− 9 44.4	0 15 16.74	+ 6 12 11.9	n8.864	0.675	+3.67	+22.7
25 11 39 51	51	30 , 6	+2 30.22	− 5 17.5	0 11 15.47	+ 5 57 33.9	n8.510	0.677	+3.69	+23.2

1903 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(89) *Julia*.					
h m s			m s	′ ″	h m s	° ′ ″			s	″
Sept. 19 10 28 40	52	30 . 8	+0 17.72	− 2 11.5	0 31 36.49	+30 20 43.0	n9.459	0.264	+4.17	+19.0
20 10 4 10	52	24 . 8	−0 42.52	+ 2 14.1	0 30 36.26	+30 25 38.8	n9.512	0.301	+4.18	+19.2
21 10 0 31	53	25 . 5	−4 9.09	+ 0 40.9	0 29 34.70	+30 30 15.6	n9.509	0.296	+4.20	+19.2
25 10 22 0	54	30 . 6	+3 9.98	+ 5 52.5	0 25 15.56	+30 44 37.6	n9.384	0.205	+4.22	+20.9
25 10 51 36	55	30 . 6	+2 41.50	− 2 31.4	0 25 14.07	+30 44 41.5	n9.252	0.154	+4.23	+20.9
					(80) *Sappho*.					
Nov. 15 9 40 58	56	30 . 6	−0 51.74	+ 1 39.7	2 15 51.07	+11 50 53.5	n9.074	0.604	+4.19	+15.5
19 11 1 44	57	20 . 4	+3 38.92	− 5 33.7	2 13 26.61	+11 11 30.2	8.911	0.611	+4.16	+15.9
20 10 9 20	58	30 . 6	+2 11.29	+ 7 56.3	2 12 56.26	+11 2 43.9	n8.200	0.611	+4.16	+15.8
22 9 11 0	58	30 . 6	+1 14.69	− 9 11.2	2 11 59.66	+10 45 36.3	n9.062	0.619	+4.16	+15.7
25 10 54 52	59	30 . 6	+0 10.59	+ 1 27.9	2 10 44.37	+10 20 53.2	9.078	0.625	+4.14	+15.6
25 10 54 52	60	30 . 6	+0 9.83	+ 1 20.2	2 10 44.52	+10 20 53.7	9.078	0.625	+4.14	+15.6
					(23) *Thalia*.					
Nov. 6 9 48 33	61	35 . 7	−0 42.36	+ 3 52.6	2 26 54.57	+ 5 58 20.9	n9.279	0.683	+4.03	+14.2
8 9 37 18	62	30 . 6	−0 46.11	+ 7 5.5	2 24 50.32	+ 5 57 5.4	n9.285	0.683	+4.03	+14.3
10 10 22 29	63	30 . 6	+1 31.15	+ 4 39.4	2 22 44.65	+ 5 56 18.7	n8.941	0.679	+4.04	+14.7
12 10 19 87	63	30 . 6	−0 30.66	+ 4 21.1	2 20 42.85	+ 5 56 0.4	n8.872	0.678	+4.05	+14.7
14 9 47 5	64	30 . 6	+1 7.38	+ 1 43.9	2 18 44.68	+ 5 56 9.7	n9.072	0.680	+4.04	+14.8
14 10 4 54	65	30 . 6	+1 2.07	+ 1 54.2	2 18 44.05	+ 5 56 12.0	n8.920	0.679	+4.04	+14.8
					(28) *Bellona*.					
Nov. 10 11 7 53	66	30 . 6	+0 6.47	+10 18.9	3 6 4.28	+ 2 7 25.3	n8.917	0.720	+4.00	+10.7
14 10 49 50	67	30 . 6	+2 1.75	+ 0 47.7	3 2 36.92	+ 1 51 45.9	n8.906	0.722	+4.02	+10.8
14 11 12 13	68	30 . 6	+1 26.18	− 7 17.8	3 2 36.04	+ 1 51 39.7	n8.556	0.722	+4.02	+10.8
15 10 26 18	69	35 . 7	+0 3.93	+ 5 42.6	3 1 46.00	+ 1 48 16.2	n9.068	0.723	+4.02	+10.6
19 11 37 17	70	30 . 6	−1 21.99	+ 6 30.1	2 58 19.10	+ 1 35 45.7	8.791	0.725	+4.04	+10.4
					(64) *Angelina*.					
Nov. 22 10 22 41	71	29 . 6	+0 42.91	− 3 12.4	2 53 41.62	+18 47 31.3	n8.758	0.479	+4.52	+12.4
25 11 39 54	72	30 . 6	+0 36.74	−10 55.3	2 51 2.09	+18 34 51.2	9.128	0.493	+4.52	+12.8
27 9 40 42	73	30 . 6	+2 30.22	+ 7 29.1	2 49 26.63	+18 27 9.0	n8.977	0.490	+4.50	+13.2
30 8 59 24	73	25 . 5	+0 10.27	− 4 14.0	2 47 6.68	+18 15 25.8	n9.176	0.502	+4.50	+13.1
30 9 16 50	74	29 . 6	−1 17.79	+ 0 53.2	2 47 6.04	+18 15 20.0	n9.060	0.496	+4.50	+13.0
					(14) *Irene*.					
Nov. 30 10 5 27	75	30 . 6	−0 26.63	+ 3 11.2	4 29 5.39	+17 3 10.2	n9.338	0.540	+4.64	+ 1.3
Dec. 3 10 33 53	76	30 . 6	+0 5.06	+ 1 34.8	4 25 52.55	+17 5 19.2	n9.126	0.521	+4.68	+ 1.6
3 10 32 59	77	30 . 6	−0 25.59	+ 5 17.8	4 25 52.61	+17 5 19.2	n9.131	0.522	+4.68	+ 1.5
5 11 27 54	78	30 . 6	+0 25.11	+ 2 37.6	4 23 42.33	+17 6 56.9	n7.130	0.510	+4.70	+ 1.8
5 11 45 21	79	30 . 6	−1 23.94	− 4 19.9	4 23 41.76	+17 6 51.5	8.546	0.511	+4.70	+ 1.6
					(53) *Kalypso*.					
Nov. 20 11 10 33	80	34 . 7	−0 4.83	− 0 22.6	5 6 51.20	+13 48 2.1	n9.257	0.583	+4.36	2.4
20 11 58 21	81	24 . 8	−0 37.81	− 0 15.7	5 6 50.58	+13 47 58.3	n9.465	0.578	+4.35	2.5
22 11 31 32	82	30 . 6	+1 28.75	− 3 27.6	5 5 20.79	+13 14 8.7	n9.255	0.584	+4.40	2.2
26 10 45 56	83	28 . 6	+0 25.88	− 6 20.3	5 2 5.90	+13 37 22.6	n9.560	0.595	+4.46	2.0
Dec. 7 11 6 5	84	30 . 6	+1 30.98	− 2 0.1	4 51 59.72	+13 26 30.1	n8.942	0.578	+4.61	1.4
7 11 23 58	85	30 . 6	+0 59.40	+ 4 51.0	4 51 59.17	+13 26 31.4	n8.709	0.576	+4.60	1.4
11 10 44 4	86	30 . 6	+1 47.42	3 30.9	4 48 13.37	+13 25 56.6	8.967	0.578	+4.65	1.0
					(198) *Ampella*.					
Nov. 26 11 46 20	87	35 . 7	0 56.86	+ 2 26.0	5 9 55.77	+27 38 55.5	n9.452	0.258	+4.95	4.3
27 11 7 22	88	30 . 6	+1 54.34	− 6 16.8	5 8 50.86	+27 33 4.4	n9.330	0.298	+4.98	3.7
Dec. 6 11 4 16	89	29 . 6	−0 25.55	− 0 33.2	4 58 28.50	+26 34 33.8	n9.086	0.287	+5.08	2.4
7 9 51 12	90	28 . 5	+1 32.04	2 24.7	4 57 22.82	+26 27 58.8	n9.418	0.362	+5.09	2.0
7 10 16 36	91	30 . 6	+1 32.89	− 3 45.0	4 57 21.63	+26 27 52.1	n9.325	0.331	+5.09	2.0
11 9 37 7	92	30 . 6	+0 26.13	+ 3 27.7	4 52 50.18	+25 59 51.3	n9.396	0.366	+5.11	1.3
11 9 51 10	93	30 . 6	−0 54.59	6 25.1	4 52 49.71	+25 59 50.2	n9.333	0.346	+5.12	1.5

1903 Washington M.T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(81) *Terpsichore.*						
Dec. 16 13h 10m 24s	94	35 , 7	+0m 3s.29	+ 6′ 9″.2	4h 12m 16s.78	+35° 33′ 56″.6	9.471	0.031	+5s.62	+ 0″.7
17 11 51 46	95	30 , 6	+2 9.52	− 2 17.2	4 41 20.75	+35 31 45.9	9.110	9.776	+5.63	+ 1.2
22 9 8 5	96	29 , 6	1 18.07	− 8 17.0	4 36 19.16	+35 18 27.7	n9.317	9.906	+5.65	+ 1.8
				(5) *Astraea.*						
Dec. 13 9 28 36	97	30 , 6	+0 33.78	+ 0 9.1	5 11 55.22	+14 33 8.3	n9.421	0.591	+1.70	3.9
14 9 14 51	97	30 , 6	−0 26.94	+ 0 20.6	5 10 54.52	+14 33 19.7	n9.418	0.595	+1.72	4.0
22 10 13 13	98	29 , 6	−2 38.86	− 9 23.7	5 2 53.49	+14 38 35.9	n8.990	0.559	+4.79	− 3.7
23 10 23 28	99	30 , 6	+2 40.56	− 3 40.7	5 1 55.91	+14 39 16.3	n8.825	0.556	+1.78	− 2.9
28 8 40 59	100	30 , 6	−0 7.10	+ 9 17.9	4 57 30.08	+14 46 52.0	n9.343	0.576	+1.79	− 3.0
				(111) *Kassandra.*						
Dec. 15 9 18 51	101	30 , 6	−1 17.68	− 3 15.5	5 20 28.09	+15 7 7.6	n9.453	0.588	+1.71	− 5.3
17 10 51 3	102	30 , 6	+1 42.60	+ 7 10.0	5 18 25.00	+15 3 9.2	n8.965	0.552	+1.76	− 4.8
22 11 19 25	103	30 , 6	−1 35.38	− 2 40.6	5 13 32.95	+14 51 53.9	8.891	0.553	+1.80	− 4.8
28 9 41 4	98	30 , 6	+2 14.21	+ 0 9.9	5 8 16.58	+14 48 9.3	n9.078	0.559	+4.81	− 3.9
				(119) *Althaea.*						
Dec. 13 10 5 40	103	30 , 6	−1 8.51	+ 2 21.2	5 13 59.71	+14 59 56.2	n9.308	0.569	+4.72	− 4.3
15 10 18 6	104	30 , 6	+2 19.11	+ 1 10.7	5 12 0.80	+14 58 11.9	n9.206	0.562	+4.74	− 3.9
16 12 19 32	104	29 , 6	+1 15.05	+ 0 21.4	5 10 56.75	+14 57 22.5	8.986	0.554	+4.75	− 4.0
22 11 1 32	98	29 , 6	−0 18.13	+ 6 37.2	5 5 11.22	+14 54 36.8	n7.513	0.550	+4.79	− 3.7
23 11 15 18	105	29 , 6	+1 3.06	− 1 12.5	5 1 18.09	+14 54 25.6	8.547	0.551	+4.79	− 3.4

Mean Places of Comparison-Stars for the beginning of the year.

✱	α	δ	Authority	✱	α	δ	Authority
1	22h 24m 32s.89	−12° 1′ 9″.6	Camb., U.S., A.G.Zones	33	23h 11m 43s.41	+ 4° 1′ 35″.4	Albany, A.G. 8155
2	22 24 37.79	−12 1 7.1	.. ..	34	23 37 12.32	+ 3 39 14.1	 8142
3	22 23 22.37	−11 51 13.7	.. ..	35	23 34 44.45	+ 3 37 17.8	 8131
4	22 24 54.91	−11 42 1.1	.. ..	36	23 33 29.22	+ 3 12 19.0	 8127
5	22 21 32.48	−11 43 16.2	Rad. 1890, 6026	37	23 34 10.16	+ 3 3 13.9	 8130
6	22 15 17.76	−10 42 54.7	Camb., U.S., A.G.Zones	38	23 50 28.31	−12 29 39.6	Camb., U.S., A.G.Zones
7	22 10 21.73	−10 35 36.8	.. ..	39	23 47 18.39	−12 33 22.6	.. ..
8	22 9 52.75	−10 30 43.0	.. ..	40	23 51 2.17	−13 1 35.6	.. ..
9	21 55 2.50	+ 0 27 34.9	Nicolajew, A.G. 5546	41	23 41 34.03	−13 17 38.2	.. ..
10	21 52 14.01	+ 0 13 16.9	 5536	42	23 45 55.66	−13 21 39.3	.. ..
11	21 52 7.92	+ 0 8 27.5	 5534	43	23 56 11.65	− 0 36 50.9	Nicolajew, A.G. 5936
12	21 40 33.09	+ 0 14 1.5	 5506	44	23 54 12.81	− 1 2 22.1	 5926
13	21 45 34.87	+ 0 18 2.3	 5518	45	23 52 13.00	− 1 11 19.8	 5923
14	21 40 51.79	+ 0 17 41.2	½ (Munich I, 29431 + Munich II, 11969)	46	23 54 11.64	− 1 7 22.2	 5925
15	22 1 51.58	−22 33 12.9	Algiers, A.G. Zones	47	23 51 29.56	− 1 28 20.8	 5918
16	21 56 48.17	−22 34 6.3	.. .. ..	48	0 17 1.64	+ 6 13 35.3	Leipzig II, A.G. 97
17	21 52 3.26	−22 13 33.6	Cordoba Vol. 8, 1584	49	0 15 48.21	+ 6 12 7.1	 91
18	22 20 21.58	− 0 30 9.8	Nicolajew, A.G. 5641	50	0 12 45.63	+ 6 21 33.6	 72
19	22 22 12.89	− 0 48 16.1	 5647	51	0 8 41.56	+ 6 2 28.2	 43
20	22 16 38.80	− 1 52 34.2	Newcomb's Fund. Catal.	52	0 31 14.60	+30 23 5.5	½ (Camb., Eng., A.G. 350 + Leiden, A.G. 191)
21	22 14 14.83	− 1 53 25.1	Nicolajew, A.G. 5619	53	0 33 39.59	+30 29 15.5	½ (Camb., Eng., A.G. 37[illegible] + Leiden, A.G. 20[illegible])
22	23 15 13.82	− 1 26 48.7	Rad. 1890, 6239	54	0 22 1.36	+30 38 24.2	Leiden, A.G. 130
23	23 8 39.21	− 4 20 31.6	Strassburg, A.G. Zones	55	0 22 28.34	+30 46 52.0	Leiden, A.G. 135
24	23 7 12.61	− 4 27 54.1	.. .. ..	56	2 16 38.62	+11 48 58.3	Leipzig I, A.G. 685
25	23 5 16.80	−12 0 19.7	Camb., U.S., A.G. Zones	57	2 9 43.53	+11 16 48.0	 660
26	23 3 16.37	−12 7 25.2	.. ..	58	2 10 40.81	+10 54 31.8	 668
27	23 2 19.76	−12 19 53.2	.. ..	59	2 10 29.64	+10 19 9.7	 665
28	23 4 34.43	−12 34 21.8	.. ..	60	2 10 30.55	+10 19 17.9	 666
29	23 34 32.06	− 5 29 10.4	Strassburg, A.G. Zones	61	2 27 32.90	+ 5 54 14.1	Leipzig II, A.G. 944
30	23 32 37.03	− 5 38 40.3	Wien, A.G. Zones	62	2 25 32.40	+ 5 49 45.6	 935
31	23 34 24.43	− 5 59 49.6	.. .. ..	63	2 21 9.46	+ 5 51 24.6	 908
32	23 30 48.65	− 6 10 35.5	.. .. ..	64	2 17 33.26	+ 5 54 11.0	 884

*	α	δ	Authority
	h m s	° ′ ″	
65	2 17 37.94	+ 5 54 3.0	Leipzig II. A.G. 885
66	3 5 53.81	+ 1 56 55.7	Albany, A.G. 911
67	3 0 31.15	+ 1 50 17.1	‟ ‟ 879
68	3 1 5.84	+ 1 58 16.7	‟ ‟ 881
69	3 1 38.05	+ 1 42 23.0	‟ ‟ 889
70	2 59 37.05	+ 1 29 5.2	‟ ‟ 871
71	2 52 54.19	+18 50 31.3	Berlin A. A.G. 796
72	2 50 20.83	+18 45 33.7	‟ ‟ ‟ 786
73	2 46 51.91	+18 19 26.7	‟ ‟ ‟ 772
74	2 48 19.33	+18 14 13.8	‟ ‟ ‟ 778
75	4 29 27.38	+16 59 57.7	‟ ‟ ‟ 1227
76	4 25 42.81	+17 3 42.8	‟ ‟ ‟ 1205
77	4 26 13.52	+16 59 59.9	‟ ‟ ‟ 1207
78	4 23 12.52	+17 4 17.5	‟ ‟ ‟ 1191
79	4 25 1.00	+17 11 12.8	‟ ‟ ‟ 1201
80	5 6 51.67	+13 48 27.1	Leipzig I. A.G. 1550
81	5 7 24.01	+13 48 16.5	‟ ‟ ‟ 1555
82	5 3 47.64	+13 47 38.5	‟ ‟ ‟ 1528
83	5 1 35.56	+13 43 44.9	‟ ‟ ‟ 1511
84	4 50 21.13	+13 28 31.6	‟ ‟ ‟ 1123
85	4 50 55.17	+13 21 41.8	‟ ‟ ‟ 1126
86	4 46 21.30	+13 29 28.5	Leipzig I. A.G. 1397
87	5 10 47.68	+27 36 33.8	Camb., Eng., A.G. 2388
88	5 6 51.54	+27 39 24.9	‟ ‟ ‟ 2350
89	4 58 18.97	+26 35 9.4	‟ ‟ ‟ 2284
90	4 55 15.69	+26 30 25.5	‟ ‟ ‟ 2265
91	4 55 13.65	+26 31 39.1	‟ ‟ ‟ 2264
92	4 52 19.24	+25 56 24.9	‟ ‟ ‟ 2242
93	4 53 39.18	+26 6 17.1	‟ ‟ ‟ 2249
94	4 42 7.87	+35 27 16.7	Lund, A.G. 2369
95	4 39 5.60	+35 34 1.9	‟ ‟ 2352
96	4 38 31.58	+35 26 42.9	‟ ‟ 2345
97	5 11 16.71	+14 33 3.1	Leipzig I. A.G. 1578
98	5 5 27.56	+14 48 3.3	‟ ‟ ‟ 1541
99	4 59 10.60	+14 43 29.9	‟ ‟ ‟ 1483
100	4 57 32.39	+14 37 37.1	‟ ‟ ‟ 1476
101	5 21 41.03	+15 10 28.4	½ (Berlin A. A.G. 1727 + Leipzig I. A.G. 1637)
102	5 16 37.64	+14 56 1.0	½ (Berlin A. A.G. 1486 + Leipzig I. A.G. 1622)
103	5 15 3.53	+14 57 39.3	½ (Berlin A. A.G. 1474 + Leipzig I. A.G. 1609)
104	5 9 36.95	+14 57 5.1	½ (Berlin A. A.G. 1446 + Leipzig I. A.G. 1571)
105	5 3 10.24	+14 58 11.5	½ (Berlin A. A.G. 1404 + Leipzig I. A.G. 1521)

The star places from the Strassburg, Cambridge (U.S.) and Algiers Zones were furnished through the courtesy of the Directors of the observatories at those places.

Planets (15), (27), (67), (51), (63), (89), (80), (23), (81), (5), (114) and (119) were found photographically by Mr. G. H. PETERS.

OBSERVATIONS OF BROOKS'S COMET (1889 V) = *d* 1903.

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY.

By C. W. FREDERICK. [Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.

1903	Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
	h m s			m s	′ ″	h m s	° ′ ″			s	″
Aug. 20	11 37 39	1	t27 , 6	−1 22.66	− 4 55.4	21 1 27.31	27 1 26.5	8.820	0.903	+3.73	+21.2
21	11 20 21	2	t23 , 5	+1 55.95	+ 2 38.7	21 0 13.51	−27 4 13.0	8.577	0.903	+3.74	+20.9
Sept. 13	10 21 33	3	t23 , 5	+1 22.49	− 8 25.3	20 50 5.85	−26 4 37.9	9.111	0.895	+3.59	+18.8
14	10 1 40	3	t30 , 6	+1 16.68	− 3 12.1	20 50 0.04	−25 59 51.7	8.980	0.897	+3.59	+18.8
15	9 17 29	3	t28 , 6	+1 12.70	+ 1 18.5	20 49 56.05	−25 51 51.2	8.864	0.898	+3.58	+18.7
25	9 42 36	1	t30 , 6	−1 5.15	0 16.0	20 51 11.12	−24 56 15.8	9.158	0.890	+3.44	+18.3
Oct. 12	8 23 58	5	t30 , 6	1 15.51	2 19.9	21 1 8.17	22 15 31.1	8.982	0.885	+3.18	+18.2
13	8 0 6	5	d 9 , 8	−0 24.18	+ 6 17.6	21 1 59.48	22 36 53.7	8.710	0.886	+3.16	+18.1
19	7 38 55	6	d 8 , 8	+0 11.65	1 16.8	21 7 15.96	−21 42 1.3	8.640	0.883	+3.07	+18.1
Nov. 8	6 51 13	7	t39 , 8	−1 58.72	10 1.4	21 33 15.18	−18 12 7.4	8.743	0.866	+2.82	+18.7
9	6 32 16	7	d 8 , 8	−0 30.86	+ 1 15.4	21 31 43.03	−18 0 47.6	8.294	0.866	+2.81	+18.7
9	7 13 35	8	t39 , 8	−1 6.55	1 21.3	21 31 45.63	−18 0 24.9	9.024	0.863	+2.81	+18.7
18	7 16 43	9	t26 , 6	+1 26.61	+ 2 50.1	21 48 49.41	−16 12 31.1	9.186	0.851	+2.71	+18.8
19	6 13 12	10	d10 , 10	+0 4.66	2 29.6	21 50 25.28	16 0 19.3	8.949	0.854	+2.70	+19.0
19	7 11 12	11	t33 , 7	−1 7.02	+ 4 15.5	21 50 27.37	16 0 1.8	9.185	0.850	+2.71	+18.8
20	6 14 29	12	t31 , 7	+2 18.10	5 10.7	21 52 4.04	15 47 48.4	8.992	0.852	+2.69	+18.9
Dec. 6	6 21 31	13	d10 , 10	−0 28.29	+ 7 21.4	22 19 56.03	12 16 21.9	9.403	0.831	+2.62	+19.1
7	6 36 15	14	t30 , 6	−2 49.14	1 36.0	22 21 46.38	12 2 26.5	9.186	0.828	+2.63	+19.1
11	6 12 45	15	d10 , 10	+0 32.48	5 17.8	22 29 8.79	11 6 12.5	9.258	0.820	+2.60	+19.1
13	6 12 48	16	t29 , 6	1 3.23	1 56.0	22 32 50.32	10 38 5.2	9.120	0.821	+2.60	+19.0
15	6 33 21	17	t30 , 6	+2 12.41	4 1.4	22 36 36.96	10 9 11.7	9.252	0.815	+2.58	+19.1
17	6 27 16	18	t29 , 6	+2 26.51	0 51.7	22 40 23.57	9 40 16.8	9.245	0.812	+2.58	+19.1
22	6 27 27	19	d12 , 12	+0 8.74	+ 1 20.4	22 49 56.60	8 26 55.8	9.286	0.803	+2.58	+19.0
Jan. 5	6 15 5	20	d12 , 12	0 25.60	3 2.7	23 17 21.79	4 55 31.9	9.432	0.775	0.50	− 1.1
14	6 51 11	22	d12 , 10	+0 2.53	+ 2 57.1	23 35 21.39	2 36 30.7	9.486	0.758	0.47	1.8
15	6 43 9	23	t10 , 3	+2 2.55	4 2.0	23 37 22.08	2 21 16.9	9.472	0.756	0.47	1.7
18	6 59 22	24	t32 , 6	+3 5.47	3 47.7	23 43 26.19	1 34 22.7	9.517	0.750	0.47	1.8
19	6 47 19	25	t26 , 6	+2 9.91	0 25.8	23 45 26.51	1 18 56.1	9.497	0.749	0.46	2.0
19	7 18 6	26	t30 , 6	2 17.97	+ 1 57.4	23 45 29.10	1 18 37.5	9.551	0.748	0.44	2.2
Feb. 3	7 5 50	27	t25 , 6	+0 51.51	+ 1 8.6	0 16 5.01	+ 2 34 7.6	9.575	0.726	0.41	3.2
4	7 1 30	28	t30 , 8	1 29.25	0 54.4	0 18 8.23	+ 2 49 35.8	9.572	0.724	0.40	3.3
6	7 15 2	29	t22 , 6	+0 56.34	+ 2 53.3	0 22 16.44	+ 3 20 27.9	9.593	0.723	0.40	3.4
8	7 23 58	30	t21 , 6	−1 44.55	+ 0 21.5	0 26 24.74	+ 3 51 17.7	9.607	0.722	0.40	3.6
11	7 14 18	31	t38 , 5	−0 29.88	+ 1 8.4	0 32 35.54	+ 4 37 4.5	9.604	0.717	0.38	3.8
15	7 30 16	32	d 4 , 1	−0 13.85	+ 1 1.8	0 40 54.16	+ 5 37 47.8	9.626	0.717	0.38	4.0

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority
	h m s	° ′ ″	
1	21 2 16.24	−26 59 52.3	C.G.C. 28982
2	20 58 43.82	27 7 12.6	C.G.C. 28872
3	20 48 39.77	25 56 31.4	C.G.C. 28662
4	20 52 12.83	−24 56 18.1	C.G.C. 28724
5	21 2 20.50	−22 13 29.1	C.G.C. 28969
6	21 7 31.24	−21 41 16.8	Algiers, A.G. Zones
7	21 35 11.08	18 2 21.7	Algiers, A.G. Zones
8	21 35 19.37	−17 59 22.3	Algiers, A.G. Zones
9	21 47 20.09	−16 15 40.0	Washington, A.G. Zones
10	21 50 17.92	15 58 8.7	Washington, A.G. Zones
11	21 51 31.68	−16 1 36.1	Washington, A.G. Zones
12	21 49 13.25	−15 42 56.6	Washington, A.G. Zones
13	22 20 21.70	−12 24 2.4	Camb., U.S., A.G. Zones
14	22 24 32.89	−12 1 9.6	Camb., U.S., A.G. Zones
15	22 28 33.71	−11 1 13.8	Camb., U.S., A.G. Zones
16	22 33 50.95	−10 36 28.2	Camb., U.S., A.G. Zones
17	22 34 21.97	10 5 29.1	Camb., U.S., A.G. Zones
18	22 37 51.48	− 9 39 11.2	½ (Cambridge, U.S. A.G. Zones + Wien, A.G. Zones)
19	22 49 15.24	− 8 31 34.9	Wien, A.G. Zones
20	23 17 47.89	4 52 28.1	Strassburg, A.G. Zones
21	23 34 27.71	− 2 11 11.4	Strassburg, A.G. Zones
22	23 35 19.33	2 39 26.0	Mic. Comp. with *21
23	23 35 20.00	2 17 13.2	Strassburg, A.G. Zones
24	23 40 21.49	− 1 30 33.2	Nicolajew, A.G. 5887
25	23 43 17.03	1 18 28.3	Nicolajew, A.G. 5895
26	23 47 47.51	1 20 32.7	Nicolajew, A.G. 5902
27	0 15 13.94	+ 2 30 2.2	Albany, A.G. 55
28	0 19 37.88	+ 2 50 33.5	Albany, A.G. 72
29	0 21 20.50	+ 3 17 38.0	Albany, A.G. 79
30	0 24 40.59	+ 3 50 59.8	Albany, A.G. 87
31	0 33 5.80	+ 4 35 59.9	Albany, A.G. 135
32	0 44 8.39	+ 5 33 50.0	Leipzig II, A.G. 253

The first two observations are reprinted from *A.J.* 547. The star places from the Strassburg, Cambridge (U.S.), and Algiers, A.G. Zones, were furnished through the courtesy of the Directors of the observatories at those places.

REMARKS: — *Aug.* 20, Comet difficult; 21, very difficult, poor observation. — *Sept.* 15, Difficult; 25, very difficult, poor observation. — *Oct.* 19, Well seen, good observation. — *Nov.* 8, Rather difficult, comet appeared double, possibly a faint star in the neighborhood; 18, very poor observation; 19, first observation good, second not so good. — *Dec.* 6, Comet brightest of entire season, almost an easy object, nucleus visible at times, but observation rather poor on account of difficulty with the illumination: 13, fairly well seen, good observation: 15, difficult, but good observation: 17, very difficult; 22, very difficult, moonlight, poor observation, comet observed by turning illumination off and on. — *Jan.* 5, Comet invisible half the time, but came out very brightly at intervals, fairly good observation; 14, difficult, poor observation; 15, very difficult, observation almost worthless: 18, 19, Difficult. — *Feb.* 3, Very difficult, poor observation: 6, 8, very poor observations: 11, observation fairly good, a very transparent sky. The comet appeared surprisingly bright for so low an altitude. It was still discernible when five hours down, being then only fifteen degrees above the horizon. *Feb.* 15, Very difficult. Attempts to observe the comet on *Feb.* 16 and 17 proved failures.

THE NEBULAR HYPOTHESIS OF LAPLACE.

Probably this hypothesis has given LAPLACE more popular fame than all his great labors and discoveries in celestial mechanics. LAPLACE is rich in scientific works, and we ought to be fair to him in regard to this hypothesis. In publishing it he says: "*I present this hypothesis with the distrust which everything ought to inspire that is not the result of observation and calculation.*"

1904 *March* 8. A. HALL.

THE MISSING STAR DM. +19°2773.

BY F. KÜSTNER.

The star DM. +19°2773, noted as missing by Mr. DANIEL in *A.J.* 555, has already been earlier so noted by SAFARIK, as also its neighbor +19°2761; compare *A.N.* 2871. SCHÖNFELD has examined the original records of the *Durchmusterung*, and found everything clear and correct, as is also communicated in *A.N.* 2874. By a new revision of the originals I find the data of SCHÖNFELD confirmed, and have only to add that also in zone 386, 1854 March 5, KRUGER covered the place centrally, and that both stars are wanting in it. The place was difficult of observation, since *Arcturus* was in the field of view of the comet-seeker.

CORRIGENDA.

No. 557, p. 41. In dates of Sunspot table, May 26 and June 24, *for* 12h *put* 0h; *for* June 5 11h *put* 4 23h; *for* Dec. 21 11h *put* 20 23h. Author not responsible.

CONTENTS.

PUBLISHED AT 10 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed March 21.*
Entered February 2, 1904, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 559.

VOL. XXIV. **BOSTON, 1904 APRIL 5.** **NO. 7**

OBSERVATIONS OF THE SATELLITES OF *SATURN* IN 1903,

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By C. W. FREDERICK.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

An unusual amount of cloudy weather during the months of June and August reduced the number of observations that could be made. A magnifying power of four hundred diameters was used ordinarily, though for the faint satellites a power of two hundred diameters was sometimes used. A bright wire illumination was employed altogether, as there is no provision for a light field with the 26-inch equatorial.

Position angles were always taken about the inner satellite of each pair. They were measured with the longitudinal wire. In making the separate settings the micrometer was moved alternately forward and backward in position angle when bringing the wire up to coincidence with the pair. Ordinarily eight settings were made, half before and half after the measurements in distance. Also eight measurements of distance were made, usually four on each side of coincidence. The micrometer was always used head upward in order that the spring inside the box might act *with* gravity. Then, when making measurements the fixed wire was always moved *upward* to coincidence with its object, and the movable wire alternately *upward* and *downward*.

The times were taken from a clock in the dome, which is automatically set to Washington Mean Time every day at noon. The time of each measurement was noted to the nearest five seconds, so the mean time of an observation should be correct within about two seconds.

The micrometer equivalent adopted for the reductions was $9''.9316 - 0''.00005\,(t^\circ - 50^\circ F.)$. This was derived from the determinations made by Prof. T. J. J. SEE, *Publications of the U.S. Naval Observatory*, Second Series, Vol. III, page A, VI.

The measurements were corrected for refraction. Tables of differential refraction were computed for eighteen, and twenty degrees south declination. The first table was used up to July 10, the second from this date on to the end of the season. Interpolations were sometimes made between the two tables. The effect of temperature was allowed for by diminishing a given correction one per cent for each 5° *F.*, above 50° *F.*, the temperature for which the tables were computed.

The orientation of the micrometer was made the subject of careful study. The variation of the parallel produced by the instrumental constants was computed by the formula given in *A.J.* 552, page 224, eq. (7). This formula gives the deviation, λ, from a parallel, p_m, determined by the trail of an equatorial star near the meridian when the telescope is east of the pier. The values of λ were computed for nineteen degrees south, and every thirty minutes in hour-angle. Position angles were then corrected by the formula,

$$p = p' + \lambda - p_m$$

in which p' is the observed, and p the corrected position angle. The value of p_m was adopted from a consideration of all the parallel determinations made through the entire season. The micrometer was not removed or disturbed in any way, except on June 30, when the box was removed by unfastening one of the guides. It was replaced without greatly disturbing the orientation. Parallel determinations were usually made by trailing *Vega* on the longitudinal wire (the wire used in measuring position angles). For their reduction we have

$$p_m = p_1 + \lambda + \Delta r + \Delta h - 90^\circ$$

in which p_1 is the observed parallel, Δr is a correction for refraction, and Δh is a correction for the motion of *Saturn*. The following results were obtained.

Dates	No. Determ.	(p_m)	Mean of Resid.	Adopt Index Corr. ( )
May 9 to June 3	8	+0.192	±0.008	
June 20 to June 29	5	+0.212	±0.009	+0.20
June 30 to Sept. 22	21	+0.201	±0.007	
Sept. 22 to Oct. 14	5	+0.177	±0.004	+0.18

In measuring position angles only one vernier was read, therefore it was necessary to apply a correction for eccentricity. The maximum value of this correction was 0.01

In the last column of the printed results is given the state of the seeing as to its probable effect on the accuracy of the measurements. The following abbreviations are used, *b* = bad seeing, *p* = poor, *f* = fair, *g* = good, *e* = excellent. Other remarks are indicated by a number, referring to notes printed at the end of the observations.

Tethys–Rhea.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	May 8	16 18 43	95.20	16 17 42	43.07	*f*1	27	Aug. 9	10 33 49	312.62	10 34 0	54.70	*f*
2	10	16 19 18	287.15	16 19 18	84.79	*p*1	28	11	10 11 42	95.85	10 11 30	103.68	*b*
3	13	16 2 22	151.13	16 2 20	38.71	*g*	29	17	10 37 20	228.30	10 38 7	29.28	*f*3
4	21	15 15 8	77.13	15 14 35	30.23	*f*	30	18	10 2 38	289.59	10 3 11	107.35	*f*
5	June 2	15 36 25	313.53	15 36 21	37.73	*g*	31	20	9 55 16	93.43	9 55 12	43.63	*b*
6	3	14 15 33	353.16	14 15 20	13.95	*p*	32	21	9 33 43	160.54	9 33 50	43.50	*f*
7	15	15 23 15	285.14	15 23 11	74.80	*p*	33	22	10 22 18	285.38	10 22 12	98.72	*b*
8	18	13 50 25	136.79	13 50 23	46.50	*f*	34	23	10 30 40	334.60	10 30 52	13.10	*b*
9	20	14 44 32	279.13	14 44 57	105.55	*g*	35	24	9 52 26	76.74	9 52 14	74.98	*b*
10	21	12 54 43	65.00	12 54 39	60.07	*p*	36	Sept. 2	8 28 26	71.67	8 28 18	27.34	*f*
11	29	13 10 2	273.68	13 10 0	15.14	*f*	37	3	8 15 3	109.21	8 14 58	99.50	*g*
12	30	12 24 6	341.34	12 23 25	39.39	*g*	38	4	8 30 8	267.08	8 30 24	95.10	*f*
13	July 2	15 35 18	143.65	15 35 32	14.69	*b*	39	10	8 20 52	320.92	8 21 11	57.65	*f*4
14	6	14 32 59	101.91	14 32 53	90.10	*p*	40	11	8 8 8	97.72	8 8 21	69.70	*f*
15	7	13 27 14	254.01	13 27 12	79.57	*p*	41	12	7 51 8	100.60	7 51 17	58.38	*p*
16	9	13 32 30	312.83	13 33 16	34.76	*e*	42	13	7 47 38	243.33	7 47 43	65.90	*g*
17	15	11 58 12	107.24	11 57 54	29.57	*p*	43	14	7 21 5	300.19	7 21 11	61.75	*g*
18	18	12 7 27	70.64	12 7 9	26.38	*b*	44	15	7 48 20	162.89	7 48 14	15.06	*f*
19	21	10 52 26	278.08	10 52 22	71.14	*f*	45	20	7 36 38	88.39	7 36 51	105.98	*f*
20	22	11 55 46	282.61	11 55 20	63.67	*b*	46	21	7 38 34	141.27	7 38 12	21.01	*p*
21	23	10 59 23	69.58	10 59 15	73.90	*b*	47	25	8 57 40	85.61	8 57 35	57.51	*g*
22	24	11 30 53	125.90	11 30 51	51.08	*f*	48	26	7 31 51	105.35	7 31 49	76.62	*f*
23	27	10 58 22	55.75	10 58 12	48.26	*b*	49	29	7 35 53	69.27	7 36 15	68.75	*g*
24	28	12 26 15	91.75	12 26 11	35.13	*g*	50	30	7 31 26	128.70	7 31 28	58.73	*f*
25	30	11 43 56	272.23	11 43 38	114.78	*f*2	51	Oct. 7	6 54 34	1.28	6 54 22	21.95	*f*
26	Aug. 5	10 48 3	335.74	10 48 0	47.75	*b*	52	14	6 31 38	154.08	6 31 43	12.40	*g*

(1) Comparisons 10, 10. (2) Magnifying power 200. (3) Clouds interrupt. (4) Comparisons 8, 9.

Tethys–Dione.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	May 9	16 10 23	102.18	16 9 46	67.29	*p*1	19	July 24	11 21 31	279.21	11 21 20	61.62	*f*
2	10	15 51 53	271.21	15 51 48	27.19	*p*1	20	27	11 44 20	66.55	11 42 42	46.37	*b*
3	13	16 21 4	256.24	16 21 3	61.19	*g*	21	28	12 38 27	201.16	12 38 20	13.83	*g*
4	June 2	15 18 28	31.88	15 18 19	27.36	*f*	22	30	13 17 7	358.02	13 16 10	21.45	*p*2
5	3	14 33 36	130.97	14 33 45	23.79	*p*	23	Aug. 9	10 44 18	292.75	10 44 9	42.97	*p*
6	18	14 7 47	270.04	14 7 47	42.22	*f*	24	17	10 50 15	303.33	10 50 23	16.78	*f*
7	20	13 19 16	231.71	13 19 29	38.34	*p*	25	18	10 13 39	312.89	10 13 10	45.31	*f*
8	21	14 0 40	355.17	14 0 39	18.79	*p*	26	20	10 40 2	285.17	10 40 26	88.52	*f*
9	29	12 56 32	259.37	12 56 33	26.94	*g*	27	21	9 43 26	94.55	9 43 34	62.70	*p*
10	30	12 38 55	44.32	12 38 40	30.02	*f*	28	22	10 58 57	249.84	10 59 0	9.48	*b*3
11	July 6	14 50 43	99.13	14 50 50	76.97	*p*	29	23	10 45 18	271.09	10 46 28	40.59	*b*
12	7	13 41 14	268.59	13 41 8	90.52	*b*	30	24	10 1 24	75.48	10 0 58	56.35	*b*
13	8	14 50 55	82.48	14 51 4	77.30	*f*	31	Sept. 2	8 19 14	243.40	8 19 4	38.04	*f*
14	9	14 36 1	241.52	14 36 11	34.54	*b*	32	3	8 5 8	334.34	8 5 0	10.35	*g*
15	15	11 45 56	272.23	11 46 12	75.37	*p*	33	10	8 55 20	97.61	8 55 26	25.97	*f*
16	18	11 57 50	249.30	11 58 5	40.13	*p*	34	11	8 19 38	261.82	8 19 50	55.59	*g*
17	21	11 3 1	296.32	11 3 2	60.73	*g*	35	12	8 23 51	64.43	8 23 40	53.70	*b*
18	23	10 30 16	126.34	10 31 21	9.33	*b*	36	20	8 23 19	81.45	8 23 33	61.69	*f*

(1) Comparisons 10, 10. (2) Power 200. (3) Comparisons 9, 8 clouds.

Dione–Rhea.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	May 28	15 28 13	270.31	15 29 8	115.31	*f*	5	July 6	13 11 48	121.77	13 11 41	14.44	*p*1
2	June 15	15 42 10	282.58	15 42 24	24.63	*p*	6	9	15 29 41	22.82	15 29 20	43.63	*b*
3	21	14 21 14	87.49	14 21 21	64.71	*p*	7	15	12 18 4	97.34	12 17 48	104.98	*b*
4	30	13 23 14	301.76	13 24 0	32.93	*p*	8	20	11 6 47	170.60	11 6 44	7.58	*b*

Dione Rhea.—Cont.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
9	July 21	11 13 12	224.50	11 13 12	22.92	p	19	Aug. 21	10 32 0	238.75	10 32 10	65.78	b
10	22	12 10 5	283.81	12 10 21	101.87	b	20	22	11 10 21	290.51	11 9 31	87.00	b
11	23	10 45 12	61.54	10 45 18	67.63	b	21	23	10 57 44	72.90	10 57 25	35.95	p
12	24	11 40 54	112.03	11 40 40	109.08	b	22	Sept. 2	8 37 11	68.11	8 37 22	67.15	f
13	27	11 59 27	18.71	11 58 34	7.21	b	23	3	8 29 19	113.82	8 29 33	105.12	g
14	28	13 13 2	75.18	13 13 41	44.60	f	24	4	8 59 28	263.97	9 0 3	109.90	f
15	30	13 38 28	264.83	13 38 11	116.24	p2	25	10	8 15 36	309.73	8 45 11	74.91	f
16	Aug. 9	11 8 55	7.20	11 8 44	19.92	b	26	11	8 30 9	91.03	8 29 37	123.48	g
17	17	11 32 27	255.21	11 32 21	16.52	g	27	12	8 35 20	165.18	8 35 5	35.41	f
18	20	10 51 24	101.38	10 51 46	133.60	p	28	20	8 12 28	100.33	8 12 15	47.76	f

(1) Comparisons 9, 8. (2) Power 200.

Enceladus–Tethys.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	June 3	15 21 53	92.67	15 22 21	70.88	f	16	Sept. 4	9 56 45	89.13	9 58 8	39.34	p6
2	18	14 29 28	81.42	14 29 25	58.17	p1	17	10	9 41 19	135.68	9 42 0	36.82	p7
3	20	14 58 57	77.98	15 1 34	5.48	p2	18	11	9 2 10	40.86	9 2 31	31.61	g1
4	29	13 57 10	34.53	13 57 16	17.49	b1	19	12	8 48 35	262.58	8 48 15	43.52	f
5	30	15 23 2	124.76	15 23 13	6.20	g3	20	13	8 22 48	84.90	8 22 22	10.22	g8
6	July 6	13 30 34	160.57	13 39 37	16.23	p4	21	14	7 49 41	133.13	7 49 50	20.80	g
		13 47 35	168.17	. . . . .	. .		22	15	9 15 28	63.26	9 15 38	38.57	p
7	15	13 10 5	116.33	13 10 26	6.58	p1	23	18	8 18 7	290.77	8 48 1	11.02	b9
8	21	11 56 39	118.06	11 56 29	32.54	g	24	20	7 50 27	269.70	7 50 27	67.57	f
9	23	11 52 9	257.18	11 52 24	55.80	p1	25	21	8 39 34	107.74	8 38 17	69.45	p
10	24	11 57 9	94.62	11 56 58	50.03	p1	26	22	9 5 50	325.17	9 5 48	24.18	p
11	28	12 16 22	100.49	12 16 16	77.48	g	27	25	7 23 58	117.47	7 23 59	59.18	g3
12	30	11 25 20	98.93	11 21 15	7.90	f5	28	26	8 21 56	335.81	8 23 4	30.22	b2
13	Aug. 17	10 23 33	287.13	10 22 10	72.16	f1	29	29	7 6 52	113.62	7 6 34	30.34	f
14	Sept. 2	9 49 50	116.62	9 49 42	39.63	f	30	30	7 9 9	336.01	7 8 16	28.21	p
15	3	9 6 40	299.67	9 6 56	9.07	f1	31	Oct. 14	6 55 52	101.34	6 55 16	34.35	f

(1) *Enceladus* very difficult. (2) Comparisons 4, 4: clouds stop observation. (3) Disk of *Saturn* obliterated by pasting a strip of black paper to the metal rim in front of the eyepiece. (4) Comparisons 4, 8, 4. (5) Comparisons 10, 8; power 200. (6) Power 200. (7) Difficult; power 200; cannot see *Enceladus* with higher power. (8) Comparisons 8, 9. (9) All other measures on this date were thrown away.

Mimas–Tethys.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	July 28	11 31 30	96.81	11 31 0	72.24	g1	3	Sept. 14	9 29 28	235.25	9 31 34	7.05	g3
2	28	11 51 55	97.62	11 51 0	72.17	g2							

It was intended to combine *Mimas* with *Rhea* and *Dione* in addition to *Tethys*, but the weather conditions prevented a series being obtained. (1) *Mimas* very difficult; power 400. (2) Difficult; power 200. (3) Difficult; power 170.

Tethys Titan.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	May 13	16 36 45	306.71	16 36 16	73.39	g1	24	Aug. 20	10 6 57	78.45	10 7 18	135.40	b
2	20	15 46 21	111.88	15 45 39	163.33	b2	25	21	9 53 13	98.05	9 52 50	213.45	b
3	June 2	15 55 31	91.27	15 55 27	210.13	f	26	22	9 58 47	97.26	9 58 10	164.90	f
4	3	14 57 47	102.63	14 57 16	117.17	b	27	23	10 7 15	114.78	10 7 32	173.16	f
5	21	14 13 45	111.60	14 13 35	160.25	p	28	24	9 43 21	118.07	9 43 20	106.58	f
6	29	13 21 14	297.69	13 21 13	89.78	f	29	Sept. 2	8 18 7	315.26	8 48 11	95.07	f
7	30	14 1 55	325.05	14 1 44	91.48	g	30	3	8 51 58	61.39	8 52 10	105.14	g
8	July 6	14 8 9	103.49	14 8 21	192.10	p	31	4	9 10 15	58.95	9 10 42	87.87	[illegible]
9	7	14 6 11	133.51	14 7 9	95.45	b	32	10	8 9 13	160.51	8 9 36	53.40	f
10	9	14 14 56	231.00	14 14 40	92.05	b3	33	11	8 46 42	211.70	8 46 49	85.89	[illegible]
11	13	13 12 33	279.93	13 12 37	226.40	b	34	12	8 0 41	255.10	8 0 42	102.37	f
12	15	14 35 32	296.18	14 35 35	161.36	f	35	13	7 59 0	259.29	7 58 57	168.17	[illegible]
13	18	12 19 31	81.00	12 19 13	126.34	b	36	14	7 30 37	276.80	7 31 1	159.45	g
14	21	14 26 28	95.89	14 26 40	189.47	g	37	15	7 27 4	276.32	7 28 38	200.05	[illegible]
15	23	11 35 0	112.58	11 34 47	137.21	p	38	20	7 26 10	79.60	7 26 21	160.67	f
16	24	12 12 13	173.31	12 11 40	73.23	g	39	24	7 18 31	81.12	7 18 26	118.21	[illegible]
17	27	11 11 11	259.57	11 11 27	140.90	b	40	25	8 39 9	127.67	8 39 8	81.51	[illegible]
18	28	13 24 4	273.52	13 24 20	226.60	f	41	26	7 7 29	150.49	7 7 2	94.05	f
19	30	12 4 29	286.79	12 4 14	201.72	f4	42	29	6 53 53	269.15	6 53 29	149.40	[illegible]
20	Aug. 9	10 55 18	135.77	10 56 31	76.54	p	43	30	6 52 57	268.09	6 53 7	181.01	[illegible]
21	11	10 24 25	244.22	10 24 35	79.91	b	44	Oct. 7	6 31 17	88.19	6 31 42	198.53	f
22	17	11 2 30	19.22	11 2 30	54.81	p	45	14	8 2 23	259.25	8 2 38	115.76	f
23	18	9 50 16	5.52	9 50 6	65.60	f							

(1) Daylight. (2) Comparisons 7, 8. (3) Seeing suddenly become bad. (4) Power 200.

Rhea–Titan.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s		h m s	″			1903	h m s		h m s	″	
1	May 10	16 35 6	276.29	16 39 26	95.79	*g*1	25	Aug. 21	10 17 56	87.09	10 18 3	201.75	*p*
2	19	15 32 13	103.24	15 31 43	235.00	*b*	26	22	10 9 28	100.23	10 9 37	264.70	*f*
3	28	15 45 23	326.97	15 15 5	53.09	*f*2	27	23	10 17 30	117.43	10 17 56	182.15	*f*
4	June 2	16 15 51	97.43	16 15 19	236.62	*f*3	28	24	10 12 10	163.08	10 12 24	69.93	*p*
5	3	15 9 43	107.77	15 10 1	151.33	*p*	29	Sept. 2	8 58 52	302.21	8 58 48	110.41	*f*
6	15	16 1 38	69.72	16 1 18	122.09	*p*	30	3	8 40 22	1.51	8 40 32	80.05	*g*
7	21	15 1 2	135.14	15 1 2	111.76	*f*	31	4	9 21 15	74.58	9 21 40	178.77	*p*
8	29	13 38 48	319.02	13 37 46	51.34	*f*	32	6	9 41 40	103.65	9 41 25	154.60	*b*
9	July 6	15 15 34	105.17	15 16 0	101.68	*p*	33	10	8 34 47	150.76	8 34 56	108.46	*f*
10	7	14 33 14	107.20	14 34 18	158.70	*p*	34	11	9 17 12	241.50	9 17 6	129.95	*g*
11	9	14 59 26	216.10	15 0 12	96.25	*b*4	35	12	8 10 52	264.51	8 11 13	157.65	*p*
12	15	12 40 27	296.08	12 40 24	185.77	*p*	36	13	8 8 42	268.86	8 8 30	104.31	*g*
13	20	11 16 59	83.99	11 16 55	161.58	*b*	37	14	7 40 20	263.37	7 40 37	107.28	*g*
14	21	11 38 32	96.74	11 38 25	258.44	*g*	38	15	7 37 30	272.38	7 37 36	195.09	*f*
15	23	11 12 21	144.91	11 12 32	94.50	*p*	39	20	7 15 18	64.47	7 15 56	57.83	*f*
16	24	12 25 40	213.12	12 25 12	51.46	*f*	40	21	7 27 21	71.29	7 27 6	109.38	*p*
17	27	11 27 0	252.92	11 27 15	158.88	*b*	41	25	8 49 0	170.84	8 49 14	54.68	*g*
18	30	12 57 15	304.15	12 57 32	94.11	*f*5	42	26	7 20 31	206.01	7 20 46	68.70	*f*
19	Aug. 7	11 1 50	102.32	11 1 48	124.91	*b*	43	29	7 46 12	262.98	7 46 47	213.06	*g*
20	9	11 22 18	136.44	11 22 7	127.28	*p*	44	30	7 40 18	278.02	7 40 12	232.68	*f*
21	11	10 37 32	262.53	10 37 42	177.18	*b*	45	Oct. 7	6 41 45	94.87	6 42 0	199.38	*f*
22	17	11 17 34	31.45	11 17 0	82.78	*g*	46	12	7 33 38	173.67	7 34 10	88.74	*b*
23	18	10 26 18	75.16	10 26 20	112.25	*f*	47	14	6 43 10	261.90	6 42 57	124.67	*g*
24	20	10 18 54	71.61	10 18 53	93.84	*p*							

(1) Comparisons 4, 8. Daylight stopped measure. (2) Comparisons 7, 8. (3) Daylight. (4) Clouds interrupt. (5) Power 200.

Tethys–Iapetus.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	July 6	12 46 30	176.39	12 47 30	37.69	*p*	10	Aug. 17	10 6 36	75.12	10 6 50	150.10	*g*
2	7	12 52 50	212.02	12 52 35	109.29	*p*	11	18	9 39 0	61.50	9 38 42	116.28	*f*
3	8	14 9 48	239.29	14 9 56	80.76	*b*	12	20	10 28 28	75.59	10 28 43	205.60	*f*
4	9	12 43 4	258.00	12 42 24	193.74	*e*	13	21	10 6 10	87.04	10 7 19	290.74	*b*
5	13	12 45 15	267.45	12 45 50	349.27	*b*	14	Sept. 21	7 52 40	183.91	7 51 30	54.47	*b*
6	15	11 16 3	269.22	11 45 54	412.89	*p*1	15	22	7 44 40	152.43	7 44 47	80.35	*p*
7	18	11 29 9	267.26	11 29 19	438.01	*p*2	16	24	7 18 21	216.59	7 18 16	84.47	*b*
8	21	12 14 46	272.57	12 14 37	512.86	*g*2	17	25	7 7 38	256.86	7 7 18	155.52	*f*
9	23	12 11 51	272.97	12 12 31	523.37	*p*2	18	26	6 58 2	245.97	6 58 1	153.22	*g*

(1) Slid eyepiece back and forth. (2) Single distances (measurements all on one side of coincidence); slid eyepiece.

Titan Iapetus.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	July 2	14 54 8	157.82	14 53 52	94.23	*b*	23	Sept. 2	9 24 34	99.98	9 24 6	553.80	*f*1
2	6	12 21 30	270.92	12 21 16	180.20	*p*	24	3	9 35 55	100.94	9 36 38	480.65	*f*7
3	7	13 9 57	275.45	13 9 41	174.75	*p*	25	10	9 20 40	89.65	9 21 0	401.82	*p*2
4	8	14 30 11	278.18	14 30 42	151.44	*p*	26	11	9 47 5	91.26	9 47 15	445.78	*g*2
5	9	13 2 9	278.02	13 2 4	124.02	*e*	27	12	9 10 14	93.34	9 10 30	479.88	*f*8
6	13	12 20 18	246.78	12 20 16	134.98	*p*	28	13	8 40 33	95.98	8 40 48	496.87	*g*2
7	15	10 52 20	253.93	10 53 40	270.24	*b*	29	14	8 3 48	99.09	8 4 0	491.73	*g*2
8	23	12 40 11	277.22	12 39 45	655.07	*p*1	30	15	8 6 43	102.88	8 7 0	458.24	*f*2
9	24	12 51 53	278.55	12 52 41	596.94	*f*1	31	19	8 9 24	150.89	8 9 34	136.57	*b*
10	27	12 15 37	277.82	12 16 2	408.70	*f*2	32	20	7 3 44	198.38	7 4 22	105.52	*p*
11	28	13 40 35	275.15	13 40 6	370.54	*f*2	33	21	7 8 6	238.30	7 8 23	140.57	*b*
12	30	12 21 18	269.63	12 21 15	377.20	*f*3	34	22	7 31 0	255.99	7 31 8	187.62	*g*
13	Aug. 11	9 59 25	329.28	9 59 30	122.31	*p*	35	23	7 48 42	265.03	7 48 35	217.04	*b*
14	12	11 11 9	24.60	11 12 0	87.60	*b*4	36	24	6 55 7	270.22	6 55 14	224.78	*b*
15	17	9 46 55	96.80	9 47 23	129.05	*g*	37	25	6 54 40	273.95	6 54 36	213.21	*f*
16	18	9 29 46	95.82	9 29 42	97.18	*f*	38	26	6 47 0	275.84	6 47 2	188.20	*g*
17	20	9 43 46	69.63	9 43 49	68.38	*b*	39	29	6 44 10	262.74	6 44 14	123.00	*g*
18	21	9 23 50	60.89	9 24 1	91.28	*f*	40	30	6 39 54	254.79	6 41 18	136.70	*f*
19	22	9 47 33	62.52	9 47 26	137.88	*p*	41	Oct. 13	6 59 16	281.12	6 59 23	480.81	*b*2
20	23	9 55 19	67.78	9 55 2	204.11	*p*	42	14	7 45 40	281.07	7 46 16	415.00	*g*2
21	24	9 31 30	73.33	9 31 54	285.24	*f*5	43	20	7 16 24	270.30	7 17 4	416.81	*p*2
22	Sept. 1	8 39 9	98.35	8 47 22	620.32	*b*6							

(1) Measurements made by placing the two outside movable wires successively in coincidence each with its object; slid eyepiece back and forth. (2) Measurements made by single distances. (3) Power 200; single distances. (4) Comparisons 7, 8: clouds interrupting. (5) Comparisons 8, 9. (6) *Iapetus* very difficult: clouds stop measurements: comparisons 4, 8; measures made as in (1). (7) Very difficult: single distances. (8) Power 170; single distances.

Tethys–Hyperion.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	Sept. 13	9 21 36	92.76	9 21 33	167.67	*g*1	4	Sept. 22	8 11 36	240.36	8 10 50	129.00	*p*
2	14	8 25 20	92.86	8 25 15	224.45	*g*1	5	25	7 47 10	275.20	7 47 20	257.71	*f*1
3	15	8 32 11	107.77	8 31 31	172.22	*f*1	6	26	7 50 34	273.28	7 50 28	212.81	*f*

(1) *Hyperion* identified by motion.

Titan–Hyperion.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1903	h m s	°	h m s	″			1903	h m s	°	h m s	″	
1	July 24	13 28 45	289.35	13 10 7	290.60	*f*1	9	Sept. 22	8 39 30	263.95	8 39 0	333.23	*p*2
2	Aug. 17	11 54 7	273.85	11 57 37	124.08	*g*	10	25	8 15 45	282.96	8 15 58	326.67	*f*
3	Sept. 11	10 19 30	53.92	10 18 0	180.76	*g*	11	Oct. 13	7 28 40	268.71	7 28 40	81.58	*b*3
4	12	9 37 35	75.38	9 37 20	269.00	*f*	12	14	7 19 27	268.02	7 19 54	64.68	*f*
5	13	9 5 55	86.64	9 6 0	341.68	*g*2	13	18	7 5 0	261.86	7 6 26	81.22	*b*
6	14	8 18 8	94.57	8 47 58	380.16	*g*2	14	20	6 45 26	272.73	6 44 19	130.13	*b*
7	15	8 52 7	102.06	8 52 25	375.00	*f*2	15	21	7 0 48	278.46	7 1 0	144.94	*p*4
8	21	8 18 46	253.88	8 19 6	274.22	*f*							

Hyperion could not be seen with *Saturn* in the field, except by pasting a strip of black paper to the metal rim in front of the eye-piece to cut out the light of *Saturn*. (1) Very difficult; *Hyperion* disappeared before the observation could be finished: comparisons 4, 8. (2) Single distances. (3) Possibly not *Hyperion*. (4) *Hyperion* exceedingly difficult.

ELEMENTS OF (1903 *NF*),

BY W. T. CARRIGAN AND E. D. TILLYER.

The elements given below are based on the middle observation of the series and two normals formed from the four extreme places. The residuals shown are those furnished by the comparison of places computed directly from the elements with the adopted fundamental positions. It was considered inadvisable, in view of the limited number of observations, as well as the small interval of time covered, to attempt a further reduction of these residuals. The computation was made by the method of A. O. LEUSCHNER, and the ease with which the results were obtained by this method is, in our opinion, a convincing proof of its great value, not only in special cases, such as the one here dealt with, but also in cases to which the older methods are entirely applicable.

The only element which shows any remarkable feature is the eccentricity, which so far exceeds in magnitude that of any asteroid orbit hitherto computed that one would naturally be inclined to regard it with suspicion. However, the eccentricity issues as it stands from the computation, and we have satisfied ourselves by trial that any ordinary change of the residuals would not affect it materially.

Epoch 1903 December 17.73093 G.M.T.

$$\left.\begin{aligned} \Omega &= 233°\ 21'\ 0''.1 \\ i &= 17\ \ 49\ \ 44.3 \\ \pi &= 336\ \ 59\ \ 12.6 \end{aligned}\right\}\ 1903.0$$

$$M_0 = 42°\ 59'\ 39''.2$$
$$e = 0.5585231$$
$$\log a = 0.5530944$$
$$\mu = 525''.25$$

$$\left.\begin{aligned} x &= r\,(9.9864850)\sin(65°\ 37'\ 58''.1 + v) \\ y &= r\,(9.9900915)\sin(332\ \ 29\ \ 18.5 + v) \\ z &= r\,(9.5104837)\sin(\ \ 23\ \ 21\ \ 48.0 + v) \end{aligned}\right\}\ 1903.0$$

O – C

$\cos\delta\,\Delta\alpha$	$\Delta\delta$
s	″
−0.03	+0.3
+0.03	±0.0
−0.04	±0.0

CORRECTIONS TO OBSERVATIONS OF MINOR PLANETS (*A.J.* 556, p. 33).

BY MARTIN EBELL.

1. The observations of (73) *Klytia* do not belong *Klytia*, but to (207) *Hedda*.

2. The observations of a new planet, PETERS (1903 Sept. 28), are only a continuation of the observations, on same page, of (163) *Erigone*. The circular orbit on p. 34 also shows the identity.

Kiel, 1904 *March* 1.

COMPARISON OF THE NEW TABLES OF *JUPITER* AND *SATURN*,

WITH THE GREENWICH OBSERVATIONS OF 1889-1900,

BY G. W. HILL.

The New Tables of *Jupiter* and *Saturn* are founded upon a discussion of observations which ended with the year 1888. They could not, however, be generally used for the calculation of the Ephemerides till 1901. This leaves a gap of twelve years of observations uncompared with the new theories. It seemed that a useful service would be done for astronomy by supplying the lacking comparisons. In this work I have confined myself to the Greenwich observations as the published positions made at other places are desultory in character. Desiring to form normals as near the time of opposition as possible, I have not included observations when the time of culmination was earlier than 10^h. During some portion of the summer of 1891 the instrument appears to have been dismounted; thus only a weak normal for *Jupiter* could be formed for this year.

It seemed desirable to reduce the normal positions to the standard of Professor NEWCOMB's *Catalogue of Fundamental Stars* (*Astronomical Papers*, Vol. VIII, Part II). With regard to the right-ascensions, in this memoir (p. 228) $+0^s.019$ is given as the correction for the Greenwich Catalogue of 1890. But, being apprehensive that this quantity was not applicable to the whole period 1889-1900, I have determined it at nine different epochs, getting results which range from $+0^s.011$ to $+0^s.059$. The correction seems to have augmented from the beginning to the end of the period. The systematic correction for the declinations is taken from the table on p. 236 of the same paper, given as applicable to the Greenwich Catalogue of 1890. A comparison of the declinations in the Greenwich volume for 1900 with those of the *Fundamental Catalogue* showed that these systematic corrections have persisted without marked change till the later date.

The columns in the following exhibit scarcely need explanation. In the second column the dates of the first and last observation are given in order that the observations used may be readily identified.

Date Greenw. M.N.	Interval of Observation	No. of Obsns.	Mean Diff. from N.A. (s)	Syst. Corr. (s)	Mean Diff. from N.A. (″)	Syst. Corr. (″)	Positions Resulting from Observation α (h m s)	δ (° ′ ″)
			Jupiter.					
1889 June 27	May 21–Aug. 3	20–20	–.032	+.041	+0.01	+.75	18 13 15.289	–23 15 20.84
1890 Aug. 8	July 3–Sept. 3	25–26	+.095	+.046	–0.19	+.69	20 34 41.141	–19 31 17.50
1891 Aug. 22	July 24–Oct. 17	7–6	+.233	+.046	+0.28	+.48	23 6 37.239	– 7 17 11.14
1892 Oct. 13	Sept. 8–Nov. 18	24–24	+.271	+.046	+1.21	+.35	1 15 31.330	+ 6 15 5.86
1893 Nov. 19	Oct. 26–Dec. 23	26–27	+.327	+.056	+0.71	+.28	3 36 45.083	+18 14 33.19
1894 Dec. 31	Nov. 15–Jan. 30	29–29	+.216	+.056	+0.58	+.25	6 0 34.102	+23 14 49.63
1896 Feb. 15	Jan. 14–Mar. 1	15–16	+.116	+.050	–0.06	+.27	8 15 5.466	+20 33 23.81
1897 Mar. 10	Jan. 22–Apr. 8	21–21	+.020	+.050	+1.27	+.33	10 23 8.330	+11 31 28.00
1898 Apr. 8	Feb. 26–May 7	27–30	+.040	+.059	+1.10	+.42	12 16 2.969	– 0 0 13.18
1899 May 9	Mar. 29–June 2	31–31	+.089	+.057	+0.93	+.53	14 7 31.066	–11 23 21.94
1900 June 1	Apr. 17–July 6	26–26	+.170	+.054	+0.01	+.70	16 14 36.364	–20 20 49.79
			Saturn.					
1889 Feb. 16	Jan. 8–Mar. 15	19–19	–.138	+.046	+0.21	+.29	9 15 23.788	+17 8 56.20
1890 Mar. 3	Jan. 23–Apr. 5	22–23	–.108	+.046	+0.59	+.32	10 8 59.738	+13 15 37.31
1891 Mar. 17	Feb. 12–Apr. 18	20–20	–.063	+.046	+0.55	+.35	10 59 53.383	+ 8 46 31.80
1892 Apr. 6	Mar. 8–Apr. 30	26–27	–.112	+.046	+1.08	+.39	11 46 24.704	+ 4 11 51.87
1893 Apr. 11	Mar. 4–May 13	41–38	–.230	+.056	+1.97	+.42	12 35 27.096	– 0 52 56.81
1894 Apr. 20	Mar. 15–May 24	24–26	–.281	+.056	+2.18	+.46	13 22 23.815	– 5 41 19.36
1895 Apr. 30	Mar. 13–June 5	27–27	–.311	+.048	+2.37	+.50	14 8 43.764	–10 7 23.03
1896 May 8	Mar. 28–June 15	37–37	–.385	+.050	+2.36	+.58	14 55 36.495	–14 4 53.06
1897 May 16	Apr. 22–June 21	27–27	–.460	+.050	+2.19	+.65	15 43 28.440	–17 24 52.96
1898 June 12	May 12–July 8	16–16	–.418	+.059	+1.32	+.70	16 26 21.771	–19 47 6.38
1899 June 23	May 15–July 20	30–30	–.370	+.057	+1.38	+.72	17 14 55.847	–21 34 32.40
1900 July 8	May 26–Aug. 4	28–28	–.304	+.054	+0.51	+.74	18 2 56.220	–22 28 34.65

As the next step the heliocentric positions of the planets for the dates of their normals were obtained from the New Tables, and the positions of the *Sun* for the same dates from Professor NEWCOMB's Tables, neglecting, however, the small terms of nutation. Thus were obtained the apparent geocentric positions, which, with the residuals of the observations are noted below. The latter, although of a systematic character in the case of *Saturn*, are small enough to render probable the conclusion that the New Tables will represent well the future motion of the planets.

Jupiter.

	α	δ	Obs. — Cal. $\Delta\alpha$	Obs. — Cal. $\Delta\delta$
	h m s	° ′ ″	s	″
1889 June 27	18 13 15.331	−23 15 21.44	−0.042	+0.60
1890 Aug. 8	20 31 41.165	−19 31 17.32	−0.024	−0.18
1891 Aug. 22	23 6 37.219	− 7 17 10.52	+0.020	−0.62
1892 Oct. 13	1 15 31.333	+ 6 15 6.35	−0.003	−0.49
1893 Nov. 19	3 36 45.028	+18 14 33.32	+0.055	−0.13
1894 Dec. 31	6 0 34.087	+23 11 49.49	+0.015	+0.14
1896 Feb. 15	8 15 5.447	+20 33 24.07	+0.019	−0.26
1897 Mar. 10	10 23 8.369	+11 31 27.14	−0.039	+0.86
1898 Apr. 8	12 16 2.941	− 0 0 13.84	+0.028	+0.66
1899 May 9	14 7 31.041	−11 23 25.77	+0.025	+0.83
1900 June 1	16 11 36.344	−20 20 50.69	+0.020	+0.90

Saturn.

	α	δ	Obs. — Cal. $\Delta\alpha$	Obs. — Cal. $\Delta\delta$
	h m s	° ′ ″	s	″
1889 Feb. 16	9 15 23.797	+17 8 56.55	−0.009	−0.35
1890 Mar. 3	10 8 59.687	+13 15 37.77	+0.051	−0.46
1891 Mar. 17	10 59 53.369	+ 8 46 31.74	+0.014	+0.06
1892 Apr. 6	11 46 21.667	+ 4 11 51.59	+0.037	+0.28
1893 Apr. 11	12 35 27.108	− 0 52 57.24	−0.012	+0.43
1894 Apr. 20	13 22 23.830	− 5 41 19.46	−0.015	+0.10
1895 Apr. 30	14 8 43.789	−10 7 23.19	−0.025	+0.16
1896 May 8	14 55 36.558	−14 1 53.81	−0.063	+0.75
1897 May 16	15 43 28.515	−17 24 53.65	−0.075	+0.69
1898 June 12	16 26 21.838	−19 47 6.86	−0.067	+0.48
1899 June 23	17 14 55.918	−21 34 33.59	−0.071	+1.19
1900 July 8	18 2 56.227	−22 38 35.48	−0.007	+0.83

OBSERVATIONS OF COMET 1903 II (= *d* 1902, *GIACOBINI*).

MADE WITH THE 26-INCH REFRACTOR OF THE LEANDER McCORMICK OBSERVATORY OF THE UNIVERSITY OF VIRGINIA.

By ORMOND STONE AND G. FREDERIC PADDOCK.

1903 Charl. M.T.	∗	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.		Obs.
h m s			m s	′ ″	h m s	° ′ ″			s	″	
Jan. 30 9 25 2	1	*d*8 , 8	−0 0.29	2 18.3	6 12 35.86	+13 17 8.2	*n*8.927	0.566	+1.95	−12.6	P
11 26 58	1	*d*8 , 8	−0 3.19	1 7.1	6 12 32.97	+13 18 49.1	9.218	0.571	+1.96	12.6	S
Feb. 4 9 16 9	2	8 , 8	−0 36.30	+0 45.1	6 40 13.26	+14 56 21.6	8.763	0.536	+1.91	−12.3	P
5 8 19 19	3	10 , 8	−0 49.49	1 56.8	6 39 49.24	+15 15 23.7	*n*9.211	0.542	+1.91	12.5	P
10 42 50	3	9 , 8	−0 51.78	+0 14.6	6 39 46.95	+15 17 15.1	9.118	0.557	+1.91	−12.3	S
13 8 46 1	4	9 , 4	+1 19.07	+0 52.8	6 37 21.22	+17 19 23.2	*n*8.620	0.186	+1.88	−11.5	S
17 8 58 1	5	8 , 8	+1 0.17	+2 5.0	6 36 12.99	+19 3 17.5	8.273	0.152	+1.83	11.2	S
19 8 23 0	6	5 , 4	0 29.11	1 59.0	6 36 33.58	+19 38 12.7	*n*8.593	0.140	+1.82	−11.1	S
8 30 5	7	5 ,	−0 39.83	. . .	6 36 33.66	. . . . .	*n*8.380	. . .	+1.81	. . .	S
8 39 18	7	5 ,	−0 39.97	. . .	6 36 33.52	. . . . .	*n*7.621	. . .	+1.81	. . .	P
8 46 10	6	5 , 5	−0 29.21	1 36.3	6 36 33.48	+19 39 5.4	8.026	0.138	+1.82	11.1	P
Mar. 1 11 2 21	8	10 , 10	0 31.28	3 30.5	6 37 21.71	+22 30 2.2	9.549	0.492	+1.70	10.0	P
Apr. 18 9 25 17	9	7 , 4	−1 14.89	+6 56.9	7 16 36.99	+31 11 32.5	9.653	0.403	+1.04	7.0	S

Mean Places of Comparison-Stars for the beginning of the year.

∗	α	δ	Authority	∗	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	6 12 34.20	+13 20 9.1	DM. +13 1413*	6	6 37 0.87	+19 40 52.8	Berlin A, A.G. 2335
2	6 40 47.65	+14 55 48.8	Leipzig I, A.G. 2477	7	6 37 11.68	+19 34 57.0	Berlin A, A.G. 2338
3	6 40 36.82	+15 17 12.8	Berlin A, A.G. 2369	8	6 37 51.29	+22 33 12.7	Berlin B, A.G. 2551
4	6 36 0.27	+17 48 11.9	Berlin A, A.G. 2321	9	7 17 50.81	+31 37 42.6	Leiden, A.G. 3113
5	6 35 40.99	+19 1 23.7	Berlin A, A.G. 2319				

d indicates direct micrometrical measurement. Correction for refraction has been applied.

* Micrometrically connected with Leipzig I, A.G. 2514

NOTES ON SOME LONG-PERIOD VARIABLE STARS.

By A. STANLEY WILLIAMS.

The following notes are in continuation of those published in the *A.J.* 529.* The observations on which they are based were all, however, made with the $6\frac{1}{2}$-inch reflector, and none with the $2\frac{3}{4}$-inch refractor, partly in order to secure uniformity, and partly because of observation being so much pleasanter and easier with the reflector than with the refractor. The observations have been reduced in the manner described by HAGEN, on p. 3 of the Introduction to his "Observations of Variable Stars," the mean of both the arithmetical and geometrical proportion being taken whenever practicable. As this work has been carefully done, it is not likely that any material alteration will be made hereafter in the concluded times of maxima and minima, though the results are necessarily at present of a somewhat provisional character. The present notes had all, with one exception, been written out before the publication of the *V.J.S. Ephemerides* for 1904. In the introduction to these *Ephemerides* HARTWIG has given notes on several of the stars contained in the present list. The conclusions come to by him appear in every case to be almost identical with those derived by the writer.

(557). *RU Andromedae.*

Observations made on 20 nights, between 1903 Aug. 3 and 1904 Jan. 16, show that a fairly well marked, though not very sharply defined minimum, occurred on 1903 Aug. 25 ($12^{M}.2$), and a similar maximum on 1903 Dec. 11 ($9^{M}.9$). A minimum had also been observed here on 1902 Dec. 18 (See *A.N.* 3861), so that we have for the elements of variation:

$$\text{Maximum} = \text{J.D. } 2416460 + 250^{d}\ \text{E.}$$

the interval $M-m$ being 108 days.

562. *Y Andromedae.*

This star, the variability of which was discovered by ANDERSON, increased from $9^{M}.25$ on 1903 Sept. 2 to a sharply defined maximum on Sept. 21: and then immediately declined until by Nov. 23 it had diminished to $11^{M}.0$. The maximum brightness was $8^{M}.3$; that is one-tenth magnitude brighter than DM. $+38°305$; 14 observations between the above limiting dates.

1205. *Y Persei.*

The following dates of minimum have been derived from the observations: (1) 1902 Dec. 30 ($9^{M}.8$), from 8 observations, between 1902 Nov. 12 and 1903 Mar. 3. (2) 1903 Sept. 6 ($9^{M}.9$), from 9 observations, between Aug. 3 and Dec. 6, but there is only a single observation preceding the minimum. These dates are in good agreement with the period of about 251 days suggested in the *A.J.* 529. p. 5.

(6718). *RY Lyrae.*

This variable rose from 12^{M} on 1903 May 21 to a well defined maximum ($9^{M}.5$) on July 25, and by Oct. 17 it had decreased again to 12^{M}. Observations were made on 16 nights between the above limiting dates. The decline from maximum was very rapid at first. Comparing the above maximum with the photographic maximum of 1900 Oct. 23, referred to in the *A.N.* 3870, we get for the elements of variation:

$$\text{Maximum} = \text{J.D. } 2416321 + 335^{d}\ \text{E.}$$

(6733*a*). *RW Lyrae.*

A diagram showing some of the stars in the neighborhood of this star was published in *A.N.* 3833 (reproduced in *Popular Astronomy*, No. 102, p. 101), together with a *photographic* light-scale. The following is my provisional *visual* light-scale, with the magnitudes of the comparison-stars on the assumption that the star *b* (DM. $+43°3071$, 9.0) is $9^{M}.0$, and the value of a step is $0^{M}.08$.

Star	Scale	Mag.
b	0	9.0
e	16	10.3
h	25	11.0
k	31	11.5

In 1902 the variable was invisible in a $6\frac{1}{2}$-inch reflector between Aug. 15 and Nov. 17. In 1903 the observations (16 in number) were commenced on May 7, when the star was $11^{M}.2$, and from that date it rose somewhat rapidly to a sharply defined maximum on May 25 ($10^{M}.3$). The decrease was less rapid than the increase, a magnitude of 12.5 not being reached until Oct. 17, and there was a hump, or ill-defined secondary maximum at the commencement of August. Comparing this maximum with the photographic maximum of 1900 Oct. 6 (See *A.N.* 3833), we have for the elements of variation:

$$\text{Maximum} = \text{J.D. } 2416260 + 480^{d}\ \text{E.}$$

and these elements accord with the invisibility of the star in 1902, and with the photographic observations of 1899 and 1901. The maximum of 1903 seems to have been rather a faint one, as the visual brightness of the star at the maximum of 1900 was probably at least two magnitudes brighter. The next maximum should occur on Sept. 16 of the present year.

6816. *Z Lyrae.*

Observations were made on 27 nights, between 1903 May 19 and 1904 Jan. 9. These indicate a well-defined

* The following corrections to the notes in the *A.J.* 529 should be noted. On p. 6 the comparison-star 1 for *Y Lyrae* should be *l*. Also the date of the last maximum of *Z Lyrae* should be April 7.

maximum ($9^M.6$) for 1903 Nov. 10, three days earlier than the computed date according to the elements in *A.J.* 529, p. 6. There is a $12\frac{1}{2}^M$ star, $10''$–$12''$ n.f. the variable, and easily mistaken for the latter when faint.

6895. *RU Lyrae.*

This star rose from $12^M.5$ on 1903 May 19, at first slowly, but after July 24 rapidly, to a well defined maximum ($9^M.4$) on Sept. 5; and by Oct. 17 it had declined to $11^M.5$. Comparison of this maximum with the one which I observed in 1902 (see *A.J.* 529) yields the following elements of variation:

$$\text{Maximum} = \text{J.D. } 2415983 + 380^d\,\text{E}.$$

7019. *TY Cygni.*

Twenty-six observations, between 1903 May 21 and 1904 Jan. 9, indicate a well defined maximum for 1903 Oct. 7, eight days earlier than the date computed from the elements in *A.J.* 529, p. 7. The maximum brightness was $8^M.7$, the variable being then a full half magnitude brighter than DM. $+27°3433$.

7505. *UX Cygni.*

This star was invisible in a $6\frac{1}{2}$-inch reflector in 1902 between July 11 and Nov. 17. In 1903 observations were commenced on May 7. The variable was then one-fifth magnitude fainter than DM. $+29°4233$ ($9^M.5$), and from that date it faded slowly until by the middle of August it had decreased to about $11^M.5$. The maximum evidently occurred some time before May 7 — about 13 days earlier if the light change was similar to what it was in 1901. This would make the date of maximum about 1903 Mar. 25; and comparing this with the photographically observed maximum of 1901 Oct. 23 (*A.J.* 529), we get 518 days as a first approximation to the period of variation. This period would accord with the invisibility of the star in 1902; whereas, one of half the length would imply a maximum for July 9 of that year, at which time, however, the variable was invisible in a $6\frac{1}{2}$-inch reflector.

It should be mentioned that there is a $10\frac{1}{2}^M$ star about $30''$ north preceding, and liable to be mistaken for the variable when the latter is faint. This $10\frac{1}{2}^M$ star is photographically a little faint, so that it did not at first seem likely that it could be identical with the $9^M.5$ star DM. $+29°1231$; but as the star is just visible in a $2\frac{1}{4}$-inch refractor, it is not impossible that it, and not the variable, is actually identical with the DM. star. Measures are required to decide this question. The next maximum of *UX Cygni* should occur about Aug. 24 of the present year.

7571a. *TW Cygni.*

Observations were made on 11 nights in 1903, between May 21 and Oct. 16. The star rose from $10^M.5$ on the former date to a well defined maximum on Aug. 3, six days later than the computed date of maximum according to the elements in the *A.J.* 529, p. 8. The decrease was somewhat more rapid than the increase, the star having sunk to $11^M.8$ by Oct. 16. At its greatest brightness the variable attained to $9^M.0$, being then slightly brighter than DM. $+28°3986$.

6783. *RX Lyrae.*

A little chart of the stars in the neighborhood of this variable is given by Seeliger in *A.N.* 3857. On May 7, 1903 the variable was estimated 5 steps brighter than the star marked (1) on this chart, and 5 steps fainter than the principal component of the double star marked (2). It decreased slowly in brightness, until by May 26 it was rated 10 steps fainter than (1). This would make it about equal in brightness to the faint companion of (2) shown on the chart. The maximum evidently occurred some time previous to May 7. Hartwig fixed the date of this for April 28 (*A.N.* 3873). A comparison of my observations with those of the last mentioned observer, and with one made by Wolf on Mar. 25, when the star was about 14^M, gave an independent determination of the date of maximum as 1903 April 27. The value of a step in the above estimates would be not far from $0^M.06$.

20 Hove Park Villas, Hove, 1904 Feb. 12.

ELEMENTS OF 6189 *U OPHIUCHI.*

By S. C. CHANDLER.

The material for the determination of the elements of *U Ophiuchi* consists of the following complete minima observed between 1882 and 1902:

Sawyer	45	complete minima
Chandler	29	" "
Yendell	26	" "
Dunér	4	" "

in addition to times of a large number of other minima which may be more or less certainly deduced from isolated observations. Of the latter class the most important are the times derived from the observations of Schaeberle and at Cordoba, which I have fully discussed in *A.J.* 161.

The elements given in *A.J.* 553, page 5,* are

$$1881 \text{ July } 17^d\,15^h\,32^m \text{ Gr} + 20^h\,7^m.6903\,\text{E} - 3^m.0\,t^2 + 0^s.3\,t^3$$

where, for brevity, t is put $= \dfrac{\text{E}}{1000}$. These elements represent well the data for the whole interval, 1863 to 1902, as is shown by the following comparison; where, in order to make clear the nature of the demonstrated inequalities, I have given in the column "Obs'd Inequality" the deviations of the observed normal epochs from the uniform period of the elements; then the sum of the computed terms in t^2 and t^3; and in the last column O – C.

* The exponents to the terms in t were unfortunately omitted in the place cited. The reader is asked to supply them, also on the same page, for 5889 *U Herculis*, to write E^2 for E^3 in the last term.

Mean E	Inequality Obs'd	Comp'd	O − C
	m	m	m
−7885	−338.1 :	−333.6	− 4.5 :
−4267	− 71.8 :	− 77.9	+ 6.1 :
+ 120	− 6.1 :	0.0	− 6.1 :
350	+ 2.0 :	− 0.4	+ 2.4 :
416	+ 6.5	− 0.5	+ 7.0
451	− 17.8 :	− 0.6	−17.2 :
457	+ 0.4	− 0.6	+ 1.0
479	+ 0.5	− 0.7	+ 1.2
517	− 3.6	− 0.8	− 2.8
517	+ 5.0	− 0.8	+ 5.8
838	+ 3.8	− 1.9	+ 5.7
839	− 5.0	− 1.9	− 3.1
888	− 3.7	− 2.2	− 1.5
891	− 10.4	− 2.2	− 8.2
914	+ 3.9	− 2.3	+ 6.2
930	− 3.9	− 2.4	− 1.5
1328	+ 6.5	− 4.6	+11.1
+1769	− 9.5	− 7.7	− 1.8
+2218	− 1.4	− 11.4	+10.3
2719	− 23.8	− 16.1	− 7.7
3051	− 24.4	− 19.4	− 5.0
3472	− 35.4	− 23.7	−11.7
3515	− 23.7	− 24.1	+ 0.4
3794	− 51.3 :	− 26.8	−24.5 :
3920	− 13.7	− 28.0	+14.3
4393	− 34.3	− 32.5	− 1.8
4420	− 31.1	− 32.7	+ 1.6
4742	− 48.4	− 35.4	−13.0
4762	− 30.7	− 35.6	+ 4.9
4807	− 16.3	− 36.0	+19.7
5213	− 22.5	− 39.1	+16.6
5223	− 25.0	− 39.1	+14.1
5684	− 22.1	− 41.9	+19.8
6143	− 43.3	− 43.6	+ 0.3
+9093	− 23.6	− 22.6	− 1.0

ELEMENTS OF 2610 *R CANIS MAJORIS*,

BY S. C. CHANDLER.

The material for the determination of the elements of *R Canis Majoris* consists of the following minima observed between 1887 and 1902:

YENDELL	18	minima
CHANDLER	8	"
SAWYER	6	"
DOBERCK	5	"

Three of SAWYER'S I have derived by combining his 1887 observations into normals. HAGEN'S 3 minima (*A.J.* 212) are uncertain from insufficiency of the duration of observed intervals. The elements of the Third Catalogue (*A.J.* 379) were retained in the "Revision, &c." (*A.J.* 553) because the observations during the last ten years do not indicate any certain correction. Comparison with the whole series of individual minima follows.

Epoch	O−C	Obs.	Epoch	O−C	Obs.	Epoch	O−C	Obs.	Epoch	O−C	Obs.
	m			m			m			m	
0	−36.5 :	S	279	− 14.8	S	1574	−37.7	Y	3483	+ 5.4	D
14	− 3.6 :	S	308	+ 29.0	C	1589	−23.0	Y	3484	− 8.4	D
21	+ 3.4	S	904	+ 6.2	Y	1618	−54.0	Y	3497	−22.9	D
22	+11.5 :	S	904	(+133.5)	H	1905	− 4.1	Y	3526	− 9.8	D
171	−10.9	C	911	(+ 80.0)	H	1912	+11.6	Y	3527	− 0.9	D
178	+11.4	C	926	(+ 9.8)	H	2523	+31.0	Y	4185	+35.4 :	Y
179	+15.8 :	C	949	− 11.1	Y	2537	+32.3	Y	4680	+43.0 :	C
192	+16.8	C	963	− 22.1	S	2545	+ 2.6	Y	4759	+17.3	Y
236	− 8.4	C	985	− 18.2	Y	2552	+12.3	Y	4774	+21.7	Y
257	+12.6 :	C	1272	+ 31.0	Y	2564	+27.2	Y	4781	+22.1	Y

OCCULTATION OF *a TAURI* 1904 MARCH 22.

Immersion, $+6^h\ 46^m\ 11^s.3$; Emersion, $8^h\ 1^m\ 17^s.1$ Madison Mean Time. Observed with the finder of the Clark equatorial telescope of the Washburn Observatory, aperture 89mm., magnifying power, 20 diameters.

Madison, 1904 *March* 29.

GEORGE C. COMSTOCK.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed April* 4.

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 560.

VOL. XXIV. BOSTON, 1904 APRIL 11. NO. 8

EPHEMERIDES OF LONG-PERIOD VARIABLES, 1903 TO 1910.

By S. C. CHANDLER.

Following is an ephemeris of dates of maxima occurring before 1910 for all the variables of long period (over 100 days) contained in the Third Catalogue (*A.J.* 379) according to the revised elements in *A.J.* 553.

The first column gives the number of the epoch; the second the date of maximum computed from the elements to the nearest tenth of a day. This degree of precision is used, not because the elements warrant it as a matter of prediction, but to allow accurate comparison with future observed data. Following the ephemeris of maximum dates is the value $M-m$, of the interval from minimum to maximum, in months and days; the subtraction of which from any of the times of maximum will give the time of minimum having the same numbering of the epoch. On the last line for each star is given the approximate range of magnitude at both maximum and minimum phases, separated by a semi-colon.

In a few cases where the elements are uncertain the ephemeris stops short of the general limit, 1910.0.

The form in which the ephemeris is given will be found convenient by the observer in arranging his working scheme of observation, since it will readily permit, by inspection, of an approximate estimate of the star's magnitude at any given time, and whether its brightness will be found increasing or decreasing, &c. This can be done roughly even when the value of $M-m$ is not stated, by bearing in mind that this interval in general is rather less than one-half the whole period. The accuracy of such an estimate of the phase will of course be assisted by some knowledge of the general form of the light-curve of the star, but even without such knowledge a guess can be made of the current phase that will be practically helpful, for purposes of identification and the like.

22. *V Sculptoris.*

16	1903 Sept. 12.0
17	1 July 3.5
18	5 Apr. 25.0
19	6 Feb. 14.5
20	Dec. 7.0
21	7 Sept. 28.5
22	8 July 20.0
23	9 May 11.5

$M-m=$ . . . .
9. ; <12.

62. *S. Sculptoris.*

57	1903 Nov. 21.
58	4 Nov. 21.
59	5 Nov. 22.
60	6 Nov. 23.
61	7 Nov. 21.
62	8 Nov. 21.
63	9 Nov. 25.

$M-m=5^{m}\ 9^{d}$
6. ; 10.

103. *T Andromedae.*

62	1903 June 1.
63	4 Mar. 11.
64	Dec. 17.
65	5 Sept. 21.
66	6 July 2.
67	7 Apr. 9.
68	8 Jan. 15.
69	Oct. 22.
70	9 July 30.

$M-m=4^{m}\ 19^{d}$
8. ; 13.?

107. *T Cassiopeae.*

27	1904 Feb. 3.5
28	5 Apr. 22.0
29	6 July 9.5
30	7 Sept. 26.0
31	8 Dec. 12.5

$M-m=8^{m}\ 3^{d}$
7. 8. ; 11. 12.

110. *S Tucanae.*

21	1903 Oct. 17.
22	4 June 14.
23	5 Feb. 10.
24	Oct. 9.
25	6 June 7.
26	7 Feb. 3.
27	Oct. 2.
28	8 May 30.
29	9 Jan. 26.
30	Sept. 24.

$M-m=2^{m}\ 14^{d}$:
8.5 ; <11.5

112. *R Andromedae.*

40	1904 Feb. 27.0
41	5 Apr. 7.2
42	6 May 18.3
43	7 June 29.5
44	8 Aug. 10.9
45	9 Sept 21.5

$M-m=3^{m}\ 29^{d}$
5.5 8.5 ; <13.

111. *S Ceti.*

35	1903 Sept. 22.0
36	4 Aug. 7.6
37	5 June 21.2
38	6 May 10.8
39	7 Mar. 27.4
40	8 Feb. 11.0
41	Dec. 27.6
42	9 Nov. 13.2

$M-m=4^{m}\ 23^{d}$
7. 8. ; 12.

116. *T Sculptoris.*

25	1903 Oct. 5.
26	4 Apr. 24.
27	Nov. 12.
28	5 June 2.
29	Dec. 21.
30	6 July 11.
31	7 Jan. 29.
32	Aug. 19.
33	8 Mar. 8.

116. *T Sculp.* Cont.

34	1908 Sept. 26.
35	9 Apr. 16.
36	Nov. 4.

$M-m=2^{m}\ 24^{d}$
8 ; 10.

243. *U Cassiopeae.*

22	1903 July 29.
23	4 Apr. 30
24	5 Jan. 31.
25	Nov. 3
26	6 Aug. 6.
27	7 May 9.
28	8 Feb. 9
29	Nov. 11.
30	9 Aug. 14.

$M-m=3^{m}\ 9^{d}$
8. 9. ; <15.

291. *W Cassiopeae.*

14	1903 Apr. 10.
15	4 May 18.
16	5 June 26.
17	6 Aug. 4.
18	7 Sept 12.
19	8 Oct. 20.
20	9 Nov. 28

$M-m=6^{m}\ 25^{d}$
8.5 ; 11.5

101. *U Sculptoris.*

19	1903 Sept 19.
20	4 Aug. 12
21	5 July 6.
22	6 May 30
23	7 Apr. 23
24	8 Mar 16
25	9 Feb 7.

$M-m=$ . . . .
8.5 ; <12.5

119. *U Andromedae.*
9 1903 July 28.
10 4 July 10.
11 5 June 23.
12 6 June 6.
13 7 May 20.
14 8 May 2.
15 9 Apr. 15.
$M - m = ...$
9. ; <13.

432. *S Cassiopeae.*
24 1903 June 20.7
25 5 Feb. 21.1
26 6 Oct. 31.0
27 8 June 29.1
$M - m = 9^m\ 6^d$
6.5–8.5 ; <13.

434. *S Piscium.*
34 1903 Aug. 18.8
35 4 Sept. 28.9
36 5 Nov. 10.3
37 6 Dec. 22.7
38 8 Feb. 3.1
39 9 Mar. 16.1
$M - m = 5^m\ 11^d$:
8.–9. ; <15.

466. *U Piscium.*
49 1903 Mar. 19.1
50 Sept. 8.0
51 4 Feb. 27.9
52 Aug. 18.8
53 5 Feb. 7.7
54 July 30.6
55 6 Jan. 19.5
56 July 11.4
57 7 Jan. 0.3
58 June 22.2
59 Dec. 12.1
60 8 June 2.0
61 Nov. 21.9
62 9 May 13.8
63 Nov. 2.7
$M - m = 2^m\ 22^d$
10. ; 15.

513. *R Piscium.*
39 1903 Aug. 27.7
40 4 Aug. 6.0
41 5 July 16.3
42 6 June 25.6
43 7 June 4.9
44 8 May 14.2
45 9 Apr. 23.5
$M - m = 4^m\ 21^d$
7.–9. ; <13.

678. *U Persei.*
15 1903 Jan. 8.
16 Nov. 22.
17 4 Oct. 5.
18 5 Aug. 19.
19 6 July 3.
20 7 May 17.
21 8 Mar. 30.
22 9 Feb. 11.
23 Dec. 26.
$M - m = 5^m\ 13^d$
7.–8. ; 11.5

715. *S Arietis.*
39 1903 May 27.8
40 4 Mar. 15.0
41 5 Jan. 1.2
42 Oct. 20.4
43 6 Aug. 8.6
44 7 May 27.8
45 8 Mar. 15.0
46 9 Jan. 1.2
47 Oct. 20.4
$M - m = 5^m\ 13^d$
9.–10. ; 14.?

782. *R Arietis.*
72 1903 June 8.9
73 Dec. 12.1
74 4 June 15.4
75 Dec. 18.7
76 5 June 23.0
77 Dec. 26.4
78 6 June 30.9
79 7 Jan. 3.5
80 July 9.0
81 8 Jan. 11.6
82 July 16.3
83 9 Jan. 19.1
84 July 24.9
$M - m = 3^m\ 0^d.5$
7.5–9. ; 11.5–13.

806. *o Ceti.*
1904 Mar. 30.2
5 Feb. 25.0
6 Jan. 22.4
Dec. 19.6
7 Nov. 15.5
8 Oct. 11.3
9 Sept. 6.9
$M - m = 4^m\ 4^d \pm$
2.–5. ; 8.–9.5

845. *R Ceti.*
79 1903 Apr. 16.
80 Sept. 30.
81 4 Mar. 15.
82 Aug. 29.
83 5 Feb. 12.
84 July 29.
85 6 Jan. 12.

845. *R Ceti.* Cont.
86 1906 June 28.
87 Dec. 12.
88 7 May 28.
89 Nov. 11.
90 8 Apr. 26.
91 Oct. 10.
92 9 Mar. 26.
93 Sept. 9.
$M - m = 2^m\ 9^d$
7.5–9. ; 13.5

893. *U Ceti.*
28 1903 Jan. 9.4
29 Sept. 2.2
30 4 Apr. 25.0
31 Dec. 16.8
32 5 Aug. 9.6
33 6 Apr. 2.4
34 Nov. 24.2
35 7 July 18.0
36 8 Mar. 9.8
37 Nov. 0.6
38 9 June 24.4
$M - m = 4^m\ 0^d$:
7. ; 12.

906. *R Trianguli.*
17 1903 Mar. 13.
18 Dec. 6.
19 4 Aug. 30.
20 5 May 25.
21 6 Feb. 17.
22 Nov. 12.
23 7 Aug. 7.
24 8 May 1.
25 9 Jan. 24.
26 Oct. 19.
$M - m = 4^m\ 1^d$
6.–7. ; 11.5

976. *T Arietis.*
35 1903 Mar. 30.
36 4 Feb. 6.
37 Dec. 15.
38 5 Oct. 24.
39 6 Sept. 2.
40 7 July 12.
41 8 May 20.
42 9 Mar. 29.
$M - m = 4^m\ 5^d$
8. ; 9.5

1018. *R Horologii.*
13 1903 Nov. 25.
14 5 Jan. 3.
15 6 Feb. 12.
16 7 Mar. 24.
17 8 May 2.
18 9 June 11.
$M - m = 5^m\ 2^d$
6. ; 10.

1113. *U Arietis.*
11 1903 Dec. 27.
12 4 Dec. 31.
13 6 Jan. 5.
14 7 Jan. 10.
15 8 Jan. 15.
16 9 Jan. 19.
$M - m = . . .$
7.–8.5 ; <11.

1166. *X Ceti.*
26 1903 Jan. 18.
27 July 19.
28 4 Jan. 17.
29 July 17.
30 5 Jan. 15.
31 July 16.
32 6 Jan. 14.
33 July 15.
34 7 Jan. 13.
35 July 14.
36 8 Jan. 12.
37 July 12.
38 9 Jan. 10.
39 July 11.
$M - m = 2^m\ 15^d$
9.5 ; <12.5

1222. *R Persei.*
72 1903 Mar. 0.5
73 Sept. 28.8
74 4 Apr. 28.1
75 Nov. 26.5
76 5 June 26.9
77 6 Jan. 25.2
78 Aug. 25.5
79 7 Mar. 25.6
80 Oct. 23.6
81 8 May 22.4
82 Dec. 20.0
83 9 July 19.4
$M - m = 3^m\ 5^d$
7.5–9. ; 13.

1357. *U Eridani.*
25 1903 Feb. 28.
26 Oct. 25.
27 4 June 20.
28 5 Feb. 14.
29 Oct. 11.
30 6 June 7.
31 7 Feb. 1.
32 Sept. 28.
33 8 May 24.
34 9 Jan. 18.
35 Sept. 14.
$M - m = 4^m\ 2^d$
8.5 ; 11.5<

1386. *T Eridani.*
20 1903 Aug. 16.
21 4 Apr. 23.
22 Dec. 30.
23 5 Sept. 7.
24 6 May 16.
25 7 Jan. 22.
26 Sept. 30.
27 8 June 7.
28 9 Feb. 13.
29 Oct. 22.
$M - m = 3^m\ 19^d$:
7. ; 11.

1577. *R Tauri.*
46 1903 Apr. 7.
47 4 Feb. 26.
48 5 Jan. 16.
49 Dec. 7.
50 6 Oct. 28.
51 7 Sept. 18.
52 8 Aug. 8.
53 9 June 29.
$M - m = 1^m\ 18^d$:
7.5–9. ; 12.5–13.5

1582. *S Tauri.*
42 1903 Feb. 4.4
43 4 Feb. 6.6
44 5 Feb. 7.6
45 6 Feb. 9.3
46 7 Feb. 10.6
47 8 Feb. 11.7
48 9 Feb. 11.4
$M - m = 2^m\ 9^d$:
9.5 ; <13.5

1623. *T Camelop.*
11 1903 Feb. 15.
12 4 Feb. 20.
13 5 Feb. 24.
14 6 Mar. 1.
15 7 Mar. 6.
16 8 Mar. 10.
17 9 Mar. 15.
$M - m = . . .$
7.–8. ; <12.

1635. *R Reticuli.*
51 1903 Feb. 15.5
52 Nov. 22.0
53 4 Aug. 27.5
54 5 June 3.0
55 6 Mar. 9.5
56 Dec. 14.0
57 7 Sept. 19.5
58 8 June 25.0
59 9 Apr. 0.5
$M - m = . . .$
7. ; <13.

1654. *R Doradus.*
18 1903 Feb. 24.
19 4 Feb. 4.
20 5 Jan. 14.
21 Dec. 25.
22 6 Dec. 5.
23 7 Nov. 15.
24 8 Oct. 25.
25 9 Oct. 5.
$M - m = 3^m\ 9^d$
5.7 ; 6.7

1662. *R Caeli.*
3 1903 Dec. 3.
4 5 Jan. 4.
5 6 Feb. 6.
6 7 Mar. 11.
7 8 Apr. 12.
8 9 May 15.
$M - m = . . .$
7.5 ; 10.5

1701. *R Pictoris.*
7 1903 Mar. 31.
8 Sept. 2.
9 4 Feb. 9.
10 July 18.
11 Dec. 25.
12 5 June 3.
13 Nov. 10.
$M - m = . . .$
8. ; 9.5

1717. *V Tauri.*
65 1903 Jan. 1.5
66 June 20.6
67 Dec. 7.7
68 4 May 25.8
69 Nov. 11.9
70 5 May 1.0
71 Oct. 18.1
72 6 Apr. 6.2
73 Sept. 23.3
74 7 Mar. 12.4
75 Aug. 29.5
76 8 Feb. 15.6
77 Aug. 3.7
78 9 Jan. 20.8
79 July 9.9
80 Dec. 27.0
$M - m = 3^m\ 1^d$
8.5–9.5 ; <13.5

1761. *R Orionis.*
47 1903 Dec. 16.5
48 4 Dec. 29.0
49 6 Jan. 11.5
50 7 Jan. 25.0
51 8 Feb. 7.5
52 9 Feb. 20.0
$M - m = 5^m\ 16^d$
9. ; 11.–13.5

1771. *R Leporis.*
33 1903 Aug. 1.0
34 4 Oct. 10.1
35 5 Dec. 20.2
36 7 Mar. 1.3
37 8 May 10.4
38 9 July 20.5
$M - m = 6^m\ 29^d$
6.–7. : 8.5?

1803. *T Leporis.*
14 1903 Sept. 25.
15 4 Sept. 19.
16 5 Sept. 14.
17 6 Sept. 9.
18 7 Sept. 4.
19 8 Aug. 29.
20 9 Aug. 24.
$M - m =$ . . .
8. : 11.

1805. *V Orionis.*
17 1903 July 21.
18 4 Apr. 13.
19 5 Jan. 5.
20 Sept. 29.
21 6 June 23.
22 7 Mar. 17.
23 Dec. 9.
24 8 Sept. 1.
25 9 May 26.
$M - m = 4^m\ 3^d$
8.4 : <13.

1850. *S Pictoris.*
7 1903 Feb. 16.0
8 4 Apr. 19.5
9 5 June 22.0
10 6 Aug. 24.5
11 7 Oct. 27.0
12 8 Dec. 28.5
$M - m =$ . . .
8.5 : <13.

1855. *R Aurigae.*
32 1903 Jan. 28.2
33 4 Apr. 29.9
34 5 Aug. 1.5
35 6 Nov. 3.9
36 8 Feb. 7.1
37 9 May 13.1
$M - m = 7^m\ 22^d$
6.5–8. : 12.5

1891. *T Columbae.*
22 1903 Apr. 17.
23 Nov. 28.
24 4 July 10.
25 5 Feb. 20.
26 Oct. 3.
27 6 May 16.
28 Dec. 27.

1894. *T Col.*–Cont.
29 1907 Aug. 9.
30 8 Mar. 21.
31 Nov. 1.
32 9 June 14.
$M - m = 3^m\ 11^d$
7.5 : 11.5

1944. *S Orionis.*
30 1904 Jan. 5.
31 5 Feb. 21.
32 6 Apr. 11.
33 7 May 28.
34 8 July 11.
35 9 Aug. 31.
$M - m = 6^m\ 11^d$
8.5–9.5 : 11.–13.

2013. *U Aurigae.*
11 1903 Apr. 8.5
12 4 May 18.0
13 5 June 27.5
14 6 Aug. 7.0
15 7 Sept. 16.5
16 8 Oct. 26.0
17 9 Dec. 5.5
$M - m =$ . . .
8.5 : 12.

2059. *S Columbae.*
17 1903 Jan. 18.
18 Dec. 11.
19 4 Nov. 2.
20 5 Sept. 25.
21 6 Aug. 18.
22 7 July 11.
23 8 June 2.
24 9 Apr. 25.
$M - m =$ . . .
8. : <10.

2100. *U Orionis.*
17 1903 May 17.
18 4 May 26.
19 5 June 5.
20 6 June 15.
21 7 June 25.
22 8 July 4.
23 9 July 14.
$M - m = 4^m\ 26^d$
6.5–7.5 : <12.

2141. *R Octantis.*
3 1903 Feb. 14.
4 4 Jan. 10.
5 Dec. 5.
6 5 Oct. 31.
7 6 Sept. 26.
$M - m =$ . . .
7.5 : <11.

2266. *V Monocerotis.*
22 1903 Feb. 11.
23 4 Jan. 9.
24 Dec. 6.
25 5 Nov. 3.
26 6 Oct. 1.
27 7 Aug. 29.
28 8 July 26.
29 9 June 23.
$M - m = 5^m\ 8^d$
6.5 : 10.7<

2415. *W Monocerotis.*
21 1903 Jan. 14.5
22 Oct. 4.0
23 4 June 22.5
24 5 Mar. 12.0
25 Nov. 29.5
26 6 Aug. 19.0
27 7 May 8.5
28 8 Jan. 26.0
29 Oct. 14.5
30 9 July 4.0
$M - m =$ . . .
8.8 : <10.

2478. *R Lyncis.*
28 1903 Oct. 18.6
29 4 Nov. 4.2
30 5 Nov. 22.0
31 6 Dec. 9.8
32 7 Dec. 27.4
33 9 Jan. 12.5
$M - m = 6^m\ 3^d$
8. : <13.

2528. *R Geminorum.*
35 1903 June 27.7
36 4 July 3.2
37 5 July 10.1
38 6 July 17.3
39 7 July 24.8
40 8 Aug. 0.6
41 9 Aug. 8.6
$M - m = 4^m\ 0^d$
6.5 8. : <13.5

2530. *V Canis min.*
14 1903 Sept. 4.
15 4 Sept. 2.
16 5 Sept. 1.
17 6 Aug. 31.
18 7 Aug. 30.
19 8 Aug. 28.
20 9 Aug. 27.
$M - m =$ . . .
10.3 : <13.7

2539. *R Canis min.*
48 1903 July 2.6
49 4 June 1.3
50 5 May 8.0
51 6 Apr. 10.7
52 7 Mar. 14.1
53 8 Feb. 15.1
54 9 Jan. 17.8
55 Dec. 21.5
$M - m = 4^m\ 8^d$
7. 8. : 9.5–10.

2583. *L₂ Puppis.*
81 1903 Apr. 27.2
82 Sept. 14.1
83 4 Feb. 1.6
84 June 20.8
85 Nov. 8.0
86 5 Mar. 28.2
87 Aug. 15.4
88 6 Jan. 2.6
89 May 22.8
90 Oct. 10.0
91 7 Feb. 27.2
92 July 17.4
93 Dec. 4.6
94 8 Apr. 22.8
95 Sept. 10.0
96 9 Jan. 28.2
97 June 17.1
98 Nov. 1.6
$M - m = 1^m\ 29^d$
3.5 : 6.3

2625. *V Geminorum.*
31 1903 July 14.
32 4 Apr. 15.
33 5 Jan. 16.
34 Oct. 19
35 6 July 22.
36 7 Apr. 24.
37 8 Jan. 25.
38 Oct. 27.
39 9 July 30.
$M - m = 4^m\ 10^d$
8.–9. : 12. 14.

2684. *S Canis min.*
44 1903 Feb. 10.0
45 4 Jan. 2.5
46 Nov. 21.4
47 5 Oct. 17.9
48 6 Sept. 11.1
49 7 Aug. 6.1
50 8 July 1.0
51 9 May 27.7
$M - m = 5^m\ 12^d$
7.–8 : 10.5 12.5

2691. *T Canis min.*
38 1903 Oct. 12.6
39 4 Aug. 30.3
40 5 July 19.0
41 6 June 6.7
42 7 Apr. 25.4
43 8 Mar. 13.1
44 9 Jan. 29.8
45 Dec. 18.5
$M - m =$ . . .
9.–10.5 : <13.5

2735. *U Canis min.*
21 1903 Sept. 12.
22 4 Oct. 26.
23 5 Dec. 10.
24 7 Jan. 24.
25 8 Mar. 9.
26 9 Apr. 23.
$M - m = 5^m\ 23^d$
8.5–9. : 12.5–13.5

2742. *S Geminorum.*
64 1903 Aug. 22.2
65 4 June 11.0
66 5 Apr. 0.8
67 6 Jan. 19.6
68 Nov. 9.4
69 7 Aug. 30.2
70 8 June 19.0
71 9 Apr. 8.8
$M - m = 3^m\ 29^d$
8.5 : <13.5.

2776. *W Puppis.*
21 1903 Feb. 19.2
22 June 20.0
23 Oct. 18.8
24 4 Feb. 16.6
25 June 16.4
26 Oct. 15.2
27 5 Feb. 13.2
28 June 13.8
29 Oct. 12.6
30 6 Feb. 10.4
$M - m = 1^m\ 24^d$
7.6 : 12.0

2780. *T Geminorum.*
69 1903 May 13.4
70 4 Feb. 25.5
71 Dec. 9.6
72 5 Sept. 23.7
73 6 July 8.8
74 7 Apr. 22.9
75 8 Feb. 5.0
76 Nov. 19.1
77 9 Sept. 3.2
$M - m =$ . . .
8.5 : <13.5

2857. *U Puppis.*
26 1903 Aug. 11.
27 4 June 21.
28 5 May 2
29 6 Mar. 13
30 7 Jan. 22.
31 Dec. 2
32 8 Oct. 15
33 9 Aug. 21
$M - m =$ . . .
8.5–9. : <14

2946. *R Cancri.*
52 1903 Nov. 17.7
53 4 Nov. 12.7
54 5 Nov. 8.2
55 6 Nov. 2.8
56 7 Oct. 28.0
57 8 Oct. 20.5
58 9 Oct. 13.6
$M - m = 4^m\ 5^d$
6.–8. : <11.7

2976. *V Cancri.*
43 1903 June 2.3
44 4 Mar. 0.4
45 Nov. 27.5
46 5 Aug. 26.6
47 6 May 25.7
48 7 Feb. 21.8
49 Nov. 20.9
50 8 Aug. 19.0
51 9 May 18.1
$M - m = 3^m\ 25^d$
7.–8. : <12.

3060. *U Cancri.*
60 1903 May 27.
61 4 Mar. 27.
62 5 Jan. 26
63 Nov. 27.
64 6 Sept. 28
65 7 July 30.
66 8 May 30.
67 9 Mar. 31
$M - m =$ . . .
8.5–10.5 : <14

3128. *R Pyxidis.*
14 1903 July 21
15 4 July 13.
16 5 July 3.
17 6 June 23
18 7 June 13
19 8 June 2
20 9 May 23
$M - m =$ . . .
8 : <11.

3170. *S Hydrae.*
66 1903 June 9.
67 4 Feb. 20.
68 Nov. 2.
69 5 July 16.
70 6 Mar. 29.
71 Dec. 10.
72 7 Aug. 23.
73 8 May 5.
74 9 Jan. 16.
75 Sept. 29.
$M - m = 3^m\ 9^d$
7.5–8.5 ; <12.

3184. *T Hydrae.*
57 1903 Mar. 26.6
58 4 Jan. 9.4
59 Oct. 24.2
60 5 Aug. 9.0
61 6 May 24.8
62 7 Mar. 9.6
63 Dec. 23.4
64 8 Oct. 7.2
65 9 July 23.0
$M - m = . . .$
7.–8. ; <13.

3186. *T Cancri.*
Minima.
35 1904 Apr. 5.
36 5 July 31.
37 6 Nov. 25.
38 8 Mar. 21.
39 9 July 16.
$M - m = . . .$
8. : 9.5–10.5

3264. *W Cancri.*
12 1903 June 24.
13 4 July 11.
14 5 July 29.
15 6 Aug. 16.
16 7 Sept. 3.
17 8 Sept. 20.
18 9 Oct. 8.
$M - m = . . .$
9.–9.5 ; <13.

3418. *R Carinae.*
38 1903 Oct. 1.3
39 4 Aug. 5.5
40 5 June 11.3
41 6 Apr. 17.7
42 7 Feb. 22.7
43 8 Jan. 0.3
44 Nov. 6.3
45 9 Sept. 15.8
$M - m = 4^m\ 14^d$
4.5–5.5 ; 9.5–10.

3425. *X Hydrae.*
14 1903 July 28.
15 4 May 19.
16 5 Mar. 11.
17 6 Jan. 1.
18 Oct. 24.
19 7 Aug. 16.
20 8 June 7.
21 9 Mar. 30.
$M - m = . . .$
8.5 : 12.

3477. *R Leonis min.*
38 1903 Sept. 16.1
39 4 Sept. 23.5
40 5 Oct. 2.2
41 6 Oct. 11.0
42 7 Oct. 20.0
43 8 Oct. 28.0
44 9 Nov. 5.8
$M - m = 5^m\ 13^d$
6.–8. : 13.

3493. *R Leonis.*
171 1903 Oct. 7.8
172 4 Aug. 15.6
173 5 June 24.4
174 6 May 3.2
175 7 Mar. 12.0
176 8 Jan. 18.8
177 Nov. 26.6
178 9 Nov. 5.4
$M - m = 4^m\ 22^d$
5.–6.5 ; 9.5–10.

3567. *U Leonis.*
28 1903 Mar. 17.8
29 Dec. 15.9
30 4 Sept. 14.0
31 5 June 14.1
32 6 Mar. 14.2
33 Dec. 12.3
34 7 Sept. 11.4
35 8 June 10.5
36 9 Mar. 10.6
37 Dec. 8.7
$M - m = . . .$
8.5 : <13.5

3637. *S Carinae.*
76 1903 Apr. 18.2
77 Sept. 13.9
78 4 Feb. 9.6
79 July 7.3
80 Dec. 3.0
81 5 May 0.7
82 Sept. 26.4
83 6 Feb. 22.1
84 July 20.8
85 Dec. 16.5
86 7 May 14.2
87 Oct. 9.9

3637. *S Car.*—Cont.
88 1908 Mar. 6.6
89 Aug. 2.3
90 Dec. 29.0
91 9 May 26.7
92 Oct. 22.4
$M - m = 2^m\ 25^d$
6. ; 9.

3662. *Z Carinae.*
3 1903 Dec. 9.
4 5 Jan. 6.
5 6 Feb. 4.
6 7 Mar. 5.
$M - m = . . .$
9.5 ; 12.

3825. *R Ursae Maj.*
61 1903 Sept. 21.3
62 4 July 19.9
63 5 May 19.4
64 6 Mar. 18.9
65 7 Jan. 16.4
66 Nov. 15.5
67 8 Sept. 13.6
68 9 July 13.5
$M - m = 3^m\ 19^d$
6.–8. ; 12.5–13.

3991. *S Leonis.*
82 1903 June 25.0
83 4 Jan. 0.5
84 July 8.0
85 5 Jan. 13.5
86 July 22.0
87 6 Jan. 27.5
88 Aug. 5.0
89 7 Feb. 10.5
90 Aug. 19.0
91 8 Feb. 24.5
92 Sept. 1.0
93 9 Mar. 9.5
94 Sept. 15.0
$M - m = 3^m\ 2^d$:
9.–10. : <13.

4225. *X Centauri.*
16 1903 Mar. 0.
17 4 Jan. 8.
18 Nov. 17.
19 5 Sept. 27.
20 6 Aug. 7.
21 7 June 17.
22 8 Apr. 26.
23 9 Mar. 6.
$M - m = 4^m\ 18^d$
7.7 ; 12.4

4260. *W Centauri.*
26 1903 Oct. 21.
27 4 May 14.
28 Dec. 3.
29 5 June 24.
30 6 Jan. 13.
31 Aug. 4.
32 7 Feb. 23.
33 Sept. 14.
34 8 Apr. 4.
35 Oct. 24.
36 9 May 15.
37 Dec. 4.
$M - m = . . .$
8.5 : 13.

4315. *R Comae.*
47 1903 July 12.6
48 4 July 8.4
49 5 July 5.2
50 6 July 2.0
51 7 June 28.8
52 8 June 24.6
53 9 June 21.4
$M - m = 3^m\ 29^d$:
7.5–8. ; <13.5

4377. *T Virginis.*
45 1903 Feb. 22.5
46 4 Jan. 28.0
47 5 Jan. 1.5
48 Dec. 7.0
49 6 Nov. 11.5
50 7 Oct. 17.0
51 8 Sept. 20.5
52 9 Aug. 26.0
$M - m = 5^m\ 0^d$:
8.–9. ; 10.–13.5

4407. *R Corvi.*
40 1903 Apr. 11.0
41 4 Feb. 23.5
42 5 Jan. 7.0
43 Nov. 21.5
44 6 Oct. 6.0
45 7 Aug. 20.5
46 8 July 4.0
47 9 May 18.5
$M - m = . . .$
7.–7.5 ; <11.5

4488. *U Centauri.*
6 1903 Aug. 19.8
7 4 Mar. 23.6
8 Oct. 26.4
9 5 June 0.2
10 6 Jan. 3.0
$M - m = 3^m\ 15^d$?
9. : 11.3

4492. *Y Virginis.*
34 1903 July 23.2
35 4 Feb. 27.0
36 Oct. 2.8
37 5 May 9.6
38 Dec. 11.4
39 6 July 21.2
40 7 Feb. 25.0
41 Oct. 1.8
42 8 May 7.6
43 Dec. 12.4
44 9 July 19.2
$M - m = 2^m\ 24^d$:
8.–9.5 : 11.5–13.

4511. *T Ursae Maj.*
60 1903 Jan. 2.8
61 Sept. 17.2
62 4 June 1.2
63 5 Feb. 14.6
64 Nov. 0.4
65 6 July 17.7
66 7 Apr. 3.2
67 Dec. 19.1
68 8 Sept. 4.2
69 9 May 22.5
$M - m = 3^m\ 16^d$
6.–8.5 ; 12.–13.

4521. *R Virginis.*
236 1903 May 7.4
237 Sept. 29.7
238 4 Feb. 22.1
239 July 16.5
240 Dec. 9.0
241 5 May 3.5
242 Sept. 26.2
243 6 Feb. 18.8
244 July 14.6
245 Dec. 7.4
246 7 May 2.3
247 Sept. 25.2
248 8 Feb. 19.2
249 July 13.3
250 Dec. 6.4
251 9 May 1.6
252 Sept. 24.6
$M - m = 2^m\ 7^d.5$
6.5–8. : 9.5–11.

4557. *S Ursae Maj.*
69 1903 Mar. 9.8
70 Oct. 19.5
71 4 May 30.2
72 5 Jan. 9.3
73 Aug. 21.4
74 6 Apr. 2.8
75 Nov. 13.5
76 7 June 26.4
77 8 Feb. 6.7
78 Sept. 19.2
79 9 May 3.0
$M - m = 3^m\ 17^d$
7.–8. : 10.–11.

4596. *U Virginis.*
65 1903 Mar. 24.5
66 Oct. 26.6
67 4 May 19.7
68 Dec. 11.8
69 5 July 5.9
70 6 Jan. 28.0
71 Aug. 22.1
72 7 Mar. 16.1
73 Oct. 8.2
74 8 May 1.2
75 Nov. 23.2
76 9 June 17.3
$M - m = 2^m\ 27^d$
8. : 12.5

4816. *V Virginis.*
63 1903 May 3.0
64 4 Jan. 8.5
65 Sept. 15.0
66 5 May 23.5
67 6 Jan. 29.0
68 Oct. 6.5
69 7 June 14.0
70 8 Feb. 19.5
71 Oct. 27.0
72 9 July 4.5
$M - m = . . .$
8.–9. ; <13.

4826. *R Hydrae.*
10 1903 Jan. 15.6
11 4 Mar. 8.6
12 5 Apr. 30.0
13 6 June 21.0
14 7 Aug. 11.5
15 8 Oct. 0.4
16 9 Nov. 20.0
$M - m = 6^m\ 7^d$
3.5–5.5 ; 9.7

4847. *S Virginis.*
50 1903 Aug. 25.0
51 4 Sept. 4.9
52 5 Sept. 16.8
53 6 Sept. 28.7
54 7 Oct. 10.6
55 8 Oct. 21.5
56 9 Nov. 2.4
$M - m = 5^m\ 5^d$:
6.–8. : 12.5

4940. *W Hydrae.*
14 1903 Nov. 28.
15 4 Dec. 6.
16 5 Dec. 25.
17 7 Jan. 13.
18 8 Feb. 1.
19 9 Feb. 19.
$M - m = . . .$
6.7 : 8.0

4948. *R Can. Venat.*
17 1903 Oct. 15.
18 4 Sept. 12.
19 5 Aug. 11.
20 6 July 10.
21 7 June 8.
22 8 May 6.
23 9 Apr. 4.
$M - m = . . .$
6.–7. : 11.5

5037. *RR Virginis.*
40 1903 Feb. 17.
41 Sept. 22.
42 4 Apr. 26.
43 Nov. 29.
44 5 July 4.
45 6 Feb. 6.
46 Sept. 11.
47 7 Apr. 16.
48 Nov. 19.
49 8 June 23.
50 9 Jan. 26.
51 Aug. 31.
$M - m = . . .$
11.–12. : <14.

5070. *Z Virginis.*
27 1903 Feb. 7.5
28 Dec. 12.0
29 4 Oct. 14.5
30 5 Aug. 18.0
31 6 June 21.5
32 7 Apr. 25.0
33 8 Feb. 26.5
34 Dec. 30.0
35 9 Nov. 2.5
$M - m = . . .$
9.5–11. ; <14

5095. *R Centauri.*
72 1903 Jan. 11.0
73 June 23.5
74 Dec. 1.0
75 4 May 9.5
76 Oct. 17.0
77 5 Mar. 26.5
78 Sept. 3.0
79 6 Feb. 10.5
80 July 21.0
81 Dec. 28.5
82 7 June 7.0
83 Nov. 11.5
84 8 Apr. 23.0
85 Oct. 0.5
86 9 Mar. 10.0
87 Aug. 17.5
$M - m = 2^m\ 0^d$:
6.0–6.3 ; 8.7–9.8

5157. *S Bootis.*
54 1903 Feb. 27.1
55 Nov. 29.5
56 4 Aug. 12.9
57 5 May 6.4
58 6 Jan. 28.0
59 Oct. 21.8
60 7 July 15.4
61 8 Apr. 7.4
62 Dec. 30.5
63 9 Sept. 23.7
$M - m = 1^m\ 10^d$
7.7–8.5 : 12.5–13.

5174. *RS Virginis.*
13 1903 Jan. 10.
14 Dec. 31.
15 4 Dec. 20.
16 5 Dec. 10.
17 6 Nov. 30.
18 7 Nov. 20.
19 8 Nov. 9.
20 9 Oct. 30.
$M - m = . . .$
8.2 : 12.?

5190. *R Camelop.*
45 1903 Feb. 8.5
46 Nov. 8.5
47 4 Aug. 7.2
48 5 May 6.8
49 6 Feb. 3.1
50 Nov. 2.4
51 7 Aug. 0.9
52 8 Apr. 28.4
53 9 Jan. 24.6
54 Oct. 22.5
$M - m = 1^m\ 20^d$
7.–8.5 : 12.–13.5

5194. *V Bootis.*
27 1903 Aug. 1.
28 4 Apr. 16.
29 Dec. 28.
30 5 Sept. 10.
31 6 May 24.
32 7 Feb. 4.
33 Oct. 18.
34 8 June 30.
35 9 Mar. 13.
36 Nov. 24.
$M - m = 3^m\ 11^d$
7.–7.5 ; 9.–10.5

5237. *R Bootis.*
73 1903 Feb. 4.4
74 Sept. 16.2
75 4 Apr. 27.2
76 Dec. 6.7
77 5 July 18.1
78 6 Feb. 26.3
79 Oct. 7.3

5237. *R Bootis.* Con.
80 1907 May 18.0
81 Dec. 26.5
82 8 Aug. 4.9
83 9 Mar. 15.2
84 Oct. 23.5
$M - m = 3^m\ 10^d.5$
6.–8. : 11.–12.

5249. *V Librae.*
30 1903 Apr. 30.0
31 4 Jan. 10.2
32 Sept. 21.4
33 5 June 3.6
34 6 Feb. 13.8
35 Oct. 27.0
36 7 July 9.2
37 8 Mar. 20.4
38 Dec. 0.6
39 9 Aug. 12.8
$M - m = 3^m\ 28^d$:
9.3 : 12.2

5321. *S Lupi.*
13 1903 Nov. 17.
14 4 Oct. 27.
15 5 Oct. 7.
16 6 Sept. 17.
17 7 Aug. 28.
18 8 Aug. 7.
19 9 July 18.
$M - m = . . .$
9.7 ; <12.

5338. *U Bootis.*
47 1903 Jan. 5.5
48 July 2.0
49 Dec. 26.5
50 4 June 21.0
51 Dec. 15.5
52 5 June 11.0
53 Dec. 5.5
54 6 June 1.0
55 Nov. 25.5
56 7 May 22.0
57 Nov. 15.5
58 8 May 11.0
59 Nov. 4.5
60 9 May 1.0
61 Oct. 25.5
$M - m = 3^m\ 4^d$
9.–10. : 12.–13.5

5396. *S Apodis.*
4 1903 Sept. 30.
5 4 July 24.
6 5 May 18.
7 6 Mar. 12.
$M - m = . . .$
9. : <11.5

5405. *RT Librae.*
13 1903 July 15.
14 4 Mar. 23.
15 Nov. 30.
16 5 Aug. 9.
17 6 Apr. 18.
18 Dec. 26.
19 7 Sept. 4.
20 8 May 13.
21 9 Jan. 20.
22 Sept. 29.
$M - m = . . .$
8.5 : 11.7<

5430. *T Librae.*
38 1903 Feb. 3.
39 Sept. 29.
40 4 May 24.
41 5 Jan. 17.
42 Sept. 12.
43 6 May 8.
44 7 Jan. 1.
45 Aug. 27.
46 8 Apr. 21.
47 Dec. 15.
48 9 Aug. 10.
$M - m = 3^m\ 14^d$
9.–10. : <14.7

5438. *Y Librae.*
56 1903 Mar. 7.
57 Dec. 4.
58 4 Sept. 1.
59 5 May 31.
60 6 Feb. 27.
61 Nov. 26.
62 7 Aug. 25.
63 8 May 23.
64 9 Feb. 19.
65 Nov. 18.
$M - m = . . .$
8.5 : 12.

5491. *S Librae.*
55 1903 May 22.6
56 Nov. 30.6
57 4 June 9.7
58 Dec. 18.8
59 5 June 28.9
60 6 Jan. 7.0
61 July 18.1
62 7 Jan. 26.2
63 Aug. 6.3
64 8 Feb. 14.4
65 Aug. 24.5
66 9 Mar. 4.6
67 Sept. 12.7
$M - m = 3^m\ 2^d$
7.5–8.3 ; <13.

5501. *S Serpentis.*
75 1903 Sept. 7.5
76 4 Sept. 18.1
77 5 Sept. 29.6
78 6 Oct. 11.0
79 7 Oct. 22.3
80 8 Nov. 1.1
81 9 Nov. 12.4
$M - m = . . .$
7.5–8.5 : 12.5?

5504. *S Coronae.*
43 1903 Mar. 11.3
44 4 Mar. 5.5
45 5 Mar. 0.4
46 6 Feb. 23.0
47 7 Feb. 17.6
48 8 Feb. 12.2
49 9 Feb. 5.8
$M - m = 3^m\ 29^d$
6.–8. : 12.–12.5

5541. *RS Librae.*
23 1903 June 7.
24 4 Jan. 14.
25 Aug. 22.
26 5 Mar. 31.
27 Nov. 7.
28 6 June 16.
29 7 Jan. 23.
30 Sept. 1.
31 8 Apr. 9.
32 Nov. 16.
33 9 June 25.
$M - m = 4^m\ 8^d$:
8.2 : 13.

5583. *X Librae.*
55 1903 Mar. 7.0
56 Aug. 17.6
57 4 Jan. 28.2
58 July 9.8
59 Dec. 20.4
60 5 June 2.0
61 Nov. 12.6
62 6 Apr. 25.2
63 Oct. 5.8
64 7 Mar. 18.4
65 Aug. 29.7
66 8 Feb. 8.6
67 July 21.2
68 9 Jan. 0.8
69 June 13.4
70 Nov. 24.0
$M - m = 2^m\ 19^d$
9.5–10. : 14

5593. *W Librae.*
44 1903 Feb. 22.0
45 Sept. 15.5
46 4 Apr. 8.0
47 Oct. 30.5

5593. *W Librae.* Con.
48 1905 May 21.0
49 Dec. 15.5
50 6 July 9.0
51 7 Jan. 30.5
52 Aug. 24.0
53 8 Mar. 16.5
54 Oct. 8.0
55 9 May 1.5
56 Nov. 23.0
$M - m = . . .$
9.8 : <14.

5601. *S Ursae min.*
14 1903 Feb. 27.
15 4 Jan. 18.
16 Dec. 8.
17 5 Oct. 29.
18 6 Sept. 19.
19 7 Aug. 10.
20 8 June 30.
21 9 May 21.
$M - m = 5^m\ 4^d$
7.5 : 11.5

5617. *U Librae.*
48 1903 Apr. 15.6
49 Nov. 27.8
50 4 July 11.0
51 5 Feb. 22.2
52 Oct. 6.4
53 6 May 20.6
54 7 Jan. 1.8
55 Aug. 16.0
56 8 Mar. 29.2
57 Nov. 10.4
58 9 June 21.6
$M - m = . . .$
9. : <14

5644. *Z Librae.*
31 1903 May 19.
32 4 Mar. 9.
33 Dec. 29.
34 5 Oct. 20.
35 6 Aug. 11.
36 7 June 2.
37 8 Mar. 23.
38 9 Jan. 12.
39 Nov. 3.
$M - m = . . .$
11. : <13.

5675. *[illegible]*
25 1903 Mar. 17.5
26 4 Mar. 8.0
27 5 Feb. 27.5
28 6 Feb. 19.0
29 7 Feb. 10.5
30 8 Feb. 2.0
31 9 Jan. 23.5
$M - m = 5^m\ 19^d$
7.2–8.5 ; 10.–12.

5677. *R Serpentis.*

78 1903 Aug. 25.6
79 4 Aug. 17.7
80 5 Aug. 13.9
81 6 Aug. 8.5
82 7 Aug. 3.1
83 8 July 27.6
84 9 July 22.1
$M - m = 5^m\ 0^d$
5.5–7.5 ; 13.

5682. *R Lupi.*

5 1903 Mar. 22.5
6 Nov. 12.0
7 4 July 3.5
8 5 Feb. 23.0
9 Oct. 15.5
10 6 June 7.0
$M - m = 3^m\ 26^d$?
9 ; <11

5688. *R Librae.*

68 1903 June 17.2
69 4 Feb. 14.6
70 Oct. 14.0
71 5 June 13.4
72 6 Feb. 10.8
73 Oct. 11.2
74 7 June 10.6
75 8 Feb. 8.0
76 Oct. 7.4
77 9 June 6.8
$M - m = . . .$
9. 10. ; <13.

5704. *RR Librae.*

24 1903 Aug. 16.8
25 4 May 19.5
26 5 Feb. 20.2
27 Nov. 23.9
28 6 Aug. 27.6
29 7 June 0.3
30 8 Mar. 3.0
31 Dec. 4.7
32 9 Sept. 7.4
$M - m = . . .$
8.5 ; 11.

5761. *Z Scorpii.*

30 1903 Oct. 4.
31 4 Oct. 8.
32 5 Oct. 13.
33 6 Oct. 18.
34 7 Oct. 23.
35 8 Oct. 27.
36 9 Nov. 1.
$M - m = . . .$
9.–9.5 ; 12.?

5768. *RR Herculis.*

13 1903 May 8.:
14 4 Jan. 1.:
15 Aug. 26.:
16 5 July 30.:
17 6 Mar. 25.:
18 Nov. 18.:
$M - m = . . .$
7.8 ; 9.5

5770. *R Herculis.*

44 1903 Nov. 11.2
45 4 Sept. 25.4
46 5 Aug. 11.2
47 6 June 26.1
48 7 May 10.4
49 8 Mar. 23.3
50 9 Feb. 3.7
$M - m = . . .$
8.–9. ; <13.

5776. *X Scorpii.*

49 1903 Jan. 0.
50 July 18.
51 4 Feb. 2.
52 Aug. 19.
53 5 Mar. 6.
54 Sept. 21.
55 6 Apr. 8.
56 Oct. 24.
57 7 May 11.
58 Nov. 26.
59 8 June 12.
60 9 Jan. 2.
61 July 15.
$M - m = . . .$
10. ; <13.

5795. *W Scorpii.*

44 1903 Feb. 1.0
45 Sept. 10.5
46 4 Apr. 19.0
47 Nov. 26.5
48 5 July 6.0
49 6 Feb. 12.5
50 Sept. 22.0
51 7 May 1.5
52 Dec. 9.0
53 8 July 17.5
54 9 Feb. 24.0
55 Oct. 3.5
$M - m = 1^m\ 8^d$:
10.–11. ; <14.7

5830. *R Scorpii.*

65 1903 Feb. 14.5
66 Sept. 26.6
67 4 May 7.7
68 Dec. 17.8
69 5 July 29.9
70 6 Mar. 11.0
71 Oct. 21.1

5830. *R Scorpii*–Con.

72 1907 June 2.2
73 8 Jan. 12.3
74 Aug. 23.1
75 9 Apr. 4.5
76 Nov. 14.6
$M - m = . . .$
9.5-10.5 ; <13.

5834. *S Scorpii.*

136 1903 Mar. 19.6
137 Sept. 12.3
138 4 Mar. 7.0
139 Aug. 30.7
140 5 Feb. 23.4
141 Aug. 19.1
142 6 Feb. 11.8
143 Aug. 7.5
144 7 Feb. 0.2
145 July 26.9
146 8 Jan. 19.6
147 July 14.3
148 9 Jan. 7.0
149 July 2.7
$M - m = . . .$
9.–10.5 : <13.

5856. *W Ophiuchi.*

24 1903 Mar. 17.2
25 4 Feb. 10.0
26 5 Jan. 4.8
27 Dec. 0.6
28 6 Oct. 26.4
29 7 Sept. 21 2
30 8 Aug. 16.0
31 July 11.8
$M - m = . . .$
9.–9.5 ; <13.5

5887. *V Ophiuchi.*

35 1903 May 12.5
36 4 Mar. 10.0
37 5 Jan. 6.5
38 Nov. 5.0
39 6 Sept. 3.5
40 7 July 3.0
41 8 May 0.5
42 9 Feb. 27.0
43 Dec. 26.5
$M - m = 5^m\ 24^d$
7.–7.5 ; 9.5–10.5

5889. *U Herculis.*

38 1903 Feb. 24.2
39 4 Mar. 27.5
40 5 Apr. 28.0
41 6 May 28.7
42 7 June 27.6
43 8 July 25.6
44 9 Aug. 22.8
$M - m = 5^m\ 19^d$
6.5–7.5 ; 11.5–12.5

5903. *Y Scorpii.*

27 1903 June 27.
28 4 June 26.
29 5 June 26.
30 6 June 26.
31 7 June 26.
32 8 June 25.
33 9 June 25.
$M - m = . . .$
10.? ; 11.

5931. *S Ophiuchi*

72 1903 Aug. 1.6
73 4 Mar. 22.4
74 Nov. 11.2
75 5 July 3.0
76 6 Feb. 21.8
77 Oct. 13.6
78 7 June 4.4
79 8 Jan. 24.2
80 Sept. 14.0
81 9 May 5.8
82 Dec. 25.6
$M - m = . . .$
8.3–9. ; <13.

5950. *W Herculis.*

31 1903 May 3.6
32 4 Feb. 12.4
33 Nov. 21.6
34 5 Aug. 30.8
35 6 June 8.0
36 7 Mar. 14.6
37 Dec. 18.2
38 8 Sept. 20.4
39 9 June 23.5
$M - m = 4^m\ 6^d$
8.–8.5 : 11.5–14.

5955. *R Draconis.*

39 1903 June 28.2
40 4 Feb. 28.5
41 Nov. 0.4
42 5 July 4.0
43 6 Mar. 6.6
44 Nov. 7.2
45 7 July 10.8
46 8 Mar. 12.4
47 Nov. 13.0
48 9 July 16.6
$M - m = 3^m\ 17^d$
6.5–8.5 ; 12.–13.

6044. *S Herculis*

55 1903 Jan. 15.5
56 Nov. 16.0
57 4 Sept. 16.1
58 5 July 19.0
59 6 May 21.6
60 7 Mar. 25.1
61 8 Jan. 27.4
62 Dec. 1.6
63 9 Oct. 7.7
$M - m = 5^m\ 0^d$
6.–7.5 ; 11.5–13.

6062. *RR Scorpii.*

21 1903 Sept. 10.
22 4 June 18.
23 5 Mar. 27.
24 6 Jan. 3.
25 Oct. 12.
26 7 July 21.
27 8 Apr. 28.
28 9 Feb. 4.
29 Nov. 13.
$M - m = 4^m\ 8^d$:
7.–7.5 : 9.5–10.

6132. *R Ophiuchi.*

55 1903 Jan. 13.0
56 Nov. 11.2
57 4 Sept. 8.4
58 5 July 7.6
59 6 May 5.8
60 7 Mar. 4.0
61 8 Jan. 0.2
62 Oct. 28.4
63 9 Aug. 26.6
$M - m = . . .$
7.–8. ; <12.

6170. *RW Scorpii.*

13 1903 Nov. 24.
14 4 Dec. 15.
15 6 Jan. 6.
16 7 Jan. 28.
17 8 Feb. 19.
18 9 Mar. 12.
$M - m = . . .$
9.5 ; 14.

6207. *Z Ophiuchi.*

11 1903 Oct. 30.
12 4 Oct. 12.
13 5 Sept. 25.
14 6 Sept. 8.
15 7 Aug. 22.
16 8 Aug. 4.
17 9 July 18.
$M - m = 5^m\ 16^d$
7.5–8. ; 11.5–12.5

6225. *RS Herculis.*

11 1903 Apr. 22.
12 Dec. 1.
13 4 July 11.
14 5 Feb. 19.
15 Sept. 30.
16 6 May 11.
17 Dec. 20.
18 7 July 31.
19 8 Mar. 10.
20 Oct. 19.
21 9 May 30.
$M - m = . . .$
8. ; 11.

6275. *S Octantis.*

5 1903 Oct. 31.
6 4 July 22.
7 5 Apr. 13.
8 6 Jan. 3.
9 Sept. 25.
10 7 June 17.
11 8 Mar. 8.
12 Nov. 28.
13 9 Aug. 20.
$M - m = 3^m\ 16^d$
8. ; <11.7

6331. *RU Scorpii.*

13 1903 Jan. 28.
14 4 Feb. 10.
15 5 Feb. 22.
16 6 Mar. 7.
17 7 Mar. 20.
18 8 Apr. 1.
19 9 Apr. 14.
$M - m = . . .$
9.3 ; 12.7

6449. *T Draconis.*

7 1903 Feb. 9.
8 4 Apr. 10.
9 5 June 10.
10 6 Aug. 10.
11 7 Sept. 20.
12 8 Dec. 9.
$M - m = 6^m\ 0^d$
8. ; 10.–11.5

6500. *R Pavonis.*

5 1903 May 16.
6 4 Jan. 0.
7 Aug. 16.
8 5 Apr. 2.
9 Nov. 17.
10 6 July 4.
$M - m = 3^m\ 18^d$?
7.5 ; 10.

6512. *T Herculis.*

78 1903 June 9.4
79 Nov. 20.7
80 4 May 3.0
81 Oct. 14.2
82 5 Mar. 27.4
83 Sept. 7.6
84 6 Feb. 18.7
85 Aug. 1.9
86 7 Jan. 13.0
87 June 26.1
88 Dec. 7.3
89 8 May 19.4
90 Oct. 30.6
91 9 Apr. 12.8
92 Sept. 24.0
$M - m = 2^m\ 18^d$
7.–8.5 ; 10.–12.5

6608. *RV Sagittarii.*
16 1903 Aug. 7.
17 4 June 22.
18 5 May 8.
19 6 Mar. 24.
20 7 Feb. 7.
21 Dec. 24.
22 8 Nov. 8.
23 9 Sept. 24.
$M - m = . . .$
8. ; 12.3

6624. *T Serpentis.*
45 1903 June 24.0
46 4 May 30.8
47 5 May 7.6
48 6 Apr. 14.4
49 7 Mar. 22.2
50 8 Feb. 27.0
51 9 Feb. 2.8
$M - m = . . .$
7. ; 8.3

6682. *X Ophiuchi.*
19 1903 Nov. 7.
20 4 Oct. 7.
21 5 Sept. 7.
22 6 Aug. 8.
23 7 July 9.
24 8 June 8.
25 9 May 9.
$M - m = 6^m\ 25^d$
7. ; 9.

6849. *R Aquilae.*
51 1903 June 26.6
52 4 May 21.5
53 5 Apr. 16.7
54 6 Mar. 13.2
55 7 Feb. 7.1
56 8 Jan. 1.4
57 9 Dec. 1.1
58 Oct. 29.2
$M - m = 4^m\ 16^d$
6.–7.5 ; 11.–11.5

6871. *V Lyrae.*
10 1903 Dec. 6.
11 4 Dec. 15.
12 5 Dec. 25.
13 7 Jan. 4.
14 8 Jan. 11.
15 9 Jan. 23.
$M - m = 4^m\ 17^d$
9. ; <12.

6892. *RX Sagittarii.*
10 1903 Nov. 3.
11 4 Sept. 30.
12 5 Aug. 28.
13 6 July 26.
14 7 June 23.
15 8 May 20.
16 9 Apr. 17.
$M - m = . . .$
10. ; <13.3

6894. *S Lyrae.*
16 1903 Jan. 27.
17 Sept. 2.
18 4 Apr. 7.
19 Nov. 11.
20 5 June 17.
21 6 Jan. 21.
22 Aug. 27.
23 7 Apr. 2.
24 Nov. 6.
25 8 June 11.
26 9 Jan. 15.
27 Aug. 21.
$M - m = 3^m\ 24^d$
9. ; 12.

6900. *W Aquilae.*
8 1904 Jan. 16.
9 5 May 10.
10 6 Sept. 2.
11 7 Dec. 26.
12 9 Apr. 19.
$M - m . . .$
7.5 ; 11.

6903. *T Sagittarii.*
34 1903 Apr. 19.2
35 4 June 2.5
36 5 July 18.8
37 6 Sept. 4.1
$M - m = . . .$
7.5–8. ; <11.

6905. *R Sagittarii.*
50 1903 May 28.7
51 4 Feb. 20.9
52 Nov. 14.6
53 5 Aug. 8.8
54 6 May 2.6
55 7 Jan. 24.0
56 Oct. 17.2
57 8 July 9.4
58 9 Apr. 1.0
$M - m = 1^m\ 16^d$
7.–8. ; 12.5

6921. *S Sagittarii.*
58 1903 May 3.1
59 Dec. 22.2
60 4 Aug. 11.4
61 5 Apr. 1.7
62 Nov. 21.0
63 6 July 12.2
64 7 Mar. 2.3
65 7 Oct. 21.2
66 8 June 9.7
67 9 Jan. 27.9
68 Dec. 25.7
$M - m = 3^m\ 11^d$
9:10.5 ; 14.5

6923. *Z Sagittarii.*
12 1903 June 23.
13 4 Sept. 17.
14 5 Dec. 13.
15 7 Mar. 10.
16 8 June 1.
17 9 Aug. 30.
$M - m = 7^m\ 13^d$
8.5 ; <12.

7045. *R Cygni.*
42 1903 Oct. 21.1
43 4 Dec. 21.2
44 6 Feb. 21.0
45 7 Apr. 23.7
46 8 June 23.6
47 9 Aug. 21.1
$M - m = 5^m\ 5^d$
6.–8. ; <14.

7077. *T Pavonis.*
21 1903 May 15.
22 4 Jan. 14.
23 Sept. 14.
24 5 May 16.
25 6 Jan. 15.
26 Sept. 16.
27 7 May 18.
28 8 Jan. 17.
29 Sept. 17.
30 9 May 19.
$M - m = . . .$
7.5 ; 12.

7085. *RT Cygni.*
30 1903 Apr. 24.0
31 Nov. 0.5
32 4 May 9.0
33 Nov. 15.5
34 5 May 25.0
35 Dec. 1.5
36 6 June 10.0
37 Dec. 17.5
38 7 June 26.0
39 8 Jan. 2.5
40 July 11.0
41 9 Jan. 17.5
42 July 27.0
$M - m = 2^m\ 27^d$
7.–7.5 ; 10.–11.

7118. *X Aquilae.*
10 1903 Feb. 24.
11 4 Feb. 7.
12 5 Jan. 20.
13 6 Jan. 3.
14 Dec. 17.
15 7 Nov. 30.
16 8 Nov. 12.
17 9 Oct. 26.
$M - m = . . .$
8.5 ; <12.

7120. *χ Cygni.*
126 1903 Oct. 25.2
127 4 Dec. 1.0
128 6 Jan. 13.7
129 7 Feb. 23.6
130 8 Apr. 1.5
131 9 May 15.5
$M - m = 5^m\ 19^d.5$
4.–6.5 ; 13.5

7151. *RU Sagittarii.*
5 1903 Oct. 27.
6 4 June 22.
7 5 Feb. 16.
8 Oct. 13.
9 6 June 9.
10 7 Feb. 3.
$M - m = 3^m\ 3^d$
8. ; <12.5

7155. *RR Aquilae.*
8 1904 Jan. 7.
9 5 Feb. 5.
10 6 Mar. 7.
11 7 Apr. 6.
12 8 May 5.
13 9 June 4.
$M - m = . . .$
8.5 ; <12.

7162. *RS Aquilae.*
6 1903 Apr. 6.
7 4 May 16.
8 5 June 26.
9 6 Aug. 6.
10 7 Sept. 16.
11 8 Oct. 26.
12 9 Dec. 6.
$M - m = . . .$
10. ; <12.5

7192. *Z Cygni.*
22 1903 Feb. 26.
23 Nov. 18.
24 4 Aug. 9.
25 5 May 1.
26 6 Jan. 21.
27 Oct. 13.
28 7 July 5.
29 8 Mar. 26.
30 Dec. 16.
31 9 Sept. 7.
$M - m = 4^m\ 3^d$;
7.–8.5 ; 11.5–12.

7220. *S Cygni.*
43 1903 Aug. 8.7
44 4 June 28.0
45 5 May 18.4
46 6 Apr. 7.7
47 7 Feb. 26.4
48 8 Jan. 16.6
49 Dec. 6.0
50 9 Oct. 26.5
$M - m = 5^m\ 11^d$
9–11. ; <11.

7231. *R Capricorni.*
16 1903 Apr. 10.
17 4 Mar. 19.
18 5 Feb. 28.
19 6 Feb. 5.
50 7 Jan. 15.
51 Dec. 25.
52 8 Dec. 3.
53 9 Nov. 12.
$M - m = . . .$
9.–11. ; <13.

7242. *S Aquilae.*
93 1903 Mar. 22.4
94 Aug. 15.8
95 4 Jan. 9.5
96 June 1.2
97 Oct. 28.9
98 5 Mar. 21.6
99 Aug. 18.3
100 6 Jan. 12.0
101 June 7.7
102 Nov. 1.4
103 7 Mar. 28.1
104 Aug. 21.8
105 8 Jan. 15.5
106 June 10.2
107 Nov. 3.9
108 9 Mar. 30.6
109 Aug. 24.4
$M - m = 2^m\ 11^d$
8.5–10. ; 10.7–11.8
Secondary phases also.

7252. *W Capricorni.*
54 1903 Apr. 12.
55 Nov. 6.
56 4 June 1.
57 Dec. 26.
58 5 July 22.
59 6 Feb. 15.
60 Sept. 11.
61 7 Apr. 7.
62 Nov. 1.
63 8 May 27.
64 Dec. 21.
65 9 July 17.
$M - m = 2^m\ 26^d$
10.–11. ; <11.7

7260. *Z Aquilae.*
24 1903 Mar. 9.8
25 July 15.0
26 Nov. 19.2
27 4 Mar. 25.4
28 July 30.8
29 Dec. 4.8
30 5 Apr. 11.0
31 Aug. 16.2
32 Dec. 21.4

7260. *Z Aquilae.*—Con.
33 1906 Apr. 27.6
34 Sept. 1.8
35 7 Jan. 7.0
36 May 11.2
37 Sept. 18.4
38 8 Jan. 23.6
39 May 29.8
40 Oct. 4.0
41 9 Feb. 8.2
42 June 15.4
43 Oct. 20.6
$M - m = 2^m\ 4^d$
9. ; 11.

7261. *R Delphini.*
48 1903 Jan. 0.2
49 Oct. 12.1
50 4 July 23.6
51 5 May 5.7
52 6 Feb. 16.3
53 Dec. 0.5
54 7 Sept. 14.1
55 8 June 28.0
56 Apr. 12.2
$M - m = 4^m\ 8^d$;
7.5–9. ; 11–13.

7266. *RT Sagittarii.*
4 1903 Sept. 28.
5 4 July 25.
6 5 May 22.
7 6 Mar. 19.
$M - m = 1^m\ 8^d$
7.5 ; <11

7299. *U Cygni.*
25 1903 Jan. 13.5
26 4 Apr. 18.8
27 5 July 24.1
28 6 Oct. 28.4
29 8 Feb. 1.7
30 9 May 8.0
$M - m = 7^m\ 16^d$
7–8 ; 9.5–11.5

7404. *RW Cygni.*
19 1903 Feb. 1.8
20 June 20.6
21 Nov. 6.4
22 4 Mar. 24.2
23 Aug. 10.0
24 Dec. 26.8
25 5 May 14.6
26 Oct. 0.4
27 6 Feb. 16.2
28 July 5.0
$M - m = 2 . . 5$
8 ; 12

7128. *V Cygni.*
19 1903 Mar. 12.
20 4 May 3.
21 5 June 25.
22 6 Aug. 17.
23 7 Oct. 9.
24 8 Nov. 30.
$M - m = 7^m\ 7^d$
7.–9.5 ; 13.5

7131. *S Delphini.*
49 1903 Apr. 13.5
50 4 Jan. 16.0
51 Oct. 19.5
52 5 July 24.0
53 6 Apr. 27.5
54 7 Jan. 30.0
55 Nov. 3.5
56 8 Aug. 7.0
57 9 May 11.5
$M - m = 3^m\ 27^d$
8.5–9.5 ; 10.5–12.

7144. *T Delphini.*
43 1903 Sept. 16.6
44 4 Aug. 12.8
45 5 July 10.0
46 6 June 6.2
47 7 May 3.4
48 8 Mar. 29.6
49 9 Feb. 23.8
$M - m =$ . . .
8.–10. ; <13.

7455. *U Capricorni.*
82 1903 Feb. 24.0
83 Sept. 16.2
84 4 Apr. 7.3
85 Oct. 28.5
86 5 May 20.8
87 Dec. 11.0
88 6 July 3.2
89 7 Jan. 23.5
90 Aug. 15.7
91 8 Mar. 6.8
92 Sept. 27.0
93 9 Apr. 19.0
$M - m =$ . . .
10.–11. ; <13.

7158. *V Delphini.*
9 1904 Apr. 11.
10 5 Oct. 3.
11 7 Mar. 27.
12 8 Sept. 17.
$M - m =$ . . .
9. ; 12.?

7168. *T Aquarii.*
74 1903 Feb. 2.2
75 Aug. 24.4
76 4 Mar. 14.5

7168. *T Aquar.*–Con.
77 1904 Oct. 3.5
78 5 Apr. 24.3
79 Nov. 13.1
80 6 June 3.7
81 Dec. 23.2
82 7 July 13.6
83 8 Feb. 1.0
84 Aug. 21.3
85 9 Mar. 11.5
86 Sept. 29.8
$M - m = 2^m\ 27^d$
6.5–8.5 ; 12.5–13.

7482. *U Pavonis.*
15 1903 May 9.
16 4 Feb. 21.
17 Dec. 5
18 5 Sept. 19.
19 6 July 4.
20 7 Apr. 18.
$M - m =$ . . .
9.5 ; <12.5

7492. *RZ Cygni.*
13 1903 July 6.
14 4 Apr. 11.
15 5 Jan. 16.
16 Oct. 23.
17 6 July 30.
$M - m = 4^m\ 15^d$
9. ; 13.

7495. *S Indi.*
2 1903 Apr. 6.4
3 4 May 16.1
4 5 June 25.8
5 6 Aug. 5.5
$M - m =$ . . .
8.5 ; <12.5

7502. *X Delphini.*
10 1903 Apr. 15.
11 4 Jan. 17.
12 Oct. 20.
13 5 July 24.
14 6 Apr. 27.
15 7 Jan. 29.
16 Nov. 2.
17 8 Aug. 5.
18 9 May 9.
$M - m = 3^m\ 29^d$
8.0–8.5 ; <10.

7514. *T Octantis.*
6 1903 May 16.
7 Dec. 7.
8 4 June 29.
9 5 Jan. 20.
10 Aug. 13.
11 6 Mar. 6.
$M - m = 1^m\ 25^d$
9. ; <12.5

7560. *R Vulpeculae.*
100 1903 Mar. 11.7
101 July 28.2
102 Dec. 10.8
103 4 Apr. 24.2
104 Sept. 6.6
105 5 Jan. 20.0
106 June 4.4
107 Oct. 17.8
108 6 Mar. 2.2
109 July 15.6
110 Nov. 28.0
111 7 Apr. 12.5
112 Aug. 26.0
113 8 Jan. 8.6
114 May 23.1
115 Oct. 5.8
116 9 Feb. 18.7
117 July 4.4
118 Nov. 17.2
$M - m = 2^m\ 1^d$
7.5–8.5 ; 12.5–13.5

7571. *V Capricorni.*
83 1903 Apr. 13.1
84 Sept. 16.8
85 4 Feb. 20.5
86 July 26.2
87 Dec. 29.9
88 5 June 4.6
89 Nov. 8.3
90 6 Apr. 14.0
91 Sept. 17.7
92 7 Feb. 21.4
93 July 28.1
94 8 Jan. 0.8
95 June 5.5
96 Nov. 9.2
97 9 Apr. 14.9
98 Sept. 18.6
$M - m =$ . . .
9. ; 14.?

7577. *X Capricorni.*
60 1903 May 28.2
61 4 Jan. 2.8
62 Aug. 9.9
63 5 Mar. 18.4
64 Oct. 25.4
65 6 June 3.7
66 7 Jan. 11.1
67 Aug. 20.7
68 8 Mar. 29.3
69 Nov. 5.7
70 9 June 15.0
$M - m = 3^m\ 26^d$.
9.5–10.5 ; <16.

7590. *Z Capricorni.*
8 1903 Sept. 15.
9 4 Sept. 5.
10 5 Aug. 27.
11 6 Aug. 18.
12 7 Aug. 9.

7590. *Z Capri.*–Con.
13 1908 July 30.
14 9 July 21.
$M - m =$ . . .
9. ; 11.5
Another max. midway?

7609. *T Cephei.*
28 1903 Mar. 21.
29 4 Apr. 11.
30 5 May 3.
31 6 May 25.
32 7 June 16.
33 8 July 7.
34 9 July 29.
$M - m = 6^m\ 25^d$
5.–7. ; 8.5–10.5

7659. *T Capricorni.*
65 1903 Sept. 18.0
66 4 June 13.2
67 5 Mar. 9.4
68 Dec. 3.6
69 6 Aug. 29.8
70 7 May 26.0
71 8 Feb. 19.2
72 Nov. 14.4
73 9 Aug. 10.6
$M - m = 4^m\ 27^d$
9.–9.5 ; 13.5

7733. *Y Capricorni.*
31 1903 Mar. 2.
32 Sept. 24.
33 4 Apr. 17.
34 Nov. 9.
35 5 June 3.
36 Dec. 26.
37 6 July 20.
38 7 Feb. 11.
39 Sept. 5.
40 8 Mar. 29.
41 Oct. 21.
42 9 May 15.
$M - m =$ . . .
10.–11. ; 14.?

7779. *S Cephei.*
29 1904 Feb. 8.2
30 5 June 11.0
31 6 Oct. 12.9
32 8 Feb. 13.8
33 9 June 16.8
$M - m = 8^m\ 23^d$
7.5–9. ; 11.5–12.5

7813. *R Gruis.*
12 1903 Sept. 11.
13 4 Aug. 7.
14 5 July 4.
15 6 May 31.
16 7 Apr. 27.
$M - m =$ . . .
8.5 ; <12.5

7896. *V Pegasi.*
10 1903 Sept. 25.
11 4 July 24.
12 5 May 23.
13 6 Mar. 22.
14 7 Jan. 19.
15 Nov. 18.
16 8 Sept. 16.
17 9 July 16.
$M - m = 3^m\ 24^d$:
8. ; <13.

7907. *U Aquarii.*
39 1903 Feb. 21.:
40 Nov. 6.:
41 4 July 21.:
42 5 Apr. 5.:
43 Dec. 19.:
44 6 Sept. 3.:
$M - m =$ . . .
9.5–10. ; 14.?

7909. *S Piscis austr.*
17 1903 May 9.
18 4 Feb. 5.
19 Nov. 3.
20 5 Aug. 2.
21 6 May 1.
22 7 Jan. 28.
$M - m =$ . . .
8.5–9. ; <11.

7914. *T Pegasi.*
38 1903 Sept. 1.4
39 4 Sept. 9.2
40 5 Sept. 18.0
41 6 Sept. 26.8
42 7 Oct. 5.6
43 8 Oct. 13.4
44 9 Oct. 22.2
$M - m =$ . . .
8.5–9.5 ; <13.

7994. *R Piscis austr.*
38 1903 Mar. 8.
39 Dec. 25.
40 4 Oct. 12.
41 5 July 31.
42 6 May 19.
43 7 Mar. 7.
44 Dec. 24.
45 8 Oct. 11.
46 9 July 30.
$M - m =$ . . .
8.5 ; <11.?

7999. *X Aquarii.*
9 1903 Mar. 26.
10 4 Feb. 4.
11 Dec. 15.
12 5 Oct. 26.
13 6 Sept. 6.

7999. *X Aquar.*–Con.
14 1907 July 18.
15 8 May 28.
16 9 Apr. 8.
$M - m =$ . . .
8.5 ; 13.

8039. *T Gruis.*
8 1903 Feb. 20.
9 July 11.
10 Nov. 29.
11 4 Apr. 18.
12 Sept. 6.
13 5 Jan. 25.
14 June 15.
$M - m = 2^m\ 3^d$
8.5 ; 11.0

8040. *S Gruis.*
2 1903 Feb. 4.
3 4 Mar. 20.
4 5 May 4.
5 6 June 18.
$M - m =$ . . .
7. ; 12.5

8068. *S Lacertae.*
18 1903 Jan. 27.
19 Sept. 18.
20 4 May 9.
21 Dec. 29.
22 5 Aug. 20.
23 6 Apr. 11.
24 Dec. 1.
$M - m = 3^m\ 22^d$
7.5–8. ; <12.

8153. *R Lacertae.*
25 1903 Aug. 25.0
26 4 June 19.8
27 5 Apr. 15.6
28 6 Feb. 9.4
29 Dec. 6.2
30 7 Oct. 2.0
31 8 July 27.8
32 9 May 23.6
$M - m =$ . . .
9. ; <13.5

8230. *S Aquarii.*
57 1903 Aug. 10.9
58 4 May 16.6
59 5 Feb. 20.3
60 Nov. 27.0
61 6 Sept. 2.7
62 7 June 9.4
63 8 Mar. 15.1
64 Dec. 19.8
65 9 Sept. 25.5
$M - m =$ . . .
7.5–9. ; <12.5

8290. *R Pegasi.*

51	1903	Aug. 28.1
52	4	Sept. 6.1
53	5	Sept. 17.0
54	6	Sept. 29.0
55	7	Oct. 12.0
56	8	Oct. 25.1
57	9	Nov. 9.1

$M - m = 5^m\ 20^d$:
7.–8. : <13.

8324. *V Cassiopeae.*

15	1903	May 26.5
16	4	Jan. 13.0
17		Sept. 0.5
18	5	Apr. 20.0
19		Dec. 7.5
20	6	July 27.0
21	7	Mar. 15.5
22		Nov. 2.0
23	8	June 20.5
24	9	Feb. 7.0
25		Sept. 26.5

$M - m = 3^m\ 16^d$
7.–8. : 12.5

8369. *W Pegasi.*

8	1903	Apr. 8.
9	4	Mar. 14.
10	5	Feb. 18.
11	6	Jan. 25.

$M - m = \ldots$
8. : 10.

8373. *S Pegasi.*

44	1903	Mar. 6.5
45	4	Jan. 18.0
46		Dec. 0.5
47	5	Oct. 14.0
48	6	Aug. 27.5
49	7	July 11.0
50	8	May 23.5
51	9	Apr. 6.0

$M - m = 4^m\ 16^d$
7.5–8. : 12.–<13.

8512. *R Aquarii.*

86	1903	Feb. 6.1
87	4	Mar. 5.3
88	5	Apr. 1.8
89	6	Apr. 28.6
90	7	May 24.7
91	8	June 17.9
92	9	July 12.1

$M - m = \ldots$
6.–8.5 : 11.?

8588. *R Phoenicis.*

4	1903	Mar. 5.
5		Nov. 30.
6	4	Aug. 26.
7	5	May 23.
8	6	Feb. 17.
9		Nov. 11.

$M - m = 4^m\ 15^d$?
8.5? : 11.?

8591. *V Cephei.*

21	1903	Nov. 27.
22	4	Nov. 21.
23	5	Nov. 16.
24	6	Nov. 11.
25	7	Nov. 6.
26	8	Oct. 31.
27	9	Oct. 26.

$M - m = 7^m\ 7^d$
6.3 : 7.0

8594. *R Tucanae.*

4	1903	Apr. 30.
5	4	Jan. 30.
6		Oct. 31.
7	5	Aug. 2.
8	6	May 4.

$M - m = \ldots$
10. : <12.5

8597. *V Ceti.*

33	1903	Mar. 31.
34		Dec. 15.
35	4	Sept. 1.
36	5	May 20.
37	6	Feb. 5.
38		Oct. 24.
39	7	July 12.
40	8	Mar. 29.
41		Dec. 15.
42	9	Sept. 2.

$M - m = \ldots$
8.5–9.5 : 11.?

8600. *R Cassiopeae.*

42	1904	Feb. 21.5
43	5	Apr. 28.7
44	6	July 4.2
45	7	Sept. 7.9
46	8	Nov. 10.8

$M - m = 6^m\ 0^d$
5.–7. : 10.–12.

8604. *S Phoenicis.*

7	1903	Jan. 1.1
8		June 1.4
9		Oct. 30.8
10	4	Mar. 30.0
11		Aug. 28.2
12	5	Jan. 26.4
13		June 26.6

$M - m = 2^m\ 5^d$
7.2 : 8.7

8622. *W Ceti.*

7	1903	Jan. 12.
8	4	Jan. 13.
9	5	Jan. 13.
10	6	Jan. 14.
11	7	Jan. 15.
12	8	Jan. 16.
13	9	Jan. 16.

$M - m = \ldots$
8.5 : 12.

RESULTS OF OBSERVATIONS WITH THE ZENITH-TELESCOPE.

FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA.

BY C. L. DOOLITTLE.

The series of Zenith-Telescope Observations, which has been in progress at this place, was begun in October, 1896. It was intended to continue this series on the same general plan for a period of seven years. This program has been carried out with but slight variation. The first observation was made Oct. 1, 1896. The series was brought to a close Dec. 28, 1903.

The number of determinations in the successive years was as follows:

1896	456
1897	1734
1898	1582
1899	1830
1900	1719
1901	1819
1902	1786
1903	1971
Total	12897

All of these observations, excepting about 300, were made by myself.

It is now proposed to continue this work for a time on a somewhat more elaborate plan, giving special attention to the local and diurnal changes which are sometimes noticed. These apparent changes are commonly attributed to anomalous refraction, or temperature effects, which simply amounts to the statement that we are ignorant of their true cause. Whatever this may be, it seems desirable to investigate the subject. Also the wide range found in the different determinations of the Aberration Constant appears to indicate errors of a systematic character, some of which it should be possible to detect.

The values resulting from the seven years' observation at this place, and about three years at South Bethlehem, are reasonably accordant, but they are uniformly larger than those commonly employed.

The present plan is to carry on simultaneous observations with two instruments. With this in view the zenith-telescope has been thoroughly renovated, and the four-inch objective has been replaced by one of 5¼-inches aperture. The tube has been correspondingly lengthened. This makes it possible to select pairs of stars which will be free from long intervals of time between the two stars, and from large differences of zenith-distance.

The width of the observing slit has at the same time been increased to six feet.

The second instrument is to be a reflex zenith-tube, eight inches in aperture, on the same general plan as that in use at Greenwich. Mr. JOSEPH WHARTON has generously provided the means for installing this instrument, and WARNER and SWASEY have its construction well advanced. Meanwhile the zenith stars are now being observed with the zenith-telescope.

The values of the latitude which follow are in continuation of those found in No. 558 of this *Journal*.

The value of the Aberration Constant from this series is

$$20''.524 \pm .0088$$

$\varphi = 39° 58' +$

Date	I	No.	II	No.
1902 Dec. 14	1.94	4	..	..
18	1.92	10	..	..
23	2.14	10	..	..
25	2.16	10	..	..
27	1.86	10	..	..
1903 June 6	2.02	10	..	..
9	2.02	10	..	..
11	2.01	10	..	..
12	2.08	9	..	..
13	2.21	6	..	..
16	1.91	9	..	..
17	2.14	10	..	..
19	2.02	7	..	..
21	2.05	5	..	..
22	1.91	9	1.89	9
23	2.30	10	..	..
28	2.09	2	..	..
30	2.10	10	2.01	10
Feb. 2	2.09	10	1.87	10
5	2.14	10	2.10	10
6	2.02	9	1.82	2
9	2.02	8	1.99	10
10	2.06	10	..	..
11	..	..	1.90	8
12	1.99	10	2.08	10
17	2.02	10	2.02	10
18	..	..	2.16	10
19	1.96	10	..	..
20	2.22	10	2.15	10
21	..	..	2.08	8
22	1.79	8	1.99	10
23	2.15	10	2.08	10
25	2.06	10	2.03	10
26	2.17	10	2.04	9
Mar. 1	2.15	10	2.08	10
4	2.07	7	2.06	8
6	1.80	10	..	..
11	2.29	9	2.22	10
12	2.17	10	2.16	10
13	..	..	2.31	9
14	2.21	10	..	..
Apr. 9	..	..	2.18	10
10	..	..	2.19	10
17	..	..	2.25	10
18	..	..	2.32	9
1903 Apr. 19	..	..	2.22	10
20	..	..	2.57	1
21	..	..	2.34	10
22	..	..	2.16	10
26	..	..	2.67	2
27	..	..	2.22	10
29	..	..	2.29	10
30	..	..	2.05	1

Date	II	No.	III	No.
1903 May 1	2.45	10	2.32	10
2	2.14	10	..	..
5	2.26	10	2.27	10
6	2.19	10	2.14	4
7	2.45	10	2.40	10
8	..	..	2.61	10
9	2.41	10	2.43	10
10	2.22	10	2.11	10
11	2.12	10	..	..
12	2.36	10	..	..
13	2.25	10	1.95	2
15	2.44	6	2.41	10
17	2.28	9	2.23	10
20	2.37	7	2.32	10
21	2.20	9	..	..
23	2.52	9	..	..
25	2.33	10	2.16	10
26	2.21	9	..	..
28	2.15	8	2.36	10
31	2.43	9	2.29	10
June 2	2.26	10	2.21	9
3	2.38	10	2.29	10
8	..	..	2.26	1
12	2.61	6	2.45	1

Date	III	No.	IV	No.
1903 June 29	2.39	5	2.14	6
30	2.13	8	2.44	6
July 1	2.27	9	2.16	8
2	2.43	9	2.51	8
4	2.14	9	2.35	8
6	2.52	9	2.46	9
7	2.24	10	2.33	8
8	2.32	10	2.48	9
1903 July 9	2.34	10	2.31	9
15	2.27	10	2.32	9
17	2.30	10	2.17	9
19	2.37	10	2.12	1
21	2.28	10	2.18	8
23	2.10	10	2.42	9
25	2.41	7	2.16	6
26	2.49	9	..	..
27	2.28	10	2.16	9
28	2.31	7	2.13	6
Aug. 1	2.03	10	2.53	9
5	2.28	3	..	..
7	2.28	10	2.22	9
9	2.27	10	..	..
10	2.36	2	..	..
11	2.04	6	2.33	8
12	2.43	4	2.13	9
14	2.27	9	2.35	6
17	2.27	9	2.10	9
18	2.08	10	2.19	6
20	2.17	9	2.28	9
21	2.16	10	2.28	9

Date	IV	No.	I	No.
1903 Oct. 25	1.94	7	1.98	9
26	2.11	9	1.89	9
27	1.93	2	..	..
28	..	..	1.97	10
29	2.02	9	2.10	10
30	..	..	2.05	10
31	1.96	8	2.17	9
Nov. 1	2.14	9	2.14	10
2	2.01	8	1.95	9
3	2.04	9	2.01	10
6	2.02	9	2.10	10
7	1.94	8	2.09	10
8	1.90	9	2.10	10
9	2.06	9	1.97	9
10	2.14	9	2.02	10
12	2.16	9	..	..
13	1.99	1	..	..
14	1.94	9	2.00	9
15	2.01	7	1.68	1
18	2.09	9	2.06	10
19	2.08	9	2.03	10
1903 Nov. 20	1.95	8	2.09	10
21	2.10	8	..	..
23	..	..	1.85	5
25	1.96	9	2.20	10
26	2.21	9	2.08	10
27	1.98	8	1.95	10
29	1.99	8	2.02	10
30	..	..	2.04	10
Dec. 1	2.02	9	..	..
3	..	..	2.12	10
5	2.04	8	1.85	10
6	1.97	8	2.12	10
7	1.88	8	2.04	10
11	..	..	2.00	10
15	..	..	1.89	10
16	..	..	1.96	10
17	..	..	1.94	10
18	..	..	2.18	10
22	..	..	2.08	10
23	..	..	2.04	9
26	..	..	2.00	10
28	..	..	1.97	10

MEAN VALUES.

Weighted Mean Date	φ	No.
1902 Dec. 28	2.016	64
1903 Jan. 15	2.060	56
Feb. 2	2.022	137
19	2.052	127
Mar. 5	2.122	142
Apr. 19	2.251	93
May 8	2.336	182
27	2.301	167
1903 July 6	2.329	169
24	2.314	140
Aug. 14	2.242	147
Oct. 30	2.032	147
Nov. 12	2.037	185
30	2.022	170
Dec. 20	2.007	89
Whole number		2015

The observations of Oct. 25 and after, were made with instrument renovated and improved as mentioned above.

1904 *March* 20.

CORRECTED ELEMENTS OF (1903 *NF*), (see *A.J.* 559).

BY W. T. CARRIGAN AND E. D. TILLYER.

A new computation with corrected data gives:

Epoch 1903 December 17.73129 G.M.T.

$$\left.\begin{aligned} \Omega &= 230^\circ\ 45'\ 20''.6 \\ i &= 16\ 31\ 13.2 \\ \pi &= 332\ 57\ \ 7.3 \end{aligned}\right\}1903.0 \qquad \varphi = 24^\circ\ 22'\ 52''.2$$

$$M_0 = 65\ \ 0\ 35.0 \qquad \log a = 0.4742918 \qquad \mu = 689''.55$$

$$\left.\begin{aligned} x &= r(9.9891912)\sin(\ 61^\circ\ 45'\ 45''.9+v) \\ y &= r(9.9895833)\sin(328\ 53\ 33.5+v) \\ z &= r(9.4897335)\sin(\ 15\ 50\ 22.5+v) \end{aligned}\right\}1903.0$$

O−C, $\cos\delta\,\Delta\alpha$	$+0^s.01$	$\pm 0^s.00$	$+0^s.01$
O−C, $\Delta\delta$	$+0''.3$	$\pm 0''.0$	$-0''.2$

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed April 9.*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 561.

VOL. XXIV. BOSTON, 1904 MAY 2. NO. 9

SYSTEMS OF PERIPLEGMATIC ORBITS,

BY E. O. LOVETT.

The following note was inspired by Dr. HILL's memoir* on pairs of GYLDÉN's periplegmatic orbits † which appeared in a recent number of the *Astronomical Journal*. The successive sections of the note are occupied with triple and n-ple systems of plane periplegmatic orbits. There appears at the end a postscriptum which has to do with certain pairs of entangled plane orbits, periplegmatic or otherwise, whose determination depends on elliptic functions or those new transcendental functions lately discovered by PAINLEVÉ.‡ The method of discussion employed is essentially that used by HILL, and the generalizations constructed are suggested very naturally by the examples of his memoir.

1. In the plane of reference let a pole be adopted, and let v denote the longitude, and r_1, r_2, r_3 the radii vectores of three orbits in the plane. The line of departure from which v is measured may be chosen arbitrarily, but as r_1, r_2, r_3 are not in general periodic functions of v, the latter must be allowed to assume all values in the range from negative infinity to positive infinity. Let p_1, p_2, p_3 be three constants, and put

$$\frac{p_i}{r_i} - 1 = \rho_i \quad . \qquad (i = 1, 2, 3) \tag{1}$$

then if the orbital potential be assumed in the form

$$2V = -\sum_1^3 \rho_i^2 - \mu \{\rho_1^2(\rho_2+\rho_3)+\rho_2^2(\rho_3+\rho_1)+\rho_3^2(\rho_1+\rho_2)\} \tag{2}$$

the differential equations of the orbits are

(3)
$$\frac{d^2\rho_i}{dv^2}+\rho_i+\mu\{\rho_i(\rho_j+\rho_k)+\tfrac{1}{2}(\rho_j^2+\rho_k^2)\} = 0 \ . \ (ijk = 123, 231, 312)$$

On representing the radii vectores by the equations

$$r_i = \frac{\mu p_i}{\mu+\rho_i} \quad . \qquad (i = 1, 2, 3) \tag{4}$$

the equations (3) can be replaced by

(5)
$$\frac{d^2\rho_i}{dv^2}+\rho_i(1+\rho_j+\rho_k)+\tfrac{1}{2}(\rho_j^2+\rho_k^2) = 0 \quad . \quad (ijk = 123, 231, 312)$$

The latter have the Jacobian integral

(6)
$$\sum_1^3 \left\{\left(\frac{d\rho_i}{dv}\right)^2+\rho_i^2\right\}+\rho_1^2(\rho_2+\rho_3)+\rho_2^2(\rho_3+\rho_1)+\rho_3^2(\rho_1+\rho_2) = a^2$$

Representing the triad (ρ_1, ρ_2, ρ_3) by a point in ordinary space whose rectangular coordinates are (x, y, z), the quantities $\frac{d\rho_i}{dv}$ can be real only when the representative point P lies on the negative side of the surface whose equation is

$$xy(x+y)+yz(y+z)+zx(z+x)+x^2+y^2+z^2 = a^2 \tag{7}$$

To determine how this surface is cut by an arbitrary straight line through the origin, put

$$x = \rho\cos\alpha \ , \ y = \rho\cos\beta \ , \ z = \rho\cos\gamma \tag{8}$$

with the condition

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1 \tag{9}$$

The equation (7) becomes

$$k\rho^3+\rho^2-a^2 = 0 \tag{10}$$

where (11)

$$k = \cos\alpha\sin^2\alpha+\cos\beta\sin^2\beta+(\cos^2\alpha+\cos^2\beta)(1-\cos^2\alpha-\cos^2\beta)$$

in virtue of the equation of condition (9).

The necessary and sufficient condition that the cubic (10) have three unequal roots is

$$1-27a^2k^2 > 0 \tag{12}$$

To find the range of values which the arbitrary constant a may assume we have then to investigate the maxima values of the function k. Forming the partial derivatives of k and equating them to zero, we have after an easy reduction

$$\begin{aligned}(3\cos^2\alpha-1)(\cos\gamma-\cos\alpha)\{\ldots 1-\cos\gamma \ldots 2\ldots\} \\ (3\cos^2\beta-1)(\cos\gamma-\cos\beta)\{\ldots 1-\cos\gamma \ldots 2\ldots\}\end{aligned} \tag{13}$$

---

* G. W. HILL — "Examples of Periplegmatic Orbits," *The Astronomical Journal*, No. 554, Volume XXIV, No. 2, pp. 9–14; 1904 January 21.

† H. GYLDÉN — "Traité analytique des orbites absolues des huit planètes principales," tome I, livre I, chapitre I.

‡ P. PAINLEVÉ — "Sur les équations différentielles du second ordre et d'ordre supérieur dont l'intégrale générale est uniforme," *Acta Mathematica*, tome XXV, pp. 1–86.

These equations are equivalent to

$$(14)\qquad \left\{\begin{array}{l}(9\cos^4\alpha+1)\cos^2\gamma-(9\cos^4\gamma+1)\cos^2\alpha = 0\\ (9\cos^4\beta+1)\cos^2\gamma-(9\cos^4\gamma+1)\cos^2\beta = 0\end{array}\right.$$

the latter possess the solutions

$$(15)\qquad \cos^2\alpha = \cos^2\beta = \frac{1}{9\cos^2\gamma}$$

$$(16)\qquad \cos^2\alpha = \cos^2\beta = \cos^2\gamma$$

Putting the first of these in (9) we have the quadratic equation

$$(17)\qquad 9(\cos^4\gamma-\cos^2\gamma)+2 = 0$$

for $\cos^2\gamma$, whose roots are

$$(18)\qquad \cos^2\gamma_1 = \tfrac{2}{3} \quad , \quad \cos^2\gamma_2 = \tfrac{1}{3}$$

The first of these roots gives

$$(19)\qquad \cos^2\alpha = \cos^2\beta = \tfrac{1}{6} \quad , \quad \cos^2\gamma = \tfrac{2}{3}$$

and the second

$$(20)\qquad \cos^2\alpha = \cos^2\beta = \cos^2\gamma = \tfrac{1}{3}$$

which is the same as (16).

The values (19) and (20) assign to k the respective values

$$(21)\qquad \frac{7}{18}\sqrt{6} \quad , \quad \tfrac{2}{3}\sqrt{3}$$

the former of these and (12) demand that a^2 be less than $\frac{1}{5}$, while the second value of k gives $\frac{8}{49}$ as a superior limit to a^2.

It may be concluded then that the cubic surface (7) has a closed shell around the origin of coordinates if the arbitrary constant a^2 satisfy the inequalities

$$(22)\qquad 0 < a^2 < \tfrac{1}{5}$$

Since attention here is to be confined to the case where the radii vectores remain within finite limits, it will be supposed that the condition (22) holds.

The coordinates enter the equation (7) symmetrically; accordingly it will be sufficient to study the maxima and minima of any one of them. Now that the coordinate z, for example, be a maximum or minimum, it must be determined from the following equations.

(23)

$$F(x,y,z) \equiv xy(x+y)+yz(y+z)+zx(z+x)+x^2+y^2+z^2-a^2=0$$

$$(24)\qquad F'_x \equiv 2x+2xy+y^2+2zx+z^2 = 0$$

$$(25)\qquad F'_y \equiv 2y+x^2+2xy+2yz+z^2 = 0$$

From the last two we have

$$(26)\qquad (x-y)\{2(1+z)-x-y\} = 0$$

which gives either

$$(27)\qquad 2z-x-y+z = 0$$

or

$$(28)\qquad x-y = 0$$

Considering the former first, and solving (24) and (26) for x and y in terms of z we have

$$x = 1+z+u \quad , \quad y = 1+z-u \qquad (29)$$

where

$$u = \pm\sqrt{5(1+z)^2+z^2} \qquad (30)$$

Substituting (29) in (23) there results the following cubic equation for z,

$$q(z) \equiv 6z^3+15z^2+12z+4-a^2 = 0 \qquad (31)$$

Taking up now the second case, namely (28), and putting y for x in (23) and (24) we have the two equations

$$\left.\begin{array}{r}2x^3+2(1+z)x^2+2z^2x+z^2-a^2 = 0\\ 3x^2+2(1+z)x+z^2 = 0\end{array}\right\} \qquad (32)$$

After a little reduction, the eliminant of these equations can be written

$$\begin{vmatrix} -1 & 0 & 3 & 0 & 0\\ 0 & -1 & 2(1+z) & 3 & 0\\ z^2 & 0 & z^2 & 2(1+z) & 3\\ z^2-a^2 & z^2 & 0 & z^2 & 2(1+z)\\ 0 & z^2-a^2 & 0 & 0 & z^2 \end{vmatrix} = 0 \qquad (33)$$

or in expanded form

$$\begin{aligned}\psi(z) \equiv 12z^6-36z^5+11z^4+4(6+7a^2)z^3 \qquad (34)\\ +2(4-21a^2)z^2-24az^2+a^2(27a^2-8) = 0\end{aligned}$$

Consider the equation (31); it has neither positive nor multiple roots. On comparing it with the standard form

$$p_1x^3+3p_2x^2+3p_3x+p_4 = 0 \qquad (35)$$

it appears that the function

$$(p_1p_4-p_2p_3)^2-4(p_1p_3-p_2^2)(p_2p_4-p_3^2) \qquad (36)$$

is greater than zero for all permissible values of the constant a^2; hence the equation (31) cannot have three unequal real roots. Its single negative root is readily found to lie in the interval $(-\frac{4}{3}, -\frac{3}{2})$.

Attending now to the equation (34) and putting

$$\left.\begin{array}{l}\psi(-z) \equiv \sum\limits_0^6 A_i x^{6-i}\\ f_1(z) = A_0z+A_1,\ f_2(z) = A_0z^2+A_1z+A_2,\ \ldots,\\ \qquad f_6(z) = A_0z^6+A_1z^5+\ \ldots\ +A_5z+A_6\end{array}\right\} \qquad (37)$$

we have, by a well known theorem due to Thibault, the result that the equation (34) has no negative root whose numerical value is greater than unity, since all the numbers of the sequence

$$f_1(1),\ f_2(1),\ \ldots,\ f_6(1) \qquad (38)$$

are positive.

But (-1) falls below the interval $(-\frac{1}{2}, -\frac{1}{3})$ above, accordingly if we are concerned only with the superior limits (negatively) of the negative values of z it is unnecessary to consider the negative roots of equation (34).

As to the positive roots of (34), on forming the successive derivatives of $\psi(z)$ it appears that 2 is a close superior limit.

Hence the lower and upper limits to the limiting values of z are respectively

(39) $$-1.5 \quad , \quad +2$$

Returning now to the equations of motion, writing them

(40)
$$\frac{d^2\frac{\rho_i}{\mu}}{dv^2}+\frac{\rho_i}{\mu}=-\frac{1}{\mu}\left\{\rho_i(\rho_j+\rho_k)+\tfrac{1}{2}(\rho_j^2+\rho_k^2)\right\}=P_{ijk}\,, \qquad (ijk=123,\,231,\,312)$$

we see that the orbits will be periplegmatic for values of μ exceeding 12, since the values of the P_{ijk} will then fall below -1, the greatest value of

(41) $$\rho_i(\rho_j+\rho_k)+\tfrac{1}{2}(\rho_j^2+\rho_k^2)$$

being 12.

The equations of motion can be reduced to much simpler form: in fact subjecting them to the linear transformation

(42)
$$\begin{cases}\sigma_1=\rho_1+\rho_2+\rho_3 & , \quad \rho_1=\frac{1}{3}(\sigma_1+\sigma_3\sqrt{2})\\ \sigma_2=\sqrt{\frac{3}{2}}(\rho_2-\rho_3) & , \quad \rho_2=\frac{1}{2}\left\{\frac{1}{3}(2\sigma_1-\sigma_3\sqrt{2})+\sigma_2\sqrt{\frac{2}{3}}\right\}\\ \sigma_3=\sqrt{2}\,\rho_1-\sqrt{\frac{1}{2}}(\rho_2+\rho_3) & , \quad \rho_3=\frac{1}{2}\left\{\frac{1}{3}(2\sigma_1-\sigma_3\sqrt{2})-\sigma_2\sqrt{\frac{2}{3}}\right\}\end{cases}$$

they become

(43)
$$\begin{cases}\dfrac{d^2\sigma_1}{dv^2}+\sigma_1+\sigma_1^2 & =0\\[2mm] \dfrac{d^2\sigma_2}{dv^2}+\sigma_2+\frac{1}{2}\sqrt{2}\,\sigma_2\sigma_3 & =0\\[2mm] \dfrac{d^2\sigma_3}{dv^2}+\sigma_3+\frac{1}{4}\sqrt{2}\,(\sigma_2^2-\sigma_3^2) & =0\end{cases}$$

The first of these equations can be completely integrated by elliptic functions.

The Jacobian integral becomes in the new variables

(44) $$\left\{\sum_i\right\}\left(\frac{d\sigma_i}{dv}\right)^2+\sigma_i^2\left\{+\tfrac{2}{3}\sigma_1^3+\tfrac{1}{6}\sqrt{2}\,\sigma_3(3\sigma_2^2-\sigma_3^2)=3a^2\right.$$

The transformed system possesses the particular solution

(45) $$\sigma_1=\sigma_2=0$$

which leads to a periodic solution of the motion. In this case we have the single differential equation

(46) $$\left(\frac{d\sigma}{dv}\right)^2=3a^2-\sigma^2+\tfrac{1}{6}\sqrt{2}\,\sigma^3$$

On putting

$$u=-\tfrac{1}{4}\sqrt{2}\,\sigma \quad , \quad \tfrac{1}{2}C=[illegible] \tag{47}$$

the last equation becomes

$$\left(\frac{du}{dv}\right)^2=\tfrac{1}{2}C\;[illegible]\;u^3 \tag{48}$$

which is identical with an equation integrated by Dr. Hill in the memoir cited, by the aid of elliptic integrals.

Setting

$$\sigma_2=2\sqrt{2}\,\xi \quad , \quad \sigma_3=2\sqrt{2}\,\eta \tag{49}$$

the last two equations of (43) become

$$\left.\begin{aligned}\frac{d^2\xi}{dv^2}+\xi+2\xi\eta &=0\\ \frac{d^2\eta}{dv^2}+\eta+\xi^2-\eta^2 &=0\end{aligned}\right\} \tag{50}$$

By ordinary methods this system could be reduced to an equivalent one in which only first-order derivatives appear, and any of the known processes be employed for their approximate integration. In their present form the equations are in convenient shape for the successful application of Lindstedt's series. To this end we should assume

$$\xi=\sum_{-\infty}^{+\infty}\sum_{i}A_{i,i'}\,e^{[illegible]} \quad , \quad \eta=\sum_{-\infty}^{+\infty}\sum_{i}A'_{i,i'}\,e^{[illegible]} \tag{51}$$

where A, A', k, k' are constants, and i, i' are integers ranging over all values from $-\infty$ to $+\infty$. Substituting these values in (50) we find that the constants A, A', k, k' must satisfy the conditions

$$\begin{aligned}&[(ik+i'k')^2+1]\,A_{i,i'}+2\sum_{-\infty}^{+\infty}\sum A_{[illegible]}\,A'_{[illegible]}=0\\ &[(ik+i'k')^2+1]\,A'_{i,i'}+\sum_{-\infty}^{+\infty}\sum\left\{A_{[illegible]}\,A_{[illegible]}-A'_{[illegible]}\,A'_{[illegible]}\right\}=0\end{aligned} \tag{52}$$

for each combination of the indices i and i'. Further, since ξ and η as periodic functions of v involve only cosines, we have the additional relations

$$A_{[illegible]}=A_{[illegible]} \quad , \quad A'_{[illegible]}=A'_{[illegible]} \tag{53}$$

2. Hill's interesting example in the case of two orbits can be made available in that of three in a great variety of ways. For this purpose it is only necessary to use linear substitutions properly chosen either to be orthogonal or to transform the lineal element of the plane into a multiple of itself.

Thus, for example, consider the problem of three orbits characterized by the orbital potential function

$$V=[illegible]\;\tfrac{1}{2}(X^2+Y^2)+[illegible]\,X^3\;[illegible]\;Y^2+Y[illegible] \tag{54}$$

where (55)

$$X=[illegible]\;(\rho_1+\rho_2+\rho_3)\;,\quad Y=[illegible]\;(\rho_2-\rho_3)\;[illegible]$$

On forming the equations of motion, and operating on them with the substitution (55), we obtain the system,

$$(56)\quad \left\{\begin{aligned} \frac{d^2X}{dv^2} + X + 2X^2 \qquad &= 0 \\ \frac{d^2Y}{dv^2} + Y + 3Y^2 - Z^2 &= 0 \\ \frac{d^2Z}{dv^2} + Z - 2YZ \qquad &= 0 \end{aligned}\right.$$

the first of whose equations is rigorously integrable by elliptic functions; and the last two are readily reduced by linear transformation to the pair studied by HILL to whose approximate integration he applied LINDSTEDT's series and DELAUNAY's transformations.

3. The following generalization of the preceding results to the problem of n plane orbits may not be without interest.

Putting as before

$$(57)\qquad \frac{p_i}{r_i} - 1 = \rho_i \quad , \quad (i = 1, 2, \ldots, n)$$

let the orbital potential function be of the form

$$(58)\qquad 2V = -\sum_1^n \rho_i^2 \{ 1 + \mu(\rho_{i+1} + \rho_{i+2} + \ldots + \rho_n + \rho_1 + \rho_2 + \ldots + \rho_{i-1}) \}$$

Then the differential equations of motion are

$$(59)\qquad \frac{d^2\rho_i}{dv^2} + \rho_i + \mu\left\{ \rho_i \sum_1^n {}_j\, \rho_j + \tfrac{1}{2} \sum_1^n {}_j\, \rho_j^2 \right\} = 0 \, , \; (i = 1, 2, \ldots, n \, ; \, j \pm i)$$

Representing the radii vectores by

$$(60)\qquad r_i = \frac{\mu p_i}{\mu + \rho_i} \quad , \quad (i = 1, 2, \ldots, n)$$

the parameter μ disappears, and the equations become

$$(61)\qquad \frac{d^2\rho_i}{dv^2} + \rho_i\left(1 + \sum_1^n {}_j\, \rho_j\right) + \tfrac{1}{2} \sum_1^n {}_j\, \rho_j^2 = 0 \quad , \; (i = 1, 2, \ldots, n \, ; \, j \pm i)$$

The Jacobian integral of this system is

$$(62)\qquad \sum_1^n {}_i \left\{ \left(\frac{d\rho_i}{dv}\right)^2 + \rho_i^2 \right\} + \sum_1^n {}_i\, \rho_i^2 \left\{ \sum_1^n {}_j\, \rho_j \right\} = a^2 \quad , \quad (j \pm i)$$

Designating the ρ's as point-coordinates in an ordinary n-dimensional space, in order that the velocities be real it is necessary that the representative point should lie on the negative side of the surface whose equation is

$$(63)\qquad \sum_1^n {}_i\, x_i^2 \left(1 + \sum_1^n {}_j\, x_j\right) - a^2 = 0 \quad , \quad (j \pm i)$$

Setting

$$(64)\qquad x_i = \rho \cos \alpha_i \quad , \quad (i = 1, 2, \ldots, n)$$

with the condition

$$(65)\qquad \sum_1^n {}_i \cos^2 \alpha_i = 1$$

the manner in which an arbitrary straight line through the origin cuts the surface (63) is determined by the nature of the roots of the cubic

$$k\rho^3 + \rho^2 - a^2 = 0 \qquad (66)$$

where

$$(67)\qquad \left.\begin{aligned} k &= \sum_1^n {}_i \cos^2 \alpha_i \left(\sum_1^n {}_j \cos \alpha_j\right) \qquad\qquad (j \pm i) \\ &= \sum_1^n {}_i \sum {}_h \cos \alpha_i \cos \alpha_h \, (\cos \alpha_i + \cos \alpha_h) \\ &= \sum_1^n {}_i \left\{ \cos \alpha_i (1 - \cos^2 \alpha_i) + \cos^2 \alpha_i \sqrt{1 - \sum_1^{n-1} {}_i \cos^2 \alpha_i} \right\} \end{aligned}\right\}$$

in virtue of (65).

The cubic (66) will have three unequal real roots if the condition

$$4 - 27a^2k^2 > 0 \qquad (68)$$

is satisfied.

Forming the partial derivatives of k with regard to $\alpha_1, \alpha_2, \ldots, \alpha_{n-1}$, and equating them to zero, we find the following conditions for maxima and minima of k: either

$$\sin \alpha_i = 0 \quad , \quad (i = 1, 2, \ldots, n-1) \qquad (69)$$

or

$$(3\cos^2\alpha_i - 1)\sqrt{1 - \cos^2\alpha_n} + \cos \alpha_i [3(1 - \cos^2\alpha_n) - 2] = 0 \qquad (70)$$
$$(i = 1, 2, \ldots, n-1)$$

The resolution of the latter system gives either

$$\cos^2\alpha_i = \frac{1}{9\cos^2\alpha_n} \quad , \quad (i = 1, 2, \ldots, n-1) \qquad (71)$$

or

$$\cos^2\alpha_1 = \cos^2\alpha_2 = \ldots = \cos^2\alpha_{n-1} = \cos^2\alpha_n \qquad (72)$$

The first of these on substitution in (65) lead to

$$9(\cos^4\alpha_n - \cos^2\alpha_n) + n - 1 = 0 \qquad (73)$$

whose solution is

$$\cos^2\alpha_n = \tfrac{1}{2} \pm \tfrac{1}{6}\sqrt{9 - 4(n-1)} \qquad (74)$$

which has been already found for $n = 3$, and which becomes imaginary for $n > 3$.

The values (72) and the equation (65) give

$$\cos^2\alpha_n = \frac{1}{n} \qquad (75)$$

which substituted in (67) demands that

$$k = (n-1)\sqrt{\frac{1}{n}} \qquad (76)$$

Hence the constant of integration a^2 must satisfy the inequalities

$$0 < a^2 < \frac{4}{27} \frac{n}{(n-1)^2} \qquad (77)$$

in order that a branch of the surface (63) completely enclose the origin.

On putting $n=2$, and 3, we have from (77) the particular values of the constant found by Dr. HILL and above.

The equation (63) is symmetrical with respect to all the coordinates, so it will be sufficient to determine the maxima of any one of them, say x_n, and for convenience we write z in place of x_n. We have then to solve for z the system of equations

(78)
$$q \equiv z^2\left(1+\sum_1^{n-1} x_i\right) + z\sum_1^{n-1} x_i^2 + \sum_1^{n-1}\left\{x_j^2\left(1+\sum_1^{n-1} x_k\right)\right\} - a^2 = 0 \quad (k \neq j)$$

(79)
$$\frac{\partial q}{\partial x_i} = z^2+2zx_i+\sum_1^{n-1} x_j^2 + 2x_i\left(1+\sum_1^{n-1} x_j\right) = 0$$

From equations (79) are immediately obtained the following

(80)
$$q_{x_i} - q_{x_{i'}} \equiv (x_i - x_{i'})\left\{2\left(z+\sum_1^{n-1} x_j + 1\right) - x_i - x_{i'}\right\} = 0$$
$$(i, i' = 1, 2, \ldots, n-1;\ j \neq i, j \neq i')$$

These last equations require either

(81)
$$2\left(z+\sum_1^{i-1} x_j + 1\right) - x_i - x_{i'} = 0$$
$$(i, i' = 1, 2, \ldots, n-1;\ j \neq i, j \neq i')$$

or

(82)
$$x_1 = x_2 = \ldots = x_{n-2} = x_{n-1}$$

The equations (81) are equivalent to the following:

(83)
$$2[z+1+(n-3)x_j+1] - x_j - x_k = 0$$
$$x_1 = x_2 = \ldots = x_{j-1} = x_j = x_{j+1} = \ldots = x_{k-1} = x_{k+1} = x_{k+2} = \ldots = x_{n-1}$$

and in this case we have for the determination of z the following three equations:

(84)
$$\left\{\begin{array}{l} 2(1+z) + (2n-7)x_j - x_k = 0 \\ \dfrac{\partial q}{\partial x_k} \equiv z^2+2zx_k+(n-2)x_j^2+2x_k[1+(n-2)x_j] = 0 \\ q \equiv z^2[1+(n-2)x_j+x_k]+z[(n-2)x_j^2+x_k^2]+(n-2)(n-3)x_j^3+ \\ \quad +(n-2)(x_j+x_k)x_jx_k+(n-2)x_j^2+x_k^2-a^2 = 0 \end{array}\right.$$

The first two of these equations yield the following values for x_j and x_k in terms of z:

(85)
$$x_j = \alpha(z+1)+\omega \quad , \quad x_k = \beta(z+1)+(2n-7)\omega$$

where

(86)
$$\left\{\begin{array}{l} \alpha = \dfrac{11-4n}{(n-2)(4n-13)} \quad , \quad \beta = \dfrac{8n-25}{(n-2)(4n-13)} \\ \omega = \dfrac{\sqrt{(17-4n)(z+1)^2-(n-2)(4n-13)z^2}}{(n-2)(4n-13)} \end{array}\right.$$

for $n=5$, these values give (29) and (30); for $n>4$ the values (85) become imaginary, for this reason the single equation for z need not be written out here which results from the substitution of (85) in the last of equations (84).

Considering the second set of conditions, namely (82), we have

$$\left.\begin{array}{l} q \equiv (n-1)(n-2)x_i^3+(n-1)(z+1)x_i^2 \\ \qquad\qquad +(n-1)z^2x_i+z^2-a^2 = 0 \\ q_{x_i} \equiv 3(n-2)x_i^2+2(z+1)x_i+z^2 = 0 \end{array}\right\} \quad (87)$$

Neglecting the constant factor $2-i$ the eliminant of these equations is

$$\begin{vmatrix} n-1 & 0 & 3 & 0 & 0 \\ (n-1)(z+1) & (n-1)(n-2) & 2(z+1) & 3(n-2) & 0 \\ (n-1)z^2 & (n-1)(z+1) & z^2 & 2(z+1) & 3(n-2) \\ z^2-a^2 & (n-1)z^2 & 0 & z^2 & 2(z+1) \\ 0 & z^2-a^2 & 0 & 0 & z^2 \end{vmatrix} = 0 \quad (88)$$

which in expanded form is

$$F(z) \equiv \sum_0^6 c_i z^{6-i} = 0 \quad (89)$$

where

(90)
$$c_0 = (n-1)^2(4n-9) \quad , \quad c_1 = 2(n-1)(21-10n)$$
$$c_2 = 27(n-2)^2+(n-1)(49-19n),$$
$$c_3 = 2(n-1)[6+(9n-20)a^2]$$
$$c_4 = 2\{2(n-1)+3a^2[(n-1)(3n-8)-9(n-2)^2]\},$$
$$c_5 = -12(n-1)a^2 \quad , \quad c_6 = a^2[27(n-2)^2a^2-4(n-1)]$$

On making $n=3$ these coefficients assume the values of those of equation (31).

In assigning limits to the roots of the equation (89) it is advantageous to know the algebraic signs and relative magnitudes of the coefficients (90).

The indeterminate n obviously assumes only positive integral values; for all such values of n the coefficients c_2 and c_3 are positive, c_5 and c_6 are negative; for $n=$ or >3, the coefficient c_0 is positive and c_1 negative. The coefficient c_4 is positive or negative.

Putting

$$a^2 = \alpha\,\frac{1}{27}\,\frac{n}{(n-1)^2} \quad , \quad 0<\alpha<1 \quad (91)$$

the difference between c_3 and c_4 may be written

$$3(3-10n)(n-1)^3+4\alpha n(6n^2-25n+28) \quad (92)$$

neglecting unambiguous constant factors. This difference is negative for $n=3$; it is always negative since its vanishing would demand

$$\alpha = \frac{3(10n-3)(n-1)^3}{4n(6n^2-25n+28)} \quad (93)$$

the latter fraction is greater than unity, since each of the fractions

$$(94) \qquad \frac{3(10n-3)}{4n} \quad \cdot \quad \frac{(n-1)^2}{6n^2-25n+28}$$

is improper for all positive integral values of n; but by hypothesis α is less than unity.

Accordingly c_4 is the largest (numerically) of the negative coefficients, since it is also numerically greater than c_6.

Then a superior limit to the positive roots of equation (85) is

$$(95) \qquad 1+\frac{2(n-1)(10n-21)}{(n-1)^2(4n-9)} = 1+\frac{2(10n-21)}{(n-1)(4n-9)}$$

To assign limits to the negative roots it is necessary to examine the relative values of c_3 and c_4. The difference c_3-c_4 may be written

$$(96) \qquad (n-1)[81(n-1)^2+2\alpha n(9n-20)] \\ -54(n-1)^3-6\alpha n[6n^2-25n+28]$$

omitting a positive factor. The vanishing of this difference leads to

$$(97) \qquad \alpha = \frac{1}{2n}\,\frac{27(n-1)^3}{16n-9n^2-64}$$

which is less than zero for all positive values of n greater than zero, which is a contradiction. We may then conclude that $F(-z)$ has no negative coefficient whose numerical value is greater than the absolute value of c_3.

Then a superior limit to the absolute values of the negative roots is given by

$$(98)$$
$$1+\sqrt[3]{\frac{81(n-1)^2+2\alpha n(9n-20)}{27(n-1)^3(4n-9)}} \\ = 1+\frac{1}{3(n-1)}\sqrt[3]{\frac{81(n-1)^2+2\alpha n(9n-20)}{4n-9}}$$

Further, the superior limit (95) is numerically greater than the superior limit (98).

Writing the equations of motion (61) in the form

$$(99)$$
$$\frac{\frac{d^2\rho_i}{\mu}}{dt^2}+\frac{\rho_i}{\mu} = -\frac{1}{\mu}\left\{\rho_i\sum_1^n{}_j\,\rho_j+\tfrac{1}{2}\sum_1^n{}_j\,\rho_j^2\right\} = P_i,$$
$$(i = 1, 2, \ldots, n;\ j \pm i)$$

the necessary and sufficient condition that the orbits be periplegmatic is that no P_i should fall below -1. It is clear that no P exceeds

$$(100) \qquad \tfrac{1}{2}(n-1)\left\{1+\frac{2(10n-21)}{(n-1)(4n-9)}\right\}$$

hence if

$$(101) \qquad \mu > \tfrac{1}{2}\,\frac{[(n-1)(4n-9)+2(10n-21)]^2}{(n-1)(4n-9)^2}$$

the orbits of the system studied are periplegmatic. It should be added that the fraction in this formula (101) is not necessarily the inferior limit to the parameter μ, that is, the quality of being periplegmatic may obtain for smaller values of μ than those given by (101).

4. There may be interest in the examples below where the orbits are entangled — that is, so related that one cannot be determined without the virtual determination of the other — and yet capable of complete construction by the aid of elliptic functions. The systems may be generalized in a number of ways, both for pairs and several orbits.

Thus, if the potential is of the form

$$6V = -3(\rho_1^2+\rho_2^2)-2\mu(\rho_1^3+2\rho_1\rho_2^2) \qquad (102)$$

as in the preceding problems the equations of motion may be written

$$\left.\begin{aligned} \frac{d^2\rho_1}{dt^2}+\rho_1+\rho_1^2+\rho_2^2 &= 0 \\ \frac{d^2\rho_2}{dt^2}+\rho_2+2\rho_1\rho_2 &= 0 \end{aligned}\right\} \qquad (103)$$

The curve enclosing the area of real velocities has the equation

$$x^2+y^2+\tfrac{2}{3}x^3+2xy^2 = \alpha^2 \qquad (104)$$

which has a branch enclosing the origin if

$$0<\alpha^2<\tfrac{1}{6} \qquad (105)$$

The maxima values of x are roots of the equation

$$2x^3+3(x^2-\alpha^2) = 0 \qquad (106)$$

which for all permissible values of α^2 has roots in the intervals

$$(-\tfrac{3}{2}, -1) \quad , \quad (-\tfrac{1}{2}, 0) \quad , \quad (0, \tfrac{1}{2}) \qquad (107)$$

The maxima values of y are roots of the equation

$$16(y^2)^3-12(y^2)^2+2y^2+3\alpha^2(3\alpha^2-1) = 0 \qquad (108)$$

this equation in y^2 has only positive roots to which $\frac{5}{8}$ is a superior limit for all admissible values of α^2; the number of these values of y^2 is one or three according as

$$[3^5.4\alpha^4(3\alpha^2-1)^2-1] \gtrless 0 \qquad (109)$$

The corresponding orbits will be periplegmatic if

$$\mu > \tfrac{2}{3} \qquad (110)$$

Putting

$$\rho_1+\rho_2 = \xi \quad , \quad \rho_1-\rho_2 = \eta \qquad (111)$$

the equations (103) become

$$\left.\begin{aligned} \frac{d^2\xi}{dt^2}+\xi+\xi^2 &= 0 \\ \frac{d^2\eta}{dt^2}+\eta+\eta^2 &= 0 \end{aligned}\right\} \qquad (112)$$

which are immediately integrable rigorously by elliptic functions.

A great variety of more general problems analogous to this last one can be designed by putting V in the form

(113) $$V = V_1 + V_2 + V_3$$

where V_m designates a homogeneous form of the mth degree: choosing as coefficients certain functions easily constructed of twelve constants,

(114) $$a_1,\ b_1,\ c_1,\quad a_2,\ b_2,\ c_2,\quad \alpha_1,\ \beta_1,\ \gamma_1,\quad \alpha_2,\ \beta_2,\ \gamma_2$$

arbitrarily chosen to satisfy the relations

(115) $$\begin{cases} a_1b_2 - a_2b_1 \neq 0 \quad , \quad a_1a_2 + b_1b_2 = 0 \\ (2\alpha_1c_1 + \beta_1 + 1)\,a_1b_2 - (2\alpha_2c_2 + \beta_2 + 1)\,a_2b_1 = 0 \\ (2\alpha_1c_1 + \beta_1 + 1)^2 - (2\alpha_2c_2 + \beta_2 + 1)^2 = 0 \end{cases}$$

the substitution

(116) $$X = a_1\rho_1 + b_1\rho_2 + c_1 \quad , \quad Y = a_2\rho_1 + b_2\rho_2 + c_2$$

will then reduce the equations of motion to the form

(117) $$\begin{cases} \dfrac{d^2X}{dt^2} + \alpha_1X^2 + \beta_1X + \gamma_1 = 0 \\ \dfrac{d^2Y}{dt^2} + \alpha_2Y^2 + \beta_2Y + \gamma_2 = 0 \end{cases}$$

whose integration offers no difficulty.

5. The forms to be assigned to V are as various as there are functions. It might be kept a rational integral function of the ρ's alone, but its degree increased; as auxiliary to the discussion would then be demanded a study of curves and surfaces of higher degree, and the resolution of the corresponding algebraic equations would be attended with as grave inconvenience as the approximate integration of the differential equations.

Another obvious extension would be the admission of the independent argument t into the function V. Its presence would add immediately to the difficulties. Thus, for example, let t enter the potential linearly and assume the latter in either of the forms

(118) $$\begin{cases} V = -\tfrac{1}{2}(\rho_1^2 + \rho_2^2) - a(\rho_1^3 + \rho_2^3) + h(\rho_1 + \rho_2)\,t \\ W = -\tfrac{1}{2}(\rho_1^2 + \rho_2^2) - g(\rho_1^4 + \rho_2^4) + h(\rho_1^2 + \rho_2^2)\,t \end{cases}$$

after suitable linear transformation of the variables the integration of the equations of motion can be reduced to that of equations of the form

(119) $$\begin{cases} \dfrac{d^2X}{dt^2} = 6X^2 + t \\ \dfrac{d^2X}{dt^2} = 2X^3 + tX + \alpha \quad , \quad (\alpha \text{ a numerical constant}) \end{cases}$$

simple as are these equations neither of them can be integrated in finite form in terms of known algebraic or transcendental functions; in fact, PAINLEVÉ has shown (*loc. cit.*) that the above equations define new uniform functions of t meromorphic throughout the plane. It may not be out of place here to remark that PAINLEVÉ has found a third category of differential equations of the second order defining new uniform transcendents; to use the latter for purposes of illustration in this connection it would be necessary to allow the argument t to enter the potential function through an exponential function, to say nothing of other complications.

Princeton, New Jersey.

OBSERVATIONS OF COMET a 1904 (BROOKS),

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

(Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.)

1904 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$	log $p\Delta$	[illegible]	[illegible]
			m s	′ ″	h m s	° ′ ″			s	″
Apr. 17 10 50 56	1	24 , 8	−0 39.05	1 39.3	16 55 48.87	+44 51 50.1	n9.768	0.275	+1.58	7.3
17 15 47 0	1	30 , 6	−1 13.42	+6 8.1	16 55 11.51	+44 59 37.6	9.004	9.954	+1.59	−7.2
18 11 29 0	2	29 , 6	−1 28.10	5 15.6	16 52 56.79	+45 30 18.6	n9.723	9.994	+1.62	7.1
19 10 15 49	3	29 , 6	+3 41.26	2 38.2	16 50 8.42	+46 6 16.4	n9.770	0.202	+1.67	6.8
20 10 31 51	4	35 , 7	−0 5.10	−6 11.3	16 47 10.97	+46 12 22.7	n9.778	0.206	+1.70	6.6
21 11 10 46	5	29 , 6	2 17.92	+3 3.2	16 44 1.71	+47 19 18.5	n9.734	9.864	+1.73	6.4
22 12 24 58	6	30 , 6	+3 57.38	+2 40.1	16 40 41.09	+47 56 25.0	n9.571	n9.761	+1.79	6.1
23 15 9 28	7	30 , 6	3 0.75	0 16.6	16 37 1.73	+48 34 47.9	9.081	n0.154	+1.81	5.9

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority
	h m s	° ′ ″	
1	16 56 26.34	+44 53 36.7	Bonn, A.G. 10878
2	16 54 23.27	+45 36 11.3	Bonn, A.G. 10847
3	16 46 25.49	+46 9 1.4	½ (Madras [illegible] + Greenwich [illegible])
4	16 47 11.37	+46 18 40.6	Bonn, A.G. 10765
5	16 46 17.90	+47 16 21.7	Bonn, A.G. 10763
6	16 36 41.92	+47 53 51.0	Bonn, A.G. 10666
7	16 40 0.67	+48 35 10.4	Bonn, A.G. 10690

ELEMENTS AND EPHEMERIS OF COMET a 1904 (BROOKS).

BY EVERETT I. YOWELL, NAVAL OBSERVATORY,

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

The following elements were deduced from observations made at the U. S. Naval Observatory, April 17, 19 and 21.

$$T = \text{1904 March } 4.7144 \text{ G.M.T.}$$

$$\left.\begin{aligned} \pi &= 328^\circ\ 17'\ \ 9''.9 \\ \Omega &= 275\ 37\ 12.9 \\ i &= 125\ \ 5\ \ 1.1 \end{aligned}\right\} 1904.0$$

$$\log q = 0.431614$$

HELIOCENTRIC COORDINATES.

$$x = r[9.763676]\sin(222^\circ\ 57'\ \ 1''.1+v)$$
$$y = r[9.994703]\sin(300\ 12\ 42.8+v)$$
$$z = r[9.918576]\sin(\ 24\ \ 7\ 41.2+v)$$

EPHEMERIS.

G.M.T.	α	δ	log Δ	Br.
	h m s	° ′ ″		
May 1.5	16 8 16.7	+52 31 53	0.35528	0.99
2.5	4 12.7	52 58 49	0.35637	
3.5	16 0 34.0	53 24 42	0.35751	
4.5	15 56 20.9	53 49 31	0.35880	0.99
5.5	52 3.5	54 13 16	0.36014	0.98
6.5	47 42.1	54 35 54	0.36156	
7.5	43 17.1	54 57 25	0.36305	
8.5	38 48.8	55 17 47	0.36462	
9.5	34 17.6	55 37 1	0.36626	0.98
10.5	29 43.7	55 55 5	0.36796	0.97
11.5	25 7.7	56 11 58	0.36974	
12.5	20 29.9	56 27 41	0.37158	
13.5	15 50.8	56 42 14	0.37348	
14.5	11 10.8	56 55 36	0.37544	0.97
15.5	15 6 30.3	57 7 49	0.37746	0.96
19.5	14 47 52.7	+57 45 25	0.38607	0.95

The brightness at the date of discovery is adopted as the unit. Comparison with an observation made here April 23 gives as corrections to the ephemeris: $\Delta\alpha = -1^s$, $\Delta\delta = +15''$.

COMET a 1904.

[Extract from RITCHIE's Circular No. 136.]

On April 17 the announcement of the discovery of a comet by Mr. W. R. BROOKS was received *via* Harvard College Observatory, the position given being, April 16, 9h 50m Washington Time; R.A. 16h 58m 10s; Decl. +44° 10′. A position from Professor SEARES of Columbia, Mo., was received on April 18 and was circulated among American astronomers for the second position. Other positions, as hereunder noted, have been received through the courtesy of Directors CAMPBELL, SEARES and Commander ROBINSON, and a series of ante-discovery places from the Harvard photographic plates has been communicated by Professor E. C. PICKERING.

1904 Gr. M.T.	α	δ	Observer
	h m s	° ′ ″	
Mar. 15.899	17 56 8	+20 21 –	Harvard †
Apr. 1.889	17 18 44	34 29 –	Harvard †
5.850	17 20 20	37 7 –	Harvard †
13.825	17 4 4	42 28 –	Harvard †
16.855	16 56 28	44 23 –	Harvard †
17.65787	16 55 50.18	44 51 30.1	U.S.N. Obs.
17.6592	16 55 49.6	44 51 32	Seares
17.66607	16 55 48.87	44 51 50.1	U.S.N. Obs.
17.7132	16 55 40.8	44 53 37	Aitken
17.87169	16 55 14.51	44 59 37.6	U.S.N. Obs.
18.6289	16 53 7.6	+45 27 56	Seares

† These positions are referred to equinox 1855.0.

OBSERVATIONS OF BROOKS'S COMET a 1904,

MADE AT THE GOODSELL OBSERVATORY WITH THE 16-INCH REFRACTOR,

BY H. C. WILSON.

1904 Northfield M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$	
h m s			m s	′ ″	h m s	° ′ ″		
Apr. 18 10 19 45	1	9 , 4	−1 15.63	−6 9.0	16 52 57.20	+45 30 10.8	n9.754	0.458
18 10 19 45	2	9 , 6	−1 27.03	−5 52.9	16 52 57.85	+45 30 13.9	n9.754	0.458
20 9 48 53	3	9 , 6	−0 6.91	−5 41.1	16 17 18.36	+46 42 51.3	n9.773	0.488

Mean Places of Comparison-Stars for the beginning of the year.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	16 54 11.32	+1.51	+45 36 26.9	−7.1	DM. 45°2171
2	16 54 23.37	+1.51	+45 36 13.9	−7.1	Arg. N.Z. 126, 28–29
3	16 17 23.57	+1.70	+46 48 43.0	−6.7	Arg. N.Z. 126, 21

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. Closed April 30.
Entered February 2, 1904, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 562.

VOL. XXIV. BOSTON, 1904 MAY 20. NO. 10

THE ORBIT OF THE FIFTH SATELLITE OF *JUPITER*.

BY EMILY ELISABETH DOBBIN.

While the writer was a student at the Yerkes Observatory, in the summer of 1902, Prof. BARNARD requested her assistance in testing the relative worth of COHN'S(1) and TISSERAND'S(2) values of the orbital elements of the Fifth Satellite of *Jupiter*. The method used has been explained in connection with the publication of his measures of that year,(3) and the results of my work are incorporated in the same paper. These results aroused a conviction that the orbit which Mr. COHN had obtained from measures made in 1892, 1893 and 1894, by Prof. BARNARD and Prof. STRUVE, was not satisfactory when used in connection with observations of the later years 1898, 1899 and 1902; and furthermore, that the method we were using, while sufficient as a test, was not adequate to the derivation of an accurate set of elements. Prof. BARNARD suggested that I undertake a complete solution of the problem, using all the measures obtained by him subsequent to the work of Mr. COHN, including the then unpublished work of 1902. This problem had been carried nearly to completion when the prospect of additional observations in the summer of 1903 postponed its publication. Thus, I have been able to use this longest and most excellent series of observations.

Only the observations of Prof. BARNARD are considered in this paper, though I am aware of the extensive labors of Dr. R. G. AITKEN(4) of the Lick Observatory. The homogeneity gained by excluding such observations seems to more than balance any loss in quantity.

Reduction of Observations. The observations of 1898 and 1899 are published in *A.J.* 472, Vol. XX, and those of 1902 in Vol. XXIII, No. 514. The measures of the first two years are corrected for the accepted value of the micrometer screw, mentioned in the last paragraph of the latter paper. The diameter used was that obtained by the observer for each evening of observation, based on his own series of measures on previous years.(5) Combining this value with the micrometrical measures of the Fifth Satellite gives the obvious advantage of identity of observer, method, and instrument. The time of observation was corrected for planetary aberration, and the phase correction made when necessary.

In finding normal places effort was made to have the intervals of time about twelve minutes, though rarely the series will appear interrupted. It is to be noted with interest that on some nights the average number of settings made within the interval of time is twelve, and on other nights the average runs lower. The measures of September 9, 1902, are of wonderful rapidity, as many as 18 frequently made in eight minutes. The work of 1903 is a marvel of regularity, as 42 out of the 73 normal places rest on 9 settings each.

Comparison of Observations with Ephemeris. After reducing the observations, an ephemeris was computed which gave the distance of the satellite from *Jupiter's* center for the times corresponding to the normal places. The formula of the late Mr. MARTH(6) was used with COHN'S elements as the basis of a circular orbit, which the small eccentricity readily permits. In order to determine the inclination of the orbit and longitude of the node with any more accuracy than has already been done, a great many measures of the ordinate are necessary, and these are not at hand. Only 29 were made in 1898, 10 in 1899, and 8 in 1902. As these few could furnish no more than 10 normal places, it is not worth while to attempt any refinement of correction by their use. Prof. BARNARD realizes the importance of the y-component of the satellite's position, and expresses

(1) *Astr. Nachr.*, Bd. 142, s. 289.
(2) *Comptes Rendus*, CXIX, p. 581.
(3) *Astr. Journ.*, No. 544, p. 152.
(4) *L.O.B.*, No. 51.
(5) *Astr. Journ.*, No. 325, p. 102.
(6) *Astr. Nachr.*, Bd. 157, s. 265.
(7) *Mon. Not.*, Vol. LI, p. 506.

the intention of devoting more time to its measurement hereafter. Such data I hope to use later for the second part of the problem, but in the present paper only the x-coordinate is used, and only the elements in the orbital plane can be considered.

The plane of *Jupiter's* equator was selected as the fundamental plane, with origin at the planet's center. Then,

$$x = a \sin(v - L)$$

in which

a = mean distance of the satellite from *Jupiter's* center in the geocentric distance of *Jupiter*.
ρ = geocentric distance of *Jupiter*.
(ρ) = assumed mean distance of planet from *Sun*.
(a) = mean distance of satellite from *Jupiter's* center as seen at mean distance of planet from *Sun*.
n = daily motion of satellite in its orbit.
v = longitude of satellite in its orbit, starting from its ascending node on *Jupiter's* equator.
L = Jovicentric right-ascension of *Earth* on *Jupiter's* equator.

Evidently for each time of normal place,

$$a = \frac{(a)(\rho)}{\rho}$$

where (a) = 18″.065, the theoretical value of Cohn.
(ρ) = 5.2028.
ρ, obtained from the *American Ephemeris.*

Also, v_0 = 226°.40 Nov. 1, 1892, from Cohn.
n = 722°.6316.

L, obtained from *Mon. Not.* as computed and published by Mr. Crommelin.

Before proceeding to the Ephemeris, attention is called to the following errors in the publication of the work of 1902 (*A.J.* 541), all of which have been approved by Prof. Barnard:

Page 155, Aug. 21, omission of last observation,
11h 50m 50s , 36″.92 , 61″.40 , 2
" 155, Aug. 25, *for* 10h 49m 47s *read* 10h 50m 47s
" 155, Aug. 25, " 11 30 14 " 11 31 14
" 156, Sept. 9, " 10 59 23 " 10 59 21
" 154, 19th line " +82°.0 " +882°.0

The east and west elongations of 1899 as published by the *Conn. des Temps* are interchanged.

In the following comparison of observation with calculation, Wt. indicates the number of settings employed in the normal place, C the value of x obtained from the ephemeris, and R the residual obtained after substituting in each equation of condition the values derived from the solution of the normal equations of that year.

Date	G.M.T.	Wt.	x Observed	x Computed	O—C	R
1898	h m s		″	″	″	″
Mar. 2	17 35 44	3	+49.78	+50.78	−1.00	−0.43
	18 7 48	6	54.59	54.70	−0.11	+0.02
	18 19 46	4	54.64	55.07	−0.43	−0.12
	18 41 41	5	53.77	54.19	−0.42	−0.13
	18 57 38	6	52.41	52.29	+0.12	+0.23
6	16 50 22	5	45.32	45.61	−0.29	−0.17
	17 7 18	7	49.50	49.75	−0.25	−0.15
	17 24 24	10	52.88	52.83	+0.05	+0.16
	17 42 49	8	54.90	54.82	+0.08	+0.17
	17 55 59	8	55.29	55.37	−0.08	+0.07
	18 9 40	9	55.00	55.16	−0.16	+0.02
	18 24 18	8	53.75	54.06	−0.31	+0.11
	18 41 11	8	51.51	51.70	−0.19	+0.01
	18 56 19	5	48.33	48.62	−0.29	+0.06
	19 12 44	6	43.80	44.31	−0.51	−0.21
Apr. 5	14 44 49	8	54.65	54.05	+0.60	+0.40
	14 58 8	7	55.49	55.35	+0.14	+0.06
	15 8 28	5	55.70	55.84	−0.14	−0.12
	15 20 30	11	55.69	55.83	−0.14	−0.12
	15 31 35	12	55.28	55.27	+0.01	+0.05
	15 44 5	11	53.95	54.02	−0.07	0.00
	15 54 41	7	52.78	52.46	+0.32	+0.33
	16 6 55	7	49.97	50.09	−0.12	−0.02
	16 17 19	9	47.21	47.62	−0.41	−0.27
20	14 14 4	9	53.64	53.95	−0.31	−0.15
	14 26 22	11	52.44	52.53	−0.09	+0.08
	14 39 24	12	50.08	50.35	−0.27	−0.08
	14 49 3	11	48.15	48.32	−0.17	+0.04
	15 4 58	13	44.25	44.22	+0.03	+0.25
26	13 2 59	11	+53.57	+53.77	−0.20	−0.18
1898	h m s		″	″	″	″
Apr. 26	13 29 54	12	+53.80	+54.15	−0.35	−0.21
	13 41 7	9	53.07	53.42	−0.35	−0.15
	13 54 0	9	51.88	51.95	−0.07	+0.13
	14 8 29	5	48.96	49.50	−0.54	−0.20
	14 17 32	9	+47.25	+47.57	−0.32	−0.11
1899						
Apr. 18	19 25 45	5	+54.80	+55.72	−0.92	−0.54
	19 34 29	4	54.21	54.91	−0.70	−0.29
	19 43 5	4	53.73	53.80	−0.07	+0.10
20	18 22 22	6	53.02	53.76	−0.74	−0.61
	18 29 17	4	54.64	54.70	−0.06	−0.06
	18 58 9	3	56.69	56.41	+0.28	+0.16
	19 1 49	2	56.56	56.37	+0.19	+0.10
25	17 29 8	11	48.93	48.65	+0.28	0.00
	17 37 31	8	51.00	51.06	−0.06	−0.05
	17 48 34	9	52.94	52.45	+0.49	+0.25
	17 59 42	10	54.58	54.24	+0.34	+0.21
	18 12 53	12	56.13	55.72	+0.41	+0.35
	18 41 26	8	56.13	56.27	−0.14	−0.04
	18 52 1	8	55.53	55.58	−0.05	+0.08
	19 5 20	9	53.71	54.04	−0.33	−0.09
	19 18 10	12	51.27	51.86	−0.59	−0.31
	19 28 34	9	48.85	49.61	−0.76	−0.36
	19 38 57	9	46.46	46.95	−0.49	−0.10
	19 50 38	9	43.10	43.50	−0.40	+0.02
May 1	16 38 17	9	43.67	43.57	+0.10	−0.24
	16 51 4	6	47.41	47.30	+0.11	−0.15
	17 3 17	8	50.47	50.24	+0.23	−0.03
	17 15 51	8	52.99	52.46	+0.53	+0.26
	17 26 25	8	+54.62	+54.43	+0.19	+0.04

Date	G.M.T.	Wt.	x Observed	x Computed	O—C	R
1899	h m s		″	″	″	″
May 1	17 36 35	10	+55.79	+55.54	+0.25	+0.15
	17 46 41	11	56.30	56.20	+0.10	+0.07
	17 57 28	10	56.66	56.43	+0.23	+0.26
	18 8 33	9	56.24	56.14	+0.10	+0.12
	18 19 41	9	55.95	55.32	+0.63	+0.67
	18 29 57	10	53.86	54.09	−0.23	−0.03
	18 39 40	7	52.49	52.53	−0.04	+0.16
	18 51 32	9	49.84	50.11	−0.27	+0.02
	19 3 7	9	46.89	47.22	−0.33	+0.01
	19 13 34	5	43.96	44.20	−0.24	+0.10
23	14 58 29	9	47.92	48.13	−0.21	−0.48
	15 18 13	7	51.88	51.71	+0.17	−0.06
	15 30 27	11	53.67	53.46	+0.21	+0.03
	15 40 23	11	54.58	54.44	+0.14	+0.02
	15 52 2	12	55.41	55.05	+0.36	+0.29
	16 3 52	9	54.83	55.09	−0.26	−0.20
	16 14 7	9	54.67	54.65	+0.02	+0.08
	16 21 17	5	53.84	54.07	−0.23	−0.08
June 13	14 28 8	9	51.54	51.47	+0.07	+0.18
	14 37 9	7	50.48	50.41	+0.07	+0.21
	14 48 5	9	48.45	48.73	−0.28	−0.03
16	13 48 49	12	52.14	52.30	−0.16	−0.14
	14 0 59	12	52.09	51.95	+0.14	+0.22
	14 12 19	8	51.04	51.09	−0.05	+0.07
	14 21 44	8	49.71	49.98	−0.27	−0.06
	14 33 10	12	48.04	48.20	−0.16	+0.09
18	14 24 29	7	47.02	47.61	−0.59	−0.26
19	13 49 4	8	50.80	51.29	−0.49	−0.30
	14 1 57	8	49.94	50.09	−0.15	+0.03
	14 11 17	5	48.17	48.82	−0.65	−0.29
	14 21 58	6	+46.50	+46.96	−0.46	−0.12
1902 July 21	19 7 35	6	−53.51	−53.29	−0.22	−0.10
	19 16 31	6	55.83	55.48	−0.35	−0.27
28	18 12 4	12	48.94	48.49	−0.45	−0.18
	18 21 41	10	51.92	51.50	−0.42	−0.22
	18 30 35	10	53.98	53.96	−0.02	+0.11
	18 46 2	9	57.67	57.46	−0.21	−0.11
	19 8 29	12	60.33	60.66	+0.33	+0.26
	19 17 20	11	61.43	61.28	−0.15	−0.27
	19 26 36	12	61.17	61.54	+0.37	+0.17
	19 36 55	9	61.05	61.34	+0.29	+0.06
	19 46 50	9	60.13	60.69	+0.56	+0.28
	19 56 35	9	59.22	59.60	+0.38	+0.04
	20 5 32	7	57.55	58.21	+0.66	+0.20
Aug. 5	18 22 0	6	60.01	60.56	+0.55	+0.33
	18 39 47	6	61.72	61.64	−0.08	−0.16
	18 49 12	9	61.42	61.61	+0.19	+0.03
	18 55 38	7	61.15	61.35	+0.20	+0.02
	19 4 38	10	60.57	60.66	+0.09	−0.17
21	16 14 21	9	52.83	52.45	−0.38	−0.06
	16 23 0	6	55.26	54.68	−0.58	−0.37
	16 38 9	9	58.36	57.81	−0.55	−0.34
	16 45 27	6	59.04	58.96	−0.08	+0.02
	16 57 32	9	60.52	60.33	−0.19	−0.12
	17 7 19	9	61.13	60.94	−0.18	−0.17
	17 15 13	5	61.49	61.12	−0.37	−0.30
25	16 7 29	9	55.89	55.83	−0.06	+0.20
	16 18 10	12	57.61	57.84	+0.23	+0.47
	16 29 14	12	59.21	59.44	+0.23	+0.37
	16 41 6	9	−60.20	−60.42	+0.22	+0.26
1902 Aug. 25	16 52 57	9	−60.94	−60.81	−0.14	−0.14
	17 4 47	12	60.55	60.54	−0.01	−0.10
	17 15 55	12	59.42	59.69	+0.27	+0.11
	17 26 39	10	58.28	58.34	+0.05	−0.15
Sept. 9	14 51 8	12	55.38	55.31	−0.07	+0.27
	15 0 55	15	57.27	56.86	−0.41	−0.09
	15 11 58	15	58.54	58.20	−0.34	−0.09
	15 22 6	16	58.98	58.91	−0.07	+0.06
	15 31 40	18	59.24	59.16	−0.08	−0.01
	15 41 21	18	59.05	58.98	−0.07	−0.08
	15 49 43	15	58.51	58.49	−0.02	−0.08
	16 0 49	15	57.25	57.34	+0.10	−0.03
	16 9 4	10	56.11	56.15	+0.04	−0.12
	16 19 55	16	54.05	54.13	+0.08	−0.09
Oct. 7	12 56 12	12	54.61	54.67	+0.06	+0.05
	13 5 42	6	54.48	54.77	+0.29	+0.30
	13 17 0	9	54.13	54.41	+0.28	+0.23
	13 28 31	12	53.37	53.50	+0.13	0.00
	13 39 4	8	−52.24	−52.19	−0.15	+0.31
19[illegible] July 21	18 18 22	6	+50.82	+48.69	+2.13	+1.72
	18 34 50	6	54.34	52.66	+1.68	+1.38
	18 58 53	6	56.60	56.48	+0.12	0.00
	19 8 49	9	57.09	57.35	−0.26	−0.20
	19 19 21	9	57.71	57.78	−0.07	−0.02
	19 28 17	9	57.78	57.76	+0.02	+0.07
	19 36 47	9	57.26	57.44	−0.18	+0.03
	19 45 51	9	56.41	56.73	−0.32	−0.03
	19 54 22	9	55.64	55.75	−0.11	+0.15
	20 5 56	11	53.54	53.90	−0.36	+0.11
Aug. 11	17 13 14	9	59.69	60.25	−0.56	−0.51
	17 22 53	9	60.51	60.83	−0.32	−0.19
	17 31 58	5	60.58	60.97	−0.39	−0.20
17	16 21 23	9	58.21	58.50	−0.29	+0.20
	16 29 35	9	59.49	59.75	−0.26	+0.18
	16 37 52	9	60.27	60.70	−0.43	−0.20
	16 44 31	9	60.94	61.24	−0.30	−0.18
	16 50 54	9	61.21	61.55	−0.34	−0.22
	16 56 59	9	61.81	61.67	+0.14	−0.03
	17 6 10	9	61.38	61.52	−0.14	+0.09
	17 14 00	9	60.90	61.08	−0.18	+0.11
	17 21 5	9	60.27	60.44	−0.17	+0.08
	17 30 7	9	58.96	59.28	−0.32	−0.10
	17 37 35	9	57.06	58.04	−0.98	−0.43
	17 45 23	7	56.64	56.04	+0.60	+0.03
24	16 14 54	6	61.31	62.27	−0.96	−0.48
	16 31 58	6	61.22	61.99	−0.77	−0.54
	16 49 24	6	59.87	60.27	−0.40	−0.03
	17 3 59	8	57.56	57.74	−0.18	+0.30
31	15 3 49	9	59.69	59.40	+0.29	+0.17
	15 10 54	9	60.80	60.55	+0.25	+0.16
	15 17 24	9	61.46	61.40	+0.06	+0.01
	15 24 10	9	62.41	62.07	+0.34	+0.28
	15 30 9	9	62.38	62.48	−0.10	−0.06
	15 36 1	9	62.79	62.72	+0.07	−0.08
	15 42 29	10	62.94	62.79	+0.15	−0.12
	15 49 7	8	62.65	62.65	0.00	−0.16
	15 55 33	9	62.32	62.32	0.00	−0.20
	16 1 43	8	61.60	61.81	−0.21	+0.03
	16 7 52	9	61.09	61.12	−0.03	+0.18
	16 15 2	9	+59.80	+60.10	0.30	+0.04

Date	G.M.T.	Wt.	x Observed	x Computed	O−C	R
1903	h m s		″	″	″	″
Aug. 31	16 19 19	5	+59.01	+59.38	−0.34	+0.03
Sept. 1	15 19 22	10	62.31	62.16	+0.15	+0.15
	15 31 26	9	62.75	62.65	+0.10	+0.14
	15 41 22	9	62.70	62.82	−0.12	−0.01
	15 53 19	9	62.46	62.40	+0.06	+0.24
	15 55 28	6	61.83	62.25	−0.42	−0.22
21	13 27 51	9	62.34	62.03	+0.31	−0.38
	13 34 26	9	62.86	62.50	+0.36	+0.20
	13 43 25	10	63.27	62.80	+0.47	+0.35
	13 51 16	9	62.74	62.74	0.00	−0.06
	13 58 51	9	62.55	62.41	+0.14	+0.10
	14 7 21	8	61.75	61.71	+0.04	0.00
	14 14 26	8	61.03	60.86	+0.17	−0.10
	14 23 45	12	59.33	59.38	−0.05	+0.08
22	13 32 16	11	62.95	62.59	+0.36	+0.20
	13 44 55	12	+62.55	+62.70	−0.15	−0.12
Sept. 22	13 59 20	11	+61.49	+61.89	−0.40	−0.28
28	14 14 0	11	51.97	52.07	−0.10	+0.22
	14 24 50	8	+48.41	+48.47	−0.06	+0.21
Oct. 13	17 29 18	10	−59.84	−60.19	+0.35	+0.22
	17 36 50	9	60.19	60.59	+0.40	+0.30
	17 48 44	11	−60.40	−60.68	+0.28	+0.10
20	12 0 9	9	+52.90	+53.62	−0.72	−0.34
	12 5 39	5	51.72	52.28	−0.56	−0.23
26	12 11 21	6	38.77	39.39	−0.62	−0.31
	12 15 51	5	+37.01	+37.64	−0.63	−0.30
	16 8 58	9	−57.47	−57.31	−0.16	−0.15
	16 15 5	8	57.52	57.92	+0.40	+0.23
	16 20 53	9	58.30	58.35	+0.05	−0.03
	16 28 28	9	58.37	58.68	+0.31	+0.18
	16 38 24	9	58.34	58.73	+0.39	+0.19
	16 50 4	9	−57.72	−58.35	+0.63	+0.36

Inspection of the residuals O−C displays the fact that the satellite is ahead of its ephemeris, as O−C is positive before and negative after eastern elongations, the signs being reversed on the west. Prof. BARNARD noted the same fact in comparing computed times of elongation based on his observations for the year 1902, with the times given in the *Conn. des Temps*, which are based on the periodic time derived by Dr. COHN.

The large residuals at the beginning and end of an evening's observation extending through an elongation, such as 1898 March 6, are easily understood upon consideration of the conditions of observation. These measures are made when the satellite is near the planet, and suffer somewhat from the luminosity of *Jupiter*, even though dulled by the smoked mica used; moreover, the satellite is moving rapidly across the line of vision, so that the time-factor enters very heavily here. But when the satellite is at elongation, moving tangent to the line of sight, it appears to stand almost perfectly still for some minutes, so that the time-factor may be quite erroneous here and escape notice. Manifestly then, the discrepancy between observation and calculation at elongation is more independent of observer's errors than other measures, and depends more strongly upon errors in the time-elements chosen, and upon the disregard of the ellipticity of the orbit. The latter consideration is of greatest importance when either axis of the ellipse is coincident with the x-axis, that is, perpendicular to the line of sight.

Equations of Condition. Upon the residuals O−C are to depend differential corrections to the elements derived by Dr. COHN, as well as a value of the eccentricity and the motion of the line of apsides caused by the oblateness of *Jupiter*.

Let e denote the eccentricity, and P the longitude of perijove, referred to the same origin as v and L, and if we place

$$h = e \sin P$$
$$k = e \cos P$$

then will the differential equation for x become

$$dx = a\cos(v-L)\,dv_0 + x\frac{d(a)}{(a)} - a\{\cos(v-L)\cos v + \cos L\}\,h + a\{\cos(v-L)\sin v + \sin L\}\,k$$

Introducing the variation of P by placing $P = P_0 + dP$, the equation becomes

$$dx = a\cos(v-L)\,dv_0 + x\frac{d(a)}{(a)} - a\{\cos(v-L)\cos(v-dP) + \cos(L-dP)\}\,h + a\{\cos(v-L)\sin(v-dP) + \sin(L-dP)\}\,k$$

For brevity's sake the coefficient of dv_0 may be called l, that of $\frac{d(a)}{(a)}$ is b, that of h is c, and the coefficient of k is denoted by d.

The equation of condition now reads

$$dx = l\,dv_0 + b\frac{d(a)}{(a)} + ch + dk$$

To formulate these equations for the normal places Dr. COHN's value for $dP = 911°.7$ yearly was used, though from Prof. BARNARD's results it seemed that two applications of the differential method might be necessary to obtain a satisfactory correction. The equations of condition for each year were used for a set of normal equations, the results of 1898 and 1899, giving suggestions for the work of 1902 and 1903, as they pointed out the direction of needed changes. The number of settings included in each normal place was the basis of weighting used.

Normal Equations and their Solution. As a Schuster-calculating machine and a Burroughs' Arithmometer were available in the solution of these equations very little use was made of logarithmic tables, and the results, as well as the coefficients, are given in natural numbers. The combining of the 35 equations of 1898 by the method of least-squares gives the following set of normal equations:

$$\begin{aligned}
&+6870.69\,\frac{d(a)}{(a)} - 5213.92\,dv_0 - 3852.18\,h + 2329.82\,k = -16.5655\\
&-5213.92 \quad +19842.65\,dv_0 + 7646.85\,h + 1382.88\,k = +24.6948\\
&-3852.18 \quad +\ 7646.85\,dv_0 + 5979.12\,h + 1475.06\,k = +\ 8.8510\\
&+2329.82 \quad +\ 1382.88\,dv_0 + 1475.06\,h + 1256.41\,k = -\ 3.1172
\end{aligned}$$

The solution of these equations for the epoch 1898 March 30.0 G.M.T. gives

$$\begin{aligned}
dv_0 &= +0^\circ.075 & \pm 0^\circ.032\\
d(a) &= -0''.194 & \pm 0''.015\\
e &= \ \ 0.00401 & \pm 0.00153\\
P_0 &= -56^\circ\,44' & \pm 9^\circ\,24'
\end{aligned}$$

the latter two being derived from

$$\begin{aligned}
h &= -0.00335 & \pm 0.00112\\
k &= +0.00220 & \pm 0.00101
\end{aligned}$$

The 55 equations of 1899 give the following normal equations:

$$\begin{aligned}
&+10709.81\,\frac{d(a)}{(a)} - 2690.98\,dv_0 + 1865.71\,h + 7598.21\,k = -10.2654\\
&-\ 2690.98 \quad +31629.01\,dv_0 + 5801.22\,h - 1337.92\,k = +75.0579\\
&+\ 1865.71 \quad +\ 5801.22\,dv_0 + 7879.77\,h + 4098.72\,k = +11.2679\\
&+\ 7598.21 \quad -\ 1337.92\,dv_0 + 4098.72\,h + 8220.31\,k = -\ 5.0136
\end{aligned}$$

The solution gives

$$\begin{aligned}
h &= -0.000663 & \pm 0.000082\\
k &= +0.000956 & \pm 0.000123\\
dv_0 &= +0^\circ.611 & \pm 0^\circ.023\\
d(a) &= -0''.043 & \pm 0''.033
\end{aligned}$$

whence for the epoch 1899 May $19^d.8$,

$$\begin{aligned}
e &= 0.00116 & \pm 0.00118\\
P_0 &= -34^\circ\,45' & \pm 0^\circ\,56'
\end{aligned}$$

This very small value of e is unsatisfactory when viewed alone, but the observations of 1899 extend over two months only, including but three elongations:

April 25, May 1, and May 23

Combining the solutions of 1898 and 1899 we obtain

$$\begin{aligned}
e &= \ \ 0.00226 & \pm 0.0015\\
dP &= \ \ 2^\circ.451 \text{ daily} & \pm 0^\circ.027\\
d(a) &= -0''.102 & \pm 0''.038\\
dv_0 &= +0^\circ.615 & \pm 0^\circ.026\\
(a) &= \ \ 47''.963
\end{aligned}$$

The corrected elements were used in the ephemeris for 1902 and 1903.

$$\begin{aligned}
(a) &= \ \ 47''.963\\
v_0 &= 227^\circ.02\\
dP &= \ \ 2^\circ.451
\end{aligned}$$

With these values the 18 equations of condition of 1902 result in the following normal equations for A [illegible] 29 7:

$$\begin{aligned}
&+16241.93\,\frac{d(a)}{(a)} + 5458.07\,dv_0 - 10289.69\,h + 3090.42\,k = -\ 8.1592\\
&+\ 5458.07 \quad +29677.70\,dv_0 - 2901.64\,h + 9661.14\,k = +56.7969\\
&-10289.69 \quad -\ 2901.64\,dv_0 + 10494.08\,h - 599.50\,k = -\ 1.7957\\
&+\ 3090.42 \quad +\ 9661.14\,dv_0 - 599.50\,h + 10768.02\,k = +\ 7.1240
\end{aligned}$$

The solution of these is

$$\begin{aligned}
h &= -0.001006 & \pm 0.000503\\
k &= -0.002079 & \pm 0.000983\\
dv_0 &= +0^\circ.155 & \pm 0^\circ.062\\
d(a) &= -0''.058 & \pm 0''.006
\end{aligned}$$

From h and k are obtained

$$\begin{aligned}
P_0 &= 205^\circ\,49' & \pm 2^\circ\,8'\\
e &= 0.002310 & \pm 0.00121
\end{aligned}$$

The normal equations similarly formed of the 73 equations of 1903 are:

$$\begin{aligned}
&+24585.08\,\frac{d(a)}{(a)} - 417.79\,dv_0 + 6411.19\,h - 14410.62\,k = -11.0200\\
&-\ \ 417.79 \quad +431.32\,dv_0 - 511.46\,h - 710.50\,k = +\ 9.8335\\
&+\ 6411.19 \quad -511.46\,dv_0 + 13237.12\,h - 811.36\,k = -\ 3.0407\\
&-14410.62 \quad -710.50\,dv_0 - 811.36\,h + 17515.13\,k = +30.1586
\end{aligned}$$

The solution of these equations gives for the epoch Sept. $8^d.25$,

$$\begin{aligned}
d(a) &= +0''.035 & \pm 0''.009\\
dv_0 &= +0^\circ.046 & \pm 0^\circ.038\\
h &= +0.000810 & \pm 0.000263\\
k &= +0.003608 & \pm 0.004082\\
P_0 &= +12^\circ\,39' & \pm 4^\circ\,18'\\
e &= \ \ 0.00370 & \pm 0.00126
\end{aligned}$$

Combining the results of 1902 and 1903 we obtain

$$\begin{aligned}
d(a) &= -0''.002 & \pm 0''.011\\
dv_0 &= +0^\circ.089 & \pm 0^\circ.028\\
e &= \ \ 0.00315 & \pm 0.00144\\
dP &= 2^\circ.366 \text{ daily or } 864^\circ.182 \text{ yearly}
\end{aligned}$$

Conclusion.

Bringing together the solutions for the four years to the epoch 1903 Sept. 8.25, we have the following set of elements:

$$\begin{aligned}
(a) &= \ \ 47''.961\\
e &= \ \ \ 0.00308\\
dP &= \ \ 2^\circ.429 \text{ daily or } 887^\circ.20 \text{ yearly}\\
v &= 227^\circ.10 \quad , \quad v - L = 193^\circ.80\\
n &= 722^\circ.6316 \text{ daily}
\end{aligned}$$

The most noticeable feature of these results is the smallness of the eccentricity, as compared with that obtained by Dr. COHN and M. TISSERAND. Whether the change is real, and denotes a progressive perturbation which will end in reducing the ellipse to a perfect circle, or is an accidental one, can only be decided after another decade or more. Might it not be that the epochs accidently fall at times when the eccentricity is at low ebb? Dr. MOULTON[7] has shown that the oblateness of a central body will change both the eccentricity and major axis equally and in opposite directions during a complete revolution. As though to verify this plausible explanation we have a very constant value for (a) at the epochs chosen, and one which differs considerably from both the semi-axes of the ellipse — which Prof. BARNARD gives as 47″.606 and 48″.215.

The method of using the differential corrections derived from 1898 and 1899 in the equations of the later years was a fortuitous means of actually obtaining a second set from a preliminary work without repetition. The outcome with respect to the motion of perijove is peculiarly a verification of M. TISSERAND's and Prof. BARNARD's value.

This motion when substituted in the KEPLER equation for the semi-major axis as used by Dr. COHN, gives a value of 48″.066. Concerning this element obtained theoretically, I think the last word has been said by Dr. COHN and Prof. BARNARD, and have nothing to add until some new determination of the mass of *Jupiter* and of his satellites is made.

Evidently the lag in the ephemeris of the Fifth Satellite noted early in the present work is caused by the initial value of the longitude of node, which is too small by 0°.70. At the daily rate, $n = 722°.6316$, this would correspond to a lag of $1^{m}.4$, an amount extraordinarily near the error in 1902. It would seem that the orbit should be moved forward 0°.70, or $1^{m}.4$.

It is my pleasure to express my indebtedness to my instructors for their assistance and guidance in this work: To Prof. F. R. MOULTON for the general plan of attack suggested early in the effort, and to Prof. KURT LAVES for specific directions concerning weighting, reductions, etc. Besides these, I am particularly indebted to Prof. BARNARD for repeated kindnesses and encouragement all along the line. All responsibility for the numerical work belongs alone to me.

(7) MOULTON's *Celestial Mechanics*, p. 233.

Yerkes Observatory, Williams Bay, Wisconsin.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY.

BY W. WALTER DINWIDDIE.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1903 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
					(271) *Penthesilia.*					
Nov. 15 10^h 57^m 46^s	1	t30 , 7	+5^m 8^s.46	−0′ 19″.2	4^h 10^m 54^s.27	+26° 40′ 57″.2	n9.322	0.323	+4^s.82	+ 3″.6
26 12 35 0	2	10 , 10	+ 21.89	+ 6.7	4 0 34.61	+26 17 27.9	9.078	0.296	+4.94	+ 4.9
26 13 14 38	3	t30 , 6	−4 27.07	+3 40.0	4 0 33.24	+26 17 26.0	9.308	0.332	+4.94	+ 4.2
26 13 58 49	4	t30 , 6	+2 23.79	+3 16.7	4 0 31.55	+26 17 20.3	9.462	0.386	+4.94	+ 5.1
					(417) *Suevia.*					
Nov. 19 10 12 59	5	t29 , 6	+1 8.67	+0 56.1	3 56 3.02	+14 2 54.6	n9.342	0.587	+4.42	+ 5.4
20 12 42 53	6	t30 , 6	+2 12.58	+3 29.2	3 55 4.24	+13 58 3.4	8.954	0.570	+4.43	+ 5.6
22 11 7 1	7	t28 , 6	− 37.91	+7 34.0	3 53 20.87	+13 49 40.1	n8.938	0.572	+4.44	+ 5.4
					(225) *Henrietta.*					
Nov. 19 11 19 30	8	10 , 10	+ 5.44	+1 54.9	3 51 43.19	+ 2 48 33.7	n8.902	0.713	+4.11	+ 5.8
20 10 52 33	9	t35 , 7	−1 0.07	+1 22.5	3 50 59.96	+ 2 43 24.8	n9.091	0.714	+4.12	+ 5.7
22 12 11 53	10	t29 , 6	+2 42.50	+4 14.0	3 49 28.64	+ 2 32 53.1	8.721	0.715	+4.13	+ 6.0
					(339) *Dorothea.*					
Nov. 20 11 49 39	11	t30 , 6	+1 6.84	+2 19.5	4 12 1.48	+ 7 21 32.2	n8.722	0.661	+4.24	+ 3.8
22 13 2 18	12	t33 , 7	+ 25.12	+3 46.2	4 10 19.80	+ 7 14 3.9	9.048	0.664	+4.26	+ 3.8
25 11 21 25	13	t30 , 6	+1 3.72	+ 25.0	4 7 53.58	+ 7 4 9.5	n8.790	0.665	+4.29	+ 4.0
25 11 40 42	14	t28 , 6	+1 56.76	+1 52.6	4 7 52.96	+ 7 4 6.3	n8.360	0.664	+4.29	+ 4.1
					(161) *Athor.*					
Nov. 20 13 34 22	15	10 , 10	− 5.70	−2 34.3	4 14 0.96	+31 59 57.9	9.248	0.091	+5.14	+ 2.4
22 14 55 24	16	t30 , 6	− 41.45	−8 43.0	4 11 26.84	+32 0 34.3	9.558	0.294	+5.17	+ 2.8
25 13 52 56	17	t30 , 6	+1 35.20	+3 52.6	4 7 43.81	+32 0 6.2	9.435	0.184	+5.20	+ 4.0
					(382) *Dodona.*					
Nov. 25 15 25 5	18	t30 , 6	−1 23.16	−5 10.8	3 56 11.66	+30 21 38.7	9.644	0.450	+5.12	+ 5.2
Dec. 7 8 12 58	19	t30 , 6	+1 59.01	−6 12.2	3 46 0.36	+29 42 35.2	n9.507	0.319	+5.16	+ 7.7
13 10 2 40	20	t29 , 6	+0 43.63	−2 54.1	3 41 15.85	+29 19 25.1	n8.443	0.159	+5.14	+ 8.5

1903-4 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(371) *Burgundia.*					
Nov. 27 9 37 28	21	10 , 10	− 17.67	+5 49.7	4 24 23.27	+15 31 27.1	n9.411	0.581	+1.56	+ 2.0
27 10 1 8	22	30 , 6	− 48.49	+2 26.9	4 24 22.58	+15 31 22.3	n9.375	0.569	+1.56	+ 1.9
Dec. 6 10 36 55	23	30 , 6	+1 6.34	+ 39.2	4 16 16.90	+11 52 29.2	n8.919	0.554	+1.62	+ 2.7
6 10 59 43	24	30 , 6	−1 2.40	+2 40.7	4 16 15.98	+11 52 25.9	n8.558	0.551	+1.62	+ 2.4
					(350) *Ornamenta.*					
Nov. 30 11 40 4	25	30 , 6	−1 4.13	−9 0.0	4 56 43.73	+ 9 5 35.2	n8.918	0.640	+1.40	− 1.6
Dec. 7 11 9 48	26	10 , 10	− 9.23	−2 50.6	4 49 28.58	+ 9 56 59.2	n8.869	0.628	+1.50	− 1.3
14 9 13 26	27	33 , 7	+3 10.04	−4 35.1	4 42 17.59	+10 53 19.9	n9.244	0.623	+1.58	− 0.6
					(426) [1897 *DH*].					
Nov. 30 13 17 58	28	10 , 8	+1 31.04	+4 4.8	4 3 14.33	+47 31 23.7	9.497	n9.885	+6.28	+ 5.1
Dec. 3 12 18 33	29	30 , 6	+4 15.41	+1 10.9	3 59 42.26	+47 15 19.6	9.283	n0.039	+6.28	+ 6.5
					[1903 *NF*].					
Dec. 15 11 27 41	30	10 , 10	+ 9.94	−9 18.1	5 42 1.99	+ 9 20 30.4	n8.963	0.636	+4.57	− 7.4
15 11 57 25	31	10 , 10	+ 23.54	−8 40.3	5 42 0.78	+ 9 20 26.0	n8.303	0.635	+4.57	− 7.4
17 12 41 17	32	10 , 10	− 18.93	−3 38.5	5 39 59.14	+ 9 12 50.4	8.946	0.638	+4.58	− 7.6
18 13 42 8	33	33 , 7	+2 45.45	+ 22.1	5 38 57.05	+ 9 9 9.0	9.330	0.650	+4.60	− 7.2
18 14 6 10	34	20 , 4	+3 23.87	−3 44.6	5 38 56.19	+ 9 9 6.4	9.409	0.655	+4.60	− 7.1
					(65) *Cybele.*					
Dec. 16 11 30 29	35	10 , 10	+ 5.60	−2 29.2	6 22 36.35	+19 2 13.4	n9.184	0.488	+4.81	−12.2
Jan. 5 10 39 54	36	30 , 6	−2 9.22	+2 48.3	6 7 4.49	+19 14 5.9	n8.799	0.448	+1.56	−10.4
					(42) *Isis.*					
Dec. 17 10 3 8	37	30 , 6	+1 2.60	−0 41.3	7 1 15.81	+25 38 1.9	n9.583	0.479	+4.93	−16.9
17 10 20 52	38	29 , 6	− 37.34	+2 15.5	7 1 15.12	+25 38 5.3	n9.551	0.454	+4.92	−17.1
Jan. 5 12 10 30	39	29 , 6	+ 18.74	+0 12.9	6 40 8.38	+26 53 25.4	8.794	0.262	+1.60	−11.0
					(490) [1902 *JP*].					
Dec. 17 11 21 27	40	38 , 8	− 21.73	+2 2.9	5 52 27.69	+ 9 57 50.1	n8.989	0.629	+4.60	− 8.8
18 14 53 3	40	30 , 6	−1 19.19	+1 39.9	5 51 30.24	+ 9 57 27.0	9.493	0.657	+4.61	− 8.9
					(492) [1902 *JR*].					
Dec. 18 9 18 4	41	30 , 6	+ 53.88	+ 9.4	6 22 1.15	+25 0 16.9	n9.584	0.491	+5.06	−12.5
18 9 35 13	42	30 , 6	+1 1.44	−1 5.0	6 22 0.35	+25 0 20.1	n9.554	0.468	+5.06	−12.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	4 5 40.99	+26 41 12.8	Camb. (Eng.) A.G.2011	22	4 25 6.51	+15 28 53.5	Berlin A. A.G. 1202
2	4 0 7.78	+26 17 16.3	Camb. (Eng.) A.G.1987	23	4 15 5.94	+11 51 47.3	Leipzig I. A.G. 1266
3	4 4 55.37	+26 13 41.8	Camb. (Eng.) A.G.2008	24	4 17 13.76	+11 49 12.8	Leipzig I. A.G. 1278
4	3 58 2.82	+26 13 58.5	Camb. (Eng.) A.G.1972	25	4 57 43.46	+ 9 14 36.8	Leipzig II. A.G. 1949
5	3 54 49.93	+14 1 53.1	Leipzig I. A.G. 1168	26	4 49 33.31	+ 9 59 51.1	Leipzig II. A.G. 1866
6	3 52 47.23	+13 54 28.6	Leipzig I. A.G. 1152	27	4 39 2.97	+10 57 55.6	Leipzig I. A.G. 1367
7	3 53 54.34	+13 42 0.7	Leipzig I. A.G. 1159	28	4 1 37.01	+47 27 13.8	Fund. Catal. 260
8	3 51 33.64	+ 2 46 33.0	Albany. A.G. 1151	29	3 55 20.57	+47 14 2.2	Bonn A.G. 3322
9	3 51 55.91	+ 2 41 56.6	Albany. A.G. 1152	30	5 41 47.48	+ 9 29 55.9	Leipzig II. A.G. 2399
10	3 46 12.01	+ 2 28 33.1	Albany. A.G. 1127	31	5 42 0.78	+ 9 20 26.0	Leipzig II. A.G. 2396
11	4 10 50.40	+ 7 19 8.9	Leipzig II. A.G. 1568	32	5 40 13.49	+ 9 16 36.5	Leipzig II. A.G. 2382
12	4 9 50.42	+ 7 10 13.9	Leipzig II. A.G. 1558	33	5 36 7.00	+ 9 8 51.1	Leipzig II. A.G. 2367
13	4 6 15.57	+ 7 3 40.5	Leipzig II. A.G. 1539	34	5 35 27.72	+ 9 12 58.1	Leipzig II. A.G. 2363
14	4 5 51.91	+ 7 2 9.6	Leipzig II. A.G. 1533	35	6 22 25.94	+19 5 24.8	Berlin A. A.G. 2162
15	4 14 1.52	+32 2 29.8	Leiden. A.G. 1648	36	6 9 12.45	+19 11 28.0	Berlin A. A.G. 2014
16	4 12 3.12	+32 9 14.5	Leiden. A.G. 1632	37	7 0 8.28	+25 39 0.1	Camb. (Eng.) A.G.3736
17	4 6 3.41	+31 56 9.6	Leiden. A.G. 1588	38	7 1 47.54	+25 36 6.9	Camb. (Eng.) A.G.3749
18	3 57 29.70	+30 26 44.3	Leiden. A.G. 1524	39	6 39 48.04	+26 53 23.5	Camb. (Eng.) A.G.3478
19	3 43 56.19	+29 48 39.7	Camb. (Eng.) A.G.1862	40	5 52 44.82	+ 9 55 56.0	Leipzig II. A.G. 2526
20	3 40 27.08	+29 22 10.7	Camb. (Eng.) A.G.1840	41	6 21 2.21	+25 0 20.0	[illegible]
21	4 24 36.38	+15 25 35.4	Berlin A. A.G. 1199	42	6 20 53.85	+25 1 37.9	Camb. (Eng.) A.G.3229

Planets 271, 339, 161, 382, 374, 350, 426, [1903 *NF*], 490, and 492 were found photographically by Mr. G. H. PETERS, 417 and 225 by myself.

Magnitude of (382) *Dodona* was estimated at 13.5, and (350) *Ornamenta* 11.5 on Dec. 7.

Observations published in *A.J.* 556 under the head of [1903 Sept. 28] are observations of (163) *Erigone*. Attention was called to this fact by Dr. W. LUTHER of Düsseldorf.

OBSERVATIONS OF COMET a 1904 (BROOKS),

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

BY J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1904 Washington M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Apr. 30 11 41 20	1	30 , 5	+0 11.27	− 2 43.7	16 11 59.28	+52 9 38.3	n9.557	n0.134	+2.07	−3.4
May 1 9 4 33	2	30 , 6	+2 51.65	− 0 29.8	16 8 25.79	+52 34 24.0	n9.838	0.065	+2.09	−3.0
2 10 4 7	3	10 , 2	+2 2.06	+ 3 8.6	16 4 10.77	+53 2 20.9	n9.761	n9.409	+2.11	−2.6
3 11 13 21	4	30 , 6	+2 36.66	+ 4 4.9	15 59 49.42	+53 29 23.9	n9.584	n0.176	+2.14	−2.2
4 8 57 52	5	20 , 4	−4 55.25	+ 9 51.4	15 55 59.23	+53 51 47.8	n9.835	9.808	+2.16	−2.0
5 10 18 12	6	30 , 6	−1 35.51	− 1 47.8	15 51 26.55	+54 16 42.0	n9.700	n0.029	+2.18	−1.5
7 8 32 10	7	30 , 6	+2 42.89	+ 8 46.6	15 42 58.31	+54 59 7.4	n9.848	9.732	+2.21	−0.6
8 8 35 58	8	35 , 7	+0 28.89	+ 3 28.9	15 38 28.44	+55 19 32.5	n9.839	9.379	+2.23	−0.2
9 10 40 16	9	9 , 2	+2 33.52	+ 3 4.9	15 33 32.86	+55 40 21.5	n9.518	n0.300	+2.24	+0.3
10 8 42 21	10	30 , 6	−0 45.69	+ 2 29.0	15 29 21.04	+55 56 49.6	n9.816	n9.499	+2.25	+0.7
11 10 13 57	11	30 , 6	+0 37.20	+11 30.3	15 24 26.81	+56 14 40.4	n9.589	n0.294	+2.25	+1.2
12 10 32 1	12	15 , 3	+6 15.73	+ 8 36.1	15 19 44.19	+56 30 28.7	n9.481	n0.360	+2.24	+1.8
13 10 42 46	13	30 , 5	−4 17.32	− 6 17.8	15 15 2.73	+56 44 58.5	n9.377	n0.394	+2.27	+2.1
15 9 42 59	14	30 , 6	+0 40.01	+ 5 29.3	15 5 52.41	+57 9 46.8	n9.588	n0.330	+2.23	+3.1
16 11 46 42	15	29 , 6	−2 12.95	− 7 26.0	15 0 47.00	+57 21 38.9	8.943	n0.444	+2.23	+3.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	16 11 45.94	+52 12 25.4	Camb., U.S., A.G. 4947	9	15 30 57.10	+55 37 16.3	Hels.-Gotha A.G. 8408
2	16 5 32.05	+52 34 56.8	" " " 4926	10	15 30 4.48	+55 54 19.9	" " " 8402
3	16 2 6.60	+52 59 14.9	" " " 4911	11	15 23 47.36	+56 3 8.9	" " " 8373
4	15 57 10.62	+53 25 21.2	" " " 4884	12	15 13 26.52	+56 21 50.8	" " " 8291
5	16 0 52.32	+53 41 58.4	" " " 4904	13	15 19 47.78	+56 51 14.2	" " " 8338
6	15 52 59.88	+54 18 31.3	" " " 4869	14	15 5 10.20	+57 4 14.4	" " " 8240
7	15 40 13.24	+54 50 21.4	Camb. U.S., A.G. 4822 and Hels. Gotha A.G. 8474	15	15 2 57.72	+57 29 1.4	" " " 8227
8	15 37 57.32	+55 16 3.8	Hels.-Gotha A.G. 8455				

ELEMENTS AND EPHEMERIS OF COMET a 1904 (BROOKS).

BY EVERETT I. YOWELL.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

The following parabolic elements were obtained from normal places, formed by combining observations made at the U.S. Naval Observatory on April 17 and 18, 22 and 23, 30 and May 1.

$$T = 1904 \text{ March } 5.76884 \text{ G.M.T.}$$

$$\left.\begin{aligned} \pi &= 328^\circ\ 43'\ 47''.6 \\ \Omega &= 275\ \ 41\ \ 22.8 \\ i &= 125\ \ \ 6\ \ 17.4 \end{aligned}\right\} 1904.0$$

$$\log q = 0.431988$$

HELIOCENTRIC COORDINATES.

$$\begin{aligned} x &= r[9.764001] \sin(223^\circ\ 12'\ 40''.0 + v) \\ y &= r[9.994756] \sin(300\ \ 33\ \ 10.4 + v) \\ z &= r[9.918342] \sin(\ \ 24\ \ 29\ \ 22.0 + v) \end{aligned}$$

The brightness at the date of discovery is adopted as the unit in the following ephemeris. Comparison with an observation made here May 16 gives as corrections to the ephemeris: $\Delta\alpha = -3^s$, $\Delta\delta = +15''$.

EPHEMERIS.

G.M.T.	α	δ	log Δ	Br.
	h m s	° ′ ″		
May 19.5	14 47 45.8	+57 45 53	0.38595	0.80
20.5	43 9.3	52 32	0.38821	0.79
21.5	38 34.9	57 58 8	0.39052	0.78
22.5	32 3.0	58 2 42	0.39286	0.77
23.5	29 34.0	6 16	0.39525	0.76
24.5	25 8.3	8 53	0.39766	0.75
25.5	20 46.2	10 33	0.40011	0.74
26.5	16 27.9	11 20	0.40259	0.73
27.5	12 13.7	11 15	0.40510	0.72
28.5	8 4.0	10 19	0.40763	0.71
29.5	14 3 58.9	8 36	0.41018	0.70
30.5	13 59 58.7	6 7	0.41276	0.69
31.5	13 56 3.4	+58 2 55	0.41536	0.68

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed May 19.*

Entered February 2 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 563.

VOL. XXIV. BOSTON, 1904 JUNE 1. NO. 11

PERIOD OF 6760 κ *PAVONIS*.

BY ALEXANDER W. ROBERTS.

In *A.J.* 553, p. 7, Dr. CHANDLER indicates that uncertainty* exists with regard to the exact period of the southern variable κ *Pavonis*.

When his judgment came under my notice I was engaged in reducing the Lovedale observation of κ *Pavonis*. It is therefore both convenient and opportune to state briefly the main results of this reduction in as far as they bear on the question of the star's period.

My observations of κ *Pavonis* were begun in May, 1891, and have been continued regularly, with the exception of a break in 1897, down to the present time. During the thirteen years, 1891–1903, 1043 observations have been secured, and 190 maxima determined.

These observations are so uniformly distributed over the thirteen years, 1891–1903, that no uncertainty in enumerating the number of light-phases of κ *Pavonis*, during this period, can possibly arise.

It will make for brevity and simplicity if all the observations recorded in any one year be reduced to a mean light-curve for that year. In carrying out these reductions I have taken 9.091 days as the mean period. It will be evident that a slightly different period would have little or no appreciable effect upon the mean light-curve for any one year.

The mean minima and maxima thus determined are as follows:

TABLE I. MAXIMA.

E	Date	O	C	O—C (d)
−341	1891 July 15.5	241 1929.5	1929.5	+0.0
301	1892 July 13.2	2293.2	2293.4	+0.1
260	1893 July 21.2	2666.2	2665.8	+0.4
224	1894 June 13.3	2993.3	2993.1	+0.2
187	1895 May 15.3	3329.3	3329.5	−0.2
142	1896 June 27.2	3738.2	3738.6	−0.4
52	1898 Sept. 23.8	4556.8	4556.8	+0.0
− 27	1899 May 9.0	4784.0	4784.0	+0.0
+ 21	1900 July 19.7	5220.7	5220.4	+0.3
60	1901 July 9.5	5575.5	5575.0	+0.5
99	1902 June 28.8	5929.8	5929.5	+0.3
+145	1903 Aug. 20.7	241 6347.7	6347.7	+0.0

* The uncertainty spoken of was between the periods $9^d.102$ and $9^d.092$. Mr. ROBERTS's article satisfactorily removes the doubt.

ED.

TABLE II. MINIMA

E	Date	O	C	O—C (d)
−341	1891 July 20.1	241 1934.1	1934.3	−0.2
301	1892 July 18.0	2298.0	2297.9	+0.1
260	1893 July 26.1	2671.1	2670.6	+0.5
224	1894 June 18.0	2998.0	2997.9	+0.1
187	1895 May 20.0	3334.0	3334.3	−0.3
142	1896 July 1.7	3742.7	3743.4	−0.7
52	1898 Sept. 28.6	4561.6	4561.6	+0.0
− 27	1899 May 13.7	4788.7	4788.8	−0.1
+ 21	1900 July 24.2	5225.2	5225.2	+0.0
60	1901 July 14.2	5580.2	5579.8	+0.4
99	1902 July 3.7	5934.7	5934.3	+0.4
+145	1903 Aug. 25.5	241 6352.5	6352.5	+0.0

The first three columns of the foregoing tables need no explanation.

The dates given in the fourth column are computed from the following elements:

$$T_M = 241\,5029.5 + (9^d.0910)\,E$$
$$T_m = 241\,5034.3 + (9^d.0910)\,E$$

where T_M and T_m are for Tables I and II respectively.

It is evident that the residuals given in column O − C of both tables, residuals computed on the basis of an invariable period, vary according to definite law. Charting down the values we find that there is at once exhibited a curve of the simple character represented by the trigonometrical expression,

$$m \cos(E\theta - M)$$

A graphical determination indicates that this periodical inequality runs through its full cycle in eight years.

Whether this systematic variation in the length of the period is due to subjective causes, *i.e.* position-error, atmospheric absorption, fluctuations in magnitude of one or more of the comparison-stars, and is therefore only apparent; or whether it has an actual objective existence, I am not able as yet to determine.

The causes which underlie many of the minute but yet appreciable inequalities that affect the period of not a few variable stars are exceedingly obscure.

Now if

$$T_M = \text{any observed maximum of } \kappa\ Pavonis$$
$$p + \Delta p = \text{period of star}$$
$$t + \Delta t = \text{epoch of maximum passage}$$
$$m \cos(\mathrm{E}\theta - M) = \text{secular variation in period}$$

then for any maximum we have the following equation of condition:

$$(p+\Delta p) + \mathrm{E}\,(t+\Delta t) + m\cos(\mathrm{E}\theta - M) = T_M$$

or

$$\Delta p + \mathrm{E}\Delta t + m\cos(\mathrm{E}\theta - M) = T_M - (p+\mathrm{E}t)$$

that is,

$$\Delta p + \mathrm{E}.\Delta t + \cos\mathrm{E}\theta\,(m\cos M) + \sin\mathrm{E}\theta\,(m\sin M) = \mathrm{O}-\mathrm{C}$$

The same will hold good for any minimum determination.

It has already been stated that the secular irregularity of period runs through a full cycle of variation in 8 years: this gives as the value of θ,

$$\theta = 1^\circ.13$$

We have thus data for the construction of suitable equations of condition for the determination of the principal elements of variation of this star.

Forming and solving these equations of condition in the ordinary way, we obtain the following results:

	For Maxima	For Minima
$\Delta\delta$	$+0^d.04$	$-0^d.05$
Δp	+0.0005	+0.0006
m	+0.39	+0.46
M	52° 26′	56° 20′

Combining both sets of values, we obtain as the elements which best represent the Lovedale observations of κ *Pavonis*, during the period 1891–1903, the following:

$$T_M = 241\,5029.54 + 9^d.09155\,\mathrm{E} + 0^d.43\cos(\mathrm{E}\,1^\circ.13 - 54^\circ)$$
$$T_m = 241\,5034.25 + 9^d.09155\,\mathrm{E} + 0^d.43\cos(\mathrm{E}\,1^\circ.13 - 54^\circ)$$
$$M - m = 4^d.71 \quad , \quad m - M = 4^d.38$$

The practical equality of the ascending and descending periods of κ *Pavonis* has been referred to in previous notes on this star's type of variation.

As we are assured of the constancy (within certain well-defined limits) of the mean period during the thirteen years, 1891–1903, we may with a certain measure of confidence relate the Lovedale measures to those made at Cordoba in 1871, 1872 and 1873.

There are three maxima and three minima of κ *Pavonis* recorded in the *Uranometria Argentina*, viz:

Maxima	J.D.		Minima	J.D.
1871 Dec. 31	= 240 4793	,	1871 Nov. 29	= 240 4761
1872 Oct. 6	= 240 5073	,	1872 Dec. 25	= 240 5153
1873 July 26	= 240 5366	,	1873 July 3	= 240 5343

As no hour is recorded, we may take the dates as they stand, reducing them by means of the period $9^d.091$ to one mean maximum and minimum phase.

For future reference we may indicate this reduction:

Maxima.	Minima.
240 4793 + $345^d.5$ = 240 5138.5	240 4761 + $381^d.8$ = 240 5142.8
240 5073 − 63.6 = 240 5136.6	240 5153 − 9.1 = 240 5143.9
240 5366 − 227.3 = 240 5138.7	240 5343 − 200.0 = 240 5143.0
Mean max. = 240 5137.9	Mean min. = 240 5143.2

The intervals between the Cordoba mean values and the Lovedale mean values, are accordingly,

	Maxima		Minima	
	$241\,5029^d.5$	,	$241\,5034^d.3$	
	240 5137.9	,	240 5143.2	
Interval	9891.6		9891.1	Mean $9891^d.4$

If no *per saltum* change of period has taken place during the years 1873–1891, the Lovedale observations indicate that in the interval, 9891.4 days, there have occurred 1088 periods, which yields as the mean duration of a single period, 9.0911 days.

Any period differing $\pm 0^d.001$ from this value is incompatible with the Lovedale observations.

It is hoped that the present consideration may tend towards establishing a satisfactory period for κ *Pavonis*. It is in this spirit that I have put on record now, rather than at a later date, my observations of the star.

If this paper does not remove existing difficulties it will at least afford an opportunity to astronomers of comparing the Lovedale observations with any others which they may have in their possession.

One is tempted to speculate on the causes which give rise to a phenomenon so interesting as the secular variation in the period of κ *Pavonis*.

It will readily occur to the mind of most that if we regard κ *Pavonis* as a bright companion circling round a massive dark central star, in a period of about eight years, and in an orbit whose plane is practically in the line of sight we have all the circumstances which would produce variation of the character determined.

The difficulties, probably insuperable, is the magnitude of the orbit, and the massiveness of the central body which such a light equation as ten hours would imply.

Yet if we assume κ *Pavonis* to revolve in an orbit 70 times the size of the earth's orbit, and round a central body 5,000 times more massive than the sun, we have a complete explanation of the secular variation of period.

Unless κ *Pavonis* is at an immense distance from us, heliometer observations should testify (or otherwise) to the eight-yearly vibration of the star, and spectroscopic observations, at no distant date, might yield evidence of its orbital movement.

OBSERVED MAXIMA AND MINIMA OF VARIABLE STARS, 1900–1903.

BY PAUL S. YENDELL.

4805. *W Virginis.*

Thirteen observations of *W Virginis*, from 1900 Mar. 25, to May 22, and thirteen in 1902, from April 28 to June 9, indicate maxima and minima as follows:

Max.	Wt.	Min.	Wt.
1900 Mar. 31.5	4	1900 May 26.9	2
1902 May 31.9	4	1902 May 8.6	4

5912. *g Herculis.*

I have ten observations, from May 30 to July 28, and one on Sept. 5. When first observed the star's light was estimated at $5^{\mathrm{M}}.2$. On June 5 it had increased to $5^{\mathrm{M}}.0$, at which light it remained until June 14, diminishing to a minimum of $6^{\mathrm{M}}.0$ on July 10. A single observation on September 5 showed the star at $6^{\mathrm{M}}.0$.

6404. *Y Ophiuchi.*

Thirty-three observations, from 1902 May 28 to Sept. 23, show three maxima and three minima, as follows:

Max.	Wt.	Min.	Wt.
1902 May 31.6	3	1902 June 29.1	3
July 5.2	4	July 31.4	3
Aug. 24.6	2	Sept. 6	1

6472. *W Sagittarii.*

Thirty observations, from 1902 May 28 to Sept. 23, indicate maxima and minima as follows, by the application of a mean light-curve.

Max.	Obs.	Min.	Obs.
1902 June 10.09	1	1902 July 1.38	1
18.05	1	31.41	2
July 1.23	1	Aug. 8.70	2
11.15	3		
Aug. 3.02	1		
25.46	2		
Sept. 2.34	1		

6573. *Y Sagittarii.*

Thirty observations, from 1902 May 13 to Sept. 23, indicate maxima and minima as follows.

By the mean light-curve:

Max.	Obs.	Min.	Obs.
1902 May 27.60	2	1902 May 13.09	1
June 24.11	1	June 6.58	1
29.51	1	July 3.36	3
July 6.38	1	Aug. 1.62	2
Aug. 8.93	1	7.60	1
21.74	1	25.39	1
31.72	1	Sept. 5.33	1
Sept. 7.35	1	22.33	1

By the single curves, three minima:

July 9.7 wt. 3 Aug. 25.3 wt. 2 Sept. 5.3 wt. 2.

6613. *d Serpentis.*

This star was observed twenty-three times, from 1902 May 30 to Sept. 5. The observations yield three maxima, as follows:

1902 May 31.6	wt. 2
July 1.2	3
Aug. 2.5	4

I find no confirmation in my later observations of the short period, nor of the *β Lyrae* type of variation suggested by my earlier ones. The star seems rather to be of the type of *R Scuti*, with a period of something like a month, and of very varying light-range, its variations in some seasons having been within reasonable errors of observation, so that at times I have been in doubt of the star's actual variability.

6636. *U Sagittarii.*

Thirty-one observations, from 1902 May 13 to Sept. 23, yield the following maxima and minima by the use of a mean light-curve:

Max.	Obs.	Min.	Obs.
1902 May 29.83	3	1902 May 12.60	1
June 5.16	1	June 9.46	1
25.86	2	29.07	1
July 2.36	2	July 6.37	1
9.11	2	Aug. 8.12	1
Aug. 6.71	1	22.68	1

6733. *R Scuti.*

Thirty-nine observations of this star, from 1902 May 9 to Oct. 23, show maxima and minima as follows: a minimum of $6^{\mathrm{M}}.5$ on May 15; a maximum of $5^{\mathrm{M}}.1$ on June 24; a minimum of $6^{\mathrm{M}}.2$ about July 12; a maximum of $5^{\mathrm{M}}.4$ on August 7; and a minimum of $6^{\mathrm{M}}.6$ on Sept. 6. At the last observation the star was at $5^{\mathrm{M}}.5$.

6749. *S Scuti.*

I observed this star seventeen times in 1902, from May 29 to Oct. 23. At the first observation it was faint, about 6¾ mag.; it rose to 6 mag. by the beginning of July. It remained without apparent change until the beginning of September. A detached observation on Oct. 23 found it at about 6¼ mag.

6758. *β Lyrae.*

Forty-nine observations, from 1902 May 8 to Oct. 23, show the following principal minima:

Min.	Obs.	Min.	Obs.
1902 May 17.62	1	1902 July 27.94	1
June 6.10	2	Aug. 9.22	1
July 1.49	4	22.46	1
13.49	1	Sept. 4.68	2

6791. *R Lyrae.*

I have thirty-one observations of *R Lyrae*, from May 2 to Sept. 23, 1902. At the first observation the star was estimated to be $4^{\mathrm{M}}.9$. It rose quickly to a maximum of $4^{\mathrm{M}}.3$ on May 15, and fell away almost as quickly to $4^{\mathrm{M}}.8$, at which brightness it was found on May 30, and continued dropping slowly to $5^{\mathrm{M}}.0$, at which light a minimum was observed on Aug. 7. From this it rose steadily to a max-

mum of $4^M.25$ on Sept. 7. At the last observation, on Sept. 23, it had declined to 5^M.

6984. *U Aquilae.*

U Aquilae was observed during the season of 1902 twenty-six times, from May 30 to Sept. 23. From these observations the following times of maximum and minimum are deduced. The minima marked *e* were deduced by the aid of a mean light-curve, the rest from the single curves.

Max.	Obs.	Min.	Obs.	Wt.
1902 May 31.78	1	1902 June 5.61	1*e*	
June 7.99	1	July 3.6		3
July 7.18	1	10.9		1
13.59	1	30.6		3
Aug. 3.48	1	Aug. 7.4		2
24.02	2	22.35	1*e*	
Sept. 22.57	1	Sept. 5.33		3

7034. *U Vulpeculae.*

This star is a rather difficult and unsatisfactory object, being too faint for comfortable observation with the field-glass, and the comparison-stars too distant and scattered for convenient use in the telescope. I have thirty-seven observations, from 1902 May 8 to Oct. 23. The use of a mean light-curve indicates maxima and minima as follows:

Max.	Obs.	Min.	Obs.
1902 May 5.62	1	1902 May 28.14	1
29.32	2	June 29.00	2
June 6.88	1	July 39.77	3
Aug. 2.36	1	Aug. 8.74	2
Sept. 11.40	1	23.85	1
Oct. 20.32	1	Oct. 25.48	1

7085a. *SU Cygni.*

Forty-six observations of *SU Cygni* show the following maxima and minima on the application of a mean light-curve. The very short period of the star causes many of these to depend on a single observation.

Max.	Obs.	Min.	Obs.
1902 May 13.60	1	1902 May 5.12	1
June 5.57	1	8.83	1
9.93	1	June 1.17	2
17.65	1	14.43	1
20.52	1	23.40	1
24.49	1	27.40	1
July 3.08	2	July 1.91	1
6.47	1	29.48	1
10.93	1	Aug. 9.42	1
Aug. 2.46	1		
Sept. 2.78	2		
Oct. 22.06	2		

7378. *SZ Cygni.*

I have had this star under more or less continuous observation during the seasons of 1900 to 1903, the observations numbering eighty-three.

By the use of a mean light-curve formed from all the observations up to January, 1903, I have deduced the following maxima.

Max.	Obs.	Max.	Obs.
1900 Nov. 4.41	1	1902 Sept. 10.33	1
1901 Oct. 15.60	2	Oct. 21.83	1
Nov. 11.38	1	1903 May 25.10	2
1902 July 26.74	3	Oct. 23.88	3
Aug. 10.24	2		

By the single curves:

Max.	Wt.	Min.	Wt.
1902 June 27.5	5	1902 June 20.8	4
July 12.2	4	July 6.2	4
Aug. 26.1	4		

7437. *X Cygni.*

Forty-seven observations of *X Cygni*, from 1902 May 9 to Oct. 23, show the following maxima and minima:

Max.	Wt.	Min.	Wt.
1902 June 7.9	2	1902 June 17.5	5
24.7	4	July 4.2	5
July 10.3	3		
Aug. 26.4	2		

7483. *T Vulpeculae.*

Twenty-six observations of *T Vulpeculae*, from May 9 to Sept. 2, 1902, show the following maxima and minima by the use of the mean curve:

Max.	Obs.	Min.	Obs.
1902 May 10.6	1	1902 May 29.6	3
14.8	1	June 9.4	1
June 23.7	1	July 1.4	1
July 29.6	1	5.3	2
Aug. 7.0	1	10.4	1
		14.5	1
		31.4	1
		Aug. 23.4	1

7539. *TX Cygni.*

This star and *SZ Cygni* are two of Mr. WILLIAMS's stars of which I believe I am the only observer in this country. I have thirty-seven observations of it, from May 11 to Oct. 26, 1902, and sixteen, from May 21 to Oct. 26, 1903.

These observations indicate a period of 14.728 days, which agrees very fairly with that of Mr. WILLIAMS (*A.N.*, 3769, 13). They show the following maxima and minima. Those marked with a *e* were deduced by the aid of a mean light-curve, the others from the single curves.

Max.	Wt.	Obs.	Min.	Wt.	Obs.
1902 May 10.59		1*e*	1902 May 31.7	5	
24.69		2*e*	June 1.70		2*e*
June 8.0	5		July 1.4	5	
July 7.1	5		Aug. 28.57		3*e*
Aug. 5.98		2*e*			
20.22		1*e*	Sept. 11.5		1*e*
Sept. 18.19		2*e*			
1903 May 26.83		2*e*	1903 July 20.4		1*e*
July 24.34		1*e*			
Oct. 21.37		1*e*	Oct. 14.4		1*e*

The type of light-curve of this star and of 7378 *SZ Cygni*, is substantially that of *T Vulpeculae*.

Dorchester, 1904 *May* 1.

NOTES ON VARIABLE STARS. — No. 40.

By HENRY M. PARKHURST.

Subtangent Process. A variation of the subtangent method (*A.J.*, No. 540), occurs in obtaining the minimum of 7560, which was beyond the reach of my telescope. The substance of the computation, derived from the given observations for four selected days is as follows:

The first tangent is derived from the observations of Sept. 13 and 21: mean, 6376.0. The minimum is assumed to occur at 6376.0 + t. Interval AB, 9.0 = +1.21; μ_1 = +.13.

The second tangent is derived from the observations of November 11 and 12: mean, 6431.0. The minimum occurs at 6431.0 − 55 + t. Interval CD, 1.0 = −.40; μ_2 = −.40.

The vertex, or minimum, is obtained from the intersection of the two tangents; taking the mean magnitude of each pair, reckoning the time from the first mean time given:

$$11.83 + .13t = 10.70 - .40(t - 55); \text{ or}$$
$$.53t = -11.83 + 10.70 + 22 = 20.87.$$
$$t = 39; \text{ minimum} = 6415.$$

From the two tangents we obtain for the magnitude at minimum, $11.83 + 39\mu_1 = 17.0$; $10.7 + 16\mu_2 = 17.1$.

RESULTS OF OBSERVATIONS.

No.	Star	Phase	Observed Date Julian	Calendar	E	Corr.	Wt.	Mag.	Factors			Remarks
6207	*Z Ophiuchi*	Max.	6448	Nov. 29	11	+ 30	9	7.7	—	—	—	Subtangent process, from mean of last two observations
6452	*RY Herculis*	Max.	6416	Oct. 28	—		7	8.6		—	—	218±?
6682	*X Ophiuchi*	Max.	6419	Oct. 31	19	— 7	9	6.90	1.82	0.98	28	Elements 3d catalogue.
6892	*RX Sagittarii*	Max.	6408	Oct. 20	10	— 14	9	10.15	1.11	1.36	34	329? *A.J.* 553.
7162	*RS Aquilae*	—	—	—		—	—	—	—	—	—	Irregular? Not seen.
7252	*W Capricorni*	Max.	6408	Oct. 20	55	—	E	—	—		—	Invisible.
7261	*R Delphini*	Max.	6420	Nov. 1	49	— 41	9	8.70	1.39	1.46	29	Correction diminishing.
7268	*RT Capricorni*	Max.	6386	Sept. 28	—	—	—	—	—	—	—	
7431	*S Delphini*	Min.	6418	Oct. 31	50	—	E	—				Interruption. [380±
7435	*Y Aquarii*	Max.	6389	Oct. 1	24	+ 8	9	8.91	4.0	2.0	34	Adding to epoch 60±; period
7444	*T Delphini*	Max.	6382	Sept. 24	43	+ 8	6	9.4	—	—	—	
7450	*V Aquarii*	Max.	6402	Oct. 14	20	— 158	9	8.44	6.0	4.0	64	No regular changes apparent.
7468	*T Aquarii*	Max.	6344	Aug. 17	75	— 7	1	—	—	—	—	Probably earlier.
7502	*X Delphini*	Max.	6505	Jan. 25	11	+ 8	1		—	—	—	*A.J.* 553.
7560	*R Vulpeculae*	Min.	6415	Oct. 27	102	+ 1	7	17.0	—	—	—	
7594	*RS Aquarii*	Max.	6394	Oct. 6	7	+ 13	9	9.77	1.20	1.35	24	4883 + 214 E.
7619	*RR Aquarii*	Max.	6392	Oct. 4	7	— 69	9	8.58	1.57	2.41	27	5128 + 190.5 E.
7659	*T Capricorni*	Max.	6107	Dec. 23	64	0	1	8.1	—			
8290	*R Pegasi*	Max.	6355	Aug. 28	51		E					
8369	*W Pegasi*	Min.	6407	Oct. 19	9	+ 23	8	11.5	—		—	
8373	*S Pegasi*	Max.	6506	Jan. 26	45	9	2	7.6	—			Subtangent process, *A.J.* 553
8597	*V Ceti*	Max.	6452	Dec. 3	34	— 12	5	9.5				Probably earlier.

INDIVIDUAL OBSERVATIONS.

Including Observations by ARTHUR C. PERRY.

6207 *Z Ophiuchi.*

(Cont. from 476. Comp. Star-333.)

Julian	Calendar 1903	Mag.
6344.6	Aug. 17	11.7
6371.5	Sept. 13	10.6
6380.5	22	9.90_2
6387.5	29	8.15_2
6402.5	Oct. 14	8.68_2
6407.5	19	8.70_2
6412.5	24	8.13_2
6416.5	28	7.90_2
6426.5	Nov. 7	8.52_2
6438.5	19	7.42_2

6452 *RY Herculis.*

(Continued from 490.)

Julian	Calendar 1903	Mag.
6266.6	May 31	11.9
6345.6	Aug. 18	10.6
6347.6	20	11.13_2
6369.5	Sept. 11	11.0
6371.5	13	10.1
6374.5	16	10.2
6376.5	18	10.15_4
6380.5	22	10.55_2
6384.5	26	10.06_2
6386.5	28	10.05_2

6452 *RY Hercul.*—Cont.

Julian	Calendar 1903	Mag.
6387.5	Sept. 29	10.00_2
6402.5	Oct. 14	8.89_2
6407.5	19	9.05_2
6412.5	24	8.16
6416.5	28	8.58_2
6425.5	Nov. 6	8.74_2
6437.5	18	9.3

6682 *X Ophiuchi.*

(Continued from 490.)

Julian	Calendar 1903	Mag.
6345.6	Aug. 18	7.1

6682 *X Ophiuchi.*—Cont.

Julian	Calendar 1903	Mag.
6362.5	Sept. 4	6.8
6372.5	14	8.37_2
6382.5	24	7.15_2
6388.5	30	7.02_2
6391.5	Oct. 3	7.33
6407.5	19	7.10_2
6413.5	25	7.05_2
6417.5	29	6.62_2
6426.5	Nov. 7	6.71_2
6430.5	11	7.89_2
6437.5	18	6.8

6892 *RX Sagittarii.*

(Cont. from 535. Comp. Stars 421.)

Julian	Calendar 1903	Mag.
6342.6	Aug. 15	13?
6350.6	23	12?
6368.5	Sept. 10	10.6
6369.6	11	10.58_2
6371.5	13	10.78_2
6372.5	14	11.13_2
6379.5	21	10.94_2
6383.5	25	10.81_2
6388.5	30	10.67
6401.5	Oct. 13	9.79_2

6892 *RX Sagitt.*—Cont.

Julian	Calendar	Mag.
	1903	
6409.5	Oct. 21	10.17_2
6417.5	29	10.10_2
6426.5	Nov. 7	10.62_2
6430.5	11	10.91_2

7162 *RS Aquilae.*

(Cont. from 535, Comp. Stars 464)

Julian	Calendar	Mag.
	1903	
6371.6	Sept. 13 to	
6430.5	Nov. 11	12]
	5 dates	
6408.6	Oct. 20	10]P

7252 *W Capricorni.*

(Continued from 464)

Julian	Calendar	Mag.
	1901	
5609.6	Aug. 12	12]
	1903	
6376.6	Sept. 18 to	
6430.5	Nov. 11	12]
	4 dates	

7261 *R Delphini.*

(Continued from 535)

Julian	Calendar	Mag.
	1903	
6372.6	Sept. 14	11.6]
6391.5	Oct. 3	8.10_2
6401.5	13	8.69_2
6409.5	21	8.93_2
6413.5	25	8.99_2
6418.5	30	8.34_2
6426.5	Nov. 7	8.90_2
6431.5	12	8.53_2
6446.5	27	9.5
6462.5	Dec. 13	10.36_2

7268 *RT Capricorni.*

Julian	Calendar	Mag.
	1902	
6089	Dec. 5	8.3
	1903	
6376.6	Sept. 18	8.4
6384.5	26	8.1
6386.5	28	7.91_2 red
6387.5	29	7.55_2
6388.5	30	7.81_2
6391.5	Oct. 3	7.39_3
6401.5	13	8.43_2

7268 *RT Capric.* Cont.

Julian	Calendar	Mag.
	1903	
6408.5	Oct. 20	7.96_2
6413.5	25	7.75_2
6423.5	Nov. 4	8.00
6430.5	11	7.92_2
6446.5	27	7.8

7431 *S Delphini.*

(Continued from 425)

Julian	Calendar	Mag.
	1903	
6372.6	Sept. 14	11.4
6377.5	19	10.9
6383.5	25	9.40_2
6384.5	26	10.51_2
6402.5	Oct. 14	11.08_2
6437.5	Nov. 18	9.0

7435 *Y Aquarii.*

(Continued from 498)

Julian	Calendar	Mag.
	1901	
5632.6	Sept. 4	8.8
5633.5	5	9.09_2
5643.5	15	9.37_2
5663.5	Oct. 5	10.21_2
5671.5	13	10.5
	1902	
6085	Dec. 1	9.9]
6102	18	10.8]
6112	28	10.8]
	1903	
6361.6	Sept. 6	9.10_2
6376.6	18	8.91_2
6382.5	24	8.74_2
6388.5	30	9.22_2
6401.5	Oct. 13	8.65_2
6407.5	19	8.88_2
6408.5	20	9.32_2
6415.5	27	9.44_2
6426.5	Nov. 7	9.97_2

7444 *T Delphini.*

(Continued from 311)

Julian	Calendar	Mag.
	1903	
6372.6	Sept. 14	10.1
6377.5	19	9.6
6383.5	25	9.16_2
6384.5	26	9.50_2
6402.5	Oct. 14	10.88_2
6437.5	Nov. 18	10.5

7450 *V Aquarii.*

(Continued from 425)

Julian	Calendar	Mag.
	1900	
5304.5	Oct. 11	8.8P
	1902	
6093	Dec. 9	8.8
6102	18	8.8
6112	28	9.0
	1903	
6116	Jan. 1	9.1
6344.6	Aug. 17	8.9
6347.6	20	8.51
6348.6	21	8.73
6362.6	Sept. 4	8.7
6364.6	6	8.15_2
6376.6	18	8.68_2
6382.5	24	8.62_2
6388.5	30	8.36_2
6401.5	Oct. 13	8.43_2
6407.5	19	8.41_2
6408.5	20	8.64_2
6415.5	27	8.16_2
6426.5	Nov. 7	8.89_2

7468 *T Aquarii.*

(Continued from 464)

Julian	Calendar	Mag.
	1900	
5304.5	Oct. 11	7.4P
	1901	
5632.6	Sept. 4	8.7
	1902	
6093	Dec. 9	8.6
6102	18	8.0
6112	28	8.0
	1903	
6116	Jan. 1	8.1
6344.6	Aug. 17	8.5
6347.6	20	8.79
6362.6	Sept. 4	9.4
6364.6	6	9.4

7502 *X Delphini.*

Cont from 482, Comp. Stars 438

Julian	Calendar	Mag.
	1901	
5630.6	Sept. 2	11.0]
5643.6	15	9.6
5649.5	21	9.3
5661.5	Oct. 3	8.3
5663.5	5	8.3
5669.5	11	8.48_2

7502 *X Delphini.*—Cont.

Julian	Calendar	Mag.
	1901	
5671.5	Oct. 13	8.29_2
5677.5	19	8.15_2
	1902	
6085	Dec. 1 to	
6115	31	11.5]
	5 dates	

7560 *R Vulpeculae.*

(Continued from 431)

Julian	Calendar	Mag.
	1903	
6369.6	Sept. 11	10.4
6371.5	13	11.2
6371.5	13	11.23_2
6372.5	14	11.08_2
6380.5	22	12.11_2
6407.5	Oct. 19	13]
6430.5	Nov. 11	10.9
6431.5	12	10.5

7594 *RS Aquarii.*

Julian	Calendar	Mag.
	1902	
6114	Dec. 30	10]
	1903	
6347.6	Aug. 20	10.33
6348.6	21	10.55
6371.6	Sept. 13	9.5
6378.5	20	9.70_2
6384.5	26	9.99
6386.5	28	10.23
6387.5	29	9.35_2
6401.5	Oct. 13	9.8
6406.5	18	10.07_2
6408.5	20	9.79_2
6427.5	Nov. 8	11.2]

7619 *RR Aquarii.*

Julian	Calendar	Mag.
	1903	
6371.6	Sept. 13	8.9
6378.5	20	8.88_2
6386.5	28	8.79_2
6387.5	29	8.10_2
6401.5	Oct. 13	8.9
6406.5	18	8.58_2
6408.5	20	8.61_2
6427.5	Nov. 8	9.29_2
6456.5	Dec. 7	10.0

7659 *T Capricorni.*

(Continued from 498)

Julian	Calendar	Mag.
	1902	
6107	Dec. 23	8.1

8290 *R Pegasi.*

(Continued from 535)

Julian	Calendar	Mag.
	1903	
6428.5	Nov. 9	10.5
6429.5	10	10.55_2
6437.5	18	11.0P
6439.5	20	10.34_2
6456.5	Dec. 7	10.12_2
6472.5	23	10.9:
	1904	
6493.5	Jan. 13	10.5]

8369 *W Pegasi.*

Cont from 498, Comp Stars 498

Julian	Calendar	Mag.
	1903	
6372.6	Sept. 14	11.1
6382.5	24	11.10_2
6406.5	Oct. 18	11.4
6413.5	25	11.4
6428.5	Nov. 9	11.5
6456.5	Dec. 7	10.8
6472.5	23	10.4
	1904	
6493.5	Jan. 13	9.8

8373 *S Pegasi.*

Cont from 535. Comp. Stars 464)

Julian	Calendar	Mag.
	1903	
6406.5	Oct. 18	10.31_2
6428.5	Nov. 9	10.31_2
6437.6	18	9.5P
6439.5	20	9.90_2
6456.5	Dec. 7	9.90_2
6472.5	23	8.7
	1904	
6493.5	Jan. 13	7.8

8597 *V Ceti.*

(Continued from 464)

Julian	Calendar	Mag.
	1903	
6406.5	Oct. 18	11.5
6408.6	20	10.0]P
6428.5	Nov. 9	9.52_2
6437.6	18	9.6P
6455.5	Dec. 6	9.6
6456.5	7	9.6
6472.5	23	10.0]

Comparison-Stars, 1893–1904.

6452 *RY Herculis.*

Star	DM.	Mag.	n
T	+19°3489	9.24	25
W	+19°3484	9.77	15
X	+19°3483	9.99	6
a	2*n*9*f* *T*	10.54	9
b	5*n*2*f* *T*	10.92	7
c	4*p* *T*	11.07	5
e	2*n*2*f* *b*	11.5	0

7594 *RS Aquarii.*

Star	DM.	Mag.	n
I	−4°5382	8.0	0
W	−4°5386	9.87	8
X	−4°5384	9.93	7
Y	−4°5380	9.93	13
1*Y*	−4°5383	9.78	9
h	2*s*1*f* *X*	10.73	9
q	3*n* *I*	11.89	2

7619 *RR Aquarii.*

Star	DM.	Mag.	n
L	−3°5155	7.93	4
Q	−3°5162	8.7	0
U	−3°5156	9.40	7
X	−3°5158	9.54	2
1*X*	−3°5163	9.65	1
Y	−3°5157	9.8	0
1*Y*	−3°5161	9.77	1

7944 *T Pegasi.*

Star	DM.	Mag.	n
E	+11°4724	8.02	3
N	+11°4726	8.74	6
Q	+11°4736	8.48	28
S	+11°4735	9.13	33
Z	+11°4733	10.12	4
2*Z*	+11°4731	9.65	5
3*Z*	+11°4737	9.44	18

EPHEMERIS OF COMET a 1904 (*BROOKS*),

By EVERETT I. YOWELL.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

This ephemeris is a continuation of that published in *A.J.* 562. Observations made here give the following corrections for May 25: $\Delta\alpha = -5^s$, $\Delta\delta = +15''$.

G.M.T.	α	δ	log Δ	Br.
	h m s	° ′ ″		
June 1.5	13 52 13.3	+57 59 1	0.41797	0.67
2.5	48 28.5	54 27	0.42060	
3.5	44 49.1	49 17	0.42324	0.65
4.5	41 15.2	43 31	0.42589	
5.5	37 46.8	37 12	0.42855	0.64
6.5	34 24.0	30 22	0.43122	
7.5	31 6.9	23 3	0.43389	0.62
8.5	27 55.3	15 16	0.43657	
9.5	24 49.3	57 7 4	0.43926	0.60
10.5	21 49.0	56 58 27	0.44194	
11.5	18 54.2	49 29	0.44462	0.58
12.5	16 4.9	40 11	0.44730	
13.5	13 21.1	30 34	0.44998	0.56
14.5	13 10 42.6	+56 20 40	0.45266	
June 15.5	13 8 9.4	+56 10 31	0.45532	0.55
16.5	5 41.5	56 0 7	0.45798	
17.5	3 18.6	55 49 30	0.46064	0.53
18.5	13 1 0.8	38 42	0.46328	
19.5	12 58 47.9	27 43	0.46591	0.52
20.5	56 39.8	16 36	0.46753	
21.5	54 36.4	55 5 20	0.47014	0.51
22.5	52 37.6	54 53 57	0.47374	
23.5	50 43.2	42 28	0.47632	0.49
24.5	48 53.2	30 55	0.47889	
25.5	47 7.4	19 17	0.48144	0.48
26.5	45 25.8	54 7 35	0.48397	
27.5	43 48.2	53 55 51	0.48649	0.46
28.5	42 14.5	44 5	0.48899	
29.5	40 44.6	32 18	0.49147	0.45
30.5	12 39 18.3	+53 20 30	0.49394	

In *A.J.* 562, the α for May 22.5 should read 34^m instead of 32^m.

OBSERVATIONS OF 9.1904 *ORIONIS*,

By ZACCHEUS DANIEL.

In *A.N.* 3935, Dr. WILHELM LUTHER announced that the star *Hagen* 41, 4^s following 2100 *U Orionis*, varies about one magnitude. It is q, $10^M.18$, in the Harvard sequence of comparison stars for that variable.

This star was observed here as a comparison star of *U Orionis* in connection with the work done for the Carnegie Institution. Fourteen observations were obtained with PICKERING's equalizing wedge photometer attached to the 58.4 cm. (23-inch) refractor. Two of these observations, marked R, were made by Professor W. M. REED. Another comparison star used was *Harvard* p (*Hagen* 21 = DM. +20°1179). Two settings were made on each comparison star before the variable was observed and the same number after; the stars being observed in reverse order. The agreement between the two sets was usually good. On 1903 Sept. 18, three settings of those made on p and six on q were used. The second series on p was rejected for discordance. The observation on 1904 Feb. 15 was stopped by clouds before the second series could be made. The magnitudes are based on the Harvard magnitude, 9.66, for p.

Princeton University, Princeton, N.J., 1904 *May* 25.

These observations indicate a variation of $0^M.74$, but they do not agree with the period of 41.4 days suggested by LUTHER. Ten other periods were tried, but none was found that would satisfy the observations.

A photometric observation made by Dr. E. JOST, of Gotha, on 1903 Jan. 17, is taken from *A.N.* 3909 and added here for comparison.

G.M.T.	J.D.	$q-p$	Mag.	
1903 Jan. 17	6132.226	+0.37	10.03	Jost)
Sept. 18	376.854	0.34	10.00	fair
Oct. 14	402.890	0.31	9.97R	fair
19	407.746	0.74	10.40	fair
Nov. 6	425.658	0.43	10.09	fair
18	437.830	0.46	10.12	fair
26	445.890	0.30	9.96	fair
Dec. 7	456.699	0.80	10.46	good
11	460.828	0.53	10.19	fair
15	464.828	0.67	10.33R	good
1904 Jan. 15	495.651	0.28	9.94	good
27	507.628	0.94	10.60	fair
Feb. 9	520.678	1.02	10.68	good
15	526.661	0.76	10.42	(clouds
Mar. 4	6544.590	+0.34	10.00	fair

ECLIPSE OF SATELLITE II OF *JUPITER* BY SATELLITE III,

By CHARLES P. OLIVIER.

While observing *Jupiter* on the night of 1902 August 22, with the 26-inch telescope of this observatory, two of the satellites began to near each other rapidly and it appeared that an eclipse was about to occur. The following observations were made, Mean Local Time being used:

	h m s	
Fist contact	12 59 7	Power 560
Appeared round	13 1 27	" 850
Nearly separated	13 6 22	" 850
Last contact	13 7 0*	" 850
Wide space between	13 7 36	" 850

* Last contact observed about 8s late.

The second and third contacts could not be observed from the unsteadiness of the disks, which were very indistinct in outline. Also from the two satellites being nearly the same color and not greatly different in brightness. The eclipse lasted, making correction of 8s to last contact, $7^m 15^s$. All the observations were made very difficult from the seeing, which was only fair, and the small disks of the satellites. Satellite III had eclipsed II.

Leander McCormick Observatory, University of Virginia

OBSERVATIONS OF 49.1903 *ORIONIS*,

By ZACCHEUS DANIEL.

The variability of 49.1903 *Orionis* was discovered by Dr. MAX WOLF and announced by him in *A.N.* 3899. He has also given a chart of the region in *A.N.* 3935. This star is the brightest of all the variables recently discovered in the *Orion* Nebula. Its position is

$$\alpha = 5^h\ 36^m\ 36^s.0 \quad ; \quad \delta = -1^\circ\ 11'\ 17''\ (1900)$$

Through the kindness of Dr. WOLF, I obtained an advance copy of his chart and on March 9 began to observe the variable. It was then about four magnitudes fainter than S.DM. $-4^\circ 1216$, $9^M.5$, but on April 4 and 5 was almost equal to that star, thus confirming the variability beyond doubt. On the last two dates the variable appeared slightly red.

Following are the differences in magnitude between the variable and three stars near it as observed with the photometer attached to the 58.4 cm. refractor. They depend on four settings on each star. The star a is S.DM. $-4^\circ 1216$ and c is the star adjacent to the variable and just north of it on WOLF's chart.

G.M.T.	$v-a$	$v-b$	$v-c$
1904 March 9.6	+4.13	+0.92	−0.20
16.6	+3.26	+0.01	−0.95
April 4.6	+0.12	−2.80	−4.26
5.6	+0.14	−2.75	. .

Princeton, New Jersey, 1904 *April* 11.

NOTE ON A PROBABLY NEW MINOR PLANET.

[From a letter of Rear-Admiral C. M. CHESTER, Superintendent Naval Observatory.]

An asteroid of the 11.7 magnitude, whose position is not given in the *Berliner Jahrbuch*, was photographed by Mr. G. H. PETERS, on May 11, with the 6-inch camera.

The following micrometer observation was made by Mr. W. W. DINWIDDIE, with the 26-inch equatorial, on May 12:

1904 Wash. M.T.	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Place	
May $12^d\ 14^h\ 4^m\ 49^s$	t30.6	$-1^m\ 3^s.56$	$+2'\ 31''.6$	$15^h\ 55^m\ 40^s.15$	$-19^\circ\ 8'\ 30''.5$	9.275	0.861	$+2^s.58$	$+0''.7$

Mean Place of Comparison-Star for the beginning of the year.

α	δ	Authority
$15^h\ 56^m\ 41^s.13$	$-19^\circ\ 11'\ 2''.8$	Cin. Zone Catal. 2690

The daily motion is $-49^s.5$ and $-1''.2$.

Washington, 1904 *May* 14.

CONTENTS.

PUBLISHED AT 10 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass *Closed May* 31.
Entered February 2 1903, *at Boston, Mass., as second-class matter, under Act of Congress of March* 3, 1879

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 564.

VOL. XXIV. BOSTON, 1904 JUNE 20. NO. 12

OBSERVATIONS ON THE COLOR OF CERTAIN VARIABLE STARS.

By PAUL S. YENDELL.

The observations whose results are here detailed were undertaken for the purpose of extending CHANDLER's examination of the colors of the variable stars (*A.J.*, VIII, 137) to the more recently discovered variables.

I had at the time a screen-glass on hand which nearly answered the description of the one used by CHANDLER, and in the year 1891 made a few observations of two or three stars with it. Its absorption proved so slight that it was soon discarded, and a screen of a full blue color, in the form of a double eyeglass, was substituted. The use of this form of screen made it convenient to extend the comparisons to the stars observed with the binocular and with the naked eye. An examination of the spectrum of the light transmitted by this glass showed a large absorption throughout the whole of the red and yellow.

Using this screen, observations were begun in 1892, and carried on as opportunity offered, pretty continuously until 1897. My list included about a hundred and twenty variable and suspected stars, among which were a few of ESPIN's red stars, and of THOME's suspected southern stars. These were observed from one to twenty times each, the average number of comparisons for each star being about four. From the nature of the case, it was unavoidable that I was able to secure but one observation each of many of the stars. These are given with the rest in the final table for what they are worth, the number of comparisons going to make up each estimate being stated for every star.

In April, 1893, a preliminary examination was made of the observations on thirty-five stars, among which were a number of those contained in CHANDLER's list in the paper above referred to, which were observed for the purpose of establishing the relation between CHANDLER's decimal scale and the absorptions shown by my screen. As deduced from these observations, this relation was apparently simple. Color estimates of a number of the newer variables, reduced on this basis, were furnished in 1896 for CHANDLER's Second Catalogue.

The pressure of other work interrupted these observations in 1894, but they were carried on in a rather desultory manner until 1901, when my observatory was broken into, and among other things, my screen was stolen. This brought the series of observations to a close.

The subject having recently been recalled to my attention, I have undertaken a final reduction of the observations. These proved to have accumulated to the number of four hundred and fifty-two comparisons of one hundred and seventeen stars, making an average of nearly four to each star, although, as remarked above, there are many among them of which there was but one comparison each. Fifty-one of the CHANDLER stars were in the Second Catalogue, and were observed with the purpose of establishing a relation between my absorption step-values and CHANDLER's decimal scale, to which scale I proposed to reduce my results. From the observations of these CHANDLER stars fourteen normals were formed, which are shown in the following table. The 0 and 10 points are assumed to coincide. Unfortunately, nearly all CHANDLER's values up to 1.0, as also the group of 4's, which proved entirely discordant, were eye-estimates, stated only to the unit, and had of necessity to be disregarded in drawing the curve of the relation of my absorption step-values to the decimal scale. The normals formed from these values are bracketed in the table. The column C contains the values on the CHANDLER scale, D my absorption step-values, *r* the probable error of each mean in the column D, *n* the number of stars constituting each mean, and *d* its departure from the curve as drawn.

C	D	*r*	*n*	*d*
(0)	0.14	. .	5	−0.14
(1)	1.48	. .	5	+ .33
1.25	2.72	0.3	4	−1.30
2.04	2.48	0.3	5	−0.14
2.65	2.75	. .	2	+0.05
3.36	3.06	0.6	5	−0.13
(4)	2.48	. .	5	1.83
5.04	3.85	0.2	3	−0.27
5.9	3.80	0.7	3	+0.75
6.47	4.25	0.4	3	+0.38
7.15	5.52	. .	2	−0.60
8.97	7.73	0.2	3	−0.20
9.4	7.6	. .	2	0.00
9.4	8.47	. .	2	0.10

The curve drawn from these normals is rather interesting, its curvature being reversed at about 1 of my absorption-scale, and showing that my screen absorbed rather less of the yellow rays, and more of the red, than did CHANDLER'S.

I had hoped to have been able to show an approximate relation between the decimal color-scale and that of MUELLER and KEMPF used in the Potsdam Photometric Catalogue, but owing to their exclusion of known variables from their work only a single star was found common to all three lists, and the idea had to be given up.

The scale used in the reductions is shown in the following table. The column Y contains my absorption step-values, which correspond to the values on CHANDLER'S decimal scale shown in the column C, according to the curve drawn from the means shown in the former table:

Y	C	Y	C	Y	C
0.0	0.0	3.5	4.15	7.0	8.77
0.5	0.30	3.75	5.00	7.5	9.05
1.0	0.70	4.0	5.76	8.0	9.30
1.5	1.12	4.5	6.62	8.5	9.51
2.0	1.60	5.0	7.27	9.0	9.73
2.5	2.20	5.5	7.75	9.5	9.88
3.0	3.00	6.0	8.15	10.0	10.00
3.25	3.58	6.5	8.50		

The subjoined table contains a list of the stars, with their colors expressed in the above scale. The first and second columns give the numbers and names of the stars, the third, Y, their colors, as deduced from my observations; the fourth, r, the probable error of each of my values which depends on three or more observations; the fifth, Obs., the number of comparisons from which each of my color-values was deduced; and the sixth, C, and seventh, Obs., the corresponding colors and numbers, from CHANDLER'S paper, first above referred to.

		Y	r	Obs.	C	Obs.
103	T Andromedae	5.2	0.9	4	. .	. .
107	T Cassiopeae	6.8	0.3	6	7.3	6
112	R Andromedae	1.1	. .	1	5.0	12
243	U Cassiopeae	1.8	1.2	3	6	. .
432	S Cassiopeae	5.9	0.5	5	6.7	3
678	U Persei	5.2	0.7	5	. .	. .
782	R Arietis	0.0	. .	1	1.8	1
793	T Persei	7.3	. .	1	4	. .
806	o Ceti	1.6	. .	1	5.9	14
906	R Trianguli	3.9	0.1	11	. .	. .
980	V Persei	4.9	0.6	8	. .	. .
	α Tauri	8.2	. .	1	. .	. .
1279	U Camelopardalis	8.4	0.5	3	. .	. .
1623	T Camelopardalis	3.4	0.9	6	. .	. .
1717	V Tauri	7.7	. .	2	3.3	9
1771	R Leporis	9.5	0.1	4	9.4	13
1805	V Orionis	3.6	. .	2	. .	. .
1984	S Camelopardalis	8.0	0.3	8	. .	. .
2013	U Aurigae	8.0	0.1	9	. .	. .
2098	α Orionis	9.3	. .	1	. .	. .
2100	U Orionis	6 4	0.6	3	7	. .
2509	ζ Geminorum	0.0	. .	1	. .	. .
2528	R Geminorum	5.8	0.7	6	5.7	5
2539	R Canis minoris	0.0	. .	1	5.5	13
(2574)	— Geminorum	0.3	. .	1	. .	. .
2676	U Monocerotis	1.0	0.4	6	3	. .
2857	U Puppis	4.9	1.1	3	3.2	4
3170	S Hydrae	4.1	. .	2	2.1	12
3186	T Cancri	8.2	. .	2	7.4	17
3493	R Leonis	7.6	0.1	7	6.9	18
3825	R Ursae Majoris	3.7	. .	2	1.6	13
3881	V Hydrae	9.2	0.1	8	9	. .
3890	W Leonis	1.1	. .	1	. .	. .
3934	R Crateris	8.7	. .	2	8.1	13
4300	X Virginis	5.8	. .	1	. .	. .
4511	T Ursae Majoris	3.3	0.8	5	2.0	18
4521	R Virginis	0.7	. .	1	1.3	11
4557	S Ursae Majoris	3.1	1.1	6	3.2	19
4816	V Virginis	4.9	1.1	3	2.7	7
4826	R Hydrae	7.3	. .	1	5.9	6
4847	S Virginis	2.0	0.7	9	2.6	17
4940	W Hydrae	8.7	0.3	5	7	. .
(5159)	— Bootis	7.3	. .	1	. .	. .
5174	RS Virginis	0.3	. .	2	. .	. .
5190	R Camelopardalis	3.0	. .	1	2.1	6
5438	Y Librae	1.6	. .	1	. .	. .
5504	S Coronae	5.5	0.8	3	4.9	3
5675	V Coronae	5.8	. .	1	5.9	11
5758	X Herculis	3.3	0.4	12	7	. .
5889	U Herculis	7.9	. .	2	6.5	4
6062	RR Scorpii	1.2	. .	2	. .	. .
6225	RS Herculis	5.8	. .	1	. .	. .
6368	X Sagittarii	1.2	0.1	3	1	. .
6472	W Sagittarii	0.6	0.1	7	1	. .
6512	T Herculis	2.3	. .	2	1.4	12
6549	W Lyrae	1.6	. .	1	. .	. .
6573	Y Sagittarii	0.0	0.0	3	0	. .
6633	V Sagittarii	2.2	. .	2	0.6	7
6636	U Sagittarii	4.6	0.3	12	3.7	4
6653	T Lyrae	8.9	1.3	4	. .	. .
6682	X Ophiuchi	1.6	. .	1	. .	. .
(6716)	— Lyrae	7.3	1.3	3	. .	. .
6733	R Scuti	3.7	0.6	9	4	. .
6749	S Scuti	7.4	1.0	3	. .	. .
6758	β Lyrae	0.6	0.3	5	1	. .
6794	R Lyrae	1.6	0.1	13	4	. .
6834	V Aquilae	7.1	0.5	11	.	. .
6849	R Aquilae	3.0	. .	1	5.5	8
6905	R Sagittarii	1.8	. .	2	3.6	3
6943	T Sagittae	6.6	. .	1	. .	. .
6984	U Aquilae	0.0	. .	1	0	. .
7085	RT Cygni	4.7	1.2	4	. .	. .
7118	X Aquilae	3.1	. .	2	. .	. .
7124	η Aquilae	0.7	. .	1	2	. .
7149	S Sagittae	0.6	0.1	4	0	. .
7192	Z Cygni	8 1	1.0	5	7	. .
7247	RX Cygni	0.0	. .	2	. .	. .
7257	R Sagittae	0.5	. .	2	0.8	11
7259	RS Cygni	10.2	0.1	22	7	. .
7261	R Delphini	0.7	. .	1	4.0	3

		Y	r	Obs.	C	Obs.
7299	U Cygni	8.4	0.4	3	9.3	15
7428	V Cygni	9.0	. .	1	8.3	10
7437	X Cygni	0.3	. .	2	0	. .
7450	V Aquarii	0.2	. .	2	. .	. .
7456	RR Cygni	6.0	0.6	3	6	. .
7459	T Cygni	1.3	0.1	7	1	. .
7483	T Vulpeculae	0.0	. .	1	0	. .
7560	R Vulpeculae	8.4	. .	1	2.0	7
7751	W Cygni	4.4	0.5	12	5	. .
7779	S Cephei	9.0	. .	1	9.4	16
7783	RU Cygni	6.2	0.5	11	. .	. .
7795	RV Cygni	9.7	0.1	18	. .	. .
7803	μ Cephei	6.8	0.2	11	. .	. .
8005	S Draconis	7.4	0.3	5	. .	. .
8073	δ Cephei	1.5	0.2	8	2	. .
8116	W Cephei	3.4	0.5	10	. .	. .
8153	R Lacertae	0.0	. .	1	1.8	8
8230	S Aquarii	0.0	. .	1	4.0	1
8321	V Cassiopeae	1.8	0.6	1	2	. .
8373	S Pegasi	3.0	. .	2	1.7	13
8591	V Cephei	1.1	0.3	5	. .	. .
	Espin-Bir. 673(1)	7.0	0.5	7	. .	. .
	Birm. 658	9.0	. .	2	. .	. .
	Birm. 558	1.6	. .	1	. .	. .
	Birm. 475	6.6	. .	2	. .	. .
	DM. +38°2412	5.8	. .	1	. .	. .
	DM. +33°4056(2)	1.6	. .	1	. .	. .
	DM. +17°1973(3)	7.3	. .	1	. .	. .
	S.DM. −23°2771(4)	5.8	. .	2	. .	. .
	S.DM. −22°1777	0.7	. .	1	. .	. .
	S.DM. −22°1812	0.0	. .	1	. .	. .
	S.DM. −22°1865	3.9	0.4	4	. .	. .

(1) The Espin-Birmingham and Birmingham stars are among those that have at one time or another been on my list for suspected variability.

(2) DM. +33°4056 is my comparison-star c for *Y Cygni*, which I at one time suspected of variability.

(3) = Potsdam Photometric DM, Vol. 9, 1903. R.

(4) The Southern DM. numbers are those of some of the stars whose variability was suspected by THOME, in the course of his work on the Cordoba DM.

Fifty-four of my own determinations are deduced from three or more observations each. Assembling these, the mean error of a single observation is found to be 1.9.

Comparing my own determinations with CHANDLER's, I find twelve stars common to both lists of which I have more than two observations each. Their differences are tabulated below, where the column Y−C contains the differences between the respective estimates of each star, Col. their colors by my observations, r the probable error of my determination in each case, and Obs. the number of my observations of each star.

Col.		Y−C	r	Obs.
2.0	4847 S Virginis	−0.6	0.7	9
3.3	4511 T Ursae Maj.	+1.3	0.8	5
4.6	6636 U Sagittarii	+0.9	0.3	12
4.9	4816 V Virginis	+2.2	1.1	3
4.9	2857 U Puppis	+1.7	1.1	3
5.5	5501 S Coronae	+0.6	0.8	3
5.8	107 T Cassiopeae	−1.5	0.3	6
5.8	2528 R Geminorum	+0.1	0.7	6
5.9	432 S Cassiopeae	−0.8	0.5	5
7.6	3493 R Leonis	+0.7	0.1	7
8.4	7299 U Cygni	−0.9	0.4	3
9.5	1771 R Leporis	+0.1	0.1	4

The correspondence is perhaps as satisfactory as was to be expected from the limited number of my observations, though the differences mostly exceed the values of my computed probable errors. It will be noticed that both are greatest in that part of the scale, from 3.0 to 6.0, where the step of my absorption-scale corresponds to the greatest value on CHANDLER's scale. This seems to indicate that if expressed in their step-values, these probable errors would be more nearly uniform, and that their somewhat excessive values in this part of the scale are dependent on the relation between my absorption step-values and the decimal scale, as shown in the second table.

An examination of my results in the cases of fifty-three stars available for the purpose is fairly confirmatory of CHANDLER's statement of the relation between coloration and length of period, in his paper, "On the General Relations of Variable Star Phenomena," published in this *Journal*, Vol. IX, p. 1.

I have divided these fifty-three stars into three classes, according to their types of variation. There are eight short-period stars of the *η Aquilae* and *β Lyrae* types, seven which may be called "intermediate," with periods of from forty-six to one hundred and sixty-five days, and thirty-eight which are distinctly of the long-period type.

The eight short-period stars are

6573 *Y Sagittarii*	7149 *S Sagittae*
7437 *X Cygni*	6368 *X Sagittarii*
6472 *W Sagittarii*	8073 *δ Cephei*
6758 *β Lyrae*	6636 *U Sagittarii*

Of these, the first seven, excluding *U Sagittarii*, whose color is exceptional and anomalous in this type, show a mean period of 8d.3, and a mean coloration of 0.8. There is no suggestion of any relation between color and length of period among these stars.

The seven "intermediate" stars, arranged in the order of their places in the color scale are

Color		Period
0.5	7257 *R Sagittae*	70.6
1.0	2676 *U Monocerotis*	46.1
1.6	6791 *R Lyrae*	46.4
3.3	5758 *X Herculis*	95.5
3.7	6733 *R Scuti*	71.1
4.4	7751 *W Cygni*	131.5
6.0	7456 *RR Cygni*	165.2

These stars, belonging neither to the distinctly long-period or short-period types, but having some of the characteristics of both, show a marked progression in the lengths of their periods, corresponding to that in their observed colors. Taken by themselves, they are too few for their evidence to be decisive as to the reality of the connection between the two, but considered side by side with the results deduced from the long-period stars, this progression is at least interesting.

The thirty-eight long-period stars, together with five irregular stars, in the order of their places in the color-scale, with their periods, are as follows:

Col.			Obs.	Per.
0.2	7450	V Aquarii	2	240
0.3	5174	RS Virginis	2	355
1.1	8591	V Cephei	5	360
1.2	6062	RR Scorpii	2	282
1.8	243	U Cassiopeae	3	276
1.8	8324	V Cassiopeae	4	231.5
2.0	4847	S Virginis	9	376.9
2.3	6512	T Herculis	2	165
3.0	8373	S Pegasi	2	317.5
3.1	4557	S Ursae Majoris	6	226.5
3.4	7118	X Aquilae	2	348
3.3	4511	T Ursae Majoris	5	257.2
3.4	1623	T Camelopardalis	6	370
3.6	1805	V Orionis	2	267
3.7	3825	R Ursae Majoris	2	302.1
3.9	906	R Trianguli	11	268
4.1	3170	S Hydrae	2	256
4.7	7085	RT Cygni	4	190.5
4.9	2857	U Puppis	3	315
4.9	4816	V Virginis	3	250.5
5.2	103	T Andromedae	4	265.4
5.2	678	U Persei	5	318

Col.			Obs.	Per.
5.5	5504	S Coronae	3	361.2
5.8	2528	R Geminorum	6	370.2
5.9	432	S Cassiopeae	5	610.5
6.2	7783	RU Cygni	11	396
6.4	2400	U Orionis	3	375
6.8	107	T Cassiopeae	6	445
6.8	7803	μ Cephei	11	irreg.
7.1	6834	V Aquilae	11	irreg.
7.6	3493	R Leonis	7	312.8
7.7:	1717	V Tauri	2	170
7.9	5889	U Herculis	2	411.1
8.0	1981	S Camelopardalis	8	328
8.0	2013	U Aurigae	9	405.5
8.1	7192	Z Cygni	5	265
8.2	3186	T Cancri	2	482
8.4	1279	U Camelopardalis	3	irreg.
8.7	4940	W Hydrae	5	384
9.2	3881	V Hydrae	8	long or ir.
9.5	1771	R Leporis	4	436.1
9.7	7795	RV Cygni	18	425
10.2	7259	RS Cygni	22	irreg.

Assembling these in the order of their observed colors, in groups of units, 0–1, 1–2, and so on, and forming means of the color-estimates and periods of each group, we have

MEANS.

Color	Periods	Stars	Color	Periods	Stars
0.3	298	2	5.5	392	5
1.5	294	4	6.4	408	3
2.5	338	2	7.7	305	3
3.5	286	8	8.2	364	5
4.7	248	4	9.7	427	2

Dorchester, 1904 *May* 23.

The correspondence shown here between depth of color and length of period is so marked as to point strongly to some real connection between the two, especially when taken in comparison with CHANDLER's table on p. 2 of his paper last above referred to, and is strongly confirmatory of his summing up in the former one (*A.J.*, VIII, p. 140), "The redder the tint, the longer the period." It is worthy of notice also, that from 6.8 of the color-scale upward, the stars of irregular variation become relatively numerous, the reddest star on my list, 7259 *RS Cygni*, being of this type of variation.

DECLINATIONS OF CERTAIN CIRCUMPOLAR STARS.

BY HARRIET W. BIGELOW.

Observations of the stars whose declinations are here given were requested by Dr. AUWERS in *A.N.* 3440. Both right-ascensions and declinations of these stars were observed by me during 1901–1903 with the Walker Meridian Circle of the Detroit Observatory. The right-ascensions have not yet been reduced.

A short description of the instrument, which was built by Pistor and Martins, in 1854, may be found in *A.J.* 518. The circle seems to be divided quite accurately. In each microscope there are now two pairs of threads placed one-and-a-half revolutions apart for the purpose of eliminating periodic error. The magnifying power is about 16. Settings were made with the tangent screw, as there is no zenith-distance micrometer.

An observation of a star usually consisted of three settings, symmetrical with reference to meridian transit. The plan was to obtain for each star, both at upper and lower culmination, two observations in each of the four following positions: — clamp west, direct; clamp west, reflected; clamp east, direct; clamp east, reflected. The complete carrying out of the plan was in part prevented by the difficulty in obtaining reflected observations of the fainter stars. One, *Camelopardalis s* 664, was at first accidentally omitted. Each night's observing list included one or more of the fundamental circumpolar stars of the *Berliner Jahrbuch*. Observations for nadir were taken about every three hours.

The large coefficient of cosine flexure (1″.694) is well determined. It was obtained by comparing the corresponding observations for clamp west and clamp east. The sine flexure is apparently small (0″.117). In the case of clamp west the circle readings increase from the zenith toward the north and the formula for flexure correction would be

$$\zeta = z + 0''.162 - 1''.694 \cos z + 0''.117 \sin z$$

Two tables of the declinations are given, the differential and the absolute places. In the first case the differential flexure corrections were applied. In the second table the observed declinations are given without being corrected in the separate positions for flexure. The latitude employed is that obtained from the twenty-six stars that were observed in all eight positions. From the nine zero stars was found, 42° 16′ 48″.81; from the seventeen others, 48″.74. (See also *A.J.* 518.) The value from the zero-stars was weighted one-half, giving for the latitude: —

$$42^\circ\ 16'\ 48''.76$$

For the stars not observed in all eight positions the places were corrected for flexure and combined with arbitrary weights as follows: —

Cephei 117 Hs. ¼(W.D. + 2 W.R. + E.D.) for declination above pole, combined with equal weight with position below pole.

Cephei 149 Hs. ¼(2 W.D. + W.R. + E.R.) for declination above pole, and then treated like preceding star.

Camelop. s 664. Mean of four positions above pole.

Urs. min. 33 Hs. ½(W.D. + E.D.) for position below pole, combined with half weight with observations above pole.

ε *Urs. min.* W.D. below pole combined with ¼ weight with the mean of the remaining observations.

Cephei Br. 256 taken by itself gives the latitude 49″.55.

There seems, however, no good reason for rejecting any of the observations.

I. Declinations for 1900.0 from Comparison with Zero Stars

Name of Star	Mag.	R.A.	δ	Above Pole W.D.	Above Pole W.R.	Above Pole E.D.	Above Pole E.R.	Below Pole W.D.	Below Pole W.R.	Below Pole E.D.	Below Pole E.R.	Mean
		h m s	° ′									
Cephei Br. 256	6.9	2 1 25	83 5	29.21	29.91	29.01	30.32	31.44	30.34	30.98	30.51	30.22
Cephei 117 Hs.	5.9	3 8 35	84 33	26.91	26.67	26.51	. .	26.34	26.08	27.05	26.55	26.79
Cephei 149 Hs.	5.9	3 33 55	86 19	56.82	56.40	. .	56.84	56.37	57.06	57.48	57.04	56.86
Cephei 157 Hs.	6.3	4 56 18	85 49	16.98	16.40	16.31	16.01	16.20	16.21	16.92	16.83	16.48
Cephei 158 Hs.	6.3	5 29 55	85 8	50.85	49.67	50.51	49.91	50.00	50.47	50.54	49.79	50.19
Cephei 109 Hs.	6.2	7 53 2	84 20	49.92	48.78	49.61	50.47	49.78	50.04	49.46	48.47	49.57
Urs. min. 4 B.	7.2	7 58 3	88 55	59.29	59.60	59.07	60.01	58.92	58.61	58.81	58.57	59.11
Cephei 121 Hs.	6.3	8 54 32	84 31	58.44	59.24	58.06	58.84	59.21	58.45	58.31	57.22	58.40
Camelop. s 664	7.1	11 2 30	86 10	58.33	58.09	57.78	57.79	. .	. .	. .	. .	58.00
Urs. min. 3 Hs.	6.2	12 11 23	88 15	16.00	15.57	14.93	14.93	13.90	14.15	14.89	14.42	14.85
32 H. Camelop. pr.	6.3	12 48 16	83 57	11.45	11.27	10.72	11.67	11.44	11.58	11.48	10.60	11.28
32 H. Camelop. seq.	5.5	12 48 23	83 57	22.84	23.43	22.66	23.70	23.94	24.44	24.[illegible]8	23.31	23.59
Cephei 135 Hs.	6.1	13 45 10	83 15	14.90	15.34	14.79	14.85	16.26	14.72	15.67	14.63	15.14
Urs. min. 57 B.	7.1	15 9 24	87 37	3.78	3.45	4.44	4.92	3.37	3.57	3.94	3.44	3.86
Urs. min. 33 Hs.	7.5	15 53 47	83 14	57.94	58.08	57.94	58.37	58.24	. .	57.51	. .	57.98
Cephei 3 Hs.	7.0	20 13 59	84 22	38.00	38.38	38.32	39.21	38.53	37.64	37.93	37.83	38.23
Cephei Gr. 3518	7.3	21 19 35	86 37	24.01	23.68	24.19	24.81	24.90	23.17	24.76	24.78	24.28
32 H. Cephei	5.3	22 21 18	85 36	16.54	17.28	17.04	18.40	17.91	17.30	17.22	17.14	17.32
36 H. Cephei	5.7	22 55 13	83 18	39.73	40.25	39.80	41.34	40.70	39.40	40.42	39.62	40.12
39 H. Cephei	5.9	23 27 49	86 45	21.44	20.83	20.54	21.69	20.58	21.50	20.67	20.65	20.99
Cephei 125 Hs.	6.3	23 51 46	82 38	4.32	3.42	3.70	3.97	5.45	2.78	3.72	3.96	3.88

II. Absolute Declinations for 1900.0.

Name of Star	δ	Above Pole W.D. Obs. δ	Above Pole W.R. Obs. δ	Above Pole E.D. Obs. δ	Above Pole E.R. Obs. δ	Below Pole W.D. Obs. δ	Below Pole W.R. Obs. δ	Below Pole E.D. Obs. δ	Below Pole E.R. Obs. δ	Mean
	° ′	″	″	″	″	″	″	″	″	″
43 H. Cephei	85 43	1 17.68	2 15.22	1 10.97	3 11.37	2 12.82	2 14.16	3 16.59	2 11.57	14.55
Polaris	88 46	8 29.34	5 26.11	5 21.35	3 26.48	9 21.28	5 27.21	8 29.18	7 26.15	26.64
Cephei Br. 256	83 5	3 32.42	5 29.82	2 26.87	3 30.30	2 29.66	2 31.02	3 33.05	2 30.41	30.44
Cephei 117 Hs.	84 33	2 29.59	1 27.08	3 24.10	. . .	2 24.34	3 26.63	3 29.55	1 26.03	[26.87]
Cephei 149 Hs.	86 19	2 59.51	2 55.96	. . .	2 56.91	3 53.92	2 57.65	3 60.76	2 56.23	[56.87]
Gr. 750	85 17	2 31.78	4 28.27	3 26.12	1 29.03	4 26.60	1 28.43	4 31.75	4 27.82	28.72
Cephei 157 Hs.	85 19	2 49.48	2 45.74	3 43.40	2 45.98	3 43.88	3 46.61	2 49.08	2 46.56	46.34
Cephei 158 Hs.	85 8	3 52.73	2 48.75	2 47.40	1 50.39	3 47.36	2 50.76	2 52.96	2 48.97	49.92
51 H. Cephei	87 12	2 23.61	1 19.82	3 17.24	6 21.42	3 18.32	3 20.40	2 23.32	2 20.42	20.57
Cephei 109 Hs.	81 20	2 53.02	2 48.33	2 46.88	2 50.95	2 47.45	1 50.03	2 52.48	1 48.90	49.76
Urs. min. 4 B.	88 55	2 62.28	2 58.90	5 56.36	2 59.82	3 56.36	2 60.31	1 61.61	2 58.70	59.29
Cephei 121 Hs.	84 34	2 61.38	3 58.55	3 55.10	3 59.40	3 56.01	4 58.43	4 60.11	1 57.65	58.33
1 H. Draconis	81 46	1 9.88	2 6.89	2 3.98	5 7.18	2 4.81	2 6.79	2 9.92	1 6.36	6.98
30 H. Camelop.	83 3	2 64.86	3 62.20	2 59.99	2 62.89	2 61.36	3 63.09	3 65.13	2 62.96	62.80
Camelop. s 664	86 10	2 60.04	2 58.32	2 51.66	3 58.09	. . .	. . .	. . .	. . .	[57.78]
Urs. min. 3 Hs.	88 15	3 18.26	2 14.66	4 11.91	3 15.49	3 11.31	2 14.77	3 17.62	6 13.86	14.74
32 H. Camel. pr.	83 57	3 43.53	2 40.82	3 38.06	3 41.97	2 38.20	2 41.75	2 43.12	2 40.52	41.03
32 H. Camel. seq.	83 57	3 25.07	2 22.54	3 20.31	4 24.03	3 20.78	2 24.76	1 26.20	1 22.81	23.31
Cephei 135 Hs.	83 15	3 17.56	2 14.96	3 12.30	2 15.18	3 12.86	3 15.07	4 17.31	1 14.70	14.99
Urs. min. 57 B.	87 37	3 5.97	2 2.75	3 0.73	2 4.60	3 0.67	1 4.17	2 5.19	1 3.55	3.49
Urs. min. 33 Hs.	83 14	2 60.43	2 57.48	3 54.17	1 58.50	2 55.68	. . .	3 59.68	. . .	[57.62]
ε *Urs. min.*	82 12	2 9.66	2 7.89	2 3.86	3 8.05	2 6.06	. . .	. . .	. . .	[7.63]
δ *Urs. min.*	86 36	4 50.76	2 47.63	3 44.87	2 48.35	1 46.68	2 48.73	4 50.27	3 47.36	48.08
λ *Urs. min.*	88 59	4 19.11	3 16.45	2 12.78	1 16.02	5 13.02	1 16.30	4 18.44	4 15.73	15.98
Cephei 3 Hs.	84 22	2 40.78	3 38.69	2 35.02	1 38.53	2 35.80	2 37.91	2 40.67	2 37.42	38.10
76 Draconis	82 9	3 42.54	2 39.87	2 36.87	2 41.99	2 37.93	2 40.23	2 42.91	2 40.53	40.36
Cephei Gr. 3548	86 37	2 27.92	2 24.47	3 21.80	3 24.98	2 22.02	2 24.21	3 27.53	3 24.20	24.64
32 H. Cephei	85 36	2 19.67	2 17.43	3 13.94	2 18.27	3 14.99	2 17.38	3 19.89	3 16.72	17.29
36 H. Cephei	83 48	2 42.45	5 40.30	2 37.40	4 40.72	3 37.57	2 40.49	3 41.92	2 39.41	40.04
39 H. Cephei	86 45	3 24.15	3 21.04	2 18.21	5 21.68	3 18.15	2 21.55	3 23.30	4 20.86	21.12
Cephei 125 Hs.	82 38	3 6.83	3 3.41	4 1.19	3 4.92	3 2.72	3 3.94	5 6.21	2 3.15	4.05

University of Michigan, Ann Arbor, 1904 *May* 12.

THE SECULAR PERTURBATIONS OF THE *EARTH* ARISING FROM THE ACTION OF *SATURN*.

By ERIC DOOLITTLE.

The elements which were used in the following computation are from Dr. G. W. Hill's "*New Theory of Jupiter and Saturn*," pages 192, 554, 19 and 558:

Earth.	*Saturn.*
$\pi = 100°\ 21'\ 39''.73$	$\pi' = 90°\ 6'\ 41''.37$
$i = 0\ \ 0\ \ 0.00$	$i' = 2\ 29\ 40.19$
$\Omega = \ldots\ldots$	$\Omega' = 112\ 20\ 49.05$
$e = 0.01677114$	$e' = 0.05606025$
$n = 129597''.416$	$n' = 43996''.21506$
$\log a = 0.0000000$	$\log a' = 0.9794956$
$m = 1 \div 327000$	$m' = 1 \div 3501.6$

The preliminary constants were found to have the following values:

$$I = 2°\ 29'\ 40''.19 \qquad \log k = n9.9999411$$
$$\Pi = 348\ \ 0\ \ 50.68 \qquad \log k' = n9.9996473$$
$$\Pi' = 337\ \ 45\ \ 52.32 \qquad C = +0.28595730$$
$$K = 10\ \ 13\ \ 49.89$$
$$K' = 10\ \ 16\ \ \ 6.88$$

The orbit of the *Earth* was divided into twelve parts with regard to the eccentric anomaly. As in previous cases, the approximate test formed by comparing the sums of the functions corresponding, respectively, to the odd and even points of division was not satisfied in regard to ϵ, nor with the functions immediately dependent upon this angle: with N, P, V, R_0, and the remaining functions it was, however, satisfied very exactly.

The work was carried twice through from the beginning at different times, and all known test equations were applied. Since the approximation to the roots, G' and G'', is slow in this case, these quantities were found by the method described in *A.J.*, No. 386, and their values verified by the formulas of Dr. HILL's second method. The computation of the modulus, $(\sin\theta)$, was duplicated, and also tested by the equation, $\tan^2\theta = \frac{G'+G''}{G-G'}$, since the approximate test, referred to above, fails with this angle.

The equation, $\sin q \cdot \frac{1}{2} A_1^{(i)} + \cos q \cdot B_0^{(i)} = 0$, was found to give the residual, $+0.000\,000\,000\,000\,028$.

If m' is left indefinite, the resulting values of the differential coefficients are the following:

$$\left[\frac{de}{dt}\right]_{00} = -\ 1''.5163927\ m' \qquad \text{log coeff. } n0.1808417$$

$$\left[\frac{d\pi}{dt}\right]_{00} = \left[\frac{d\chi}{dt}\right]_{00} = +\ 655.70924\ m' \qquad p2.8167113$$

$$\left[\frac{dp}{dt}\right]_{00} = -\ 18.991017\ m' \qquad n1.2785482$$

$$\left[\frac{dq}{dt}\right]_{00} = -\ 46.179399\ m' \qquad n1.6644825$$

$$\left[\frac{dL}{dt}\right]_{00} = -1514.1911\ m' \qquad n3.1802667$$

If we adopt the value of m' given above, $m' = 1 \div 3501.6$, we obtain:

$$\left[\frac{de}{dt}\right]_{00} = -0.000\,433\,057\,13$$

$$\left[\frac{d\pi}{dt}\right]_{00} = +0.187\,259\,91$$

$$\left[\frac{dp}{dt}\right]_{00} = -0.005\,423\,52\,59$$

$$\left[\frac{dq}{dt}\right]_{00} = -0.013\,188\,08\,6$$

$$\left[\frac{dL}{dt}\right]_{00} = -0.432\,514\,00$$

The results obtained by LEVERRIER are given in the *Annales de l'Observatoire de Paris*, Tome II, page 59, and Tome IV, pages 11 and 12; those of NEWCOMB are given in "*The Secular Variations of the Four Inner Planets*," pages 336 and 377; the values of $\left[\frac{dp}{dt}\right]_{00}$ and $\left[\frac{dq}{dt}\right]_{00}$ obtained by Dr. HILL are given in the "*New Theory*," pages 511 and 512. If all these results are reduced to the above value of m', they will compare with those here obtained as follows:

	LEVERRIER	NEWCOMB	HILL	Method of GAUSS
$\left[\frac{de}{dt}\right]_{00}$	$-0''.000\,44$	$-0''.000\,43$	. . . . . .	$-0''.000\,43306$
$\left[\frac{d\pi}{dt}\right]_{00}$	$+0.003\,15$	$+0.003\,14$	. . . .	$+0.003\,14056$
$\left[\frac{dp}{dt}\right]_{00}$	$-0.005\,42$	$-0.005\,42$	$-0.005\,4237$	$-0.005\,423\,526$
$\left[\frac{dq}{dt}\right]_{00}$	$-0.013\,17$	$-0.013\,18$	$-0.013\,1883$	$-0.013\,188\,086$
$\left[\frac{dL}{dt}\right]_{00}$	$-0.432\,5$	. . . .	. . . . . .	$-0.432\,514\,00$

The Flower Observatory, 1904 *May* 28.

PROVISIONAL ELEMENTS OF THE MINOR PLANET 1904 NY.

By J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

This planet was discovered by WOLF at Heidelberg, on April 20, 1904.

By means of observations made by Dr. LUTHER, at Dusseldorf, on April 23 and May 7, and one by myself, at Washington, on May 23, I have computed the following provisional elements:

1904 May 8.0 Berlin Mean Time.

$$\begin{aligned} M &= 18^\circ\ 36'\ 30''.6 \\ \pi &= 182\ 28\ 38.4 \\ \Omega &= 108\ 10\ 58.9 \\ i &= 16\ 22\ 37.9 \\ \varphi &= 10\ \ 5\ 21.3 \\ \mu &= 767''.950 \\ \log a &= 0.443146 \end{aligned}$$

(π, Ω, i:) Mean Equator and Equinox 1904.0

This planet is still being observed at the U. S. Naval Observatory. By means of these observations and others made elsewhere, the above elements will be corrected.

PHOTOGRAPHIC POSITIONS OF MINOR PLANETS,

OBSERVED WITH THE 6-INCH STAR CAMERA AT THE U.S. NAVAL OBSERVATORY,

By G. H. PETERS.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

The following asteroids were found on photographic plates exposed for this purpose by myself, and were intended to be observed micrometrically with the equatorial instruments.

Owing to cloudy weather and other circumstances, this was not done with the exception of 1903 *NT*, where the short period of observation renders the photographic position valuable. The identification has been from the opposition positions given in the *Berliner Jahrbuch*, and in some cases the coincidences are not very exact. The photographic positions are reduced to 1903.0 from measures of the plates made with the Stackpole dividing engine. Several asteroids were sought for unsuccessfully, and a list of these is appended.

Asteroid	1903	W.M.T. h m	App. α 1903.0 h m s	App. δ 1903.0 ° ′	B.J. Mag.
Ruperto Carola (353)	Apr. 28	12 22	14 29 18	− 7 8.3	15
Io (85)	May 21	11 7	14 56 5	− 7 43.1	10.8
Liberatrix (125)	June 18	10 45	17 21 12	−14 3.1	10.7
Europa (52)	June 21	9 50	17 32 23	−15 51.1	10.8
[1896 *CJ*] (111)	June 29	12 0	18 28 15	−11 38.5	10.6
Alexandra (54)	Oct. 13	12 26	1 2 57	+24 57.3	10.6
Aegle (96)	Oct. 13	12 26	1 0 40	+22 46.0	11.9
Feronia (72)	Oct. 19	10 49	1 1 57	+ 7 22.1	10.9
[1903 *NT*]	Dec. 11	10 32	5 15 58	+ 9 37.5	12

Not found: *Erato* (62), May 28; *Thisbe* (88), 2 plates, Aug. 20 and 21, (exposed by Mr. W. W. DINWIDDIE); *Orphelia* (171) and *Goberta* (316), 2 plates, Oct. 28 and Nov. 12; [1902 *LK*] (504), Dec. 11; *Carina* (491) Dec. 11, and *Nenetta* (289), Dec. 16. (All in 1903.)

EARLY DOUBLE-STAR MEASURES.

By ERIC DOOLITTLE.

The following observations were made by Dr. E. OTIS KENDALL with the 6-inch refractor of the High School Observatory at Philadelphia. Each angle is the mean of eight measures, which are remarkably accordant, and each distance is the mean of three measures. A power of "about 200" was employed; the measures were made with a bright field.

Polaris = Σ93. R.A. = $1^h\ 13^m\ 12^s$.

1844.545 207°.28 18″.686 1*n*

Neither the hour-angles nor the instrumental errors are known; the (usually very large) corrections for these errors and for refraction cannot therefore be applied. Other measures made at about this time are,

1842.33	210°.5	18″.67	2*n*	Madler
1846.23	208.9	18.39	1*n*	Mädler

γ *Virginis* = Σ1670. R.A. = $12^h\ 35^m\ 36^s$.

1844.548 11°.11 2″.294 1*n*

There is but one other measure during this year. The residuals, (O − C), from Dr. SEE's orbit are.

+1°.51 +0″.29

β *Scorpii* = *Sh*. 217. R.A. = $15^h\ 58^m\ 28^s$.

1844.550 26°.28 13″.905 1*n*

This is the only measure between 1823 and 1868. The principal star was discovered to be a close double by BURNHAM in 1879. The three stars seem to be relatively fixed, but they have a common proper motion, and are hence physically connected.

The Flower Observatory, 1904 *May* 30.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed June 18.*
Entered February 2 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 565.

VOL. XXIV. BOSTON, 1904 JULY 29. NO. 13

A METHOD OF DETERMINING THE DIRECTION OF THE *SUN'S* MOTION IN SPACE.

By W. T. CARRIGAN.

Let the right-ascension and declination of any point on the celestial sphere be designated by α and δ, and let the right-ascension and declination of some other point be A and D. Then, by the ordinary formula of spherical trigonometry the distance E between these two points, measured on the arc of a great circle, will be given by

$$(1) \qquad \cos E = \cos D \cos\delta \cos(A-\alpha) + \sin D \sin\delta$$

If the point (α, δ) move in right-ascension by the amount $\partial\alpha$, and in declination by the amount $\partial\delta$, so that its new position will be $\alpha + \partial\alpha$, $\delta + \partial\delta$, its distance E' from the point (A, D) will be given by

$$(2) \qquad \cos E' = \cos D \cos(\delta+\partial\delta) \cos(A-\alpha-\partial\alpha) + \sin D \sin(\delta+\partial\delta)$$

Developing the first term of the right-hand member of (1), we have

$$(3) \qquad \cos E = \cos D \cos\delta \cos A \cos\alpha + \cos D \cos\delta \sin A \sin\alpha + \sin D \sin\delta$$

Now let us put

$$\begin{array}{ll|ll} a &= \cos\delta\cos\alpha & r &= \cos D \cos A \\ b &= \cos\delta\sin\alpha & s &= \cos D \sin A \\ c &= \sin\delta & t &= \sin D \end{array}$$

Substituting in equation (3) the latter becomes

$$\cos E = a\,r + b\,s + c\,t$$

Similarly

$$\cos E' = (a+\partial a)\,r + (b+\partial b)\,s + (c+\partial c)\,t$$

Suppose the point (A, D) to be situated 90° distant from the point (α, δ); we then have

$$(4) \qquad a\,r + b\,s + c\,t = 0$$

which is the equation of a plane through the origin, a, b, c, being the coordinates of a point in the plane, and r, s, t, the direction-cosines of the perpendicular to the plane at the origin. If we suppose the motion of the point to take place in this plane, we also have

$$(a+\partial a)\,r + (b+\partial b)\,s + (c+\partial c)\,t = 0$$

By virtue of (4), however, this last becomes

$$r\,.\,\partial a + s\,.\,\partial b + t\,.\,\partial c = 0 \qquad (5)$$

Let α_0 and δ_0 be the right-ascension and declination respectively of a star at some adopted epoch, and assume that the variations of these coordinates are functions of the proper motions only, and that in the expressions for these variations the terms involving the squares and higher powers of the time may be neglected. Then

$$\begin{aligned} \alpha_0 &= \alpha_0 + t\,\partial\alpha \\ \delta &= \delta_0 + t\,\partial\delta \end{aligned}$$

Putting t equal to the unit of time, we have for the variations of the rectangular coordinates

$$\begin{aligned} \partial a &= -\sin\delta\cos\alpha\,\partial\delta - \cos\delta\sin\alpha\,\partial\alpha \\ \partial b &= -\sin\delta\sin\alpha\,\partial\delta + \cos\delta\cos\alpha\,\partial\alpha \\ \partial c &= \cos\delta\,\partial\delta \end{aligned}$$

Besides equations (4) and (5) we need another equation involving the coordinates of some undetermined point in the same plane. Let the coordinates of this point be x, y, z, then the equation sought will be

$$x\,r + y\,s + z\,t = 0$$

We have now three equations in r, s, t, viz.:

$$\left.\begin{aligned} a\,r + b\,s + c\,t &= 0 \\ \partial a\,.\,r + \partial b\,.\,s + \partial c\,.\,t &= 0 \\ x\,r + y\,s + z\,t &= 0 \end{aligned}\right\} \qquad (6)$$

Eliminating r, s, t, we get

$$(b\,\partial c - c\,\partial b)\,x + (c\,\partial a - a\,\partial c)\,y + (a\,\partial b - b\,\partial a)\,z = 0 \qquad (7)$$

Let us put

$$\begin{aligned} G &\equiv \sin\delta\cos\delta\,\partial\alpha \\ M' &\equiv \sin\alpha\,\partial\delta - G\cos\alpha \\ N' &\equiv \cos\alpha\,\partial\delta - G\sin\alpha \\ P' &\equiv \cos^2\delta\,\partial\alpha \end{aligned}$$

Then equation (7) may be written

$$M'x + N'y + P'z = 0$$

Dividing this last through by

$$\sqrt{M'^2 + N'^2 + P'^2} = \sqrt{\cos^2\delta\,\mu\alpha^2 + \mu\delta^2}$$

the resulting coefficients, which we will denote by M, N, P, are the direction-cosines of the perpendicular through the origin of the plane

$$(8) \qquad Mx + Ny + Pz = 0$$

Every star of which we know the right-ascension and declination at any moment, as well as the proper motion in these coordinates in unit of time, will furnish one equation of condition of the form (8); and since the point x, y, z, is to be determined as lying in the great circle in which each star moves on the sphere of unit radius, it should, if these great circles intersected in two points on a diameter of the sphere, be one of these points. If the proper motions of the stars were due entirely to the *Sun's* motion in space, the great circles in which these motions take place would all intersect rigorously in the same two points, viz.: the point toward and the point from which the *Sun* was moving at the epoch. We should, in this case, obtain values of x, y, z, which would exactly satisfy any number of equations of the form (8). It is highly improbable, however, that the stars are absolutely fixed in space, and that our *Sun* is the only body of a stellar character that is in motion. Therefore, in view of the extreme likelihood of the stars having real motions (whether independent or governed by laws unknown to us at present), we do not expect to obtain values of x, y, z, which will exactly satisfy all of our equations. To derive the most probable values of these coordinates, or, in other words, to determine the center, of gravity, as it were, of the points of intersection of all the great circles on the surface of the sphere, we must have recourse to the method of least-squares.

In the notation of this method the three resulting equations are

$$(9) \quad \left\{ \begin{aligned} [MM]x + [MN]y + [MP]z &= 0 \\ [MN]x + [NN]y + [NP]z &= 0 \\ [MP]x + [NP]y + [PP]z &= 0 \end{aligned} \right.$$

Now let us put

$$\begin{aligned} x &= \cos D' \cos A' \\ y &= \cos D' \sin A' \\ z &= \sin D' \end{aligned}$$

Since x, y, and z are functions of only two quantities, A' and D', the values of which are obtained from the ratios

$$\frac{y}{x} \text{ and } \frac{z}{x}$$

equations (9) implicitly involve only two unknowns, so we have one equation too many. In their present form, then, equations (9) will furnish three values each of A' and D'. In order that single values of A' and D' may be derived which will satisfy all three equations simultaneously, let us write the original equations in the form

$$\begin{aligned} M_1x + N_1y + P_1z &= v_1 + m_1 \\ M_2x + N_2y + P_2z &= v_2 + m_2 \\ &\ldots\ldots \end{aligned}$$

v_1, v_2, etc., being the residuals resulting from a least-square solution, and $v_1 + m_1$, $v_2 + m_2$, etc., being the residuals produced by some other solution. Multiplying through by the coefficients of x, y, z, and adding, as usual, we obtain (10)

$$\left.\begin{aligned} [MM]x+[MN]y+[MP]z-[Mm] &= [Mv] = \frac{\partial[vv]}{\partial x} \\ [MN]x+[NN]y+[NP]z-[Nm] &= [Nv] = \frac{\partial[vv]}{\partial y} \\ [MP]x+[NP]y+[PP]z-[Pm] &= [Pv] = \frac{\partial[vv]}{\partial z} \end{aligned}\right\}$$

If we now put $[Mm] = \lambda x$, $[Nm] = \lambda y$, $[Pm] = \lambda z$, we may determine, as will be shown presently, three values of λ, any one of which, substituted in equations (10), will cause these equations to yield values of the ratios

$$\frac{y}{x} \text{ and } \frac{z}{x}$$

which will satisfy all three of them simultaneously.

For convenience in writing, let us put

$$\begin{aligned} J &\equiv [MM] & Q &\equiv [MN] \\ K &\equiv [NN] & R &\equiv [MP] \\ L &\equiv [PP] & S &\equiv [NP] \end{aligned}$$

Since in a least-square solution $* * * \dfrac{\partial[vv]}{\partial x} = 0$, we may write for equations (10) the determinant equation

$$\begin{vmatrix} J-\lambda & Q & R \\ Q & K-\lambda & S \\ R & S & L-\lambda \end{vmatrix} = 0$$

The roots of this cubic are real, positive and unequal, and it may be shown that one root is numerically greater than J, K, or L, one numerically less than the least of these coefficients, and the remaining one somewhere intermediate in value.* It remains to determine from which one of these three values of λ we may obtain the most probable values of x, y and z. Differentiating equations (11), the first with respect to x, the second with respect to y, and the third with respect to z, we get

$$\begin{aligned} \frac{\partial^2[vv]}{\partial x^2} &= J - \lambda \\ \frac{\partial^2[vv]}{\partial y^2} &= K - \lambda \\ \frac{\partial^2[vv]}{\partial z^2} &= L - \lambda \end{aligned}$$

* For proof that roots of cubic are real, see SALMON'S "*Higher Algebra*," §46. Also TODHUNTER'S "*Theory of Equations*," §176.

In order that $[rr]$ may be a minimum the values of these second derivatives must be simultaneously positive. The smallest root of the cubic being the only one which will satisfy this condition, it is the one which we must use in equations (11) to derive the most probable values of

$$\frac{y}{x} \text{ and } \frac{z}{x}$$

that is, of $\tan A$ and $\tan D \sec A$.

The cubic for λ may be written

$$\lambda^3 - \eta\lambda^2 + \mu\lambda - \zeta = 0$$

where
$$\begin{aligned} \eta &= J + K + L \\ \mu &= JK + KL + JL - (Q^2 + R^2 + S^2) \\ \zeta &= JKL + 2QRS - (JS^2 + KR^2 + LQ^2) \end{aligned}$$

Whence, if we put $J' = J - \lambda$, $K' = K - \lambda$, $L' = L - \lambda$ we obtain

$$\tan A' = \frac{RS - L'Q}{K'L' - S^2} = \frac{R^2 - J'L'}{P'Q - RS} = \frac{QR - J'S}{QS - K'R}$$

$$\frac{\tan D'}{\cos A'} = \frac{Q^2 - J'K'}{K'R - QS} = \frac{K'R - QS}{S^2 - K'L'} = \frac{QR - J'S}{RS - L'Q}$$

and $\quad J'K'L' + 2QRS - (J'S^2 + K'R^2 + L'Q^2) = 0$

It is known from previous investigations that the solar apex is situated in the northern hemisphere, so that the quadrant in which A' must be taken will be determined by the sign that $\cos A'$ requires to make $\tan D'$ positive.

As a test of the applicability of the method outlined above for determining the position of the solar apex, I chose 65 stars (Epoch 1900) of magnitude 2.5 and greater, and after forming the equations of condition reduced them to the three following modified normals:

$$\begin{aligned} (32.2033 - \lambda)x &+ 1.6013\,y &- 0.7906\,z &= 0 \\ 1.6013\,x &+ (11.6674 - \lambda)y &+ 4.8788\,z &= 0 \\ -0.7906\,x &+ 4.8788\,y &+ (21.1292 - \lambda)z &= 0 \end{aligned}$$

from which resulted the cubic

$$\lambda^3 - 65.0000\,\lambda^2 + 1275.6673\,\lambda - 7098.5379 = 0$$

the roots of which are

$$\lambda = \begin{cases} 9.4626 \\ 23.19 \\ 32.35 \end{cases}$$

Using the smallest root, viz.: 9.4626, in the normal equations, I derived

$$\begin{aligned} A' &= 274^\circ\ 52' \\ D' &= +22^\circ\ 51' \end{aligned}$$

These values are well within the range of those derived by other investigators using methods entirely different from the one given in this paper. To obtain the best possible results, however, every star with a well determined proper motion should be used in the solution.

It would be of interest to investigate the degrees of probability to be attached to the different results of previous investigators in this line. This could, I think, be readily done by computing for a large number of stars the coefficients M, N, P, in equations (8), and substituting for x, y, and z, each investigator's values in turn. For the most probable values the sum of the squares of the residuals should be a minimum.

ON THE LIGHT-VARIATIONS OF 6189 *U OPHIUCHI.*

By PAUL S. YENDELL.

The variability of this star was discovered at Cordoba, in the course of the work on the *Uranometria Argentina*, and was confirmed in 1881 by SAWYER, who detected its character, and referred it to the type of *Algol*. Its history does not fall within the province of this paper, but may be found in a paper by CHANDLER, in this *Journal*, Vol. VII, p. 130, where it is entered into in detail. In the second part of the same paper, CHANDLER gives a mean light-curve of the star from his own observations. SAWYER, in Vol. VIII, p. 70, publishes a mean curve from his observations, with a graphic comparison of it with that of CHANDLER, above mentioned.

CHANDLER, *A.J.*, XXIV, pp. 6, 61, gives elements deduced from all the published material to 1904. Since the two papers first referred to, no published mean light-curve of *U Ophiuchi* has come to my notice.

I began observations of the star in 1888, and have carried them on more or less continuously until the end of the season of 1902, when they had accumulated to the number of more than four hundred. It seemed that a mean light-curve from this collection of observations might be of interest and value, both in itself and as compared with those previously published by CHANDLER and SAWYER.

At the beginning of the present year, I assembled the observations, and have treated them in a similar manner to that employed in the case of *U Cephei*, in forming the curves published in this *Journal*, Vol. XXIII, p. 213.

There proved to be four hundred and ninety-seven observations, thirty-five of which fall in the time of the star's normal light. These observations consisted mostly of comparisons with two comparison-stars, though there were frequent cases where only one was used, many including three, and some with four.

A light and magnitude scale, formed at the end of the season of 1902 from the whole body of material at hand, has been used in the formation of the mean light-curve which is the subject of this paper.

The comparison-stars and notation employed are the

same as those used by Chandler in his observations, and for the sake of convenience are displayed in the subjoined table, with their DM. numbers, positions for 1855, and their magnitudes according to the various authorities to whom I have access. The first column contains the notation used for the comparison-stars, the second, headed DM., their Bonn numbers, that headed 1855 their DM. places. In the magnitude columns DM. signifies the "*Bonn Durchmusterung*," *U.A.* the "*Uranometria Argentina*," S, Sawyer, C, Chandler, PDM. the Potsdam photometric measures, Y the scale adopted, and Lt. my light-scale.

Comparison-Stars.

		1855		Magnitudes						
	DM.	α	δ	DM.	U.A.	S	C	PDM.	Y	Lt.
		h m s	° ′							
a	−0 3221	16 58 3.9	−0 40.9	5.8	5.9	5.95	5.83	. .	5.83	16.2
b	−0 3230	17 0 44.9	−0 52.6	6.2	6.3	6.2	6.13	. .	6.12	12.2
c	−1 3282	16 59 21.7	−1 27.2	6.3	6.6	. .	6.29	. .	6.15	11.8
d	+0 3629	16 57 51.3	+0 51.7	6.3	6.2	6.25	6.42	6.05	6.40	8.2
e	+0 3283	17 8 55.8	+2 21.0	6.5	6.4	. .	6.68	6.38	6.66	4.4
f	+0 3654	17 5 29.5	+0 33.3	7.0	6.8	6.9	6.84	6.82	6.88	1.3
g	+1 3411	17 10 17.8	1 55.0	6.8	6.9	6.9	6.97	7.05	6.97	0.0
U	+1 3408	17 9 11.3	1 22.6	. .	. .	. .	. .	. .	{ 6.03 6.62	13.4 } 5.0 }

The scale of magnitudes is derived from a graphic comparison of my light-scale with the magnitudes published by Chandler. According to this scale, the value of a step is 0M.06, and is pretty constant throughout the scale.

The $T-t$ were formed from Chandler's latest elements, which he kindly furnished me in advance of publication, for the purpose of this work. These elements are

$$\text{Min. 1881 July } 17^{d}\ 15^{h}\ 32^{m} \text{ G.M.T.} + 20^{h}\ 7^{m}.6903$$

$$-3^{m}.0\,t^{2} \quad +0^{m}.3\,t^{3} \quad ; \quad \left(\text{where } t=\frac{\text{E}}{1000}\right)$$

The observations were grouped in multiples of five, according to their distribution on the curve, the grouping beginning at three hours before the computed time of minimum. From three hours after that time till three hours before it they were assumed to belong to the interval of constant light, and were grouped in fives. At the beginning of the decrease and the end of the increase the groups consist of ten, and during that part of the time of change when observations are most numerous, from two hours before to an hour and a half after minimum, each normal is formed from twenty observations, a single value at the end of the period assumed as that of normal light represents two only. It proved, however, when the normals were plotted, to indicate the beginning of the decrease.

Treating the observations as of equal weight, the probable error of a single observation is almost exactly one step (CM.98). The probable errors of the normals are small, but in most cases greater than their departures from the curve as finally drawn, as will be seen by reference to the following table. This table displays the normals. In the column $T-t$ the times are derived from the *computed* times of minimum. The column Lt. contains the values of the normals, expressed in my light-scale shown above, the column Obs. the number of observations that go to make up each normal, the column v the departure of each normal from the curve as drawn, and the column r their probable errors.

Table of Normals.

$T-t$	Lt.	Obs.	v	r
h m				
− 3 12.5	13.11	5	+0.35	±0.21
3 5.4	12.03	2	−0.42	. .
2 43.8	11.53	10	+0.20	0.33
2 25.9	10.76	10	+0.38	0.43
2 13.6	9.49	10	−0.34	0.40
1 53.4	8.82	20	+0.12	0.35
1 37.5	7.95	20	+0.13	0.28
1 21.6	7.00	20	−0.07	0.23
1 8.6	6.65	20	+0.17	0.23
0 57.6	5.81	20	−0.19	0.12
0 47.6	5.61	20	0.00	0.11
0 39.2	5.56	20	+0.19	0.16
0 30.2	5.40	20	+0.23	0.13
0 20.1	5.00	20	−0.04	0.17
0 11.9	5.20	20	+0.20	0.16
− 0 4.7	5.05	20	+0.03	0.21
+ 0 5.9	5.31	20	+0.04	0.18
0 13.8	5.52	20	−0.03	0.17
0 24.1	6.07	20	+0.06	0.21
0 35.4	6.69	20	+0.18	0.30
0 48.7	6.99	20	−0.23	0.29
1 6.7	8.55	20	+0.23	0.35
1 22.4	9.05	10	−0.22	0.31
1 39.6	10.81	10	+0.49	0.30
2 10.1	11.62	10	−0.26	0.41
+ 2 57.5	12.74	5	−0.42	0.19
+ 4 52.1	13.17	5	−0.25	0.03
7 35.7	13.32	5	−0.10	0.12
11 16.2	13.61	5	+0.19	0.06
13 17.2	13.57	5	+0.15	0.11
15 2.5	13.22	5	−0.20	0.21
16 2.4	13.43	5	+0.01	0.13
+16 34.1	13.60	5	+0.18	±0.08

As finally drawn, the curve shows a duration of seven hours, from $-3^{h}\,30^{m}$ to $+3^{h}\,30^{m}$. The end of the decrease,

however, as will appear at a glance at the above table, is much less distinctly indicated by the observations than the beginning of the decrease. The indicated duration of the light-changes is longer by an hour and a half than that shown by the curves of CHANDLER and SAWYER.

The range of variation shown is from a normal light of 13.42 to a minimum of 5.00, corresponding respectively to magnitudes 6.03 and 6.62. This is about a step less than the range shown by CHANDLER and SAWYER, and my light-curve is decidedly flatter than either of theirs. There is also a failure to indicate the inflection just after the minimum, which forms so conspicuous a feature of both their curves, especially that of CHANDLER, and on which the latter lays some stress. This inflection appears in some of the single curves, but rather slightly.

The curve shows a correction of -12^m to the computed time of minimum, corresponding to an approximate Epoch of 5380.

The readings from the mean curve are shown in the subjoined table, the times in the columns "Before" and "After" being referred to the time of minimum shown by the curve. The columns C and S contain readings from the mean curves respectively of CHANDLER and SAWYER, referred to my light-scale for the purpose of comparison. For CHANDLER'S curve, the reference was easy, as the differences of the intervals are not great; but a satisfactory relation between my light-scale and SAWYER'S was very difficult to establish, so that I finally contented myself with drawing his curve directly from the figure published in his paper above referred to. The readings in the column S are taken graphically from the curve so drawn.

TABLE OF READINGS.

$T-t$	Before	After	C Before	C After	S Before	S After
h m						
−3 30	13.42	13.42	. .	. .	. .	. .
20	13.32	13.35	. .	. .	. .	. .
10	13.10	13.25	. .	. .	. .	. .
3 0	12.76	13.07	. .	. .	. .	. .
2 50	12.30	12.82	. .	. .	. .	. .
40	11.80	12.55	13.4	13.4	. .	. .
30	11.30	12.20	13.2	13.4	. .	13.5
20	10.80	11.80	12.9	13.0	. .	13.3
10	10.30	11.37	12.3	12.5	. .	13.0
2 0	9.80	10.82	11.9	12.1	13.6	12.5
1 50	9.20	10.23	11.3	11.6	13.2	12.0
40	8.62	9.66	10.7	11.2	12.7	11.4
30	8.10	9.05	10.1	10.6	12.1	10.6
20	7.60	8.40	9.8	9.8	11.3	9.4
10	7.12	7.75	9.15	9.0	9.9	7.3
1 0	6.64	7.18	8.4	8.0	8.2	6.7
0 50	6.17	6.62	7.5	6.7	6.3	6.2
40	5.75	6.20	6.4	6.4	5.2	5.3
30	5.44	5.73	5.5	5.1	5.6	4.7
20	5.20	5.30	5.1	5.2	4.2	4.2
10	5.05	5.10	4.5	4.6	4.1	4.1
0	5.00	. .	4.2	. .	4.0	. .

The brightness observed at the different minima varies from 2st.4 to 6st.0, that is, from 6^m.80 to 6^m.55. To see if any system could be detected in these varying estimates, they were assembled to the number of thirty-five, being the observation of least light at each observed minimum, with a few which fell very nearly at the computed date and minimum light on occasions when no minimum was actually determined. When arranged in the order of their dates a simple inspection showed at once that no progressive change was indicated. Rough means of the yearly groups confirmed this inference.

The question of the influence of hour-angle on these estimates then suggested itself. The observations were found to occur at hour-angles from $3^h\ 46^m$ east to $3^h\ 53^m$ west. They consisted of comparisons with the stars d, e, f and g, principally with d and e. The star e was made use of in thirty-four, and the star d in twenty-six of the observations.

An examination of the position-angles and distances of these stars from U shows that the star e lies about 3° west of north from it, at a distance of about 4′8″. The star d lies about 9° 40′ south of west, at a distance of about 2′ 50″. When U is at hour-angle 4h east, the line from it to e is presented at an angle with the vertical of about 57° east, and the line to d at about 40° west from it. As U rises e becomes relatively higher, approaching the vertical, and d lower, receding from it, until when U passes the meridian, e is nearly vertically above it, while d is slightly lower than U. At 4h west, e stands at an angle of 63° west from the vertical, above U, and d west of U, at an angle below the horizontal line of about 30°.

Knowing the effect of position-angle on comparisons as made by the ARGELANDER method, we are led to expect from these circumstances that the estimates of intervals between U and e would increase as e approaches the vertical line above U, and would begin to decrease as it moves westward from that line, and toward the western limit mentioned. While the intervals from comparisons with d should increase continuously from the extreme eastern to the extreme western hour-angle. Also the interval $U-d$ should show a continuous increase between the same limits.

To test the actuality of these conditions, the comparisons were arranged in the order of their hour-angles, and in separate columns for each star. There proved to be thirteen observations in eastern, and twenty-two in eastern hour-angles.

The means of hourly groups were taken for each star, and also for the values shown by the complete observations. All were treated as of equal weight.

The results are shown in the following table, where the column HA shows the mean hour-angle of each hourly

group, the columns *ve* and *vd* the mean intervals between *V* and the stars *e* and *d* respectively, the columns *n* the number of comparisons in each group, and the column *de* the intervals $d-e$. The column Lt. gives the means of observed lights for each hourly group of complete observations.

	H.A. h m	*ve*	*n*	*dv*	*n*	*de*	*n*	Lt.
East	3 17	+1.2	3	+2.0	3	3.2	3	5.2
	2 25	+0.2	3	–	–	–	–	–
	2 37	–	–	+1.5	2	3.0	2	4.4
	1 22	+0.8	4	+3.5	4	4.3	4	4.8
	0 24	+1.0	3	–	–	–	–	5.1
East	0 20	–	–	+3.0	2	1.3	2	–
West	0 30	–	–	+2.8	7	3.9	7	–
	0 34	+0.3	9	–	–	–	–	4.7
	1 28	–	–	+3.0	3	3.7	3	–
	1 32	+0.2	4	–	–	–	–	5.0
	2 27	–	–	–	–	4.5	2	–
	2 29	–0.5	3	–	–	–	–	–
	2 34	–	–	+4.2	3	–	–	5.2
	3 23	–	–	+3.3	2	–	–	3.7
	3 24	–	–	–	–	4.0	2	–
West	3 30	–0.5	5	–	–	–	–	–

The column *ve* shows a continuous decrease of the interval, without any sign of the anticipated increase at the meridian. The column *dv* shows a continuous increase as anticipated, as does also the column *de*. The column Lt. shows neither any well-defined increase nor decrease, and the mean of this column is 4.8, which corresponds closely to the value deduced from the normals in forming the mean curve.

The effects indicated in the columns *ve* and *dv* tend nearly to counteract each other, showing results in the complete observations such as appear in the column Lt.

The above indications would seem to show that the difference of a step between my minimum value and those of Chandler and Sawyer may probably be attributed to subjective causes personal to the observers, or, possibly to the use of different comparison-stars near that phase.

There appears to be some question of the constancy of the normal light, as indicated by my observations. The normals show a slight maximum of two-tenths of a step above the mean about midway of the interval of constant light, but are so few and far separated, and depend on so few observations, and the value of the increase is so small, though greater than the probable error of the normals, that the reality of the change is extremely doubtful, in view of the behavior of the other stars of this type which have been examined in this respect.

The principal question raised by my observations is the difference of the indicated duration of the light-changes from that assigned by Chandler. This difference in duration is important, as it amounts to three-tenths of the whole value given by Chandler. I feel reasonably sure that my results in this respect are worthy of confidence, especially as regards the beginning of the decrease. The observations have been made with no preconceived expectations as to the star's behavior, and mostly with very vague knowledge of the probable time of minimum, often on evenings when this was entirely unknown to the observer, and on occasions when there was no expectation or purpose of following the star through its minimum phase. The whole of the curve as drawn falls well within or close without the probable errors of the normals. The normals are mostly formed from larger groups of observations than are the corresponding ones of Schönfeld in his classical discussion of the light-changes of *Algol*, and are, on the whole, about as closely represented by the curve adopted.

A progressive increase in the duration of the light-changes suggests itself as a possible explanation of the difference. To my regret, however, my observations do not afford the material for the investigation of this question, as in the earlier years they seldom covered more than three hours on any one occasion.

Dorchester, 1904 *June* 17.

OBSERVATIONS OF 2387 *NOVA GEMINORUM*.

By ZACCHEUS DANIEL.

In connection with other work done at the expense of the Carnegie Institution, fourteen observations were made on *Nova Geminorum* with Pickering's equalizing wedge photometer, attached to the 58.4 cm. (23-inch) refractor. The first three of these observations were made by Professor W. M. Reed. Usually four or five settings were made on each comparison-star before and after the *Nova*. Occasionally a number of extra settings were made to determine the brightness of the comparison-stars. In all 418 settings were obtained.

Below are the data for the comparison-stars. Under the heading H.C.O., the notation is that of Circular 70, but the magnitudes are those designated H.P. in Hagen's catalogue. The data headed J.A.P. are from a paper by J. A. Parkhurst, in *Popular Astronomy*, XI, 330. In the last three columns are given the light-scale derived from my observations, the magnitudes of the stars on the same scale adjusted to the H.C.O. system, and the number of settings on each star used in forming the scale.

COMPARISON-STARS.

H.C.O.		Hagen		J.A.P.		DM.+30°		Daniel		Set
	M		M		M		M	M	M	
g	8.76	11	8.6	d	9.02	1306	8.6	0.00	8.62	56
k	9.34	18	9.2	e	9.27	1302	9.2	0.40	9.02	31
l	10.13	31	10.2	L	9.97	1309	9.4	1.75	10.37	97
m	10.78	44	10.7	m	10.56	. . .	. .	2.38	11.00	47

Following are the observations in Greenwich mean time. The notation of the comparison-stars is that of H.C.O., Circular 70, but the magnitudes are on [illegible]. The last column gives the number of settings on the N[illegible]. The first observation was made through a photogra[illegible] shade glass whose absorption as measured here is [illegible].15. A similar glass, having an absorption of $0^M.43$, was used for part of the settings at the third observation. On April 20 and May 12 the latter glass was used on all the stars. All the other observations were made without a shade glass.

OBSERVATIONS.

1903		J.D.	$r-l$	$r-m$	$r-g$	$r-k$	Mag.	Set.
			M	M	M	M		
Mar.	31	6205.640	−1.81R	. .	. .	. .	8.56	5
Apr.	1	06.563	−1.61R	.	. .	. .	8.76	12
	10	15.577	−1.28R	. .	. .	. .	9.09	8
	20	25.582	−1.18	. .	. .	. .	9.19	5
	29	32.603	−0.22	−0.50	. .	. .	10.32	6
May	5	40.589	−0.27	−1.08	. .	. .	10.01	7
	6	41.569	−0.53	−1.26	. .	. .	9.79	8
	6	41.585	−0.28	−0.92	. .	. .	10.08	8
	8	43.576	−0.06	−0.68	+1.78	. .	10.34	5
	9	44.579	+0.04	. .	+1.90	+1.58	10.51	5
	11	46.589	+0.18	. .	. .	+1.31	10.41	5
	11	46.606	+0.26	. .	+1.89	. .	10.57	5
	12	47.592	−0.06	−0.68	+1.70	+1.23	10.30	5
	25	6260.581	+0.24	. .	. .	. .	10.61	5

Princeton University, Princeton, N.J., 1904 *June* 14.

OBSERVATIONS OF SUNSPOTS.

By ROBERT H. BAKER.

1904	d	h	New Gr	New Spts	Disap Gr	Disap Spts	Reap Gr	Reap Spts	Total Gr	Total Spts
Mar.	4	2	–	–	–	–	–	–	2	5
	5	2	–	5	–	–	–	–	2	10
	8	1	–	–	1	4	–	–	1	1
	9	2	1	3	–	–	1	1	2	1
	10	2	1	2	–	–	–	–	3	6
	17	0	–	1	–	–	–	–	2	4
	19	2	1	30	–	–	1	10	3	34
	20	2	–	–	–	–	–	–	3	25
	21	0	–	–	1	2	–	–	2	30
	24	1	1	3	–	–	1	12	4	35
	29	2	1	2	–	–	1	2	4	12
	30	2	–	4	1	2	–		3	13
April	4	22	–	–	1	1	–	–	1	15
	6	2	–	–	–	–		–	1	7
	7	22	1	2	–	–	1	2	2	3
	11	4	1	16	–	–	1	1	3	19
	12	2	–	5	1	1	–	–	2	23
	16	4	1	1	–	–	1	1	3	23
	20	5	1	1	–	–	1	1	4	14
	21	2	1	9	1	1	1	9	4	22
	21	23	1	19	–	–	–	–	5	36
	23	0	–	10	–	–	–		5	41
	23	21	–	8	–	–			4	14
	25	0	–	4	–	–	–		4	43
	26	2	–	–	2	3	–	–	2	34

1904	d	h	New Gr	New Spts	Disap Gr	Disap Spts	Reap Gr	Reap Spts	Total Gr	Total Spts
April	29	23	1	3	–	–	1	3	3	28
May	1	5	1	1	–	–	1	1	4	20
	2	2	1	3	–	–	1	1	5	15
	2	23	–	7		–		–	5	18
	4	1	–	10	2	4			3	21
	4	23	–	7	–	–	–	–	3	22
	5	23	–		–	–	–	–	3	15
	6	22		2	–			–	3	45
	8	2	1	6	1	2	1	5	3	19
	9	2		15	–	–		–	3	31
	10	0	1	13	–	–	1	1	4	42
	10	22	–	12		–		11	3	35
	12	2	2	20	–		2	5	5	49
	13	2	–	6	1	3	–	–	3	11
	14	3	–	2	–	–	–		3	30
	16	4	4	1	–	–	–	–	4	6
	20	1	–	2	1	1	–		2	6
	21	2	–	–				–	2	5
	22	2		1	–			–	2	5
	22	22							2	4
	24	2		1					2	4
	24	22	1	1	2	4	4	1	1	1
	25	22	3	11		–	1	5	4	12
	27	2	1	4		–	1	2	5	16

1904	d	h	New Gr	New Spts	Disap Gr	Disap Spts	Reap Gr	Reap Spts	Total Gr	Total Spts
May	27	23	1	3	–		1	1	5	14
	30	21			–	–		–	1	2
June	2	1	1	5	–	–	1	3	2	7
	2	23		1	–	–		–	2	8
	4	2		4	–	–		–	3	13
	5	2	–	4	–	–			3	15
	6	5	–	4	1	1	–		2	18
	9	5	1	16			1	4	4	34
	10	2	–	2	–	–			4	32
	11	2		3				–	4	26
	11	22		8	1	5			3	28
	12	21	1	1	–	–	1	1	4	21
	13	22	1	23	–	–	1	6	5	46
	15	2	–	3	1	3			4	34
	16	2		2	1	5	–	–	2	21
	18	0	1	7	–	–	1	2	3	26
	19	2	1	25	–		1	4	3	49
	19	23	–	2					3	29
	21	2		6	–		–		3	29
	22	2		8					3	33
	23	6	–	12					3	44
	23	22		–					3	23
	26	2	1	1	2	7	1	1	2	7
	26	21		2					1	8

Amherst College Observatory. Made with 6-inch Reflector. Faculae at each observation.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN PENN., WITH A 4-INCH REFRACTOR, BY A. W. QUIMBY.

1904		Time	New Grs	Total Grs	Total Spots	Fac Grs	Def
Jan.	1	1	..	3	10	1	fair
	3	9	..	3	10	1	poor
	4	11	1	3	8	2	fair
	5	9	..	3	5	2	poor
	6	1	..	2	3	..	poor
	7	10	..	2	2	..	poor
	8	3	..	2	2	..	poor
	9	2	1	4	4	2	good
	10	2	..	2	2	2	fair
	11	10	..	1	1	..	poor
	12	9	..	2	2	1	fair
	14	10	..	2	2	..	poor
	15	2	1	2	5	2	fair
	16	11	..	1	4	..	poor
	17	9	1	2	8	2	poor
	18	8	..	2	10	1	poor
	19	9	..	2	14	..	poor
	20	11	..	2	8	..	poor
	21	9	..	2	14	..	poor
	24	9	2	3	44	2	good
	25	8	..	2	23	2	poor
	26	8	..	2	13	1	poor
	27	9	..	2	8	1	fair
	28	8	..	1	2	..	poor
	30	10	..	1	2	1	fair
	31	1	..	..	..	1	fair
Feb.	1	11	..	..	..	..	poor
	2	9	1	1	2	2	fair
	3	12	..	1	2	2	fair
	4	9	1	2	2	3	poor
	5	12	1	3	8	1	poor
	6	9	..	3	10	3	fair
	7	5	..	1	6	..	v.poor
	8	9	..	2	28	..	poor
	9	9	..	1	50	..	fair
	10	1	..	1	36	..	poor
	11	8	..	1	20	1	poor
	12	9	..	1	22	1	fair
	13	8	..	1	14	1	poor
	14	2	..	1	2	..	v.poor
	15	9	..	1	2	1	good
	16	8	..	..	..	1	poor
	17	8	..	..	..	..	poor
	18	8	1	1	1	1	poor
	20	8	1	2	6	1	fair
	21	8	..	2	18	..	poor
	22	2	..	2	14	..	poor
	*23	7	..	2	10	..	poor
	24	11	..	2	8	..	poor
	25	8	..	2	10	..	poor
	26	10	..	2	6	1	poor
Mar.	1	2	..	..	..	..	v.poor
	2	9	1	1	5	1	fair

1904		Time	New Grs	Total Grs	Total Spots	Fac Grs	Def
Mar.	4	7	..	1	5	..	poor
	5	2	1	2	6	1	poor
	8	12	2	2	4	8	fair
	9	8	1	4	18	3	fair
	10	8	..	3	10	2	poor
	12	8	..	1	1	1	poor
	13	2	..	1	1	1	fair
	15	10	1	2	4	2	poor
	16	8	..	2	4	..	poor
	17	8	..	2	7	2	fair
	18	2	..	2	4	1	poor
	19	7	1	3	9	2	poor
	20	2	..	3	16	1	fair
	21	7	..	3	8	1	poor
	22	5	1	4	9	2	poor
	23	11	..	4	18	2	poor
	24	9	1	4	32	3	fair
	25	3	1	5	18	3	fair
	26	9	..	4	13	2	fair
	27	8	1	5	32	2	fair
	28	8	..	4	18	2	poor
	29	8	1	5	17	2	poor
	30	8	..	5	14	2	poor
Apr.	1	5	..	5	9	3	poor
	2	7	..	5	10	3	poor
	3	8	..	5	9	3	fair
	4	8	..	4	9	2	fair
	5	8	..	4	9	3	fair
	6	10	..	2	8	3	fair
	7	8	..	2	7	3	fair
	8	9	1	2	8	3	fair
	9	5	..	2	6	2	poor
	10	8	1	3	10	3	fair
	11	9	..	3	11	3	fair
	12	1	..	3	12	3	fair
	13	10	..	3	26	3	good
	14	9	..	2	15	1	poor
	15	9	1	3	26	2	fair
	16	10	..	3	21	2	fair
	17	9	..	3	15	2	fair
	18	8	..	3	19	2	fair
	19	8	1	4	33	3	good
	20	8	..	3	9	..	poor
	21	9	1	3	15	2	fair
	22	8	1	4	42	2	good
	23	4	..	4	51	2	fair
	24	8	..	4	62	2	good
	25	3	1	4	51	2	fair
	26	9	..	2	20	..	poor
	30	8	2	4	23	2	poor
May	1	9	..	4	21	3	fair
	2	8	..	4	12	3	fair

1904		Time	New Grs	Total Grs	Total Spots	Fac Grs	Def.
May	3	8	1	4	11	4	fair
	4	9	..	3	10	1	fair
	5	8	..	3	20	1	fair
	6	8	1	4	13	2	fair
	7	4	..	2	12	2	fair
	8	9	1	3	13	1	fair
	9	10	..	3	22	1	fair
	10	8	2	5	23	2	fair
	11	8	..	4	31	2	fair
	12	8	..	4	22	3	fair
	13	8	..	4	23	2	fair
	14	9	..	4	24	3	fair
	15	8	..	3	20	2	fair
	16	8	..	3	10	2	poor
	17	9	..	4	8	1	poor
	19	4	1	3	11	4	fair
	20	10	..	2	8	..	poor
	21	8	..	2	6	3	fair
	22	8	..	2	5	3	fair
	23	8	..	2	5	4	fair
	24	8	..	2	6	4	fair
	25	8	2	4	5	3	fair
	26	8	2	4	12	2	fair
	27	8	1	4	11	3	fair
	28	8	1	3	16	4	fair
	29	8	..	3	12	3	fair
	30	8	..	3	12	3	fair
June	3	10	..	1	3	1	fair
	4	8	2	3	11	1	fair
	5	8	..	3	12	2	fair
	6	8	..	3	12	3	fair
	7	8	1	3	17	3	good
	8	8	..	3	24	3	fair
	9	8	..	2	10	1	poor
	10	4	1	4	17	3	fair
	11	10	..	4	32	2	fair
	12	8	..	3	25	3	fair
	13	8	1	4	33	3	fair
	14	9	1	5	67	4	good
	15	8	1	5	37	3	fair
	16	8	..	3	32	4	fair
	17	6	1	4	26	..	fair
	18	8	..	3	21	..	fair
	19	8	1	4	32	1	fair
	20	7	..	4	26	1	fair
	22	8	..	3	22	1	fair
	23	7	..	3	20	..	fair
	24	8	..	3	16	2	fair
	25	8	1	4	12	2	fair
	26	9	..	2	8	..	fair
	27	8	..	2	7	1	poor
	28	7	1	2	5	1	poor

*2½-inch refractor.

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed July 28.*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 566.

VOL. XXIV. BOSTON, 1904 AUGUST 19. NO. 14

REMARKS ON THE DETERMINATION OF THE NUMBER AND MEAN PARALLAX OF STARS OF DIFFERENT MAGNITUDE AND THE ABSORPTION OF LIGHT IN SPACE,

BY J. C. KAPTEYN.

In No. 558 of this *Journal*, Prof. COMSTOCK communicated an extremely interesting investigation by which our knowledge of reliable proper motions of faint stars is extended in a most desirable way.

Although in the following lines I venture to offer some objections to one of his conclusions, I hope that they will serve the purpose of rendering still more evident the great value of such investigations.

The conclusion on which I wish to make some remarks is, that either: "The intrinsic luminosity of the stars diminishes with their apparent brightness in such a ratio that a star of the tenth magnitude possesses only one-tenth of the luminosity of the fifth magnitude"; *or*, that there is a very appreciable absorption of light. The author is evidently inclined to accept the latter alternative.

Prof. PICKERING, in his recent paper, "Distribution of Stars," *Ann. of Harv. Coll.*, 48, No. 5, finds the ratio of the increase in number of the stars of diminishing apparent brightness to deviate from the "theoretical" ratio, and seems also inclined to adopt the hypothesis of a considerable absorption of light in space.

As far as I can see, we are at the present moment unable to prove either the truth or the falseness of this hypothesis, but I think that:

a. We can prove conclusively that we are at least *not* compelled to accept the alternative of Prof. COMSTOCK;

b. We can show that an absorption of so high an amount as that found by this astronomer is highly improbable;

c. We can assign a way which, as soon as the necessary observational data will be available, must lead to a determination of the quantity of the absorption in space, free from hypotheses about the star-density at different distances from the solar system.*

* For considerations very nearly related to many of those here following, the reader is referred to SCHIAPARELLI's memoir, "*Sulle distribuzione apparente delle Stelle visibili ad occhio nudo.*"

The first point will become evident if we derive the theoretical value, both of the number of stars of a given magnitude and of their mean parallax, without introducing either the hypothesis of the same luminosity for all the stars, nor of a density in their distribution, which is the same at all distances, — hypotheses which are, evidently or probably, not in accordance with nature.

If we put, in accordance with Publication, No 11, of the Astronomical Laboratory at Groningen,

$$\delta = 2.512\ldots = \frac{\text{app. bright. of star of mag. } m}{\text{“} \quad \text{“} \quad \text{“} \quad m+1} \quad ; \quad (\log \delta = 0.4)$$

π_m = mean parallax (in seconds of arc) of the stars of the apparent magnitude m.

ρ = distance from sun ($\rho = 1$ for $\pi = 0''.1$).

N_m = number of stars between the apparent magnitudes $m - \frac{1}{2}$ and $m + \frac{1}{2}$.

$\Delta(\rho)$ = star-density = number of stars pro unit of volume at distance ρ; unit of volume = cube of which the side is distance corresponding to $\pi = 0''.1$.

h_m = apparent brightness of a star of magnitude m ($h_{5.5} = 1$).

L = luminosity = absolute quantity of light emitted by a star ($L = 1$ for sun).

M = absolute magnitude (that of sun = 5.5).

$\varphi(L)\delta L$ = probability of a star's luminosity being included between the limits L and $L + \delta L$.

$\psi(L) = \int_{L/\sqrt{\delta}}^{L\sqrt{\delta}} \varphi(z)\,\delta z$ = probability of the absolute luminosity being $= L$ within $\pm$ half a mag.

Then, if we assume $\varphi(L)$ to be independent of the distance ρ, the required formulas will be easily found to be

$$N_m = 4\pi \int_0^\infty \rho^4\,\Delta(\rho)\,\delta\rho \int_{h_{m+\frac{1}{2}}}^{h_{m-\frac{1}{2}}} \varphi(h\rho^2)\,\delta h$$

$$10\pi_m = \frac{\int_0^\infty \rho^3\,\Delta(\rho)\,\delta\rho \int_{h_{m+\frac{1}{2}}}^{h_{m-\frac{1}{2}}} \varphi(h\rho^2)\,\delta h}{\int_0^\infty \rho^4\,\Delta(\rho)\,\delta\rho \int_{h_{m+\frac{1}{2}}}^{h_{m-\frac{1}{2}}} \varphi(h\rho^2)\,\delta h}$$

Putting $h\rho^2 = z$ we have

$$\int_{h_m/\sqrt{\lambda}}^{h_m\sqrt{\lambda}} \varphi(h\rho^2)\,\delta h = \frac{1}{\rho^2}\int_{h_m\rho^2/\sqrt{\lambda}}^{h_m\rho^2\sqrt{\lambda}} \varphi(z)\,\delta z = \frac{1}{\rho^2}\psi(h_m\rho^2)$$

Therefore

$$(1)\qquad N_m = 4\pi\int_0^\infty \rho^2\,\Delta(\rho)\,\psi(h_m\rho^2)\,\delta\rho$$

$$(2)\qquad 10\pi_m = \frac{\int_0^\infty \rho\,\Delta(\rho)\,\psi(h_m\rho^2)\,\delta\rho}{\int_0^\infty \rho^2\,\Delta(\rho)\,\psi(h_m\rho^2)\,\delta\rho}$$

In these formulas the tacit assumption has been made that Δ is a function of ρ only, and not of the direction. Such an assumption is only allowable as long as we confine ourselves to the consideration of *mean* values. $\Delta(\rho)$ will thus represent the mean density at distance ρ, etc.

SCHIAPARELLI has shown that, *whatever be the distribution of the luminosities*, if we assume the star-density to be constant for all distances, we will have

$$(3)\qquad N_m = \text{const.} \times h_m^{-3/2}$$

$$(4)\qquad \pi_m = \text{const.} \times h_m^{1/2}$$

The truth of the proposition is at once evident from our formulas; for if in (1) and (2) we put

$$z = h_m\rho^2$$

they become

$$N_m = 2\pi h_m^{-3/2}\int_0^\infty \sqrt{z}\,\Delta\left(\sqrt{\frac{z}{h_m}}\right)\psi(z)\,\delta z$$

$$10\pi_m = h_m^{1/2}\,\frac{\int_0^\infty \Delta\left(\sqrt{\frac{z}{h_m}}\right)\psi(z)\,\delta z}{\int_0^\infty \sqrt{z}\,\Delta\left(\sqrt{\frac{z}{h_m}}\right)\psi(z)\,\delta z}$$

in which, if $\Delta(z)$ is constant for all values of z, the integrals become independent of h_m.

The formula (4) will be true in still another case, *viz.*: if, *whatever be the law of the densities*, we assume all the stars to have the same luminosity.

To be rigorously true, however, π_m must in this case no longer denote the mean parallax of all the stars of magnitude $m - \frac{1}{2}$ to $m + \frac{1}{2}$, but merely the parallax of the stars of the magnitude m exactly.

The truth of the proposition is evident.

Meanwhile the assumption of a constant star-density is gratuitous; that of a constant luminosity demonstrably false.

If, therefore, we refuse to admit either of these hypotheses, the equations (3) and (4) will cease to hold, and such conclusions as those of COMSTOCK and PICKERING about an appreciable absorption of light, which implicitly are based on them, lose their cogency.

We may go a step further.

In Publications No. 11 of the Astronomical Laboratory at Groningen, p. 19, I derived both the functions ψ and Δ from considerations about the proper motions together with other data, in the tacit assumption that no appreciable extinction of light exists in our stellar system.

On page 19 of that paper have been given various solutions for the quantities which, in our present notation, are

Table 5: $\dfrac{\Delta(\rho)}{\Delta(0)}$ with the argument π;

Table 6: $\log \Delta(0) + \log\psi(L)$ both with the argument, $\log L$ and with the argument M.

In order to see what can be derived from the knowledge of these functions, I have sought to represent them analytically. After a few trials I found the following forms:

$$\psi(L) = \frac{\alpha^2 \text{ mod.}}{\sqrt{\pi}\,L}\,e^{-\alpha^2(\log L - T)^2} \qquad (5)$$

$$\frac{\Delta(\rho)}{\Delta(0)} = e^{[illegible]} + \beta\rho e^{[illegible]} \qquad (6)$$

in which, for the solution V, the constants have the values,

$$\left.\begin{aligned} \Delta(0) &= 111.0 \\ T &= 1.400 \\ \alpha^2 &= 0.385 \\ \beta &= 0.0220 \\ \gamma &= 0.0052 \end{aligned}\right\} \qquad (7)$$

They represent the numbers derived from the observations as follows:

TABLE I: $\log \Delta(0) + \log\psi$.

M	$\log L$	Obs.	Comp.	O—C
−6.55	4.82	4.60–10	4.24–10	+0.36
−5.55	4.42	5.17	5.08	+ .09
−4.55	4.02	5.928	5.853	+ .075
−3.55	3.62	6.586	6.576	+ .010
−2.55	3.22	7.210	7.247	− .037
−1.55	2.82	7.815	7.863	− .048
−0.55	2.42	8.380	8.426	− .046
0.45	2.02	8.922	8.936	− .014
1.45	1.62	9.113	9.392	+ .021
2.45	1.22	9.839–10	9.795–10	+ .044
3.45	0.82	0.190	0.144	+ .046
4.45	0.42	0.478	0.440	+ .038
5.45	0.02	0.680	0.682	− .002
6.45	−0.38	0.836	0.870	− .034
7.45	−0.78	1.026	1.006	+ .020
8.45	−1.18	1.102	1.086	+ .016
9.45	−1.58	(1.10)	1.115	(− .015)
10.45	−1.98	(1.11)	1.09	(+0.02)

TABLE II. $\dfrac{\Delta(\rho)}{\Delta(0)}$.

π	ρ	$\frac{\Delta(\rho)}{\Delta(0)}$ Obs.	$\frac{\Delta(\rho)}{\Delta(0)}$ Comp.	O—C
0.00118	84.7	0.162	0.151	+0.011
.00187	53.5	.292	.303	− .011
.00296	33.8	.465	.473	− .008
.00469	21.3	.684	.667	+ .017
.00743	13.5	.852	.857	− .005
.0118	8.47	.945	.958	− .013
.0187	5.35	0.984	.990	− .006
.0296	3.38	1.000	.998	+0.002
.0469	2.13	0.980	1.000	
.0743	1.35	.957	1.000	
.118	0.85	.933	1.000	
0.204	0.49	0.929	1.000	

In the last table I leave out the last values of O—C, because, for reasons given in Publication 11, the decrease in the densities derived from the observations for small distances is probably illusory. Bearing this in mind, the representation seems to be quite satisfactory.

If now we introduce these functions into the formulas (1) and (2), replacing at the same time the brightness h_m by the apparent magnitude, by means of the relation

$$\log h_m = 2.200 - 0.1\,m \tag{8}$$

and putting

$$G = 2.200 - T = 0.800 \tag{9}$$

$$\left\{\begin{aligned} T_1 &= \int_0^\infty e^{-\beta\rho-\alpha^2(G-0.4m+2\log\rho)^2}\,\delta\rho \\ T_2 &= \int_0^\infty e^{-\beta\rho-\alpha^2(G-0.4m+2\log\rho)^2}\,\delta\rho \\ T_3 &= \int_0^\infty \frac{1}{\rho}\,e^{-\beta\rho-\alpha^2(G-0.4m+2\log\rho)^2}\,\delta\rho \\ T_4 &= \int_0^\infty \rho\,e^{-\gamma\rho^2-\alpha^2(G-0.4m+2\log\rho)^2}\,\delta\rho \end{aligned}\right. \tag{10}$$

we get

$$N_m = \frac{4\alpha^2\sqrt{\pi}\,.\,\mathrm{mod.}\,\Delta(0)}{h_m}\,(T_1+\beta T_4) \tag{11}$$

$$10\pi_m = \frac{T_3+\beta T_2}{T_1+\beta T_4} \tag{12}$$

We have to remark:

1st. That, as was shown in Publication 11, ψ is not greatly altered, if we change the data on which its derivation rests within reasonable limits. That, on the contrary, the determination of the Δ is only quite provisional; its discussion being deferred to a subsequent communication because other data have to be taken into account besides those used in Publication 11. Among these, in the first line, just such accurate determinations of the total numbers of stars of different magnitude as have been recently given by Prof. PICKERING.

2d. That, for the computation of the integrals (10) both the functions ψ and Δ must be assumed to be given by the formulas (5) and (6) for values of L and ρ ranging from *zero* to ∞, whereas, even by somewhat straining the observations (as has been done in Publication 11) ψ is only determined from $\log L = 1.82$ to $\log L = 1.98$; Δ only from $\rho = 0$ to $\rho = 85$.

For the determinations which follow, and which do not extend to apparent magnitudes below 11.0, the assumption seems permissible for ψ. For the Δ the prolongation of the observed curve beyond the limits of the observation is far more hazardous. To obviate this difficulty, to a certain extent, I will replace the value (7) of $\Delta(0)$ by the value

$$\Delta(0) = 136.9 \tag{13}$$

which is derived from the observed numbers given by PICKERING.

Still it will be well to say that, for the reasons given, the deductions which will presently be made are to be regarded as in every respect *provisional*; they are *only* meant as an illustration of the possibility of representing the data of observation, without having recourse, either to the assumption of an appreciable amount of extinction, or to empirical formulas.

The integrals (10) for the values (7) of the constants, were found by numerical integration. The results are embodied in the following table:

TABLE III. VALUE OF INTEGRALS T.

m	T_1	T_2	T_3	T_4
0.0	2.88	2.72	3.43	7.45
1.0	4.33	3.84	3.38	13.71
2.0	6.36	5.24	3.29	23.28
3.0	9.12	6.79	3.17	36.36
4.0	12.69	8.28	3.02	52.17
5.0	17.04	9.39	2.83	68.14
6.0	21.86	9.87	2.61	80.82
7.0	26.63	9.53	2.35	86.85
8.0	30.51	8.12	2.05	84.28
9.0	32.80	6.77	1.71	73.70
10.0	32.81	4.95	*1.37	57.97
11.0	30.51	3.26	1.04	40.93

Only the value for the magnitudes 0, 3, 5, 7, 9, 11 were directly computed, the rest were obtained by interpolation. The calculations were only roughly made, still I hope that no errors exceeding a few units of the last decimal will be found.

With the aid of these values I find:

TABLE IV. NUMBER OF STARS.

Mag.		N_m Obs. Pick.	Ratio	N Comp. form. (11)	Ratio	O—C	O—C (fract.)
	1.5	23		16		+ 7	+ 0.44
1.5	2.5	53		45		+ 8	+ 0.19
2.5	3.5	174	3.28	161	3.58	+ 13	+ 0.08
3.5	4.5	570	3.28	564	3.50	+ 6	+ 0.01
4.5	5.5	1 835	3.22	1 897	3.36	62	0.03
5.5	6.5	5 798	3.16	6 076	3.20	278	0.04
6.5	7.5	17 787	3.07	18 420	3.03	633	0.03
7.5	8.5	52 460	2.95	52 540	2.85	80	0.00
8.5	9.5	146 700	2.80	140 200	2.67	+ 6 500	+ 0.05
9.5	10.5	378 500	2.58	349 100	2.49	+ 29 400	+ 0.08
10.5	11.5	875 100	2.31	808 200	2.32	+ 65 900	+ 0.08

The deviations are, no doubt, still strictly systematic. Still, if we consider the provisional character of the foundations on which the computations rest, and if we bear in mind that a deviation of 0.06 in the last column corresponds to only about 0.05 magnitudes, there seems every reason to be satisfied. There can hardly be any doubt but that by small improvements in the distances, even now

possible, together with slight alterations in the value of the constants α, β, G, well within the limits of their uncertainty, we must succeed in obtaining a representation of the observed values as perfect as can be desired. Even without such an adjustment the phenomenon of the gradual diminution of the ratio

$$\frac{\text{number of stars of magnitude } m+1}{\text{" " " } m}$$

with increasing m, which is such a striking feature in PICKERING's investigation, is well shown (see columns 3 and 5 of Table IV).

In comparing the parallaxes I will reduce all the results to what would have been found, if for the velocity of the solar system had been adopted the value

18.45 kilom. pro second,

which value lies at the base of Solution V in Publication 11. The numbers of Publication 8 must be multiplied by 0.905; COMSTOCK's value for magnitude 10.5 with 1.08. We get

TABLE V. PARALLAXES.

Mag.	π Obs.	π Computed			O — C		
		Form (12)	COMSTOCK	Publ. 8	Form (12)	COMSTOCK	Publ. 8
	"	"	"	"	"	"	"
2.7	0.0383	0.0376	0.0343	0.0326	+0.0007	+0.0040	+0.0057
4.1	.0205	.0224	.0211	.0218	— .0019	— .0006	— .0013
5.1	.0147	.0159	.0155	.0163	— .0012	— .0008	— .0016
6.0	.0129	.0119	.0121	.0126	+ .0010	+ .0008	+ .0003
6.9	.0089⁵	.0092⁵	.0096	.0097	— .0003	— .0006⁵	— .0007⁵
8.6	.0063	.0059⁵	.0067	.0060	+ .0003⁵	— .0004	+ .0003
10.5	0.0048⁵	0.0039	0.0046	0.0031⁵	+0.0009⁵	+0.0002⁵	+0.0014

I have added the values derived by COMSTOCK in his paper referred to above, and the values computed with the empirical formula of Publication 8.

We see that the observations are represented at least as well by our formula (12) as by COMSTOCK's formula in which an absorption of light in space is introduced, and decidedly better than by our own empirical formula, on which the calculations of Publication 11 have been based. Already this circumstance will make a second approximation necessary for the results of that publication.

In conclusion, I think it will be admitted that, neither in the number of stars of various magnitudes, nor in their mean parallax, we have the slightest indication of an appreciable absorption of light.

Must we conclude that there is no sensible extinction?

I think not, even if the agreement found just now between theory and observation were far more perfect.

The computations of Publication 11 have been carried through in the assumption of perfect transparency of space. I have not yet tried how the results will be changed if from the outset we introduce a certain amount of extinction. As will presently appear there is a considerable difficulty in the way of such a trial.

It seems hardly doubtful, however, that if the amount assumed is small, we shall succeed nearly as well in representing the numbers N_m and π_m as in the hypothesis of no extinction at all. We shall find, however, other results both for the luminosity-curve and for the star-density; the former will be relatively little modified, the latter will be changed fundamentally; so much so, that even with an amount of the extinction such as that found by COMSTOCK, we shall be led, for the greater distances, to quite inadmissible densities.

If, therefore, we may represent the N_m and the π_m nearly equally well with largely different values of the star-densities, merely by introducing a corresponding value of the extinction, *vice versa*, it is evident that we cannot hope to determine the law of the densities from these quantities alone. It seems pretty clear that of the two unknown elements, absorption and star-density, we can make a fairly approximate determination of the one as soon as the other is given, and this even with the data at present available.

There remains to find a method which will allow the determination of the one independently of the other. The following method for the determination of the extinction fulfils this requirement. It rests on the hypothesis that $\psi(z)$ is the same at different distances from the sun; that is, it assumes that intrinsically small and bright stars are mixed in the same proportions at various distances from the sun. Not only does this hypothesis seem more plausible than any hypothesis we can make *a priori* about the law of the star-densities, but it is susceptible of proof as soon as we shall be in the possession of the necessary data of numbers and proper motions of sufficiently faint stars.

Let us suppose in the first place that there is no extinction; suppose further, that we could isolate the stars having a determined parallax π. With any determined *apparent* magnitude would correspond a determined *absolute* magnitude, and simple counts of the apparent magnitudes would, apart from a constant, depending on the star-density and the volume of the shell, at once furnish us

with the absolute-magnitude curve, by which I will denote the curve of which the abscissas are the absolute magnitudes (M), the ordinates the logarithms of the corresponding frequencies ($\log\varphi$). The curve, as found in Publication 11, is here already contained in Table I.

Now imagine in the same way isolated the stars having the parallaxes

$$\pi\delta^{\frac{1}{2}}\ ,\ \pi\delta\ ,\ \pi\delta^{\frac{3}{2}}\ \ .\ .\ .\ .$$

The first of these shells will now also furnish the absolute-magnitude curve, which, when we assume as above that the distribution of the luminosities is independent of the distance, will be identical with the former, only it will be displaced parallel to itself for just *one magnitude*. All the other shells will give identical curves; they will be displaced respectively 2 magnitudes, 3 magnitudes, etc.

In the second place, suppose that there is absorption. Then the apparent magnitude of the stars in the various shells will still be distributed in identical frequency-curves, which are nothing else than the curve of the absolute magnitudes: only the displacement will not now be *one* magnitude between two consecutive shells, but

1 mag. + absorption in magnitudes between π and $\pi\delta^{\frac{1}{2}}$
1 " + " " " " $\pi\delta^{\frac{1}{2}}$ and $\pi\delta$ etc.

Therefore, the displacement of the several curves will at once show whether there is absorption, and will give the means of measuring its amount.

Now, owing to our very restricted knowledge of the parallaxes, we are certainly unable as yet to assign to the individual stars their proper shells, but still, considerations like those of Publication 11, based on the proper motions and magnitudes, which cannot here be repeated, will allow us to determine, roughly, what number of stars of the several magnitudes will be contained in every shell. This being sufficient for the present purpose, we must be able, at least theoretically, to determine the amount of the absorption.

To show my meaning more clearly, and at the same time to see how far the data at present available will allow us to go, I will here present the number of stars contained in the spherical shells limited by the parallaxes 0".00100, 0".00158, etc. A table of such numbers is given in Publication 11 (Table II). This table, however, was obtained with the aid of the constants of what there is called Solution I. The present table is based on the improved values of Solution V.

The magnitudes below 3.1 were neglected: their number is too insignificant. Logarithms are given instead of the numbers themselves:

Table VI. Log Number of Stars.

π Limits	π Mean	Apparent Magnitudes 3.1	4.1	5.1	6.1	7.1	8.1	9.1
" " <0.00100		9.778:	0.699:	1.398	2.104	2.847	3.685	4.401
0.00100–0.00158	0.00118	0.301	0.903	1.623	2.291	2.940	3.662	4.289
.00158– .00251	.00187	0.162	1.292	1.964	2.567	3.166	3.782	4.377
.00251– .00398	.00296	0.973	1.471	2.179	2.780	3.344	3.864	4.415
.00398– .00631	.00469	1.167	1.708	2.318	2.911	3.443	3.920	4.373
.00631– .0100	.00743	1.292	1.810	2.408	2.947	3.441	3.766	4.230
.0100 – .0158	.0118	1.358	1.862	2.380	2.885	3.318	3.617	3.974
.0158 – .0251	.0187	1.328	1.852	2.279	2.730	3.093	3.332	3.620
.0251 – .0398	.0296	1.236	1.757	2.115	2.493	2.763	2.950	3.157
.0398 – .0631	.0469	1.072	1.592	1.850	2.162	2.371	2.500	2.618
.0631 – .100	.0743	0.813	1.352	1.533	1.750	1.927	1.997	2.027
0.100 – .158	.118	0.505	1.049	1.140	1.260	1.467	1.476	1.435
>0.158	0.204	0.301	0.806	0.881	1.053	1.158	1.149	1.083

According to what has been said, every line of this table gives the ordinates of the absolute-magnitude curve increased by some constant number, different for each line. We have to see how much it is displaced if we pass from one line to another. This can only be done by noting the exact place of some definite point in the successive curves. In the absence of some well-defined maximum or minimum such a determined point can only be recognized by the inclination of its tangent.

We will thus have to determine for the several curves, at what magnitude the differential quotients, instead of which we will here take the first differences, have certain determined values.

We thus begin by forming the differences of the consecutive values of any one line in the preceding table. Finding these very irregular, I have first thoroughly smoothed them. I thus obtained the following differences, D. The first and the last line, which gave precious indications for the smoothing process, are now omitted as being too uncertain:

Table VII. Difference D.

π	Apparent Magnitudes 3.6	4.6	5.6	6.6	7.6	8.6
0″.00118	0.702	0.690	0.677	0.663	0.648	0.631
.00187	.675	.660	.644	.626	.606	.584
.00296	.644	.626	.606	.584	.560	.532
.00469	.612	.591	.567	.540	.509	.474
.00743	.579	.553	.524	.492	.454	.409
.0118	.543	.512	.477	.435	.386	.330
.0187	.498	.461	.416	.364	.305	.238
.0296	.452	.405	.351	.289	.218	.135
.0469	.402	.348	.286	.214	.130	.040
.0743	.359	.298	.228	.146	.056	−.034
0.118	0.320	0.254	0.177	0.089	−0.001	−.091

Care was taken, of course, to get, in the smoothed curves, the differences of the D equal for equal values of D; besides the mean amount of the values of any one line will be found to be practically the same as that of the unsmoothed differences of Table VI, provided that the weight of the numbers corresponding to the magnitudes 3.6, 7.6, 8.6, which are the most uncertain, is assumed to be 0.36 that of the others.

By interpolation I got from this table the magnitudes corresponding to the differences $D = 130, 180, \ldots$; that is, the magnitudes corresponding to one and the same point of the absolute-magnitude-curve, characterized by a first difference 130, to the point characterized by the difference 180, etc.

We thus finally obtain the following summary:

Table VIII. Identical Points of Absolute-Magnitude-Curve.

π	$D = 130$	180	230	280	330	380	430	480	530	580	630
0″.00118	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	8.6^5
.00187	. .	. .	. .	. .	. .	. .	. .	. .	. .	8.8	6.4
.00296	. .	. .	. .	. .	. .	. .	. .	. .	8.7	6.8	4.4
.00469	. .	. .	. .	. .	. .	. .	. .	8.5	6.9	5.1	. .
.00743	. .	. .	. .	. .	. .	. .	8.1	6.9	5.4	3.6	. .
.0118	. .	. .	. .	. .	8.6	7.7	6.7	5.5	4.0	. .	. .
.0187	. .	. .	8.7	8.0	7.2	6.3	5.3	4.1	. .	. .	. .
.0296	8.6^5	8.1	7.4	6.7	5.9	5.1	4.1	. .	. .	. .	. .
.0469	7.6	7.0	6.1	5.7	4.9	4.0	. .	. .	. .	. .	. .
.0743	6.8	6.2	5.6	4.9	4.1	3.2 :	. .	. .	. .	. .	. .
0.118	6.1	5.5^5	4.9	4.2	. .	. .	. .	. .	. .	. .	. .
Greatest displac.	2.5^5	2.5^5	3.8	3.8	4.5	4.5	4.0	4.4	4.7	5.2	4.2^5
Theo. no absorpt.	3.0	3.0	4.0	4.0	4.0	4.0	3.0	3.0	3.0	3.0	2.0
" abs. Comst.	3.5	3.5	4.8	4.8	5.3	5.3	4.8	5.9	7.5	10.2	11.2

At the bottom of the table have been given, in the first line, the greatest observed displacement of the curve; in the second, the corresponding theoretical displacement required in the theory of perfect transparency of space; in the third, the theoretical displacement required if we accept the absorption found by Comstock.

The absorption for a star of parallax π (seconds of arc) has been computed by the formula

$$\text{(14)} \quad \text{absorption expressed in magnitudes} = \frac{a}{10\pi}$$

a representing, in magnitude, the loss of brightness of a star when its light traverses the unit of distance (corresponding to $\pi = 0''.1$).

The result of Comstock, expressed in this way, is

$$\text{(15)} \qquad a = 0.18$$

In reasoning on these quantities we have to bear in mind that our tables are very uncertain for the very smallest parallaxes; and further that, on account of the smallness of the curvature of our curves for these same parallaxes, evident from the smallness of the D, the values corresponding to the differences 530, 580, 630, are still far more uncertain than the rest.

It is further evident that, even apart from any possible influence of absorption, our tables show still very appreciable traces of systematic error, due no doubt partly to the imperfectness of our empirical formulas for the distances borrowed from Publication No. 8, for the greater part, however, to the very incomplete data furnished by observation. It is evident how much better results we would have been able to reach if, in the discussions of Publication 11, the data for the proper motions of the stars of the 7th, 8th and 9th magnitude had been as reliable as those of the brighter classes, and, *à fortiori*, if we had disposed of a somewhat extensive series of such determinations as those given by Comstock for stars of the magnitude 10 to 12.

As matters stand it seems unsafe to draw any more definite conclusions than the following:

Our numbers do not support the theory of an absorption of the amount found by Comstock. They are well reconcilable with an absorption of, say, one-third the amount.

I am not inclined, however, to admit even an absorption of the latter amount, at least, before we possess much more cogent reasons.

The reason, already mentioned above, is that, as soon as we admit the existence of a somewhat considerable extinction of light, we must at the same time admit a star-density which, at great distances from the sun, increases with enormous rapidity. These densities will be derived in the manner explained in Publication 11. We may, however, get a rough estimate, almost without computation, by the aid of the following

PROPOSITION.

If the absolute-magnitude-curve were an infinite straight line, then the number of stars N_m and the mean parallaxes π_m are equally well represented, whatever value a of the absorption-coefficient we adopt, provided we assume the densities at different distances from the sun to be greater than those obtained in the hypothesis $a = 0$, in the proportion of 1 to

$$\beta^{\frac{a}{10\pi}}$$

where

$$\beta = \frac{\text{number of stars of absolute mag. } M+1}{\text{,, \quad ,, \quad ,, \quad ,, \quad } M} \qquad (16)$$

which, in our supposition, will be a constant quantity.

The demonstration is extremely simple.

Imagine an infinitely thin spherical shell around the sun as a center. The apparent magnitudes of the stars in this shell will differ by a constant quantity from their absolute magnitudes. Now, as in our supposition the numbers of the stars of absolute magnitude . . . $M-1$, M, $M+1$ . . . . will form an infinite geometrical series, we shall also have first in the case of absolute transparency,

$$\left.\begin{array}{lllllllll} \text{Number of stars} & \ldots & \kappa\beta^{-1}, & \kappa, & \kappa\beta & \ldots & \kappa\beta^{p-1}, & \kappa\beta^{p}, & \kappa\beta^{p+1} \quad \ldots \\ \text{of app. mag.} & \ldots & m-1, & m, & m+1 & \ldots & m+p-1, & m+p, & m+p+1 \quad \ldots \end{array}\right\} \quad (a)$$

If these stars are viewed through a medium absorbing p magnitudes, all the magnitudes will be increased by p, and we shall have the same number of stars as before, only the number κ which belonged to the magnitude m, will now belong to magnitude $m+p$, etc. So that we will have

$$(C)$$

$$\begin{array}{lllll} \text{Number of stars} & \ldots & \kappa\beta^{-1}, & \kappa & \kappa\beta \quad \ldots \\ \text{of apparent mag.} & \ldots & m+p-1, & m+p, & m+p+1 \quad \ldots \end{array}$$

If now, however, we imagine the star-density of the shell increased in the proportion of 1 to β^p, or, according to (14) in the proportion of

$$1 \text{ to } \beta^{\frac{a}{10\pi}}$$

the numbers (C) will be multiplied by β^p, and there will be

$$\begin{array}{lllll} \text{Number of stars} & \ldots & \kappa\beta^{p-1}, & \kappa\beta^{p} & \kappa\beta^{p+1} \quad \ldots \\ \text{of apparent mag.} & \ldots & m+p-1, & m+p, & m+p+1 \quad \ldots \end{array}$$

that is, the series extending to infinity both ways, the number of stars of the several magnitudes will have become identical with the numbers in the case (a) of absolute transparency of space.

As the same reasoning holds for any spherical shell, the truth of the proposition becomes at once evident.

We take advantage of it in order to obtain an estimate of the densities, at various distances from the sun, to which we would be led by the more refined process of Publication 11, when, instead of $a = 0$, we assume a sensible value of this coefficient. For this purpose we have only to remark that, according to Table 1, the curve of the absolute magnitudes does not deviate very strongly from a straight line, for which $\beta = 3.8$, for the stars exceeding in brightness those of absolute magnitude $+2.45$.

Now, in applying the method of Publication 11, not only the total number of stars N_m, and the mean parallax π_m will be well represented, but also the respective number of stars of different apparent magnitude, *contained in spherical shells included between determined values of* π. Any theory, to be acceptable, must be able to do that. Such numbers, therefore, as those of Table VI, which have been derived from considerations completely independent of the existence or non-existence of absorption, must be *separately* represented.

If we confine our attention to stars of apparent magnitude 9.0 or brighter, the only stars of which the data furnished by observation are at all entitled to confidence, we find from the formulas,

$$\left.\begin{array}{ll} M = m+5+5\log\pi & \text{(no absorption)} \\ M = m+5+5\log\pi - \dfrac{a}{10\pi} & \text{(absorption coeff. } = a) \end{array}\right\} \quad (17)$$

that, whether there be absorption or not, the absolute brightness of these stars must exceed that of absolute magnitude 2.45 as long as $\pi < 0''.0049$.

Therefore, so far as the stars 9.0 or brighter are concerned, the absolute-magnitude-curve is approximately a straight line ($\beta = 3.8$) for all parallaxes below $0''.0049$.

For these stars, accordingly, our proposition will approximately hold. Applying it, we have only to multiply the numbers of Table II by $3.8^{\frac{a}{10\pi}}$ to find the following densities, which must at least give a fair idea of the order of the quantities to which a more rigorous process would lead.

TABLE IX. DENSITIES FOR VARIOUS VALUES OF a.

								COMSTOCK	
π	$a=0.00$	0.01	0.03	0.05	0.10	0.15	0.20	0.180	0.016
0.00118	0.162	0.50	1.8	46	13 000	3 800 000	1 000 000 000	110 000 000	0.99
.00187	0.292	0.60	2.1	10	370	13 000	160 000	110 000	0.92
.00296	0.465	0.73	1.8	4	40	400	3 900	1 600	0.96
.00469	0.681	0.91	1.6	3	12	50	200	120	1.08
.000	1.000	1.00	1.0	1	1	1	1	1	1.00

The table well shows the sort of difficulties into which we get if we assume an extinction of light of the order of that found by COMSTOCK.

We shall not even be able to ascertain whether or not the results of such an assumption lead to the right results for the parallaxes and the numbers of stars, because we do not know how to prolong the series of values for the density for still smaller parallaxes. If we try to extrapolate, we soon will be led to such extravagant values that their reality becomes a physical impossibility.

But even if we stop at parallaxes of 0''.003, it must be admitted that we have already to do with densities which are in the highest degree improbable, especially as they require us to assume for the sun a very exceptional position in space.

For smaller values of a the difficulty diminishes rapidly. Still its value must have diminished to about 0.016 before we reach somewhat uniform density. There thus can be no serious doubt but that a, if it reaches a sensible amount, must be of the order of a few hundredths. The difficulties in the way of the exact determination of so small a quantity must be evident from what precedes. Yet on this determination will depend our knowledge of the true star-density at various distances from the sun. It therefore becomes unavoidable to consider in what way those difficulties can be removed.

What we need most of all is evidently a great number of accurate proper motions of very faint stars. COMSTOCK has shown how we can obtain, even now, very reliable materials.

In a paper now going through the press, by DE SITTER and myself, were derived proper motions from photographs taken with an interval of four or five years. The results prove that with photographs taken with an interval of, say, fifteen years, we can get, with relatively very little trouble, proper motions exceeding in accuracy the proper motions of the BRADLEY stars. It may not seem too sanguine, therefore, to hope that in the near future some astronomer will undertake the work of determining accurate proper motions of some thousands of stars from magnitude 8 downward to stars as low in the scale as can be photographed in a reasonable time. This work would be of inestimable value even apart from the determination of the extinction of light, and I, for my part, most cordially agree with the opinion of Prof. COMSTOCK, that such proper motions will furnish precisely the kind of data most needed "for promoting our knowledge of the structure of the stellar system."

Groningen, 1904 *May*.

EPHEMERIS OF ENCKE'S COMET,

BY KAMINSKY AND OCOULITSCH (from *A.N.* 3962).

0h Berlin M.T.

1904	α app.	δ app.	log r	log Δ	Ab. T.
	h m s	° '			
Aug. 13	1 51 3	+21 10.2	0.3685	0.2634	15 14
15	51 41	+21 27.5	0.3650	0.2528	14 52
17	52 13	+21 45.0	0.3615	0.2421	14 30
19	52 38	+22 2.4	0.3579	0.2312	14 9
21	52 57	+22 19.8	0.3542	0.2201	13 48
23	53 9	+22 37.4	0.3502	0.2087	13 27
25	53 14	+22 55.1	0.3467	0.1970	13 5
27	53 11	+23 12.7	0.3429	0.1852	12 44
29	52 59	+23 30.4	0.3390	0.1732	12 23
31	52 39	+23 48.2	0.3350	0.1609	12 2
Sept. 1	52 25	+23 56.9	0.3330	0.1547	11 52
2	1 52 9	+24 5.7	0.3309	0.1485	11 42

1904	α app.	δ app.	log r	log Δ	Ab. T.
	h m s	° '			
Sept. 3	1 51 50	+24 14.5	0.3289	0.1422	11 32
4	51 28	+24 23.4	0.3268	0.1358	11 22
5	51 4	+24 32.3	0.3247	0.1294	11 12
6	50 36	+24 41.2	0.3226	0.1229	11 2
7	50 6	+24 50.0	0.3205	0.1163	10 52
8	49 33	+24 58.9	0.3183	0.1097	10 42
9	48 57	+25 7.6	0.3161	0.1031	10 32
10	48 18	+25 16.3	0.3139	0.0964	10 22
11	47 35	+25 25.1	0.3117	0.0896	10 12
12	46 38	+25 33.8	0.3094	0.0828	10 3
13	45 58	+25 42.5	0.3072	0.0759	9 53
14	1 45 5	+25 51.1	0.3049	0.0690	9 44

CONTENTS.

PUBLISHED AT 16 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed August 17.*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 567.

VOL. XXIV. BOSTON, 1904 SEPTEMBER 6. NO. 15

DEVELOPMENT OF FUNCTIONS IN POWER SERIES FROM SPECIAL VALUES.

By G. W. HILL.

It is often desirable to develop complicated functions in powers and products of small parameters, and frequently the readiest method for accomplishing this is the derivation of the coefficients from values of the function corresponding to definite values of the parameters. In case we mean to retain all the terms of the development corresponding to definite powers of each parameter, regardless of the order of smallness of the resulting terms, the course to be pursued is plain; but when we wish to retain only terms of certain degrees of smallness, the process to be followed is not so evident. So far as I am aware there does not exist any treatment of the subject with the mentioned limitation. A general exposition of the principles involved in the method would doubtless demand a complicated notation. But the details of the process in a special case will readily suggest what ought to be done in any other. An example from astronomy will illustrate the matter.

Let W be a function of the radii of two planets. In GYLDÉN's method of treating planetary motion it is well known that the latter are to be replaced by the substitutions

$$r = a\,\frac{1-\eta^2}{1+\eta\cos F}\,, \quad r' = a'\,\frac{1-\eta'^2}{1+\eta'\cos F'}\,,$$

where a and a' are constants having known or assumed values, but the other symbols denote variables, η and η' being always of the first order of smallness. For our purposes it will be more convenient to write these substitutions thus:

$$r = a\,\frac{1-y}{1+x}\,, \quad r' = a'\,\frac{1-y'}{1+x'}\,,$$

where x and x' are of the first order, and y and y' of the second. It is required to develop W in a series of powers and products of the four parameters x, x', y, y', it being granted that all terms of an order beyond the eighth may be neglected. The questions are: How many special values of W is it necessary to compute, and for what combination of values for the parameters, and how shall the elimination be conducted if unnecessary labor is to be avoided?

First, we note the number of terms in the development. They are

1	=	1 term of the zero order
2	=	2 terms of the first "
3 + 2.1	=	5 terms of the second "
4 + 2.2	=	8 terms of the third "
5 + 2.3 + 3.1	=	14 terms of the fourth "
6 + 2.4 + 3.2	=	20 terms of the fifth "
7 + 2.5 + 3.3 + 4.1	=	30 terms of the sixth "
8 + 2.6 + 3.4 + 4.2	=	40 terms of the seventh "
9 + 2.7 + 3.5 + 4.3 + 5.1	=	55 terms of the eighth "
Sum	=	175

It will be perceived that the number of terms in each order is not a continuous function of the degree of the order; this is because the parameters are not all of the same order. It appears then, that 175 special values of W, provided they correspond to combinations of values of the four parameters having a determinate quality, will enable us to discover the coefficients we are in quest of. The elimination necessary in this method is much facilitated when the several values of the same parameter form an arithmetical progression of which one term is zero; we accordingly adopt this restriction. Let us suppose that the common difference of the values of the parameters x and x' is d, and that of y and y' is d'. Then it is evident that our choice of combinations may be limited to those indicated by the following scheme, where to this is added an exemplification for $d = 0.02$ and $d' = 0.0025$.

$\frac{x}{d}$	$\frac{x'}{d}$	$\frac{y}{d'}$	$\frac{y'}{d'}$	x	x'	y	y'
4	4			0.08	0.08		
3	3			0.06	0.06		
2	2	2	2	0.04	0.04	0.0050	0.0050
1	1	1	1	0.02	0.02	0.0025	0.0025
0	0	0	0	0.00	0.00	0.0000	0.0000
1	1	1	1	0.02	0.02	0.0025	0.0025
2	2	2	2	0.04	0.04	0.0050	0.0050
3	3			0.06	0.06		
4	4			0.08	0.08		

Every one of these specified values for the parameters

must be included in our combinations; but we need not employ every combination arising from the preceding scheme. For the latter are in number $= 9\times9\times5\times5 = 2025$, between 11 and 12 times the number 175, which we have seen to be necessary. Also, by the limitation that no terms beyond the 8th order are to be considered, it results that certain selections of combinations do not afford independent relations between the coefficients. We must avoid such selections. This constitutes the delicate step of the problem.

A system of 175 linear equations with as many unknowns would be unmanageable; we therefore propose to break the coefficients, to be determined, into groups which can be treated separately. It is evident that, as soon as any group of coefficients is determined, it is possible to estimate the values of the terms of W which involve them, correspondent to any values we please of the four parameters. We therefore suppose that, in the treatment of any group, from the special values of W are always subtracted the correspondent special values of the terms involving the coefficients of all the preceding groups; so that the remainders constitute the special values of the function equivalent to the terms still to be determined.

We may write (using the symbol A for the general coefficient),

$$W = \Sigma A\left(\frac{x}{d}\right)^i\left(\frac{x'}{d}\right)^{i'}\left(\frac{y}{d'}\right)^j\left(\frac{y'}{d'}\right)^{j'}$$

where the exponents are integers not negative. It is preferred to determine the coefficients under this form because then the factors of the unknowns in the equations are always integers. After the A are got in this way, the coefficients, corresponding to the form

$$W = \Sigma A x^i x'^{i'} y^j y'^{j'}$$

are obtained simply by dividing each A by the factor $d^{i+i'} d'^{j+j'}$ which belongs to it.

The separation of the groups just mentioned is defined by the vanishing or non-vanishing of the exponents i, i', j, j'. Sub-groups may be formed upon the parity or imparity of the same exponent; by taking half the sum and difference of two equations in which one of the parameters receives in one case a positive value, and in the other the correspondent negative value, the values of the others remaining the same, it is evident the equations will be broken into two involving separate classes of coefficients. This operation repeated again with two more equations, and in reference to another parameter, will be called the disintegrating operation.

Each special combination of values for the parameters for each computed value of W will be indicated by writing the correspondent values of

$$\frac{x}{d}, \frac{x'}{d}, \frac{y}{d'}, \frac{y'}{d'}$$

in sequence; and we shall number these combinations in the order in which they come into use from 1 to 175. Any four combinations to be subjected to the disintegrating operation will be bracketed. When we have occasion to note a group of equations, only the multipliers of the unknowns which are integral will be set down; the sign will be indicated only when it is negative: the absolute term and the sign of equality will be omitted.

Group I is established by the condition $i = i' = j = j' = 0$, and the correspondent coefficient of W is the value for the argument

No. of Comb.	$\frac{x}{d}$	$\frac{x'}{d}$	$\frac{y}{d'}$	$\frac{y'}{d'}$
1	0	0	0	0

Group II is defined by the condition that three out of the four exponents vanish. Four sub-groups are formed by considering which one of the four has the finite value. The combinations to be used are (with the sub-groups indicated),

No. Comb.	Argument	No. Comb.	Argument	No. Comb.	Argument	No. Comb.	Argument
2	−4 0 0 0	10	0 −4 0 0	18	0 0 −2 0	22	0 0 0 −2
3	−3 0 0 0	11	0 −3 0 0	19	0 0 −1 0	23	0 0 0 −1
4	−2 0 0 0	12	0 −2 0 0	1	0 0 0 0	1	0 0 0 0
5	−1 0 0 0	13	0 −1 0 0	20	0 0 1 0	24	0 0 0 1
1	0 0 0 0	1	0 0 0 0	21	0 0 2 0	25	0 0 0 2
6	1 0 0 0	14	0 1 0 0				
7	2 0 0 0	15	0 2 0 0				
8	3 0 0 0	16	0 3 0 0				
9	4 0 0 0	17	0 4 0 0				

The 24 coefficients of the pure powers of the parameters are obtained by differencing the special values of W for the arguments in each group. In the case of the differences of odd orders we suppose half the sum of the adjacent differences to belong to the argument of the function on the same line. Let D^n denote the coefficient belonging to the n^{th} power of the parameter considered. We write in the formulas only the differences necessary in our special case; and thus:

$$1!\,D = \Delta - \tfrac{1}{6}\,\frac{\Delta^3}{2} + \tfrac{1}{30}\,\frac{\Delta^5}{2^2} - \tfrac{1}{140}\,\frac{\Delta^7}{2^3}$$

$$2!\,D^2 = \Delta^2 - \tfrac{1}{12}\,\frac{\Delta^4}{2} + \tfrac{1}{90}\,\frac{\Delta^6}{2^2} - \tfrac{1}{560}\,\frac{\Delta^8}{2^3}$$

$$3!\,D^3 = \Delta^3 - \tfrac{1}{4}\,\Delta^5 + \tfrac{7}{120}\,\Delta^7$$

$$4!\,D^4 = \Delta^4 - \tfrac{1}{6}\,\Delta^6 + \tfrac{7}{240}\,\Delta^8$$

$$5!\,D^5 = \Delta^5 - \tfrac{1}{3}\,\Delta^7$$

$$6!\,D^6 = \Delta^6 - \tfrac{1}{4}\,\Delta^8$$

$$7!\,D^7 = \Delta^7$$

$$8!\,D^8 = \Delta^8$$

Group III contains the remaining terms of W for which $j = j' = 0$. Their number is $\frac{7\cdot 8}{2} = 28$, and, involving only x and x', they are, in every [illegible] xx'. After this division, they [illegible]

$$\begin{aligned}
&A\\
&+A_1 x + A_2 x'\\
&+A_3 x^2 + A_4 xx' + A_5 x'^2\\
&+A_6 x^3 + A_7 x^2x' + A_8 xx'^2 + A_9 x'^3\\
&+A_{10} x^4 + A_{11} x^3x' + A_{12} x^2x'^2 + A_{13} xx'^3 + A_{14} x'^4\\
&+A_{15} x^5 + A_{16} x^4x' + A_{17} x^3x'^2 + A_{18} x^2x'^3 + A_{19} xx'^4 + A_{20} x'^5\\
&+A_{21} x^6 + A_{22} x^5x' + A_{23} x^4x'^2 + A_{24} x^3x'^3 + A_{25} x^2x'^4 + A_{26} xx'^5 + A_{27} x'^6
\end{aligned}$$

In deriving the values of the A we cannot suppose that either $x = 0$ or $x' = 0$, as then the division by xx' becomes nugatory. But by taking the six groups of four combinations each,

No. 26	−1	−1	0	0	No. 30	−2	−1	0	0	No. 34	−1	−2	0	0
27	1	1	0	0	31	2	1	0	0	35	1	2	0	0
28	−1	1	0	0	32	−2	1	0	0	36	−1	2	0	0
29	1	−1	0	0	33	2	−1	0	0	37	1	−2	0	0
No. 38	−2	−2	0	0	No. 42	−3	−1	0	0	No. 46	−1	−3	0	0
39	2	2	0	0	43	3	1	0	0	47	1	3	0	0
40	−2	2	0	0	44	−3	1	0	0	48	−1	3	0	0
41	2	−2	0	0	45	3	−1	0	0	49	1	−3	0	0

and applying to each of these the disintegrating operation, we have, in the first place, the following equations determining $A_1, A_6, A_8, A_{15}, A_{17}, A_{19}$:

1	1	1	1	1	1
1	4	1	16	4	1
1	1	4	1	4	16
1	4	4	16	16	16
1	9	1	81	9	1
1	1	9	1	9	81

Then by elimination in order, commencing at the left, we obtain the following groups:

1	0	5	1	0
0	1	0	1	5
1	1	5	5	5
1	0	10	1	0
0	1	0	1	10

1	0	1	5
0	0	4	5
0	5	0	0
1	0	1	10

0	4	5
5	0	0
0	0	5

4	5
0	5

By returning on this elimination we evidently can get the values of the six mentioned coefficients. Next, it is plain that the six coefficients $A_2, A_7, A_9, A_{16}, A_{18}, A_{20}$, and again the group of six, $A_4, A_{11}, A_{13}, A_{22}, A_{24}, A_{26}$ are determined by equations having the same integral coefficients as in the group just treated, the absolute terms being generally different. There still remains to be determined the group of ten coefficients, $A, A_3, A_5, A_{10}, A_{12}, A_{14}, A_{21}, A_{23}, A_{25}, A_{27}$. The six groups of four combinations afford each one equation for this purpose. Then four additional equations are necessary. We select the following four combinations for giving these equations:

No. 50	3	2	0	0
51	2	3	0	0
52	4	1	0	0
53	1	4	0	0

As we know the values of 18 coefficients of the group we can subtract from the special values of W the special values of the corresponding terms. We thus obtain 10 equations involving only the last group of 10 coefficients. They are as follows:

1	1	1	1	1	1	1	1	1	1
1	4	1	16	4	1	64	16	4	1
1	1	4	1	4	16	1	4	16	64
1	4	4	16	16	16	64	64	64	64
1	9	1	81	9	1	729	81	9	1
1	1	9	1	9	81	1	9	81	729
1	9	4	81	36	16	729	324	144	64
1	4	9	16	36	81	64	144	324	729
1	16	1	256	16	1	4096	256	16	1
1	1	16	1	16	256	1	16	256	4096

The elimination being conducted in the mentioned order, we have the groups

1	0	5	1	0	21	5	1	0
0	1	0	1	5	0	1	5	21
1	1	5	5	5	21	21	21	21
1	0	10	1	0	91	10	1	0
0	1	0	1	10	0	1	10	91
8	3	80	35	15	728	323	143	63
3	8	15	35	80	63	143	323	728
1	0	17	1	0	273	17	1	0
0	1	0	1	17	0	1	17	273

1	0	1	5	0	1	5	21
1	0	4	5	0	16	20	21
0	1	0	0	11	1	0	0
1	0	1	10	0	1	10	91
5	10	27	7	560	283	135	63
1	0	4	10	0	16	10	91
0	1	0	0	21	1	0	0
1	0	1	17	0	1	17	273

0	1	0	0	5	5	0
1	0	0	11	1	0	0
0	0	1	0	0	1	11
5	4	4	70	36	20	21
0	3	5	0	15	35	70
1	0	0	21	1	0	0
0	0	1	0	0	1	21

1	0	0	5	5	0		1	0	0	1	11		0	11	−4	−35
0	1	0	0	1	14		4	0	11	0	21		0	0	15	0
4	1	0	31	20	21		1	0	0	4	14		1	0	0	0
3	5	0	15	35	70		0	1	0	0	0		0	0	0	7
0	0	1	0	0	0		1	0	0	1	21					
0	1	0	0	1	21											

By returning on the elimination the values of the 10 coefficients are obtained.

Group IV contains those of the remaining terms of H' for which $i = i' = 0$. They are divisible by yy'; after which they take the form

$$A_0 + A_1y + A_2y' + A_3y^2 + A_4yy' + A_5y'^2$$

Here it is not allowed to suppose that either y or y' vanishes. We may take the group of six combinations

No. 54	0	0	−1	−1	No. 58	0	0	2	1
55	0	0	1	1	59	0	0	1	2
56	0	0	−1	1					
57	0	0	1	−1					

The disintegrating operation, performed on the bracketed four, gives the values of A_1, A_2, A_4 besides one equation between A_0, A_3, A_5. In the two last combinations, by subtracting the terms corresponding to the former coefficients, we have the three equations,

$$\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{array}$$

which evidently suffice for determining A_0, A_3, A_5.

Group V contains those of the remaining terms of H' for which $i' = j' = 0$. These terms are all divisible by xy, after which they take the form

$$\begin{aligned} & A_0 \\ & + A_1x \\ & + A_2x^2 + A_3y \\ & + A_4x^3 + A_5xy \\ & + A_6x^4 + A_7x^2y + A_8y^2 \\ & + A_9x^5 + A_{10}x^3y + A_{11}xy^2 \end{aligned}$$

To determine these 12 coefficients we compute the values of H' for the 8 combinations

No. 60	−1	0	−1	0	No. 64	−2	0	−1	0
61	1	0	1	0	65	2	0	1	0
62	−1	0	1	0	66	−2	0	1	0
63	1	0	−1	0	67	2	0	−1	0

The application of the disintegrating operation furnishes for determining A_3, A_7, and again for A_5, A_{10}, two equations each, which groups are identically the same as far as the numerical multipliers of the unknowns are concerned; they are

$$\begin{array}{c} 1 + 1 \\ 1 + 4 \end{array}$$

The disintegrating operation furnishes besides two equations for determining A_0, A_2, A_6, A_8 and again two for A_1, A_4, A_9, A_{11} which groups are identical as far as the coefficients of the unknowns are concerned. We therefore need four additional values of H', and choose the combinations

No. 68	1	0	2	0
69	−1	0	2	0
70	3	0	1	0
71	−3	0	1	0

From the special values of H' there can be subtracted in addition the values of the terms which correspond to the 8 previously determined coefficients of this group. Thus our equations for determining both groups of four coefficients have the form, and, with those derived by elimination, are

1	1	1	1		3	15	0		0	3
1	4	16	1		0	0	3		40	0
1	1	1	4		8	80	0			
1	9	81	1							

Groups VI, VII and VIII are defined severally by the conditions $i = j' = 0$, and $i' = j = 0$, and $i = i' = 0$; also the equations belonging to them, in respect to the integral factors multiplying the unknowns, are the same. Hence we need only set down the combinations:

Group VI.

No. 72	0	−1	−1	0	No. 76	0	−2	−1	0	No. 80	0	1	2	0
73	0	1	1	0	77	0	2	1	0	81	0	−1	2	0
74	0	−1	1	0	78	0	−2	1	0	82	0	3	1	0
75	0	1	−1	0	79	0	2	−1	0	83	0	−3	1	0

Group VII.

No. 84	−1	0	0	−1	No. 88	−2	0	0	−1	No. 92	1	0	0	2
85	1	0	0	1	89	2	0	0	1	93	−1	0	0	2
86	−1	0	0	1	90	−2	0	0	1	94	3	0	0	1
87	1	0	0	−1	91	2	0	0	−1	95	−3	0	0	1

Group VIII.

No. 96	0	−1	0	−1	No. 100	0	−2	0	−1	No. 104	0	1	0	2
97	0	1	0	1	101	0	2	0	1	105	0	−1	0	2
98	0	−1	0	1	102	0	−2	0	1	106	0	3	0	1
99	0	1	0	1	103	0	2	0	−1	107	0	−3	0	1

We come now to the terms in which only one exponent is zero. Group IX is characterized by the condition $j'=0$; the terms are all divisible by $xx'y$. After this is made the expression under consideration is

$$
\begin{aligned}
&A_0 \\
&+A_1 x + A_2 x' \\
&+A_3 x^2 + A_4 xx' + A_5 x'^2 && + A_{15} y \\
&+A_6 x^3 + A_7 x^2x' + A_8 xx'^2 + A_9 x'^3 && + A_{16} xy + A_{17} x'y \\
&+A_{10} x^4 + A_{11} x^3x' + A_{12} x^2x'^2 + A_{13} xx'^3 + A_{14} x'^4 && + A_{18} x^2y + A_{19} xx'y + A_{20} x'^2y + A_{21} y^2
\end{aligned}
$$

For determining these coefficients we need values of W for 22 combinations. Of the latter we choose first the four groups of four each:

No. 108	1	1	1	0	No. 112	2	1	1	0	No. 116	1	2	1	0	No. 120	3	1	−1	0
109	−1	1	1	0	113	−2	1	1	0	117	−1	2	1	0	121	−3	1	−1	0
110	−1	−1	1	0	114	−2	−1	1	0	118	−1	−2	1	0	122	−3	−1	−1	0
111	1	−1	1	0	115	2	−1	1	0	119	1	−2	1	0	123	3	−1	−1	0

By applying the disintegrating operation to each of the four groups we arrive at four equations determining A_1, A_6, A_8, A_{16}, and at four determining A_2, A_7, A_9, A_{17}, and at four determining A_4, A_{11}, A_{13}, A_{19}. These three groups of equations agree in having the same numerical multipliers for the unknowns. Their general statement, and the consequent steps of elimination, are

1	1	1	1		3	0	0		3	0
1	4	1	1		0	3	0		0	6
1	1	4	1		8	0	−2			
1	9	1	−1							

In addition four equations are afforded for the determination of the 10 remaining coefficients A_0, A_3, A_5, A_{10}, A_{12}, A_{14}, A_{15}, A_{18}, A_{20}, A_{21}. Six more equations are therefore needed to complete the determination of these. We choose the combinations

No. 124	2	2	1	0	No. 126	1	3	1	0	No. 128	2	1	−1	0
125	3	1	1	0	127	1	1	2	0	129	1	2	−1	0

Knowing the values of the 12 coefficients of the group previously determined it is possible to subtract from the absolute terms of the 6 last equations the special values of the terms involving these. Thus we have the following equations with the consequent steps of elimination:

1	1	1	1	1	1	1	1	1	1
1	4	1	16	1	1	1	1	1	1
1	1	4	1	4	16	1	1	1	1
1	9	1	81	9	1	−1	−1	−1	1
1	4	4	16	16	16	1	1	1	1
1	9	1	81	9	1	1	9	1	1
1	1	9	1	9	81	1	1	9	1
1	1	1	1	1	1	2	2	2	4
1	4	1	16	4	1	−1	−1	−1	1
1	1	4	1	4	16	−1	−1	−4	1

1	0	5	1	0	0	0	0	0
0	1	0	1	5	0	0	0	0
4	0	40	4	0	−1	1	−1	0
1	1	5	5	5	0	1	0	0
1	0	10	1	0	0	1	0	0
0	1	0	1	10	0	0	1	0
0	0	0	0	0	1	1	1	3
3	0	15	3	0	2	−5	−2	0
0	3	0	3	15	−2	−2	5	0

1	0	1	5	0	0	0	0
0	20	0	0	−1	−1	−1	0
1	0	4	5	0	1	0	0
0	5	0	0	0	1	0	0
1	0	1	10	0	0	1	0
0	0	0	0	1	1	1	3
0	0	0	0	−2	5	2	0
3	0	3	15	−2	2	5	0

20	0	0	−1	−1	−1	0
0	3	0	0	1	0	0
5	0	0	0	1	0	0
0	0	5	0	0	1	0
0	0	0	1	1	1	3
0	0	0	−2	−5	−2	0
0	0	0	−2	2	−5	0

3	0	0	1	0	0
0	0	−1	−5	1	0
0	5	0	0	1	0
0	0	1	1	1	3
0	0	−2	−5	−2	0
0	0	−2	−2	5	0

0	−1	5	1	0
5	0	0	1	0
0	1	1	1	3
0	2	5	2	0
0	2	2	5	0

1	5	1	0
1	1	1	3
2	5	2	0
2	2	5	0

4	0	3
5	0	0
8	3	0

0	15
1	−2

The 10 equations are therefore independent and suffice for determining the 10 coefficients.

Group X is characterized by the condition $j = 0$; the terms are all divisible by $xx'y'$. The process to be followed is identical with that which just precedes; the numerical coefficients of the equations are the same; we need only set down the combinations to be used:

No. 130	1	1	0	1	No. 134	2	1	0	1	No. 138	1	2	0	1
131	−1	1	0	1	135	−2	1	0	1	139	−1	2	0	1
132	−1	−1	0	1	136	−2	−1	0	1	140	−1	−2	0	1
133	1	−1	0	1	137	2	−1	0	1	141	1	−2	0	1

No. 142	3	1	0	−1	No. 146	2	2	0	1	No. 149	1	1	0	2
143	−3	1	0	−1	147	3	1	0	1	150	2	1	0	−1
144	−3	−1	0	−1	148	1	3	0	1	151	1	2	0	−1
145	3	−1	0	−1										

Group XI is characterized by the condition $i' = 0$; the terms are all divisible by xyy'. The division performed the expression to be treated takes the form

$$A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 y' + A_5 xy' + A_6 y + A_7 xy$$

It will be found that the 8 coefficients here involved can be obtained from the combinations

No. 152	1	0	1	1	No. 156	1	0	1	−1
153	−1	0	1	1	157	−1	0	1	−1
154	−1	0	−1	1	158	2	0	1	1
155	1	0	−1	1	159	−2	0	1	1

The first four give the values of

$$A_0 + A_2 + A_4\ ,\ A_1 + A_3 + A_5\ ,\ A_6\ ,\ A_7$$

the two following the values of

$$A_0 + A_2 - A_4\ ,\ A_1 + A_3 - A_5$$

and the two last the values of

$$A_0 + 4A_2\ ,\ A_1 + 4A_3$$

Group XII is characterized by the condition $i = 0$; the terms are all divisible by $x'yy'$. The coefficients are derived by equations of exactly the same form as in the preceding group. It is only necessary to set down the combinations:

No. 160	0	1	1	1	No. 164	0	1	1	−1
161	0	−1	1	1	165	0	−1	1	−1
162	0	−1	−1	1	166	0	2	1	1
163	0	1	−1	1	167	0	−2	1	1

The last group of terms to be considered is that of the 8 in which all the exponents have finite values. After division by $xx'yy'$ they have the form

$$A_0 + A_1 x + A_2 x^2 + A_3 x' + A_4 xx' + A_5 x'^2 + A_6 y + A_7 y'$$

The following combinations will enable us to arrive at the coefficients:

No. 168	1	1	1	1	No. 172	1	1	−1	1
169	−1	1	1	1	173	1	1	1	−1
170	−1	−1	1	1	174	2	1	1	1
171	1	−1	1	1	175	1	2	1	1

The first four give the values of

$$A_0 + A_2 + A_5 + A_6 + A_7\ ,\ A_1\ ,\ A_3\ ,\ A_4$$

The next two being added we have the values of

$$A_0 + A_2 + A_5\ ,\ A_6\ ,\ A_7$$

The addition of the two last enables us to have the values of A_0, A_2, A_5.

A few words may be added regarding the choice of the elements forming the combined arguments for the special values of W. It is not pretended that that adopted in the preceding is the best. There is a lack of symmetry which, at the moment, I am unable to remove. The latitude in the matter appears to be great; as would be anticipated when 175 things are to be chosen from 2025. It is important, however, to keep the integers multiplying d and d' as small as consists with the condition that the selected combinations afford independent equations. At least, there is no need of going outside of the scheme on page 123.

OBSERVATIONS OF TWO GREAT METEORS.

By E. E. BARNARD.

Two remarkable meteors were observed here on the nights of July 19th and 20th. They were so brilliant that they must have been seen over a wide extent of country. The observations made here, therefore, may be of value in determining the actual paths and distance of the objects.

1904 July 19^d 12^h 13^m ± 1^m (Central Standard Time)

Path from $\alpha = 14^h\ 10^m$, $\delta = +44^\circ$

to $\alpha = 13^h\ 0^m$, $\delta = +43^\circ$

This was the largest meteor I have yet seen. It was a large disc, slightly pear-shaped, and about 15′ in diameter

(estimation, one-half the diameter of the moon), with a trail streaming behind it. The flight was very rapid, the duration of visibility being about three-fourths of a second. There were no sparks, and the meteor did not seem to explode, but suddenly disappeared. There was no persistent train, but a short streak of light near the end of the path remained visible for a couple of seconds.

The meteor was intensely brilliant, and illuminated the surrounding country almost like day. No sound was heard in connection with it.

About ten minutes later a white third-magnitude meteor passed over identically the same path, with about the same duration of visibility.

Yerkes Observatory, Williams Bay, Wisconsin, 1904 July 22.

1904 July $20^d\ 12^h\ 57^m \pm 1^m$ (Central Standard Time)
Path from $\alpha = 1^h\ 40^m$, $\delta = +12^\circ$
to $\alpha = 1^h\ 10^m$, $\delta = +49^\circ$

This meteor did not exhibit any disc, but appeared like an immense star five times as brilliant as *Jupiter*, then in the east, and near its maximum brightness. The meteor was intensely brilliant and white, with a duration of about one second, or one and a half seconds. Though a streaming train accompanied the meteor, there was no persistent trail, and no sparks. There was no explosion at the end of the flight.

Fortunately, in both cases, I was looking in the direction of the meteors when they made their appearance.

OCCULTATIONS OF STARS BY THE MOON.

OBSERVED WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY, BY T. I. KING AND J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

No.	Date	Star	Phenom	Obs.	Sid. Time h m s	Wash. M.T. h m s
1	1901 Oct. 6	BD. +11°1988	DB	K	4 36 0.7	15 34 48.3
2	6	BD. +11°1988	RD	"	5 21 17.3	16 19 57.5
3	6	*κ Cancri*	RD	"	4 53 58.5	15 52 43.2
4	21	*ν Aquarii*	DD	"	2 6 47.8	12 7 1.3
5	24	22 *Piscium*	DD	"	4 13 55.5	14 2 0.4
6	25	*δ Piscium*	DD	"	4 17 12.8	14 1 21.1
7	Nov. 19	Lal. 43777	DD	"	22 50 43.9	6 57 28.4
8	30	*ω Leonis*	DB	"	7 2 13.0	14 24 21.7
9	30	*ω Leonis*	RD	"	8 24 19.4	15 46 14.7
10	1902 Jan. 14	*λ Piscium*	DD	"	3 50 5.9	8 15 50.0
11	19	*ε Tauri*	DD	"	6 47 44.0	10 53 19.5
12	19	*ε Tauri*	RB	"	7 56 31.2	12 1 55.4
13	Feb. 18	*λ Geminorum*	DD	"	11 56 27.4	14 3 15.4
14	18	*λ Geminorum*	RB	"	12 16 2.3	14 22 46.8
15	Mar. 19	*A² Cancri*	DD	"	8 29 37.8	8 42 58.0
16	July 23	Lal. 44872	DB	"	23 58 46.8	15 53 40.3
17	23	Lal. 44872	RD	"	1 18 28.6	17 13 39.0
18	Dec. 17	*A¹ Cancri*	DB	"	3 33 20.9	9 50 40.7
19	17	*A¹ Cancri*	RD	"	4 34 25.3	10 51 5.1
20	1903 Jan. 9	B.A.C. 1272	DD	"	9 49 29.2	14 31 51.5
21	12	*λ Geminorum*	RB	"	10 16 2.7	14 49 32.9
22	Feb. 8	26 *Geminorum*	DD	"	7 32 24.3	10 20 44.7
23	8	26 *Geminorum*	RB	"	8 48 39.6	11 36 44.5
24	May 4	B.D. +14°2788	DD	"	11 35 8.8	8 48 4.4
25	7	B.A.C. 4200	DD	"	16 16 39.9	13 17 1.7
26	June 3	W.B. XII, 69	DD	"	16 23 39.7	11 37 50.8
27	8	*χ Ophiuchi*	DD	"	15 39 20.8	10 33 59.6
28	8	*χ Ophiuchi*	RB	"	17 10 56.3	12 5 20.0
29	15	*θ Aquarii*	DB	"	19 0 30.2	13 27 4.6
30	15	*ρ Aquarii*	DB	"	21 30 33.5	15 56 43.3
31	15	*ρ Aquarii*	RD	"	22 1 39.0	16 27 43.7
32	July 10	B.A.C. 7063	DB	"	22 8 18.8	14 56 4.7
33	Aug. 5	*ρ¹ Sagittarii*	DD	"	18 29 17.2	9 35 25.3
34	5	*ρ¹ Sagittarii*	RB	"	20 3 25.1	11 9 17.8
35	1903 Aug. 7	9 *Aquarii*	DB	K	20 14 56.8	11 12 55.8
36	7	9 *Aquarii*	RB	"	21 9 11.0	12 7 31.0
37	Sept. 5	*ρ Aquarii*	DD	H	20 50 20.6	9 51 12.5
38	5	*ρ Aquarii*	RB	"	22 11 46.2	11 15 21.7
39	6	B.A.C. 8094	RD	"	3 41 38.4	16 40 27.0
40	12	55 *Tauri*	DB	"	2 51 49.0	15 30 9.8
41	12	55 *Tauri*	RD	"	4 19 26.2	16 54 33.1
42	12	63 *Tauri*	DB	"	5 15 26.3	17 50 21.1
43	15	*λ Geminorum*	DB	P	3 39 43.5	16 3 9.3
44	15	*λ Geminorum*	RD	P	4 48 7.6	17 11 22.2
45	Oct. 2	36 *Aquarii*	DD	K	2 29 3.6	13 15 50.5
46	28	8 *Aquarii*	DD	"	0 4 17.4	9 39 14.4
47	Nov. 2	73 *Piscium*	DD	"	5 6 21.2	14 20 49.2
48	11	*ο Leonis*	DB	"	6 56 42.6	15 35 29.4
49	11	*ο Leonis*	RD	"	8 1 50.9	16 40 27.0
50	30	96 *Piscium*	DD	H	3 26 24.5	10 51 3.4
51	30	96 *Piscium*	RB	"	4 43 39.0	12 8 5.3
52	1904 Jan. 3	1 *Cancri*	DB	H	7 9 59.8	12 20 21.2
53	3	1 *Cancri*	RD	"	8 26 37.1	13 36 45.9
54	5	B.A.C. 3398	DB	"	6 29 46.2	11 32 22.3
55	5	B.A.C. 3398	RD	"	7 9 55.3	12 12 21.8
56	25	*ξ Arietis*	DD	"	2 39 59.0	6 24 34.5
57	25	*ξ Arietis*	RB	"	4 3 1.2	7 47 23.0
58	Feb. 22	W B. II, 1033	DD	"	8 19 54.3	10 13 28.7
59	Mar. 22	B.A.C. 1406	DD	"	6 59 48.7	6 59 5.1
60	22	B.A.C. 1406	RB	"	7 28 8.9	7 27 50.5
61	22	*α Tauri*	DD	"	7 59 12.4	7 58 48.9
62	22	*α Tauri*	RB	"	9 5 25.1	9 4 50.8
63	Apr. 1	*ζ Librae*	DB	"	13 53 40.4	13 12 59.8
64	1	*ζ Librae*	RD	"	15 0 48.2	14 19 56.6
65	1	B.A.C. 4772	DB	"	14 26 44.5	13 45 58.5
66	1	B.A.C. 4772	RD	"	15 47 21.7	15 6 22.5
67	2	*σ Librae*	DB	"	15 46 16.5	15 1 24.5
68	21	*λ Geminorum*	DD	"	12 28 15.5	10 29 40.7

In the fourth column DD signifies that the star disappeared at the dark limb of the Moon;
" " " DB " " " " " " bright " " " ;
" " " RD " " " " re-appeared " " dark " " " ; and
" " " RB " " " " " " bright " " " .

Nos. 13, 14, were observed with a power 230; Nos. 4, 5, 6, 10, with a power 153; all the others with a power 370.
Nos. 1, 63, 65, poor observations. Nos. 9, 18, 26, 34, 54, 60, 64, 67, uncertain. Nos. 28, 57, probably late.
Nos. 43, 44, observed by Mr. PETERS.

SECULAR PERTURBATIONS OF THE *EARTH* FROM THE ACTION OF *URANUS*,

BY ERIC DOOLITTLE.

The elements adopted in the following computation are from Dr. G. W. HILL's *New Theory of Jupiter and Saturn*, pages 192, 551, and 109; the mass of *Uranus* has, however, been diminished to $1 \div 22800$ as suggested by Dr. HILL, (*A.J.*, No. 316).

The Earth.	*Uranus*
$\pi = 100°\ 21'\ 39''.73$	$\pi' = 168°\ 15'\ 6''.70$
$i = 0\ \ 0\ \ 0.00$	$i' = 0\ 46\ 20.54$
$\Omega = \ldots\ldots$	$\Omega' = 73\ 11\ 8.00$
$e = 0.01677144$	$e' = 0.0469236$
$n = 1295977''.416$	$n' = 15425''.752$
$\log a = 0.0000000$	$\log a' = 1.2834044$
$m = 1 \div 327000$	$m' = 1 \div 22800$

Epoch 1850.0 G.M.T.

The values of the preliminary constants are as follows:

$I = 0°\ 46'\ 20''.54$	$\log k = p9.9999609$
$II = 27\ \ 7\ 31.73$	$\log k' = p9.9999997$
$II' = 95\ \ 0\ 58.70$	$c = +0.81094500$
$K = 292\ \ 6\ 31.40$	
$K' = 292\ \ 6\ 34.66$	

The orbit of the *Earth* was divided into eight parts with regard to the eccentric anomaly. The approximate tests furnished by comparing the sums of the functions corresponding respectively to the odd and even points of division were satisfied very exactly toward the close of the computation, although in the beginning they were inapplicable. All other known test equations were also applied, and the computation was duplicated from the beginning. The equation, $\sin q \cdot \frac{1}{2}\, A_1^{(s)} + \cos q \cdot B_0^{(s)} = 0$, was found to give the residual, $-0.000\,000\,000\,000\,25$.

If m' is left indefinite, the resulting values of the differential coefficients are as follows:

$$\left[\frac{de}{dt}\right]_{00} = +\ 0''.393\,95661\,m' \qquad \text{log coeff. } p9.595\,4484$$

$$\left[\frac{d\chi}{dt}\right]_{00} = \left[\frac{d\pi}{dt}\right]_{00} = +129.13143\ \ m' \qquad p2.111\,0320$$

$$\left[\frac{dp}{dt}\right]_{00} = +\ 0.539\,88815\,m' \qquad p9.732\,3038$$

$$\left[\frac{dq}{dt}\right]_{00} = -\ 1.7895101\ \ m' \qquad n0.252\,73415$$

$$\left[\frac{dL}{dt}\right]_{00} = -184.51950\ \ m' \qquad n2.266\,0422$$

When the above value is substituted for m', the following results are obtained:

$$\left[\frac{de}{dt}\right]_{00} = +0''.000\,01727\,8801$$

$$\left[\frac{d\chi}{dt}\right]_{00} = \left[\frac{d\pi}{dt}\right]_{00} = +0.005\,66366\,05$$

$$\left[\frac{dp}{dt}\right]_{00} = +0.000\,02367\,9306$$

$$\left[\frac{dq}{dt}\right]_{00} = -0.000\,07848\,7295$$

$$\left[\frac{dL}{dt}\right]_{00} = -0.008\,09296\,04$$

The results found by LEVERRIER are given in the *Annales de l'Observatoire de Paris*, Vol. II, page 59, and Vol. IV, pages 11 and 12; those obtained by NEWCOMB are in the *Secular Variations of the Four Inner Planets*, pages 336 and 377: the values of dp and dq as computed by Dr. HILL are given in the *New Theory of Jupiter and Saturn*, pages 511 and 512. If the various results are all reduced to the above value of m', they will compare with the results of the present computation as follows:

	LEVERRIER	NEWCOMB	HILL	Method of GAUSS
$\left[\frac{de}{dt}\right]_{00}$	$+0''.00002$	$+0''.00002$	$''$	$+0''.000\,01728$
$\left[\frac{d\pi}{dt}\right]_{00}$	$+0.00009$	$+0.00010$	. . . . .	$+0.000\,09499$
$\left[\frac{dp}{dt}\right]_{00}$	$+0.00002$	$+0.00002$	$+0.0000\,237$	$+0.000\,02368$
$\left[\frac{dq}{dt}\right]_{00}$	-0.00008	-0.00008	$-0.0000\,785$	$-0.000\,07849$
$\left[\frac{dL}{dt}\right]_{00}$	-0.0081	. . . .	. . . . .	$-0.008\,09296$

The Flower Observatory, 1904 July 25.

CONTENTS.

PUBLISHED AT 10 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed September 5*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 568.

VOL. XXIV. BOSTON, 1904 NOVEMBER 2. NO. 16

OBSERVATIONS OF MINOR PLANETS AND COMET *a* 1904 (*BROOKS*),

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY.

By J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1904 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(7) *Iris*.					
Jan. 5 9 11 18	1	30 , 6	−0 33.84	− 1 41.5	7 3 13.36	+18 8 57.7	n9.521	0.566	+1.49	−11.8
7 9 56 34	2	30 , 6	−1 11.64	+ 5 8.1	7 0 56.55	+18 5 53.0	n9.379	0.529	+1.51	−11.8
7 10 22 18	3	30 , 6	−1 17.67	− 3 33.6	7 0 55.12	+18 5 51.1	n9.281	0.515	+1.51	−11.8
9 9 54 11	1	30 , 6	+1 21.00	+ 2 26.9	6 58 43.69	+18 3 1.8	n9.352	0.525	+1.53	−11.8
9 10 27 43	5	30 , 6	+2 5.38	− 3 35.1	6 58 12.11	+18 3 0.8	n9.205	0.508	+1.53	−11.8
					(315) *Teridina*.					
Jan. 5 10 12 16	6	30 , 6	−1 38.95	− 3 10.1	7 5 6.62	+ 5 44 57.2	n9.347	0.687	+1.45	−11.7
13 11 41 1	7	30 , 6	+1 56.13	+ 2 0.2	6 56 17.17	+ 5 50 55.0	8.122	0.679	+1.52	−12.6
16 10 57 16	8	24 , 8	+0 37.41	− 5 33.1	6 53 51.91	+ 5 56 27.2	n8.504	0.678	+1.54	−12.9
19 9 33 44	9	29 , 6	+0 55.11	+ 1 51.5	6 51 6.41	+ 6 3 31.8	n9.222	0.681	+1.54	−13.1
19 9 33 6	10	30 , 6	−0 12.62	− 3 54.8	6 51 6.18	+ 6 3 35.1	n9.225	0.681	+1.54	−13.1
					(182) *Elsa*.					
Jan. 13 10 15 51	11	30 , 6	+0 3.33	+ 5 12.2	6 51 12.90	+22 55 18.8	n9.156	0.399	+1.60	11.6
13 10 15 41	12	29 , 6	0 18.19	− 5 26.1	6 51 13.19	+22 55 18.7	n9.157	0.399	+1.60	11.6
14 11 10 25	13	29 , 6	+2 5.19	+ 2 7.5	6 50 10.88	+22 58 24.4	n8.208	0.378	+1.61	−11.5
15 9 24 24	14	30 , 6	+1 11.15	− 1 59.3	6 49 17.06	+23 0 15.4	n9.356	0.428	+1.62	11.4
17 9 10 18	15	30 , 6	+1 0.20	− 1 44.2	6 47 53.13	+23 5 36.9	n9.244	0.405	+1.63	11.1
					(26) *Proserpina*.					
Jan. 13 12 55 5	16	30 , 6	0 34.56	+ 1 18.3	7 6 17.80	+27 27 24.3	9.235	0.278	+1.64	11.8
16 11 50 55	17	30 , 6	−3 0.84	− 2 57.7	7 3 15.30	+27 32 34.2	8.805	0.238	+1.67	−11.7
					(37) *Fides*.					
Jan. 17 11 17 8	18	25 , 5	3 17.57	1 29.2	9 1 22.25	+21 36 0.8	9.399	0.168	+1.40	13.9
19 10 23 56	19	29 , 6	0 38.29	2 59.4	9 2 32.53	+21 43 9.3	9.511	0.506	+1.45	13.9
21 9 50 17	20	30 , 6	+0 24.91	+ 7 12.0	8 57 10.54	+22 0 17.4	9.533	0.511	+1.54	13.9
30 9 1 18	21	30 , 6	+0 2.76	+ 6 35.8	8 51 36.90	+22 20 6.1	n9.570	0.527	+1.62	14.0
Feb. 1 10 3 0	22	30 , 6	+2 13.66	+ 1 12.6	8 19 31.87	+22 26 3.8	9.103	0.452	+1.65	[illegible]
3 10 18 16	23	30 , 6	1 17.39	3 1.6	8 17 30.06	+22 31 31.2	n9.311	0.429	+1.66	13.8
6 9 57 6	24	30 , 6	+1 6.76	+ 3 7.5	8 14 32.10	+22 38 55.2	n9.337	0.432	+1.68	13.6
					(313) *Chaldaea*.					
Jan. 24 11 8 57	25	29 , 6	−0 19.82	+ 3 2.5	9 37 17.51	1 28 21.9	n9.408	0.752	+1.52	11.3
25 11 59 10	26	30 , 6	+0 59.14	− 3 11.6	9 36 10.05	1 21 28.9	n9.200	0.752	+1.54	−11.5
25 11 58 6	27	30 , 6	0 25.57	2 23.3	9 36 10.06	+ 1 21 32.6	n9.206	0.752	+1.54	11.5
30 11 0 1	28	28 , 6	1 35.64	0 39.0	9 33 23.21	0 13 15.7	n9.350	0.716	+1.60	−12.5
Feb. 3 11 11 8	29	30 , 6	+0 13.66	2 21.3	9 30 27.16	0 6 13.1	n9.218	0.711	+1.65	13.2

1904 Washington M.T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ	Red. to App. Pl.	
					(10) *Hygiea.*				
d h m s			m s	″	h m s	° ′ ″		s	″
Jan. 24 12 1 16	30	30 , 6	+1 59.69	– 2 11.3	8 19 20.73	+17 51 7.0	n8.076	0.495	+1.60	–13.4
24 12 24 29	31	30 , 6	+1 47.12	– 1 14.1	8 19 19.96	+17 51 9.8	8.566	0.496	+1.60	–13.4
25 10 34 18	32	30 , 6	+2 5.20	+ 1 11.9	8 18 32.14	+17 55 55.2	n9.251	0.515	+1.60	13.5
25 11 5 13	33	30 , 6	+2 29.85	– 1 15.5	8 18 30.95	+17 55 56.9	n9.070	0.503	+1.61	–13.5
30 9 55 16	34	30 , 6	1 28.87	– 3 18.8	8 14 16.00	+18 5 24.1	n9.321	0.520	+1.64	–13.7
Feb. 1 11 2 3	35	30 , 6	+1 3.71	– 4 19.3	8 12 32.34	+18 9 13.0	n8.748	0.492	+1.66	–13.6
					(139) *Juewa.*				
Feb. 8 10 2 55	36	30 , 6	+0 13.56	+ 2 53.7	9 42 7.26	+27 38 17.9	n9.492	0.367	+1.64	–14.2
8 10 20 32	37	10 , 10	–0 21.23	+ 5 33.2	9 42 6.41	+27 38 18.6	n9.144	0.340	+1.64	–11.2
9 10 2 2	38	30 , 6	+1 50.60	– 3 11.7	9 41 2.78	+27 39 37.3	n9.482	0.361	+1.65	–14.2
11 12 7 43	39	30 , 6	+1 3.79	+ 2 47.9	9 38 46.98	+27 40 12.4	n8.239	0.227	+1.68	–14.1
					(8) *Flora.*				
Feb. 8 11 2 44	40	30 , 6	–0 5.41	+ 0 12.8	10 44 47.43	+14 47 24.9	n9.461	0.594	+1.48	–13.4
11 11 24 42	41	30 , 6	–2 0.38	+ 1 21.8	10 42 3.09	+15 13 11.4	n9.354	0.571	+1.53	–13.7
20 10 20 14	42	25 , 5	+1 12.23	– 1 13.3	10 33 8.59	+16 30 35.4	n9.421	0.562	+1.66	–13.9
20 10 38 5	43	30 , 6	–0 44.99	+ 1 58.8	10 33 7.81	+16 30 40.9	n9.365	0.553	+1.65	–13.9
20 10 56 15	44	30 , 6	–1 21.28	– 0 9.3	10 33 7.10	+16 30 48.1	n9.297	0.544	+1.65	–13.9
					(113) *Amalthea.*				
Mar. 13 11 1 20	45	30 , 6	+1 45.23	– 0 34.1	12 28 14.24	+ 5 14 18.2	n9.366	0.693	+1.76	–11.4
13 11 24 15	46	30 , 6	–2 21.31	+ 2 12.7	12 28 13.56	+ 5 14 26.6	n9.282	0.694	+1.76	–11.3
15 11 38 23	47	30 , 6	+0 7.57	– 1 14.0	12 26 38.19	+ 5 30 31.5	n9.168	0.686	+1.78	–11.4
16 11 29 51	48	29 , 6	–0 1.59	+ 2 19.6	12 25 49.82	+ 5 38 28.7	n9.189	0.685	+1.79	–11.4
20 12 7 12	49	30 , 6	–1 42.65	– 0 31.6	12 22 27.07	+ 6 10 11.5	n8.645	0.675	+1.82	–11.4
					(349) *Dembowska.*				
Mar. 20 9 59 36	50	30 , 6	+0 3.85	+ 4 50.8	10 51 19.10	+16 30 5.0	n9.104	0.531	+1.74	–12.6
20 10 17 46	51	30 , 6	–1 40.72	+ 1 24.4	10 54 18.53	+16 30 5.9	n8.955	0.526	+1.74	–12.6
27 10 19 12	52	30 , 6	–1 40.96	– 1 2.7	10 49 20.30	+16 38 32.8	n8.302	0.519	+1.71	–12.3
27 10 39 43	53	30 , 6	+0 26.94	– 5 15.0	10 49 19.62	+16 38 35.4	8.357	0.519	+1.71	–12.3
28 10 52 28	54	30 , 6	+1 33.04	– 1 12.5	10 48 40.41	+16 39 8.8	8.769	0.521	+1.70	–12.2
28 11 14 42	55	30 , 6	+1 41.51	+ 3 4.8	10 48 39.82	+16 39 9.6	9.019	0.525	+1.70	–12.2
					(19) *Fortuna.*				
Mar. 20 11 9 28	56	30 , 6	+0 14.58	– 2 5.9	12 11 28.71	– 2 31 4.9	n9.132	0.762	+1.91	–11.7
20 11 27 14	57	29 , 6	–2 13.05	– 1 30.2	12 11 27.89	– 2 31 1.5	n9.006	0.762	+1.91	–11.6
27 11 28 22	58	30 , 6	–0 21.20	+ 2 37.6	12 5 11.78	– 1 46 53.1	n8.508	0.756	+1.93	–12.3
27 11 34 45	59	30 , 6	–0 21.96	– 4 16.6	12 5 11.54	– 1 46 48.3	n8.289	0.756	+1.93	–12.3
27 11 42 11	60	30 , 6	–0 23.82	– 4 10.4	12 5 11.31	– 1 46 47.6	n7.659	0.756	+1.93	–12.3
					(30) *Urania.*				
Apr. 5 10 15 22	61	29 , 6	+1 49.33	– 6 47.9	13 35 39.81	–13 25 57.7	n9.442	0.822	+2.16	– 7.9
14 12 14 33	62	30 , 6	–1 2.42	+ 2 34.1	13 27 0.11	–12 39 30.6	8.596	0.836	+2.21	– 8.8
16 11 4 57	63	30 , 6	+2 42.20	– 0 17.2	13 25 7.69	–12 28 54.4	n8.918	0.834	+2.21	– 9.2
16 11 33 21	64	28 , 6	+3 28.16	+ 2 43.5	13 25 6.44	–12 28 47.7	n8.397	0.836	+2.21	– 9.2
19 11 35 58	65	30 , 6	+0 34.17	+ 0 24.3	13 22 15.16	–12 12 14.4	8.017	0.834	+2.22	– 9.3
					(22) *Kalliope.*				
Apr. 7 10 24 58	66	29 , 6	–1 46.86	– 3 41.2	13 55 7.63	+ 3 5 36.0	n9.436	0.715	+1.93	– 7.7
7 11 3 40	67	30 , 6	–0 24.26	– 2 42.1	13 55 6.26	+ 3 5 36.9	n9.316	0.713	+1.93	– 7.8
14 11 7 4	68	30 , 6	–2 39.24	– 1 4.9	13 49 11.93	+ 3 23 10.8	n9.142	0.708	+1.99	– 7.6
15 10 39 58	69	30 , 6	–1 37.79	– 2 23.0	13 48 21.39	+ 3 25 15.6	n9.257	0.709	+2.00	– 7.6
17 12 18 45	70	30 , 6	–2 45.84	– 8 16.8	13 46 31.50	+ 3 29 18.3	8.498	0.705	+2.01	– 7.4
					(21) *Lutetia.*				
Apr. 14 12 39 14	71	30 , 6	–0 1.72	+ 2 56.2	13 24 44.88	– 4 21 49.0	8.964	0.778	+2.08	– 8.9
16 9 40 11	72	30 , 6	+1 6.05	– 5 1.6	13 22 58.83	– 4 12 29.9	n9.372	0.772	+2.08	– 9.0
19 9 38 54	73	30 , 6	+0 59.16	+ 4 12.9	13 20 10.33	– 3 58 1.6	n9.326	0.771	+2.09	– 9.1
19 9 57 32	74	30 , 6	–1 32.04	– 1 0.0	13 20 9.48	– 3 57 57.7	n9.251	0.772	+2.09	– 8.9
20 11 23 1	75	30 , 6	–0 19.81	– 4 36.0	13 19 10.66	– 3 52 59.8	n7.244	0.774	+2.09	– 9.0

1904 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log p Δ		Red. to App. Pl.	
				(202) *Chryseis.*						
d h m s			m s	′ ″	h m s	° ′ ″			s	″
May 4 10 30 11	76	30 , 6	+0 5.67	+0 10.1	11 36 10.65	1 37 11.8	n9.171	0.751	+2.20	1.6
11 11 43 30	77	30 , 6	+2 14.19	12 24.8	11 31 28.61	−1 16 1.4	8.778	0.752	+2.22	1.4
13 11 27 14	78	28 , 6	+0 10.75	11 51.3	14 30 4.18	1 11 26.3	8.663	0.751	+2.23	1.2
15 10 38 56	79	30 , 6	+2 12.34	9 15.0	11 28 13.77	−1 8 30.1	n8.505	0.751	+2.23	1.2
				(40) *Harmonia.*						
May 7 10 50 17	80	30 , 6	+0 16.28	1 34.7	11 47 6.05	10 10 19.5	n9.040	0.819	+2.36	1.3
8 10 29 48	81	30 , 6	+3 15.71	−1 18.4	11 46 5.61	10 7 17.4	n9.146	0.817	+2.36	1.5
11 12 20 8	82	30 , 6	−2 37.43	+1 10.5	11 42 58.39	−9 58 15.9	9.046	0.818	+2.38	1.2
				(79) *Eurynome.*						
May 15 11 27 21	83	30 , 6	−0 12.68	+2 51.8	16 12 15.91	17 6 8.3	n9.167	0.856	+2.56	−1.8
21 11 28 54	84	30 , 6	+2 38.76	+5 51.7	16 7 3.25	−16 42 37.3	n8.925	0.858	+2.63	−1.1
23 11 53 3	85	20 , 4	+3 27.91	+6 16.5	16 5 6.30	−16 31 15.1	n8.137	0.859	+2.65	−0.9
				1904 *N. Y.*						
May 19 12 5 31	86	20 , 4	+5 25.76	+2 58.9	13 33 56.49	+16 31 55.2	9.110	0.565	+1.87	3.0
20 9 17 55	87	20 , 4	−2 51.61	+2 10.5	13 33 38.02	+16 27 30.1	8.198	0.523	+1.89	2.7
22 9 20 25	88	20 , 4	+1 20.39	+5 10.1	13 33 2.13	+16 10 34.2	n8.378	0.528	+1.87	−2.6
23 9 56 31	89	25 , 5	+3 30.68	0 13.2	13 32 45.66	+16 1 28.9	8.777	0.532	+1.86	−2.6
25 8 49 36	90	30 , 6	+2 49.31	−2 3.9	13 32 19.25	+15 43 28.3	n8.790	0.538	+1.85	2.3
27 10 28 4	91	30 , 6	+1 15.60	+0 30.1	13 31 57.63	+15 23 34.6	9.195	0.554	+1.84	−2.0
28 8 57 8	92	30 , 6	−2 56.08	+1 13.7	13 31 50.14	+15 14 17.3	n8.315	0.545	+1.86	−1.8
29 8 31 32	93	30 , 6	−0 40.69	+3 26.4	13 31 43.87	+15 4 20.7	n8.770	0.549	+1.84	−1.8
June 3 9 9 12	94	30 , 6	−0 1.48	+3 37.8	13 31 33.66	+14 10 59.7	8.727	0.561	+1.81	−1.6
8 9 8 53	95	25 , 5	+1 59.62	−1 31.1	13 32 1.87	+13 14 8.4	8.962	0.581	+1.78	1.2
11 9 3 27	96	30 , 6	+0 30.81	−7 38.8	13 32 36.46	+12 38 29.1	9.012	0.591	+1.77	0.8
12 8 57 44	97	25 , 5	+2 23.01	−1 1.2	13 32 50.90	+12 26 28.4	8.995	0.594	+1.75	−0.9
15 9 13 11	98	9 , 2	+0 20.80	+2 37.8	13 33 12.93	+11 49 20.6	9.177	0.608	+1.75	0.8
18 9 12 28	99	30 , 6	−0 1.89	−2 27.1	13 34 17.61	+11 11 32.9	9.229	0.618	+1.73	0.6
				(387) *Aquitania.*						
June 3 11 12 33	100	25 , 5	−1 0.51	−3 13.1	16 56 30.30	+5 17 12.0	n9.040	0.687	+2.44	−5.0
8 11 37 36	101	25 , 5	+1 30.23	−5 16.3	16 52 9.83	+4 53 6.4	n8.084	0.690	+2.48	−5.3
11 11 22 38	102	25 , 5	+3 33.10	−2 33.8	16 49 35.55	+4 34 29.6	n8.125	0.693	+2.51	−5.8
12 10 30 3	103	30 , 6	+0 59.60	−0 15.6	16 48 16.15	+4 27 18.9	n9.034	0.696	+2.52	−6.0
				(18) *Melpomene.*						
June 11 9 16 54	104	25 , 5	−1 15.95	+0 5.8	16 37 36.50	5 28 11.7	n9.218	0.784	+2.62	−1.7
11 10 7 37	105	25 , 5	−1 16.95	+2 13.0	16 37 35.65	5 28 12.0	n9.139	0.785	+2.62	−1.8
12 9 43 29	106	20 , 4	+1 47.08	1 57.8	16 36 36.12	−5 28 9.6	n9.241	0.784	+2.62	−1.4
17 10 39 0	107	25 , 5	+3 8.68	6 8.2	16 31 39.88	5 30 34.8	n8.248	0.788	+2.61	−1.6
				(11) *Parthenope.*						
June 12 11 26 36	108	30 , 6	+0 23.97	7 56.4	17 27 50.66	17 33 26.5	n8.887	0.862	+2.85	−7.4
13 10 52 32	109	30 , 6	+2 39.94	3 29.9	17 26 52.31	17 34 18.1	9.136	0.859	+2.86	−7.2
14 11 2 26	110	30 , 6	+1 26.42	+8 18.8	17 25 51.96	17 35 12.7	9.028	0.861	+2.88	−7.3
18 10 43 31	111	25 , 5	−2 13.86	+1 17.3	17 21 51.32	17 39 30.8	9.021	0.861	+2.92	−7.2
18 11 1 55	112	20 , 4	−2 11.77	+1 7.1	17 21 53.58	17 39 31.0	n8.826	0.863	+2.92	−7.2
				(105) *Artemis.*						
June 17 12 13 1	113	30 , 6	+1 26.75	+2 27.1	18 21 9.60	+17 13 39.4	8.099	0.508	+2.54	9.1
18 11 58 34	114	30 , 6	−2 56.25	+0 19.0	18 20 22.14	+17 17 3.5	8.847	0.510	+2.55	9.3
22 11 1 42	115	25 , 5	+2 13.89	0 5.2	18 17 5.27	+17 25 55.2	9.169	0.517	+2.59	10.2
22 11 20 12	116	30 , 6	+0 32.69	+5 7.8	18 17 4.55	+17 25 56.0	n9.012	0.512	+2.59	10.2
				Comet *a* 1904 (Brooks).						
May 19 10 38 3	117	18 , 6	−1 10.53	1 10.2	14 47 1.98	+57 47 11.3	8.862	0.454	+2.16	4.7
20 9 7 59	118	25 , 5	2 31.83	+8 38.7	14 42 42.64	+57 53 11.1	n9.567	0.366	+2.14	5.0
21 9 59 39	119	25 , 5	+1 21.51	+0 21.2	14 37 58.49	+57 58 56.4	9.186	0.448	+2.08	5.5
22 9 53 10	120	20 , 4	−4 1.85	1 6.7	14 33 28.06	+58 3 25.1	n9.166	0.451	+2.10	5.7
23 10 53 30	121	25 , 5	+2 59.35	1 0.5	14 28 17.24	+58 7 0.4	9.053	0.458	+2.03	6.2
25 15 1 1	122	20 , 4	0 22.90	+4 22.0	14 19 11.63	+58 11 13.2	9.921	0.409	+1.97	7.0
June 3 10 0 52	123	30 , 6	+0 11.92	+4 37.9	13 44 13.92	+57 48 55.6	9.382	0.422	+1.56	9.1
8 9 46 36	124	30 , 6	0 58.82	−2 57.7	13 27 23.47	+57 11 32.4	9.498	0.376	+1.34	9.9
13 9 45 22	125	25 , 5	+0 26.82	+8 52.8	13 12 50.99	+56 29 32.3	9.618	0.281	+1.08	+10.3
14 9 42 58	126	25 , 5	+2 29.30	6 16.6	13 10 12.40	+56 19 38.4	9.630	0.263	+1.02	+10.3
22 9 19 20	127	25 , 5	3 21.19	+3 31.3	12 52 11.80	+54 52 15.9	9.686	0.107	+1.05	+10.4

The star places from the Strassburg and Cambridge (U. S.) Zones were furnished through the courtesy of the Directors of the Observatories at those places. Planets (182), (139), (8), (349), (50), (22), (21), (202), (40), (79), (18), (105), and 1904 N. Y., were found photographically by Mr. G. H. Peters. The right ascension of comparison-star No. 118 is one minute wrong in the Hels. Gotha A.G. Catalogue. The correct position is given in *A. N.*, Bd. 165, No 3951 52.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority
1	7h 3m 45.71s	+18 10 54.0″	Berlin A, A.G. 2644
2	7 2 36.68	+18 0 56.4	" " " 2620
3	7 2 41.28	+18 9 36.5	" " " 2622
4	6 57 21.16	+18 0 46.7	" " " 2560
5	6 56 35.20	+18 6 48.0	" " " 2547
6	7 6 44.42	+ 5 48 49.3	Leipzig II. A.G. 3593
7	6 54 49.52	+ 5 49 7.4	" " " 3416
8	6 53 12.96	+ 6 2 13.5	" " " 3391
9	6 50 9.43	+ 6 1 56.4	" " " 3343
10	6 51 17.56	+ 6 7 43.3	" " " 3365
11	6 51 37.97	+22 50 48.2	Berlin B, A.G. 2690
12	6 51 59.78	+23 1 26.7	" " " 2694
13	6 48 34.08	+22 56 28.4	" " " 2657
14	6 48 34.29	+23 5 56.1	" " " 2658
15	6 46 51.30	+23 7 32.5	" " " 2640
16	7 6 50.72	+27 23 17.8	Camb.Eng.A.G. 3815
17	7 6 14.47	+27 35 43.6	" " " 3801
18	9 8 8.42	+21 40 43.9	Berlin B, A.G. 3689
19	9 3 9.37	+21 46 22.6	" " " 3666
20	8 57 14.09	+21 53 49.0	" " " 3634
21	8 51 32.52	+22 13 44.6	" " " 3603
22	8 47 16.56	+22 22 5.1	" " " 3574
23	8 48 45.79	+22 34 49.6	" " " 3583
24	8 43 23.96	+22 36 1.3	" " " 3553
25	9 38 5.81	− 1 31 13.1	Nicolajew, A.G. 2919
26	9 35 39.07	− 1 17 35.8	" " 2912
27	9 37 4.09	− 1 18 57.8	" " 2916
28	9 34 57.25	− 0 42 24.2	Newcomb's Fund. Catal.
29	9 29 41.85	− 0 3 35.6	Nicolajew, A.G. 2891
30	8 17 19.44	+17 56 31.7	Berlin A, A.G. 3309
31	8 17 31.24	+17 58 37.3	" " " 3310
32	8 16 25.34	+17 54 23.8	" " " 3299
33	8 15 59.49	+17 57 55.9	" " " 3293
34	8 15 13.23	+18 8 56.9	" " " 3289
35	8 11 26.97	+18 13 45.9	" " " 3263
36	9 41 22.06	+27 36 8.4	Camb.Eng..A.G. 5087
37	9 42 26.03	+27 33 29.6	" " " 5090
38	9 39 10.53	+27 43 33.2	" " " 5078
39	9 37 41.51	+27 38 8.6	" " " 5068
40	10 41 51.36	+14 47 25.5	½ {Berlin A, A.G. 4278 + Leipzig I, A.G. 4128}
41	10 44 1.94	+15 12 33.3	Berlin A, A.G. 4265
42	10 31 24.70	+16 32 2.6	" " " 4199
43	10 33 51.15	+16 28 56.0	" " " 4215
44	10 34 26.73	+16 31 11.3	" " " 4220
45	12 26 27.25	+ 5 15 3.7	Leipzig II, A.G. 6151
46	12 30 33.11	+ 5 12 25.2	" " " 6176
47	12 26 28.84	+ 5 34 56.9	" " " 6153
48	12 25 49.62	+ 5 36 20.5	" " " 6148
49	12 24 7.90	+ 6 10 54.5	" " " 6136
50	10 54 13.51	+16 25 26.8	Berlin A, A.G. 4322
51	10 55 57.51	+16 28 54.1	" " " 4331
52	10 50 59.55	+16 39 47.8	" " " 4301
53	10 48 50.97	+16 44 2.7	" " " 4293
54	10 47 5.67	+16 40 33.5	" " " 4283
55	10 46 56.61	+16 36 17.0	" " " 4282
56	12 11 12.22	− 2 28 47.3	Strassburg, A.G. Zones
57	12 13 39.03	− 2 29 19.7	" " "
58	12 5 31.05	− 1 49 18.4	Nicolajew, A.G. 3364
59	12 5 31.57	− 1 42 19.4	" " 3365
60	12 5 33.20	− 1 42 24.9	" " 3366
61	13 33 48.32	−13 19 1.9	Camb., U.S., A.G. Zones
62	13 28 0.32	−12 41 55.9	" " " "
63	13 22 23.28	−12 28 28.0	" " " "
64	13 21 36.07	−12 31 22.0	" " " "
65	13 21 38.77	−12 12 29.4	Newcomb's Fund. Catal.
66	13 56 52.56	+ 3 9 27.9	Albany, A.G. 4848
67	13 55 28.59	+ 3 8 26.8	" " 4836
68	13 51 49.18	+ 3 27 23.3	" " 4828
69	13 49 57.18	+ 3 27 46.2	" " 4818
70	13 49 18.33	+ 3 37 42.5	" " 4816
71	13 24 44.52	− 4 24 36.3	Strassburg, A.G. Zones
72	13 24 50.70	− 4 7 19.3	" " "
73	13 19 9.08	− 4 2 5.4	" " "
74	13 21 39.43	− 3 56 48.8	" " "
75	13 19 28.38	− 3 48 14.8	" " "
76	14 36 32.78	− 1 37 47.3	Nicolajew, A.G. 3777
77	14 29 11.93	− 1 3 35.2	" " 3761
78	14 29 51.50	− 0 59 30.8	" " 3763
79	14 25 59.20	− 0 59 10.9	" " 3755
80	14 46 17.41	−10 9 10.5	Wien, A.G. Zones
81	14 42 47.56	−10 5 54.5	" " "
82	14 45 33.44	−10 0 22.2	" " "
83	16 12 56.03	−17 9 4.9	Washington, A.G. Zones
84	16 4 21.86	−16 48 30.1	" " "
85	16 1 35.71	−16 41 2.8	" " "
86	13 28 28.86	+16 31 59.3	Berlin A, A.G. 4953
87	13 36 30.74	+16 24 52.3	" " " 4984
88	13 31 39.87	+16 1 56.4	" " " 4967
89	13 29 13.12	+16 1 44.7	" " " 4958
90	13 29 28.09	+15 45 34.5	" " " 4960
91	13 30 40.19	+15 23 6.2	" " " 4966
92	13 34 44.36	+15 13 5.4	" " " 4975
93	13 32 22.72	+15 0 56.1	" " " 4969
94	13 31 33.33	+14 7 23.5	Leipzig I, A.G. 4886
95	13 30 0.47	+13 15 40.7	" " " 4880
96	13 32 3.88	+12 46 9.0	" " " 4888
97	13 30 26.14	+12 27 33.5	" " " 4882
98	13 33 20.38	+11 46 43.6	" " " 4895
99	13 34 50.80	+11 14 0.6	" " " 4904
100	17 0 28.37	+ 5 20 50.4	Leipzig II, A.G. 7617
101	16 47 37.12	+ 4 58 17.4	½ {Leipzig II, A.G. 7531 + Albany A.G. 5577}
102	16 45 59.94	+ 4 36 57.6	Albany, A.G. 5570
103	16 47 44.33	+ 4 27 58.5	" " 5578
104	16 38 49.83	− 5 28 22.2	Strassburg, A.G. Zones
105	16 39 19.98	− 5 30 29.8	" " "
106	16 31 46.42	− 5 23 16.2	" " "
107	16 28 28.56	− 5 24 31.2	" " "
108	17 27 23.84	−17 25 37.5	Washington A.G. Zones
109	17 24 9.51	−17 30 55.4	" " "
110	17 24 22.66	−17 44 8.8	" " "
111	17 24 5.26	−17 43 55.5	" " "
112	17 24 5.43	−17 43 45.6	" " "
113	18 19 40.31	+17 11 3.2	Berlin A, A.G. 6767
114	18 23 15.84	+17 16 35.2	" " " 6808
115	18 14 48.79	+17 25 50.2	" " " 6720
116	18 16 29.27	+17 20 38.0	" " " 6739
117	14 48 10.35	+57 48 16.8	Hels.-Gotha.A.G. 8149
118	14 45 12.33	+57 44 30.4	" " " 8117
119	14 33 34.90	+57 58 29.7	" " " 8050
120	14 37 27.81	+58 7 26.1	" " " 8071
121	14 25 45.86	+58 10 54.4	" " " 7988
122	14 19 35.56	+58 6 44.2	" " " 7949
123	13 44 0.44	+57 44 8.9	" " " 7709
124	13 28 20.95	+57 17 20.2	" " " 7608
125	13 12 23.09	+56 20 29.2	" " " 7503
126	13 7 42.08	+56 26 14.4	" " " 7473
127	12 55 31.94	+54 49 4.2	" " " 7398

SUNSPOT OBSERVATIONS,

MADE AT THE AMHERST COLLEGE OBSERVATORY,

BY ROBERT H. BAKER.

1904			New		Disapp.		Reapp.		Total		Def.
	d	h	Gr.	Spots	Gr.	Spots	Gr.	Spots	Gr.	Spots	
July	5	22	–	–	–	–	–	–	3	18	3
	6	21	–	3	–	–	–	–	3	19	2
	7	22	1	1	–	–	1	1	3	7	1
	8	6	–	1	1	2	–	1	2	5	3
	8	21	–	2	–	–	–	2	2	5	4
	9	6	–	9	–	–	–	–	3	16	5
	10	5	–	1	–	–		–	2	15	3
	10	21	–	4	–	–	–	–	2	17	1
	11	23	1	4	1	1	1	2	2	20	3
	12	5	–	8	–	–	–	–	2	28	4
	12	22	–		–	–		–	2	27	1
	13	6	–	–	–	–	–	–	2	22	3
	14	0	–	8	–	–	–	–	2	30	3
	14	20	1	1	–	–		–	3	18	2
	15	23	2	24	–	–	1	12	5	45	4
	16	5	–	29	–	–	–	–	5	73	1
	16	22	–	–	–	–	–	–	5	40	3
	18	0	–	16	–	–	–		5	62	4
	18	23	–	1	–	–	–		5	41	4
	20	0	–	–		–			5	28	4
	20	6	1	2	1	1	1	1	5	29	3
	20	21	1	7	1	4	–	2	1	26	3
	21	5	–	14	–	–		1	4	39	5
	22	0	–	1	–	–		3	3	19	2
	25	5	1	2	–	–	1	2	3	8	2
	26	0	–	1	1	1	–		2	8	1
	26	21	–	6		–	–	–	2	11	3
	27	21		10	–	–		–	2	21	2
	28	21	–	19	–	–	–		2	10	1
	29	21	–	1	–	–	–		2	34	3
	30	3	–	–	–	–			2	29	3
	31	2	–	–	–	–	–	–	1	20	2
	31	22	–	–	–	–			1	17	3
Aug.	2	21	2	5	–		1	2	3	17	4
	3	4	–	–	–	–			3	15	2
	3	23	1	1	–	–	1	1	4	14	2
	4	4	–			–			4	12	2
	4	22	–	5	1	1		–	3	16	3
	5	23		–		–			3	10	4
	6	5		2	–			–	3	8	3
	6	20		2		–		–	3	10	2
	8	2		–	1	3			2	1	2
	8	21	1	17		–	1	4	3	21	3
	9	1		–		–			3	16	3
	11	2	2	27		–	2	11	5	43	5
	12	0	–	–		–			5	34	3
	13	4		–	1	1			4	24	1
	14	3	–		–				3	20	4
	14	23	1	3		–			4	13	4
	15	5	–	1					4	13	4
	15	21	–		1	3			2	3	2
	16	5	–			–			2	3	1
	16	23				–			2	2	2
Aug.	17	5	1	3	–	–	1	3	3	5	3
	17	21					–	–	2	4	4
	18	6	–	3	–			1	2	7	5
	18	23	1	4	–	–		–	3	8	3
	19	5			–	–	–		3	5	1
	22	3	1	11	–	–	1	14	3	16	4
	23	2	–	17	1	1	–		2	32	4
	23	20	1	1	–		1	1	3	22	3
	24	5	–	12			–	1	5	34	4
	24	23	–	9		–	–	–	1	41	1
	25	5	1	28	–	–	1	3	5	69	5
	25	23	–	2	–			2	5	60	3
	26	5	–	2	–	–		2	5	54	1
	26	20	–	18	–	–			4	67	5
	27	4	–	7	–	–	–		4	74	4
	27	23	–	21	–	–	–	–	4	95	5
	28	5	–	9	–	–	–	–	4	100	5
	28	23	–		–			–	4	42	2
	29	1	–	–	–	–		–	4	45	1
	30	1	–	29		–		–	4	74	4
	30	23			–				4	45	3
	31	4			–	–		–	4	50	1
Sept.	2	0	1	1	–		1	1	4	9	1
	3	3	–	1	2	3	–	–	3	5	2
	3	22	–	2	1	1	–		2	5	3
	4	5	–	1	–		–		2	6	4
	4	21	–		1	1			1	5	3
	5	3	–		–	–	–		1	5	4
	5	23	–		–	–	–		1	4	5
	6	5	1	1	–				2	4	4
	7	5	–	1				–	1	5	5
	7	21	–		–				1	5	5
	10	22	–	–	–		–		1	3	4
	11	5	–		–				1	3	4
	11	21	–	–		–		–	1	5	5
	12	22	1	1			1	1	2	5	5
	15	4	–		1	1		–	1	1	3
	15	23	–	9	–				1	10	3
	16	5		6		–		–	1	16	4
	16	23	–						1	16	5
	17	5			–				1	12	5
	17	22	1	2	–			–	2	7	5
	18	3							2	5	5
	19	5		13					2	18	3
	19	23							2	17	4
	21	23	1	5			1	1	3	22	3
	22	5		1	–				3	20	3
	22	23		1					1	15	3
	24	23		6					3	30	5
	26	23			1	6			2	11	3
	29	22	–						3	7	1
	30	4		1	–				3	8	1

Observations made with 6-inch Reflector. Faculae except July 26, August 16, 19, and Sept. 2

ON AN APPLICATION OF THE METHOD OF LEAST-SQUARES FOR COMPARING THE PROBABILITIES OF NATURALNESS OF TWO DIFFERENT SETS OF SERIES OF HYPOTHETICAL OBSERVATION EQUATIONS, BOTH DERIVED FROM THE SAME OBSERVATIONS.

By J. MIDZUHARA.

In the present paper I shall, specially, consider the following two series of equations:

$$(1)\quad \left\{\begin{array}{l} a_1x + b_1y + c_1z + \ldots\ldots + n_1 = 0 \\ a_2x + b_2y + c_2z + \ldots\ldots + n_2 = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right.$$

$$(2)\quad \left\{\begin{array}{l} a_1x + b_1y + c_1z + \ldots + p_1w + n_1 = 0\,. \\ a_2x + b_2y + c_2z + \ldots + p_2w + n_2 = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right.$$

as the hypothetical observation equations whose probabilities of naturalness are to be compared. But it is to be noticed that the results of its discussion may be easily applied to the more general cases of any number of unknowns whatever.

Now, let us adopt the following notations:

D_1 = the determinant formed from all the coefficients of the unknown quantities in the normal equations of the equations (1);

D_2 = idem from those of the equations (2)

$D_a,\ D_b\ \ldots$ = the minors corresponding to the constituents $[aa], [bb], \ldots$ of D_1;

$D_{ab},\ D_{ac},\ \ldots$ = the minors corresponding to the constituents $[ab], [ac], \ldots$ of D_1;

$x_1,\ y_1,\ z_1,\ \ldots$ = the most probable values of $x, y, z, \ldots$ found from the equations (1);

$x_2,\ y_2,\ z_2,\ \ldots$ = idem found from the equations (2);

m = the number of observations;

$[v_1v_1]$ = the sum of the squares of the residuals of the equations (1);

$[v_2v_2]$ = idem of the equations (2);

ϵ_1 = the mean error of an observation of the equations (1);

ϵ_2 = idem of the equations (2);

$\epsilon_{x_1},\ \epsilon_{y_1},\ \epsilon_{z_1},\ \ldots$ = the mean errors of $x_1, y_1, z_1, \ldots$ respectively;

$\epsilon_{x_2},\ \epsilon_{y_2},\ \epsilon_{z_2},\ \ldots$ = the mean errors of $x_2, y_2, z_2, \ldots$ respectively;

then since

$$(3)\qquad -D_1x_1 = [an]\,D_a + [bn]\,D_{ab} + [cn]\,D_{ac} + \ldots$$

we have evidently

$$-D_1x_2 = ([an] + [ap]\,w_2)\,D_a + ([bn] + [bp]\,w_2)\,D_{ab} + ([cn] + [cp]\,w_2)\,D_{ac} + \ldots \tag{4}$$

and therefore

$$D_1(x_1 - x_2) = w_2\{[ap]\,D_a + [bp]\,D_{ab} + [cp]\,D_{ac}\ldots\} \tag{5}$$

By the same reasoning

$$-D_1y_1 = [bn]\,D_b + [an]\,D_{ab} + [cn]\,D_{bc} + \ldots \tag{6}$$

$$D_1(y_1 - y_2) = w_2\{[bp]\,D_b + [ap]\,D_{ab} + [cp]\,D_{bc} + \ldots\} \tag{7}$$

&c. &c.

Comparing these equations we see that

$$[an](x_1 - x_2) + [bn](y_1 - y_2) + [cn](z_1 - z_2) + \ldots = -w_2\{[ap]\,x_1 + [bp]\,y_1 + [cp]\,z_1\ldots\} \tag{8}$$

and we have also

$$-[pn] = [pp]\,w_2 + [ap]\,x_2 + [bp]\,y_2 + \ldots \tag{9}$$

(8), (9) being substituted in the following equation:

$$[v_1v_1] - [v_2v_2] = [an](x_1 - x_2) + [bn](y_1 - y_2) + \ldots - [pn]\,w_2 \tag{10}$$

we have

$$[v_1v_1] - [v_2v_2] = [pp]\,w_2^2 - w_2\{[ap](x_1 - x_2) + [bp](y_1 - y_2) + \ldots\} \tag{11}$$

$$= [pp]\,w_2^2 - \frac{w_2^2[ap]}{D_1}\{[ap]\,D_a + [bp]\,D_{ab} + [cp]\,D_{ac} + \ldots\}$$
$$- \frac{w_2^2[bp]}{D_1}\{[bp]\,D_b + [ap]\,D_{ab} + [cp]\,D_{bc} + \ldots\}$$
$$- \ldots\ldots\ldots\ldots \tag{12}$$

$$= \frac{w_2^2 D_2}{D_1} = w_2^2 Pw_2 \tag{13}$$

where Pw_2 is the weight of w_2.

From (13) we see that $[v_1v_1] - [v_2v_2]$ is always a positive quantity, which is another demonstration of Professor JACOBY's theorem in *A.J.*, No. 514.

Now (and hereafter), for convenience, let us suppose that the observation-equations have been transformed so that they have the relations (for this operation of transformation see my paper in *A.J.*, No. 535),

$$[aa] = [bb] = [cc] = \ldots = [pp] = m$$

then (12) may be expanded as follows:

$$[e_1e_1]-[e_2e_2]=mw_2^2\Big\{1-\frac{[ap]^2+[bp]^2+[cp]^2+\dots}{m^2}$$
$$+\frac{[ap]}{m^3}([ab][bp]+[ac][cp]+[ad][dp]+\dots)$$
$$+\frac{[bp]}{m^3}([ab][ap]+[bc][cp]+[bd][dp]+\dots)$$
$$+\frac{[cp]}{m^3}([ac][ap]+[bc][bp]+[cd][dp]+\dots)$$
$$(14)^* \qquad + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \Big\}$$

From this, since the quantities of the second order of

$$\frac{[ap]}{m},\ \frac{[bp]}{m},\ \&c.$$

are usually small, we may, simply, say that $[e_1e_1]-[e_2e_2]$ usually approximates to mw_2^2.

Also, let us denote the residuals of (1) by $e_1, e_2, e_3, \dots$ and put

$$[pe_1] = p_1e_1 + p_2e_2 + p_3e_3 + \dots$$

then since

$$(15) \qquad -w_2 = \frac{[pe_1]}{m} - \frac{[ap]}{m}(x_1-x_2) - \frac{[bp]}{m}(y_1-y_2) - \dots$$

comparing (11) and (15) we get

$$(16) \qquad [e_1e_1]-[e_2e_2] = -w_2[pe_1] = \frac{[pe_1]^2}{Pw_2}$$

$$(17) \qquad = \frac{[pe_1]^2}{m}\Big\{1+\frac{[ap]^2+[bp]^2+\dots}{m^2}+\dots\Big\}$$

This again indicates that $[e_1e_1]-[e_2e_2]$ usually approximates to $\frac{[pe_1]^2}{m}$.

Now, let μ_1 and μ_2 represent the numbers satisfying the following equations (see my paper in *A.J.*, No. 535):

$$\epsilon_1^2 = \frac{[e_1e_1]}{m-\mu_1}, \quad \epsilon_2^2 = \frac{[e_2e_2]}{m-\mu_2}$$

then, comparing these with (13), it follows:

$$(18) \qquad \epsilon_1^2-\epsilon_2^2 = \frac{[e_1e_1]-[e_2e_2]-(\mu_2-\mu_1)\epsilon_2^2}{m-\mu_1}$$

$$(18)^1 \qquad = \frac{Pw_2\{w_2^2-(\mu_2-\mu_1)\epsilon^2w_2\}}{m-\mu_1}$$

and therefore we get

Theorem I. According as $w_2^2 \gtreqless (\mu_2-\mu_1)\epsilon^2 w_2$ we have

$$\epsilon_1^2-\epsilon_2^2 \gtreqless 0$$

This is the criterion of preferring the second hypothesis to the first. Practically, however, let us suppose that m is a very large number, so that a term of m^{-2} order in $\epsilon_1^2-\epsilon_2^2$ is negligible, then by comparing (13), (14), (16), (17) and (18)[1], we may say that

Theorem II. If $\frac{w_2Pw_2}{m}$ or $\frac{[pe_1]^2}{mPw_2}$, which is sensibly equal to $\epsilon_1^2-\epsilon_2^2$, be an appreciable (that is, not negligible) quantity, the second hypothesis is preferable.

Corollary 1. If w_2^2 be an appreciable quantity, the second hypothesis is, generally, preferable.

Corollary 2. If the residuals in the first hypothesis be distributed according to the law of errors, we may say, that "the second hypothesis, in which $p_1 = p_2 = p_3 = \dots = p_m$, generally, can not exist;" but, in this case, there may be other hypotheses better than the first.

Theorem III. If each of $[ap]$, $[bp]$, &c., be comparable with $\sqrt{m}$, as in usual cases, for any appreciable value of w_2^2 or $\left(\frac{[pe_1]}{m}\right)^2$ we have, sensibly,

$$\epsilon_1^2-\epsilon_2^2 = w_2^2 = \left(\frac{[pe_1]}{m}\right)^2$$

that is to say, the larger the value of w_2^2 or $\left(\frac{[pe_1]}{m}\right)^2$ the more preferable the second hypothesis to the first.

Corollary. Let there be given a third hypothesis which is identical with the second hypothesis, except that in the former we have w_3, $[pe_1]$ instead of w_2, $[pe_1]$ in the latter, then if

$$w_3^2 \text{ or } \left(\frac{[pe_1]}{m}\right)^2 > w_2^2 \text{ or } \left(\frac{[pe_1]}{m}\right)^2$$

the former hypothesis is preferable.

From *Corollary 1* of *Theorem II* and *Theorem III* itself we have an important principle, viz:

> "to find best observation-equations it is, generally, advantageous to assume as many unknowns as possible;"

for, by doing so, we can immediately decide, from inspections of the magnitudes of the resulting values of the unknowns, which assumption is the best. Also, it is to be noticed that, the ordinary process by which we may dis-

* Or, if we put

$$\lambda^2 = \frac{[ap]^2+[bp]^2+[cp]^2+\dots}{n}$$

we have, nearly,

$$[e_1e_1]-[e_2e_2] = \frac{mw_2^2\left(1-\frac{\lambda^2}{m}\right)\left(1+\frac{n\lambda^2}{m}\right)}{1+(n-1)\frac{\lambda^2}{m}}$$

where $\quad n =$ the number of unknown quantities of (1).

cover the law of p's so that the second hypothesis may certainly exist, is an application of the principle that

> "in order that $\epsilon_1^2 - \epsilon_2^2$ may be an appreciable quantity, the most of the terms of $[pv_1]$ must have the same signs,"

as may be easily seen from *Theorem II* or *Theorem III*.

I shall now consider properties of $\epsilon^2_{x_1} - \epsilon^2_{x_2}$, $\epsilon^2_{y_1} - \epsilon^2_{y_2}$, &c. If we substitute

$$[an] = -1$$
$$[bn] = [cn] = [dn] = \ldots = 0$$

in the second members of the equations (3) and (4), we get

$$(19) \quad \frac{1}{Px_1} = \frac{D_a}{D_1}$$

$$(20) \quad \frac{1}{Px_2} = \frac{D_a}{D_1} + \frac{([ap]D_a + [bp]D_{ab} + [cp]D_{ac} + \ldots)^2}{D_1 D_2}$$

$$(21) \quad = \frac{D_a}{D_1} + \frac{(x_1 - x_2)^2}{[v_1v_1] - [v_2v_2]}$$

$$= \frac{D_a}{D_1} + \frac{[ap]^2}{m^3} - \frac{2[ap]\{[ab][bp] + [ac][cp] + \ldots\}}{m^4}$$
$$(22)^* \qquad + \ldots\ldots\ldots\ldots\ldots$$

where

$$P_{x_1} = \text{the weight of } x_1$$
$$P_{x_2} = \text{the weight of } x_2$$

By comparing (19), (20) and (21), we see that P_{x_1} is always larger than P_{x_2}, which is another demonstration of my theorem in *A.J.*, No. 521, except that in the case $x_1 = x_2$ they become equal with each other. Now since

$$\epsilon^2_{x_1} - \epsilon^2_{x_2} = \frac{\epsilon_1^2}{Px_1} - \frac{\epsilon_2^2}{Px_2} = \frac{1}{Px_1}\left\{\epsilon_1^2 - \epsilon_2^2 - Px_1\left(\frac{1}{Px_2} - \frac{1}{Px_1}\right)\epsilon_2^2\right\}$$

$$= \frac{1}{Px_1}\left\{\epsilon_1^2 - \epsilon_2^2 - \frac{[ap]^2\epsilon_2^2}{m^2} + \ldots\right\} \qquad (23)$$

$$= \frac{1}{Px_2}\left\{\epsilon_1^2 - \epsilon_2^2 - \frac{[ap]^2\epsilon_1^2}{m^2} + \ldots\right\} \qquad (24)$$

if we suppose, as before, that a term of m^{-1} order in $\epsilon_1^2 - \epsilon_2^2$ is negligible, by comparing (14), (17), (18), (23) and (24), we may say that:

Theorem IV. If each of $[ap]$, $[bp]$, &c., be comparable with $\sqrt{m}$, as in usual cases, we have, sensibly,

$$Px_1(\epsilon^2x_1 - \epsilon^2x_2) = Py_1(\epsilon^2y_1 - \epsilon^2y_2) = \&c. = \epsilon_1^2 - \epsilon_2^2;$$

that is to say, any one of $\epsilon^2_{x_1} - \epsilon^2_{x_2}$, $\epsilon^2_{y_1} - \epsilon^2_{y_2}$, &c., may be, usually, employed for the comparison of the probabilities of the two hypotheses, instead of $\epsilon_1^2 - \epsilon_2^2$.

Theorem V. If each of $[ap]$, $[bp]$, &c., $< m^{\frac{3}{4}}$, for any appreciable value of w_2^2 or

$$\left(\frac{[pv_1]}{m}\right)^2 \gtrless \left(\frac{[ap]\epsilon_1}{m}\right)^2$$

we have, generally, $\epsilon^2_{x_1} - \epsilon^2_{x_2} \gtrless 0$.

Corollary 1. If $\sqrt{m} <$ each of $[ap]$, $[bp]$, &c., $< m^{\frac{3}{4}}$, according as the difference of the numbers of the positive and the negative terms of $[pv_1]$ is larger or less than that of $[ap]$, $\epsilon^2_{x_2}$ is, generally, less or larger than $\epsilon^2_{x_1}$ respectively.

Corollary 2. If each of $[ap]$, $[bp]$, &c., $< m^{\frac{3}{4}}$, for all values of w_2^2 or $\left(\frac{[pv_1]}{m}\right)^2$ larger than $\frac{\epsilon_1^2}{m}$, we have, generally, ($\epsilon_1^2 - \epsilon_2^2$ or each of $\epsilon^2_{x_1} - \epsilon^2_{x_2}$, $\epsilon^2_{y_1} - \epsilon^2_{y_2}$, &c.), > 0.

* Or nearly

$$\frac{1}{Px_2} = \frac{D_a}{D_1} + \frac{[ap]^2}{m^3\left(1 - \frac{\lambda^2}{m}\right)\left\{1 + \frac{(n-1)\lambda^2}{m}\right\}\left\{1 + \frac{n\lambda^2}{m}\right\}}$$

Tokyo Astronomical Observatory, 1904 September.

APPROXIMATE EPHEMERIS OF ENCKE'S COMET.

By M. KAMINSKY (in *A.N.* 3973).

Berlin noon	App. α	App. δ	log r	log Δ	Berlin noon	App. α	App. δ	log r	log Δ
1904	h m s	° ′			1904	h m s	° ′		
Nov. 4	23 5 48	+23 45.1	0.1167	9.7342	Nov. 15	22 15 34	+18 33.0	0.0946	9.7040
5	23 1 3	23 20.6	.1424	.7305	16	11 18	18 1.2	.0893	.7023
6	22 56 19	22 55.1	.1380	.7270	17	7 5	17 29.0	.0840	.7008
7	51 37	22 28.7	.1335	.7237	18	22 2 56	16 56.4	.0785	.6993
8	46 57	22 1.5	.1289	.7207	19	21 58 49	16 23.7	.0730	.6978
9	42 19	21 33.5	.1243	.7178	20	54 47	15 50.9	.0673	.6965
10	37 44	21 5.0	.1195	.7151	21	50 47	15 17.8	.0615	.6953
11	33 11	20 35.6	.1147	.7125	22	46 49	14 44.3	.0555	.6942
12	28 42	20 5.8	.1098	.7102	23	42 51	14 10.6	.0495	.6931
13	24 17	19 35.3	.1048	.7080	24	21 39 4	+13 37.1	0.0433	9.6921
14	22 19 54	+19 4.5	+0.0997	9.7059					

CONTENTS.

Published at 16 Craigie St., Cambridge (Boston Postal District), Mass., semi-monthly, by S. C. Chandler. Address, Cambridge, Mass.
Associate Editors, Asaph Hall and Lewis Boss. Price, $5.00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. *Closed November 1.*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 569.

VOL. XXIV. BOSTON, 1904 NOVEMBER 23. NO. 17

STELLAR LUMINOSITY AND THE ABSORPTION OF STAR LIGHT.

BY GEORGE C. COMSTOCK.

In No. 566 of the *Astronomical Journal*, Professor KAPTEYN has expressed his dissent from one of the conclusions set forth in my paper entitled "Provisional Results of an Examination of the Proper Motions of Certain Faint Stars," *A.J.*, No. 558, and since I infer from a perusal of his argument that, in part at least, he has misapprehended my methods and results, I desire to set forth here with greater fullness than I have hitherto done the grounds for, and the limitations upon the conclusion in question. This conclusion is, in substance, that the observed parallaxes of the fixed stars in connection with the corresponding stellar magnitudes indicate either (*A*) an appreciable absorption of light in its transmission through interstellar space, or (*B*) a very rapid diminution of intrinsic luminosity with diminishing apparent brightness of the stars. Although not insisted upon in my former paper, I assume it to be sufficiently obvious that the two hypotheses here indicated are not mutually exclusive, and that both may be realized in the stellar system. If such is the case, the numerical values of the absorption and change of luminosity obtained in *A.J.* 558, must be regarded as superior limits of these respective effects rather than as actual values, since they were derived by treating each hypothesis as the sole explanation of the observed relation between parallax and stellar magnitude. In a later part of this paper there will be found some evidence tending to show that in fact both hypotheses are realized in the actual system of the stars.

For my present purpose I adopt the observed data contained in Table V of KAPTEYN's paper above cited, and for the convenience of the reader I shall follow his notation as closely as possible. It should be noted that my argument in the matter is in no way dependent upon KAPTEYN's equations 3 and 4, as he affirms, but is entirely independent of the truth or falsity of these or any other assumed relations save certain hypotheses that are hereinafter explicitly set forth and discussed. Let the unit of distance to be employed be the distance corresponding to a parallax of 0″.1. Let the luminosity of the sun be represented by the number 1, and let the stellar magnitude of the sun as seen from the distance unity be represented by the number 5.5 (KAPTEYN) on the photometric scale. Suppose a star of luminosity L to be originally placed at unit distance from the sun, and to be thence moved radially to any assigned distance r. Let π and m represent the parallax and stellar magnitude of this star corresponding to the distance r, and let ρ represent the light ratio for consecutive stellar magnitudes, $\rho^5 = 100$. From elementary considerations of the unimpeded transmission of light we obtain the following relation that must be satisfied at every point of the star's supposed trajectory,

$$\frac{L}{r^2} = \rho^{5.5-m}$$

which is equivalent to

$$L\pi^2\rho^m = [illegible]\rho^{[illegible]} \qquad (1)$$

when π is expressed in seconds of arc.

An equation of this form must obtain for every star, wherever placed, and if we isolate in one group, n stars of like magnitude, m, the mean of their corresponding equations will furnish for the group the mean relation between L, π, and m. By supposition m is constant for the group while the luminosity and parallax are variables which shall be represented by the symbols $L+y$ and $\pi+x$ where L and π are mean values for the group in question, and y and x are individual variations from the respective means. Introducing these symbols into Eq. 1 we obtain

$$L\rho^m\left(\pi^2+\frac{1}{n}\Sigma x^2\right) = [illegible]\rho^{[illegible]} - \frac{\rho^m}{n}\left(2\pi L\Sigma x+\pi^2\Sigma y+2\pi\Sigma xy+\Sigma x^2y\right) \qquad (2)$$

In this expression Σx and Σy are rigorously zero, and Σxy and Σx^2y must be small quantities, since the individual products entering into the summations will be as often positive as negative, and in the mean of any considerable number of stars their effect may be assumed insignificant.

We now put

$$(3) \qquad x^2 = \frac{1}{n}\Sigma x^2 \;, \quad 1+\left(\frac{x}{\pi}\right)^2 = \zeta$$

and passing to logarithms find, when the bracketed terms are neglected,

$$(4) \qquad \log L + 2\log\pi + 0.4\,m + \log\zeta = 0.2$$

To the discussion of this equation I desire to apply the following hypotheses, which are introduced into the problem not as supplements to the observed data, but solely as suppositions to be compared with and controlled by that data.

HYPOTHESES.

A. Within the space occupied by stars brighter than the magnitude 12.5, light is transmitted without sensible absorption.

B. Within this region the mean luminosity of the stars does not vary appreciably from magnitude to magnitude.

C. Within the same region the function above represented by ζ does not vary appreciably from magnitude to magnitude.

In accordance with these hypotheses $\bar{\pi}$ and m are the only variables entering into Eq. 4, and uniting these into the single term

$$(5) \qquad 0.4\,m + 2\log\bar{\pi} = c'$$

we write Eq. 4 in the form,

$$(5) \qquad \log L + \log\zeta - 0.2 + c' = 0$$

In this equation c' must be constant if the hypotheses *A, B, C*, correspond to the actual constitution of the stellar system, since all of the other terms are thus made constant. With KAPTEYN's values of m and $\bar{\pi}$, reproduced in Table I. below, I obtain the values of c' there given, and the better to show the general character of these quantities I have subjected them to a graphical adjustment, the results of which are given under the heading c''.

TABLE I.

m	$\bar{\pi}$ ″	c'	c''	$\log\zeta_{[L=1]}$
2.7	0.0383	8.25–10	8.13–10	2.07
4.1	.0205	8.26	8.31	1.89
5.1	.0147	8.34	8.43	1.77
6.0	.0129	8.62	8.56	1.64
6.9	.00895	8.66	8.69	1.51
8.6	.00630	9.04	9.00	1.20
10.5	0.00185	9.57	9.58	0.62

It is apparent from an inspection of these numbers that between the second and eleventh magnitudes c' can be considered a constant only by assuming the numerical data of the problem to be wholly worthless, and as I suppose KAPTEYN, equally with myself, to reject this assumption, we must attribute the observed variability of c' to an error contained in one or more of the hypotheses *A, B, C*, above enumerated. I therefore proceed to inquire what changes must be made in each of these hypotheses in order that the observed data may be reconciled with the other two hypotheses, supposed to be retained intact. We consider first:

Hypothesis C. The function ζ depends upon the law of distribution of the stars in respect of distance from the earth, the quantity x being a measure of the condensation of their actual parallaxes about the mean value $\bar{\pi}$. In the language of the modern applications of the theory of probabilities to biological problems, $\frac{x}{\pi}$ is the "coefficient of variability" of the parallaxes and the analogy of x to the Gaussian "mean error" is sufficiently apparent. Abandoning the assumption that ζ is constant for all magnitudes we seek to determine a series of values for this function that shall make c' and c'' constant. This can be done only when the mean luminosity of the stars is given, and for this purpose I shall make successively three assumptions, viz.: $\log\bar{L} = -4$, $\log\bar{L} = 0$, $\log\bar{L} = +4$, representing a hundred-million fold range of intrinsic brightness. The values of $\log\zeta$, derived from the adjusted c'', when the mean luminosity is assumed equal to that of the sun, $\log L = 0$, are shown in Table I; its values for $\log L = -4$ and $\log L = +4$, respectively, may be found by subtracting these numbers from the characteristic of the tabular $\log\zeta$.

If $+4$, or any number greater than $+1$ be subtracted from the tabular $\log\zeta$ some of the values will become negative, and this is inadmissible, since from the form of the function $\log\zeta$ must be a positive number. Indeed, the course of the tabular values of $\log\zeta$ strongly suggests that if the data were extended one step further, *e.g.* to the thirteenth magnitude, $\log\zeta$ would here become negative for $L = 1$. If hypotheses *A* and *B* are to be retained we are therefore precluded from regarding the mean luminosity of the stars as being appreciably greater than that of the sun. I shall therefore assume $L = 1$ to be the maximum possible value of this quantity, and corresponding to this assumption and to the supplementary one, $\log L = -4$, I have derived the values of $\log\frac{x}{\pi}$ given in Table II.

TABLE II.

m	$\log\frac{x}{\bar{\pi}}_{[L=1]}$	$\log\frac{x}{\bar{\pi}}_{[L=0.0001]}$	O—C ″	c'''	$\frac{a}{10\pi}$
2.7	1.03	3.03	+0.0037	8.06–10	0.5
4.1	0.94	2.94	− .0007	7.90	0.9
5.1	0.88	2.88	− .0008	7.88	1.2
6.0	0.82	2.82	+ .0009	8.04	1.5
6.9	0.75	2.75	− .0006	7.84	1.9
8.6	0.59	2.60	− .0002	7.87	2.8
10.5	0.25	2.31	+0.0002	8.05	3.9

While mathematically possible, these numbers are far too large to be accepted as representative of any actual stellar distribution. The difficulties that they present will become apparent if we seek to determine a grouping of the stars of any given magnitude, *e.g.*, the sixth, that shall preserve unchanged the observed mean parallax, and shall also give a coefficient of variability as great as 6.6, corresponding to the $\log \frac{\pi}{\pi}$ given in Table II, for $m = 6.0$, $L = 1$. These conditions can best be satisfied by dividing the stars of the sixth magnitude into two groups, a large one each member of which has the parallax 0, and a small one each of whose members has the maximum permissible parallax for such stars, the number of members of the latter group being so determined as to preserve the proper mean parallax for the entire group of stars. This maximum parallax, p, and the number of stars, n, to which it shall be attributed are determinate quantities when the mean parallax of the group, $\bar{\pi}$, and the corresponding function, ζ, are given, as is here the case. The required relations are readily shown to be,

$$p = \bar{\pi}\zeta \quad , \quad n = \frac{N}{\zeta}$$

where N is the total number of stars in the group. Thus, when the mean luminosity of the stars of the sixth magnitude is assumed equal to unity we have $\log \zeta = 1.64$, $\bar{\pi} = 0''.0129$, $p = 0''.56$ and $\frac{n}{N} = 0.023$. According to Pickering, *Annals Harvard College Observatory*, Vol. 48, Part V, Table XIV, there are some 5700 stars included between the magnitudes 5.5 and 6.5, and assuming this number to be the value of N for stars of the sixth magnitude, we find that in order to satisfy the observed mean parallax and coefficient of variability for this group, under the most favorable assumption as to the manner of distribution of the stars, there must be $n = 5700 \times 0.023 = 131$ stars of the group have parallaxes as great as $0''.56$, while all of the remaining stars are at an infinite distance from the sun. Neither the mode of distribution nor the number of large parallaxes seems credible, and the corresponding difficulties are even greater in the case of the brighter stars, *e.g.*, for the fourth magnitude it appears that 1.3 per cent. of the stars must have parallaxes as great as $1''.6$, and even greater values will be required if the mean luminosity of these stars be assumed to be less than unity. On the other hand L can not be made appreciably greater than unity and assumed constant for all magnitudes, without producing impossible values of ζ for the fainter stars.

While considerations such as the above are far from demonstrating that the function ζ is constant between the second and eleventh magnitudes, they suffice to show that no admissible variation of ζ will render constant the values of e' between these limits, and I can find no escape from the conclusion that the observed systematic variation of e' points to an error in one or both of the hypotheses A, B. Although Kapteyn denies this conclusion, I have searched his paper in vain for an alternative to it. I cannot regard as such an alternative his empirical formulas, Eq. 11 and 12, in which by means of five parameters the number of the stars and their mean parallax are represented as functions of the stellar magnitude. These appear to me entirely foreign to the purpose, and so long as his Eq. 5 and 6 professedly have no basis in theory, and the parameters have no physical significance assigned them, the proposed relations can be regarded only as interpolation formulas, that may serve a useful purpose as such, but which contain little information with respect to the structure of the stellar system.

We pass to a consideration of hypothesis B. If we seek to remove the observed variation in e' by modifying the hypothesis of constant luminosity, we may determine from Eq. 6 the required value of L corresponding to any assumed value of ζ. As only relative values of L are needed for the purpose, we may take the $\log \zeta$ of Table I as also representing $\log L$ for $\zeta = 1$. By interpolation from this table I find for the fifth and tenth magnitudes respectively, $\log L_5 = 1.78$, $\log L_{10} = 0.80$, and for the ratio of the intrinsic luminosities associated with the two magnitudes the number 9.5, corresponding to my original statement that the numerical data of the problem can be reconciled with hypothesis A only on the supposition that "the intrinsic luminosity of the stars diminishes with their apparent brightness in such a ratio that a star of the tenth magnitude possesses only one-tenth of the luminosity of a star of the fifth magnitude." If so large a diminution of intrinsic brightness be considered improbable, an exit from the numerical difficulties of the case must be sought through a modification of the only remaining hypothetical element of the problem, viz.:

Hypothesis A. If the interstellar spaces are imperfectly transparent every star appears fainter than would otherwise be the case, and if this diminution of brightness is due to a uniform absorbing medium its amount may be represented by an expression of the form $\frac{a}{10^{\pi}}$. Kapteyn. Since the parallaxes originally employed by me have been slightly modified by Kapteyn I have repeated my determination of the hypothetical absorption coefficient, using the values of m, π, e', given in Table I, and I find thus, $a = 0.184$. The resulting expression for the mean parallax corresponding to the stellar magnitude m is,

$$\log \pi = 8.972 - 10 - \frac{m}{5} + \frac{0''.00268}{\pi} \qquad 7$$

The residuals, O − C, furnished by a comparison of this formula with the observed parallaxes are given in Table II.

There are also exhibited in the same table the corrections to the observed magnitudes, $\frac{a}{10\pi}$, and the corresponding values of the quantity c', here designated by the symbol c'''. The variations of this quantity from a constant value are clearly of the same order of magnitude as the uncertainties in the data employed.

We may now recapitulate the foregoing argument as follows: The three hypotheses above designated, A, B, C, furnish a function that must remain constant for all stellar magnitudes and corresponding mean parallaxes. When the observed values of the magnitude and parallax are introduced into this function it presents systematic variations far too large to be attributed to errors in the numerical data. These variations can not be removed by any admissible modification of hypothesis C, and they must therefore be due to errors contained in the remaining hypotheses. There is presented a numerical determination of the amount of modification in each hypothesis that will serve to satisfy the observed data, and to retain unchanged the alternative hypothesis, but it is not affirmed that either of these modifications must be adopted to the exclusion of the other. A modification of both hypotheses by smaller amounts than those above determined, together with some modification of hypothesis C, may well be required to bring them into conformity with the actual structure of the stellar system, and in my judgment, such is much more likely to be the case than that either hypothesis alone should prove to be at fault. The well known presence of nebulous matter and meteoric dust in the interstellar spaces renders it improbable that as respects the transmission of light they should be absolutely transparent media. On the other hand, both intrinsic luminosity and distance are factors in determining the magnitude of any given star, and the influence of both factors is presumably represented in any considerable group of stars of like magnitude, rendering a fainter group both more distant and less luminous than a brighter one.

The *à priori* probabilities in the case are somewhat strengthened by the sequence of the numbers $\log\frac{x}{\pi}$ in Table II. It is not apparent that the distribution of the brighter stars, as respects distance, should be such as to give a markedly greater coefficient of variability than is presented by the fainter stars. I am, therefore, inclined to regard the larger values of $\log\frac{x}{\pi}$ for the brighter stars as arising from the use of relatively too small a value for L in their derivation, but the amount of such error, if any, can not be determined from the data at present available. If we accept KAPTEYN's conclusion that the maximum permissible value of the absorption coefficient is about one-third or one-fourth of the value above found, $a = 0.184$, it will indeed be possible to determine, at once, at least a rough approximation to the law of change in the luminosity, and to the distribution of the parallaxes about their mean value. But this conclusion ought not to be adopted without some consideration of the ground upon which it rests, viz.: that any absorption coefficient so great as $a = 0.184$ of necessity leads to inadmissible densities in the remoter parts of the stellar system.

The analysis leading to this result involves the following unverified hypotheses, which I state as nearly as practicable in the words of their author:

(*a*) He assumes "$\varphi(L)$ to be independent of the distance."

(*b*) "The tacit assumption has been made that Δ is a function of ρ only."

(*c*) "Both the functions ψ and Δ must be assumed to be given by the formulas 5 and 6 for values of L and ρ, ranging from zero to ∞, whereas even by somewhat straining the observations" they are determined only over a finite range of values, and even this range I can not concede to be as wide as KAPTEYN affirms.

(*d*) "The following method . . . . rests in the hypothesis that $\psi(z)$ is the same at different distances from the sun, that is, it assumes that intrinsically small and bright stars are mixed in the same proportions at various distances from the sun."

(*e*) "This determination was based on the hypothesis that the quantities

$$z = \log\frac{\pi}{\pi_0}$$

are distributed according to the law of errors."

Some of the above hypotheses seem open to serious objection, but whatever opinion may be entertained with regard to their individual plausibility, their combined result, in my judgment, can have no value other than that of a criterion by which to test the hypotheses themselves. As a question of scientific method the legitimate use of an hypothesis appears to me entirely analogous to the use of an unknown quantity in elementary algebra. It is introduced into the problem, and used for a time as if known, but ultimately its value is to be ascertained in terms of the elements otherwise known. KAPTEYN appears to have pursued the converse course, and to have determined the other elements of the problem in terms of his five hypotheses whose values still remain unknown.

But even though the validity of these hypotheses be conceded as far as the observed data extends, their application still seems open to question. Within the range of distances from the sun corresponding to the observed data upon which KAPTEYN's investigation is professedly based, the anomalies produced by the introduction of an absorbing medium such as is above supposed, are not very con-

siderable. Their serious character becomes apparent only when the empirical relations are extrapolated beyond the data upon which they are based. I suppose the last line of Table I to represent the limit of present knowledge with respect to stellar parallax and luminosity, and at this limit, which I suppose to approximate to the region of the galaxy, it imposes no serious tax upon one's credulity to believe, if necessary, that the average distance of each star from its nearest neighbor is only one-fifth, or even one-tenth, of the corresponding distance in the regions nearer to the sun. The million-fold increase of density presented by KAPTEYN as a consequence of the supposed absorbing medium is found, according to his analysis, only at distances from the sun corresponding to a parallax of 0".001 or 0".002. These numbers represent a distance from the sun about three-fold greater than the maximum that has yet been measured, and I must confess both my ignorance of the characteristics of this region and my unwillingness to extrapolate into it relations that are of doubtful validity nearer home.

I regard as entirely plausible KAPTEYN's view that the absorption of light in the interstellar spaces is represented by a coefficient, a, whose magnitude does not exceed a few one-hundredths, but I prefer to make this view depend upon the reasons above adduced for a diminishing luminosity of the stars with their diminishing brightness, rather than to base it upon the highly hypothetical argument advanced by KAPTEYN.

Washburn Observatory, 1904 October.

EPHEMERIS OF COMET a 1904 (BROOKS).*

BY HERBERT L. RICE, NAVAL OBSERVATORY.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

The following ephemeris is a continuation of that given by A. A. NIJLAND in *Astr. Nachr.*, Nos. 3952 and 3963. The parabolic elements adopted by NIJLAND are published by him in *A.N.* No. 3952.

The brilliancy of the comet remains practically constant from Oct. 10 to the end of the year, soon after which it will begin to diminish quite rapidly. Observations made with the 26-inch equatorial at the U.S. Naval Observatory, about the middle of October, indicate that good observations of this object might readily be made with the larger instruments during November and December.

EPHEMERIS OF COMET a 1904.

G.M.T. 1904	α h m s	δ ° '	log Δ	Br.
Oct. 30.5	12 36 56	+43 47.3	0.5999	0.18
Nov. 1.5	37 22	43 58.3	.5986	.18
3.5	37 46	44 10.2	.5973	.18
5.5	38 07	44 22.9	.5959	.18
7.5	38 26	44 36.5	.5945	.18
9.5	38 42	44 51.0	.5930	.18
11.5	38 55	45 6.4	.5915	.18
13.5	12 39 5	+45 22.7	0.5899	0.18
Nov. 15.5	12 39 11	+45 40.0	0.5883	0.18
17.5	39 13	45 58.2	.5866	.18
19.5	39 12	46 17.3	.5849	.18
21.5	39 7	46 37.2	.5832	.18
23.5	38 57	46 58.0	.5814	.18
25.5	38 43	47 19.7	.5796	.18
27.5	38 23	47 42.3	.5778	.18
29.5	37 58	48 5.8	.5760	.18
Dec. 1.5	37 27	48 30.2	.5742	.18
3.5	36 50	48 55.5	.5724	.18
5.5	36 7	49 21.7	.5705	.18
7.5	35 17	49 48.7	.5687	.18
9.5	34 20	50 16.6	.5669	.18
11.5	33 15	50 45.2	.5651	.18
13.5	32 1	51 14.5	.5634	.18
15.5	30 39	51 44.5	.5617	.18
17.5	29 8	52 15.2	.5601	.18
19.5	27 27	52 46.5	.5586	.18
21.5	12 25 37	+53 18.3	0.5571	0.18

Comparison with an observation made here Oct. 17 gives as corrections to the ephemeris: $\Delta\alpha = \ 2^{s}$, $\Delta\delta = -0'.4$

Mr. E. I. YOWELL, of the Naval Observatory, is making a definitive determination of the orbit of Comet a 1904 (Brooks), and requests that all yet unpublished observations of it be printed as soon as possible.

* From Supplement to No. 568.

OBSERVATIONS OF SUNSPOTS,

MADE AT BOSTON UNIVERSITY OBSERVATORY.

By ROBERT E. BRUCE.

Eastern M. T. 1903–1904		Groups N	Groups S	Spots in Groups N	Spots in Groups S	Totals Groups	Totals Spots	Def.
	d h							
Sept.	22 0	0	0	0	0	0	0	G
	23 1	1	0	5	0	1	5	G
	24 1	1	0	1	0	1	1	P
	24 21	1	0	7	0	1	7	G
	25 1	1	0	12	0	1	12	F
	25 21	1	0	4	0	1	4	G
	26 0	1	0	7	0	1	7	G
	27 21	1	1	2	3	2	5	G
	28 1	1	1	3	6	2	9	G
	28 21	0	1	0	2	1	2	G
	30 1	0	1	0	2	1	2	E
	30 21	0	1	0	1	1	1	G
Oct.	2 22	1	0	10	0	1	10	P
	5 21	1	1	16	13	2	29	P
	7 23	1	2	10	5	3	15	G
	13 21	1	1	1	54	2	55	G
	16 3	1	1	3	13	2	16	F
	18 21	1	0	1	0	1	1	P
	19 3	1	0	2	0	1	2	F
	19 21	1	1	2	7	2	9	G
	20 21	1	1	1	4	2	5	G
	22 1	1	0	3	0	1	3	G
	26 22	0	1	0	15	1	15	G
	28 1	0	1	0	17	1	17	G
	28 23	1	1	1	19	2	20	G
	30 1	1	1	5	17	2	22	G
	30 23	1	1	5	42	2	47	E
Nov.	2 3	2	2	4	20	4	24	F
	2 22	1	2	25	20	3	45	E
	3 21	1	2	25	32	3	57	G
	7 0	1	2	10	53	3	63	F
	9 3	1	2	9	86	3	95	E
	11 1	1	2	2	35	3	37	P
	11 22	0	2	0	62	2	62	G
	13 1	0	2	0	46	3	49†	G
	18 23	0	1	0	2	1	2	G
	20 1	0	1	0	6	1	6	F
	23 1	0	0	0	0	0	0	P
	23 23	0	3	0	7	3	7	E
	24 21	0	2	0	4	2	4	F
	25 23	1	2	1	5	3	6	P
	26 23	1	2	3	5	3	8	F
	28 2	1	1	3	2	2	5	F
	29 20	1	1	4	6	2	10	G
	30 22	1	2	5	2	3	7	P
Dec.	1 1	1	3	2	17	4	19	G
	6 21	2	5	4	29	7	33	G
	8 2	2	5	6	38	7	44	E
	9 22	1	3	6	27	4	33	G
	11 1	1	2	4	18	3	22	G
	12 3	1	2	3	15	3	18	F
	13 21	1	2	10	10	3	20	G
	15 3	2	1	6	2	3	8	G
	15 21	3	1	7	3	4	10	E
	17 0	3	2	6	5	5	11	G
	18 1	2	1	4	2	3	6	F
	19 1	2	1	4	6	3	10	E
	20 21	2	1	12	5	3	17	E
	22 2	2	0	10	0	2	10	E
	23 2	2	0	4	0	2	4	P
	28 0	1	2	2	5	3	7	G
	29 23	1	2	6	14	3	20	E
	31 3	0	2	0	9	2	9	G
1904 Jan.	1 1	0	3	0	21	3	21	F
	3 23	1	2	1	4	3	5	P
	5 2	1	2	1	3	3	4	P
	6 23	2	1	3	1	3	4	P
	11 1	1	0	3	0	1	3	F
	14 0	2	0	5	0	2	5	F
	15 1	2	0	6	0	2	6	G
	18 0	2	0	12	0	2	12	P
	19 0	2	0	3	0	2	3	P
	25 1	1	2	1	14	3	15	G
	27 0	0	2	0	6	2	6	F
Feb.	3 1	1	0	1	0	1	1	G
	3 23	2	0	2	0	2	2	F
	5 1	2	0	9	0	2	9	G
	8 1	2	0	28	0	2	28	F
	10 1	2	0	17	0	2	17	P
	12 1	2	0	12	0	2	12	F
	16 0	0	0	0	0	0	0	F
	17 1	1	0	1	0	1	1	F
	18 3	1	1	1	1	2	2	P
	19 23	1	1	5	6	2	11	G
	23 1	1	1	5	17	2	22	G
	24 22	1	1	3	11	2	14	G
	25 21	1	1	2	5	2	7	F
Mar.	4 3	0	2	0	7	2	7	F
	4 22	0	2	0	7	2	7	F
	8 21	2	2	9	7	4	16	E
	9 23	2	1	5	2	3	7	F
	‡11 21	1	1	1	1	2	2	G
	23 4	3	0	6	0	3	6	F
	24 2	3	2	30	9	5	39	E
	28 22	3	1	11	6	4	17	E
	30 0	3	1	8	8	4	16	E
	31 22	3	0	5	0	3	5	G
‡Apr.	1 22	3	0	7	0	3	7	G
	‡3 23	3	0	11	0	3	11	G
	5 1	2	0	15	0	2	15	E
	5 21	1	0	5	0	1	5	F
	7 3	1	1	4	1	2	5	G
	7 21	1	1	2	3	2	5	E
	11 1	2	1	2	10	3	12	F
	12 3	1	1	1	13	2	14	E

Eastern M. T. 1904	Groups N	Groups S	Spots in Groups N	Spots in Groups S	Totals Groups	Totals Spots	Def
Apr. 12 21	1	1	1	13	2	14	G
13 22	2	1	2	14	3	16	F
14 21	2	1	3	18	3	21	G
16 2	2	1	2	12	3	14	E
17 21	2	1	1	18	3	22	G
19 0	2	1	3	14	3	17	G
19 21	2	2	2	10	4	12	E
21 0	2	2	7	20	4	27	E
21 21	2	2	20	31	4	51	G
22 4	3	2	14	24	5	38	G
23 0	3	2	21	15	5	36	E
25 1	2	2	28	14	4	42	G
25 22	1	1	19	13	2	32	F
30 3	2	2	16	11	4	27	E
May 2 3	4	2	13	3	6	16	G
3 0	4	2	17	1	6	21	E
3 21	3	2	23	16	5	39	E
5 1	2	1	17	11	3	31	E
6 1	2	1	7	7	3	14	G
6 23	1	1	6	15	2	21	E
9 23	1	3	4	30	1	34	G
11 1	2	3	17	31	6	50*	G
12 1	1	2	6	17	3	23	G
13 1	2	1	18	12	4	31*	G
16 0	3	0	22	0	3	22	G
16 21	2	0	21	0	2	21	P

* Totals include a group, the position of which was not accurately determined.

‡ Observations up to and including March 11, also observations of April 1 and 3, were made with 7″.1 refractor. All other observations were with 5″ refractor.

For explanations, see *A.J.* 466.

Sixty-nine groups were observed, not counting probable recurrences, containing 675 different spots, each group being credited with the greatest number of spots counted at any one time. The latitude was determined for 66 groups containing 669 spots. These show the following distribution: 30 groups, containing 249 spots, were north of the equator, and 36 groups, containing 420 spots, were south of the equator.

There seems to be ground for concluding that six groups were observed through two rotations of the sun. The following table gives the basis for this conclusion, in some cases not fully convincing.

	Near Meridian 1st Rota.	Near Meridian 2d Rota.	Latitude 1st Rotation	Latitude 2d Rotation	Longitude 1st Rotation	Longitude 2d Rotation
1.	Mar. 8	Apr. 6	18° N.	17° N.	26	16° to 23
2.	Sept. 22	Oct. 19	13° N.	15° N.	90 to 94	93
3.	Dec. 13	Jan. 10	14° N. to 18° N.	17° N.	89 to 102	86 to 88
4.	Mar. 30	Apr. 27	15° N.	13° N. to 14° N.	108	106 to 115
5.	Feb. 22	Mar. 21	19° N.	11° N. to 19° N.	235	23[illegible] to 239
6.	Apr. 14	May 11	15° S. to 18° S.	18° S. to 22° S.	271° to 279	277 to 282°

It will be noticed from the above table that group 2 occupied practically the same position that was occupied later by group 3, one rotation intervening.

One of the large groups of the year, not included in the above table, was near the meridian Nov. 10. It extended in longitude from 161° to 181°, and in latitude from 20° S. to 24° S. At the next appearance of this region a single spot was discovered at longitude 183°, latitude 18° S.; and a group of 29 spots occupied a position at longitude 152° to 161°, latitude 15° S. to 27° S. The former disturbance had apparently been replaced by two disturbances, one ahead of its old position, and the other behind it.

The following data seem to indicate that one group, not included in the list above, persisted through four rotations:

	Near Meridian	Latitude	Longitude
1st Rotation	Oct. 8	14° N. to 15° N.	243 to 247
2d Rotation	Nov. 5	14° N. to 16° N.	229 to 250
3d Rotation	Dec. 2	16° N. to 18° N.	233 to 238
4th Rotation	Dec. 30	19° N. to 21° N.	232° to 235

It will be seen from the above that this group, if it be a recurrence, reached its maximum extent in longitude during the second rotation. It gradually receded from the equator. It is interesting to note that at a later date group 5 in the first table of recurrences occupied nearly the same region.

Observations were made with especial care upon spots with penumbral areas of regular shape, whenever such were found near either limb, in order to ascertain whether the spots had the appearance of elevations or depressions. The results were as follows: 3 had the appearance of elevations, 11 that of depressions, while 3 presented neither phenomenon. Three spots were observed when practically crossing the limb, but no one of them appeared either as a depression or elevation, although one of them when a little farther on the disk was clearly seen as a depression.

A NEW VARIABLE STAR OF SHORT PERIOD.

By J. MILLER BARR.

As the result of a series of observations made within the past three months, I am able to announce that the star 32 *Cassiopeiae* = DM. +64°127 is a rapidly-changing variable. It is one of a pair of naked-eye stars — about 49′ apart — whose approximate places for 1900 are thus given in the Harvard *Photometric Durchmusterung*:

DM. +63°149	R.A. 1h 5m.0	,	Decl. +63° 40′
32 *Cassiopeiae*	1 5.1	,	+64 29

These stars are rated as equal (5ᵐ.18) in the revised *Harvard Photometry*. ARGELANDER makes 32 *Cassiopeiae* the brighter by 0ᵐ.1, and a similar difference is [illegible] the *Photometric Durchmusterung*. The [illegible] magnitudes and spectrum-class are thus given in the [illegible] *Catalogue*: 32 *Cassiopeiae* 4.531, DM. +63°149 [illegible] 5.10

My observations of these stars were made with an ordinary binocular, by ARGELANDER'S method. Great care has been exercised in making the comparisons and I have good reason to believe that the observations are practically free from systematic errors depending on the relative positions of the stars. Occasional comparisons have also been made with DM. +63°99, rated as 5ᵐ.37 in the revised

H.P.). These show clearly that 32 *Cassiopeiae* is the variable. I find that the latter is alternately brighter and fainter than DM. +63°149, the extreme range being about seven grades, or $0^{M}.4$. The period — or more exactly, the interval between successive maxima or minima — is very nearly *eight hours*. This is the shortest period hitherto found among the brighter stars. The star has been watched, at intervals, through a complete period on five nights, viz.: Aug. 26–27, Sept. 10–11, 12–13, 13–14 and Oct. 30–31. Both the increase and decrease of light are very rapid. On Sept. 15 the variable decreased by about one-third magnitude, within twenty minutes, viz.: between $20^{h}\,30^{m}$ and $20^{h}\,50^{m}$ Eastern Standard Time. On October 2 a decrease of about $0^{M}.3$ in ten or twelve minutes was recorded. The observations of Oct. 30–31 show that the *increase* takes place with about equal rapidity.

The rapid decrease in the light of this variable renders it possible to determine, with some accuracy, the epochs at which the star appears equal in brightness to DM. +63°149. Up to the present I have secured thirteen observations of this phase,* as follows:

		h m			h m
Sept.	6	20 50	Sept.	30	20 17
	15	20 42	Oct.	2	20 16 ±
	17	20 57		14	19 35
	18	21 5		17	19 29
	19	21 35		30	18 46
	22	20 55		31	2 53
	29	20 27			

The phase $v = a$ was not directly observed on October 2, but the following observations were recorded: $20^{h}\,10^{m}$, $v 2\tfrac{1}{2} a$: $20^{h}\,20$–22^{m}, $a 2 v$. [v=32 *Cassiopeiae*, a=DM.+63°149.] The epochs are given in Eastern Standard Time. It will be seen that these observations point to an *oscillation* in the period — or possibly in the *form* of the light-curve.† Combining the observations of September 6 and October 30, the mean period for the interval is found to be $7^{h}\,59^{m}.2$. The probable error of this period will not exceed $0^{m}.1$. For the interval, Aug. 26–Oct. 30, a mean period of about $7^{h}\,59^{m}.2$ is indicated.‡ I hope, later on, to determine the period with some accuracy from a considerable number of observations. No accurate observations of the phase $v = a$, with *v increasing*, have as yet been secured. The records of October 30 show that this phase occurred about $3^{h}\,24^{m}$ later than the corresponding phase, with *v decreasing*. The time-interval between these phases is evidently variable. It was about *five hours* on Sept. 12–13 and 13–14, according to the records of those dates.

* It is believed that the *maximum* error in an observation of this kind will not exceed 15^{m}, while the probable error will fall below 5^{m}.

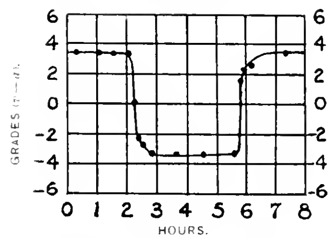
† The changes here referred to cannot be attributed to errors of observation, nor to variation in the light of the chief comparison-star, DM. +63°149.

‡ The observations of Aug. 26–27 afford only rough time-estimations for the phase $v = a$.

St. Catharine's, Ontario, Canada, 1904 Nov. 2.

My first observations of 32 *Cassiopeiae* and DM. +63°149 were made on August 20. The variation in relative brightness was discovered a few days later. Up to date I have secured 173 complete observations of these stars, besides a number of comparisons with DM. +63°99.

It may be desirable to indicate, in a few words, the writer's method of observation. No attempt is made to observe the stars *together*, but each in turn is *quickly* brought to the centre of the field; the comparisons being repeated until the observer feels satisfied that the result which he writes down is substantially correct. I have found it advantageous to compare the stars both *directly* and by slightly averted vision; the binocular, in each case, being carefully focussed. It seems evident that this procedure must yield results sensibly free from systematic errors, other than those due to the varying value of a *grade* for different intervals and magnitudes.



LIGHT-CURVE OF 32 CASSIOPEIAE.

A first approximation to the light-curve of 32 *Cassiopeiae* is shown in the annexed diagram. It is based mainly on the records of a single night (Oct. 30–31), though some additional data have been utilized in drawing the steeper parts of the curve. The black dots represent the observations of Oct. 30–31. The abscissa of *four hours* corresponds to $20^{h}\,28^{m}$ Eastern Standard Time, which is nearly the time of central *minimum*.

The provisional light-curve suggests that 32 *Cassiopeiae* may be an "eclipse-variable."

CONTENTS.

PUBLISHED AT 10 CRAIGIE ST., CAMBRIDGE (Boston Postal District), MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, CAMBRIDGE, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed November 22*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 570.

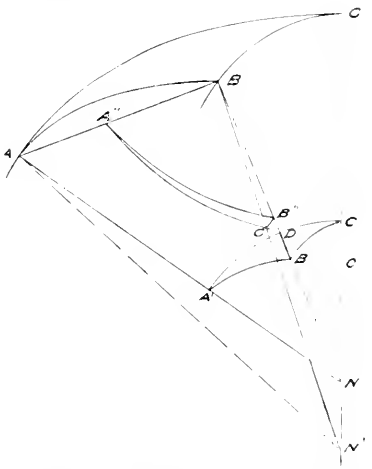
VOL. XXIV. BOSTON, 1905 JANUARY 9. NO. 18

THE COMPUTATION OF GEODETIC POSITIONS.

BY LOUIS B. STEWART.

Given the latitude and longitude of a point on the earth's surface, and the length and azimuth of the line drawn from it to a second point, to find the latitude and longitude of the second point and the azimuth of the first as seen from the second.

The following solution of the above problem appears to the writer to offer certain advantages over any other that has come to his notice, being in the convenient form of convergent series, in which a sufficient number of terms are retained to render the method applicable to longer distances than have ever occurred in any primary triangulation: and at the same time, by the omission of the smaller terms, giving a convenient solution for the moderate distances that occur most frequently in a trigonometric survey. The terms of the higher orders moreover do not assume such complicated forms as to render the method unsuitable for practical use.



While it is not claimed that the principle of the method is new, it is thought that the formulas derived have not before been published, with the exception of those which have been borrowed from well known sources to complete the problem, and extended so as to meet the requirements of the extreme distances here contemplated.

It is assumed that in reducing the lengths of the sides of a triangulation to differences of latitude and longitude the computations are carried to the nearest thousandth of a second, and on that basis an investigation is made of the minimum distances for which certain terms become appreciable.

Referring to the diagram, A represents the given point, and B that whose position is required; C is the pole, AC and BC the meridians, and AN and BN the normals at the two points; CNN' is the axis of the spheroid. ABC is a triangle on the surface of an imaginary sphere whose center is N.

φ_1 and φ_2 the geographical latitudes of A and B, respectively;

L_1 and L_2 their longitudes;

α_1 and α_2 their azimuths reckoned from the north towards the east;

$\Delta\varphi$ their difference of latitude $= \varphi_2 - \varphi_1$;

ΔL their difference of longitude $= L_2 - L_1$;

$\Delta\alpha$ the convergence of their meridians;

s the distance AB;

N_1 the length of the normal AN;

σ the angle ANB.

In the triangle ABC we have given the two sides $AC = 90° - \varphi_1$, $AB = \sigma$, and $BAC = \alpha_1$; by solving this triangle by the methods of spherical trigonometry, we obtain first approximations to the required quantities, with the exception of the difference of longitude which is given correctly for the spheroid; the difference of latitude and the reverse azimuth, however, must receive corrections to reduce them to the spheroid.

We proceed first to the determination of the difference of latitude. This may be found in two parts by drawing a perpendicular from B to AC, and finding the length of

the arc $A'D$, and the difference of latitude of D and B'. Denoting $A'D$ by u, and DB' by v, we have

$$u = \tan^{-1}(\cos\alpha_1 \tan\sigma)$$
$$v = \sin^{-1}(\sin\alpha_1 \sin\sigma)$$

then expanding, multiplying, out and arranging, these expressions become

$$\begin{aligned} u &= \cos\alpha_1 \tan\sigma - \tfrac{1}{3}(\cos\alpha_1 \tan\sigma)^3 + \tfrac{1}{5}(\cos\alpha_1 \tan\sigma)^5 - \\ &= \cos\alpha_1(\sigma + \tfrac{1}{3}\sigma^3 + \tfrac{2}{15}\sigma^5) - \tfrac{1}{3}\cos^3\alpha_1(\sigma^3 + \sigma^5) + \tfrac{1}{5}\sigma^5\cos^5\alpha_1 \\ &= \sigma\cos\alpha_1 + \tfrac{1}{3}\sigma^3\cos\alpha_1\sin^2\alpha_1 \\ &\qquad + \tfrac{2}{15}\sigma^5\cos\alpha_1\sin^2\alpha_1(1 - \tfrac{3}{2}\cos^2\alpha_1) \\ v &= \sigma\sin\alpha_1 - \tfrac{1}{6}\sigma^3\sin\alpha_1\cos^2\alpha_1 \end{aligned}$$

Then, on substituting in the first of these

$$y = \sigma\cos\alpha_1 = \frac{s}{N_1}\cos\alpha_1$$

we have, in seconds of arc,

$$(1) \qquad u = \frac{y}{\sin 1''} + \frac{y^3\tan^2\alpha_1}{3\sin 1''} + \frac{2y^5\tan^2\alpha_1}{15\sin 1''}(\tan^2\alpha_1 - \tfrac{1}{2})$$

Again, denoting the difference of latitude of D and B' by d we have

$$d = C'D - C'B'$$

also $$\cos C'B = \cos C'D \cos v$$

or $$\cos(C'D - d) = \cos C'D \cos v$$

Then expanding the cosines and sines of small angles we have

$$\cos C'D\left(1 - \frac{d^2}{2}\right) + d\sin C'D = \cos C'D\left(1 - \frac{v^2}{2} + \frac{v^4}{24}\right)$$

or $$d\tan C'D - \frac{d^2}{2} = -\frac{v^2}{2} + \frac{v^4}{24}$$

Then assuming as a first approximation

$$d\tan C'D = -\frac{v^2}{2} \quad\text{or}\quad d = -\frac{v^2}{2}\cot C'D$$

and substituting in d^2 we have

$$d\tan C'D = -\frac{v^2}{2} + \frac{v^4}{24}(1 + 3\cot^2 C'D)$$

Then introducing the value of v given above, this becomes

$$\begin{aligned} d\tan C'D = &-\tfrac{1}{2}\sigma^2\sin^2\alpha_1 + \tfrac{1}{6}\sigma^4\sin^2\alpha_1\cos^2\alpha_1 \\ &+ \tfrac{1}{24}\sigma^4\sin^4\alpha_1(1 + 3\cot^2 C'D) \end{aligned}$$

If we now denote the latitude of D by q', and substitute

$$x = \sigma\sin\alpha_1 = \frac{s}{N_1}\sin\alpha_1$$

we have, in seconds

$$d = -\frac{x^2\tan q'}{2\sin 1''} + \frac{x^2y^2\tan q'}{6\sin 1''} + \frac{x^4\tan q'}{24\sin 1''}(1 + 3\tan^2 q') \qquad (2)$$

The difference of latitude on the sphere is consequently

$$\Delta q' = u + d \qquad (3)$$

The relation between Δq, the true difference of latitude, and $\Delta q'$ must next be found. Let

s' = the length of the meridian arc included between the parallels of latitude of A and B;

R_m = the radius of curvature of the meridian corresponding to the mean of the latitudes q_m of the two points;

then taking the two expressions for s',

$$s' = N_1\,\Delta q'\left(1 - \tfrac{1}{6}\frac{e^2}{1-e^2}\,\Delta q'^2\cos^2 q_1\right)$$
$$s' = R_m\,\Delta q\,(1 + \tfrac{1}{8}e^2\,\Delta q^2\cos 2q_m)$$

(*vide* JORDAN, *Handbuch des Vermessungskunde*, Vol. 3, p. 371, and *Zachariae, Geodätische Hauptpunkte*, p. 17), and equating them, we have, on solving for Δq,

$$\Delta q = \Delta q'\frac{N_1}{R_m}\left(1 - \tfrac{1}{6}\frac{e^2}{1-e^2}\sin^2 1''.\Delta q'^2\cos^2 q_1 - \tfrac{1}{8}e^2\sin^2 1''.\Delta q^2\cos 2q_m\right) \qquad (4)$$

which gives the required relation.

In using this expression, as R_m corresponds to the mean of the true latitudes of A and B, a preliminary computation of Δq may be made by using the first term of (4) and the value of R_m corresponding to the latitude $q_1 + \frac{1}{2}\Delta q'$; and then a second approximation to the value of Δq may be made by interpolating a new value of R_m corresponding to the corrected mean latitude. Δq in the right-hand member is the value of that quantity found by the first term alone.

If $q_1 = 45°$ N. and $\Delta q' = 1°$, the sum of the last two terms in brackets amounts to

$$-0''.000608$$

and it evidently decreases as the latitude increases.

Let us next examine the terms of the higher orders in (1) and (2) to find under what conditions they may safely be neglected. The third term in (1) evidently vanishes when

$$\sec\alpha_1 = \pm\sqrt{\tfrac{3}{2}}$$

or when

$$\begin{aligned} \alpha_1 = {} & 35°\,16' \quad\text{or}\quad 144°\,44' \\ & 215\;\;16 \quad\text{or}\quad 324\;\;44 \end{aligned}$$

and also when $\alpha_1 = 0°$ or $180°$. Also, the values of α_1 for

which the term is a maximum or a minimum are given by the expression

$$\sin 2\alpha_1 = \pm \sqrt{}$$

from which we obtain the following values of α_1:

23° 27′	203° 27′
66 33	246 33
113 27	293 27
156 33	336 33

The greatest numerical value of the term under consideration, without regard to sign, is that corresponding to the second, third, sixth, or seventh of the above values of the azimuth; and if we assume

$$\varphi_1 = 45^\circ \text{ N.}$$
$$s = 100 \text{ miles}$$
$$\alpha_1 = 66^\circ 33'$$

we find the value of the term to be

$$0''.0000712$$

and as it varies as s^5 we find the value of s for which it amounts to $0''.0005$ to be

147.665 miles.

Again, the terms of the fourth order in (2) may be written in the form

$$\frac{s^4}{24\,N^4 \sin 1''}\left\{ \tan\varphi' (4 \sin^2\alpha_1 - 3\sin^4\alpha_1) + 3\tan^3\varphi' \sin^4\alpha_1 \right\}$$

Now, this expression vanishes when $\alpha_1 = 0°$ or $180°$; also assuming φ' to be constant its value is unchanged if we substitute for α_1 its supplement; therefore, φ' varying with α_1 the expression attains its maximum value when α_1 is a little less than 90°, or, with sufficient precision for our purpose, when α_1 equals 90°. This reduces it to the form

$$\frac{s^4}{24\,N^4 \sin 1''}(\tan\varphi_1 + 3\tan^3\varphi_1)$$

which may be assumed to give the maximum value of the term. From this we find that it will not exceed $0''.0005$ for distances under

13.598 miles.

We conclude, then, that if an error as large as $0''.0005$ is negligible, the terms of the fifth order may be omitted for distances under 147 miles, and also those of the fourth order for distances under 13 miles.

Proceeding next to the derivation of an expression for the difference of longitude of the two points, we follow the method of the United States Coast and Geodetic Survey. Referring again to the figure we have

$$\sin C' = \frac{\sin c' \sin A'}{\sin a'}$$

or

$$\sin \Delta L = \frac{\sin \sigma \sin \alpha_1}{\cos \varphi_2}$$

in which $\varphi_2' = \varphi_1 + \Delta\varphi'$. This may be written

$$\Delta L = \sin^{-1}\left(\frac{\sin\alpha_1}{\cos\varphi_2}\sin\sigma\right) = \sin^{-1}(m \sin\sigma)$$

which on expanding becomes

$$\Delta L = \frac{m\sigma}{\sin 1''} + \frac{(m\sigma)^3}{6\sin 1''}\left(1 - \frac{1}{m^2}\right) + \frac{(m\sigma)^5}{120 \sin 1''}\left(9 - \frac{10}{m^2} + \frac{1}{m^4}\right) \qquad (5)$$

in which

$$m = \frac{\sin\alpha_1}{\cos\varphi_2}$$

σ being expressed in radians.

In finding the convergence of the meridians of the two points we also follow the Coast Survey method. By one of NAPIER's analogies we have

$$\cot \tfrac{1}{2}(B' + A') = \frac{\cos\frac{1}{2}(b' + a')}{\cos\frac{1}{2}(b' - a')}\tan\tfrac{1}{2}C'$$

But

$$B' + A' = 360° - \alpha_2 + \alpha_1$$
$$= 180° - (\alpha_2 - 180° - \alpha_1)$$
$$= 180° - \Delta\alpha$$
$$b' + a' = 180° - (\varphi_1 + \varphi_2)$$
$$= 180° - 2\varphi_m$$
$$b' - a' = \varphi_2 - \varphi_1 = \Delta\varphi$$

$$\tan\tfrac{1}{2}\Delta\alpha = \frac{\sin\varphi_m}{\cos\frac{1}{2}\Delta\varphi}\tan\tfrac{1}{2}\Delta L = n\tan\tfrac{1}{2}\Delta L$$

or

$$\tfrac{1}{2}\Delta\alpha = \tan^{-1}(n\tan\tfrac{1}{2}\Delta L)$$

Whence, expanding and arranging as before, we have

$$\Delta\alpha = n\,\Delta L + \frac{\sin^2 1''}{12}(n\,\Delta L)^3\left(\frac{1}{n^2} - 1\right) + \frac{\sin^4 1''}{120}(n\,\Delta L)^5\left(\frac{1}{n^4} - \frac{[illegible]}{n^2} + [illegible]\right) \qquad (6)$$

in which

$$n = \frac{\sin\varphi_m}{\cos\frac{1}{2}\Delta\varphi}$$

This series is very convergent; in the example given below, in which $s = 209$ miles, the terms of the fifth order amount to only $0''.000012$.

The reverse azimuth on the sphere may now be written

$$\alpha_2 = \alpha_1 + \Delta\alpha + 180° \qquad (7)$$

This quantity, however, is the angle between the planes ABN and NBC, whereas the true azimuth of A at station B is the angle between the planes ABN' and $N'BC$. The correction to apply to the azimuth given by (7) to give the true azimuth may be obtained as follows: In the spherical triangle $A''B''C''$ given by the three directions BA, BN and BN', we have

$$A''B''C'' = \alpha_2' - 180^\circ$$
$$A''C''B'' = 360^\circ - \alpha_2$$

then denoting $B''C''$, or $\varphi_2 - \varphi_2'$, by δ_2 and the angle BAN by $90^\circ - \mu_1$, we have by spherical trigonometry,

$$\sin\alpha_2' \cot\alpha_2 = \cos\delta_2 \cos\alpha_2' + \sin\delta_2 \tan(\sigma - \mu_1)$$

then expanding the functions of the small angles δ_2 and $\sigma - \mu_1$, we have

$$\sin\alpha_2' \cot\alpha_2 - \cos\alpha_2' = \delta_2(\sigma - \mu_1) - \frac{\delta_2^2}{2}\cos\alpha_2'$$

But it may be shown that

$$\mu_1 = \frac{\sigma}{2}\left(1 + \frac{e^2}{1-e^2}\cos^2\varphi_1 \cos^2\alpha_1\right)$$

therefore substituting in the last expression we have

$$\frac{\sin(\alpha_2' - \alpha_2)}{\sin\alpha_2'} = \frac{\delta_2\sigma}{2}\left(1 - \frac{e^2}{1-e^2}\cos^2\varphi_1\cos^2\alpha_1\right) - \frac{\delta_2^2}{2}\cos\alpha_2'$$

or very nearly

$$(8)\quad \begin{cases} \alpha_2 - \alpha_2' = -\dfrac{\delta_2\sigma}{2}\sin\alpha_2'\left(1 - \dfrac{e^2}{1-e^2}\cos^2\varphi_1\cos^2\alpha_1\right) \\ \qquad\qquad + \dfrac{\delta_2^2}{2}\sin 1''\sin\alpha_2'\cos\alpha_2' \end{cases}$$

In this expression $\alpha_2 - \alpha_2'$ and δ_2 are in seconds of arc, and σ in radians; also

$$\delta_2 = \varphi_2 - \varphi_2'$$

which determines its algebraic sign.

In equation (8) the second and third terms may be written

$$\frac{\delta_2}{2}\sin 1''\sin\alpha_2'\left(\frac{e^2}{1-e^2}\sigma\cos^2\varphi_1\cos^2\alpha_1 + \delta_2\cos\alpha_2'\right)$$

δ_2 and σ both being in seconds; then, as the first term in brackets is always positive, and the second always negative, and as the expression

$$\frac{e^2}{1-e^2}\sigma\cos^2\varphi_1\frac{\cos^2\alpha_1}{\cos\alpha_2'}$$

gives an approximation to the value of δ_2, it follows that the sum of these terms is always inappreciable, and equation (8) may therefore be written

$$(9)\qquad \alpha_2 - \alpha_2' = -\frac{\delta_2\sigma}{2}\sin\alpha_2'$$

which is practically exact. In the example given below the omitted terms amount to but

$$0''.000074$$

The equations above developed will serve to determine the latitude and longitude of the distant station, and the reverse azimuth at that station, to within one or two ten-thousandths of a second, even for distances exceeding two hundred miles; but for moderate distances, not much exceeding forty miles, such as occur most frequently on trigonometric surveys, the smaller terms may be omitted, and the equations written in the abbreviated forms

$$\Delta\varphi' = \frac{y}{\sin 1''} + \frac{y^3\tan^2\alpha_1}{3\sin 1''} - \frac{x^2\tan\varphi'}{2\sin 1''} \qquad (10)$$

in which $\varphi' = \varphi_1 +$ first two terms

$$\Delta\varphi = \Delta\varphi'\frac{N_1}{R_m} \qquad (11)$$

$$\Delta L = \frac{x}{\cos\varphi_2'\sin 1''} \qquad (12)$$

$$\Delta\alpha = \frac{\Delta L\sin\varphi'_m}{\cos\frac{1}{2}\Delta\varphi'} \qquad (13)$$

$$y = \sigma\cos\alpha_1\quad ;\quad x = \sigma\sin\alpha_1$$

The logarithms of the constant terms in the above equations are as follows:

$$\log\frac{1}{\sin 1''} = 5.31442513$$
$$\frac{1}{3\sin 1''} = 4.83730$$
$$\frac{2}{15\sin 1''} = 4.43936$$
$$\frac{1}{2\sin 1''} = 5.01339951$$
$$\frac{1}{6\sin 1''} = 4.53627$$
$$\frac{1}{24\sin 1''} = 3.93421$$
$$\frac{1}{120\sin 1''} = 3.23524$$
$$\frac{\sin^2 1''}{12} = \overline{12}.29197$$
$$\frac{\sin^4 1''}{120} = 24.66312$$
$$\left(\frac{1}{6}\frac{e^2}{1-e^2}\sin^2 1''\right) = \overline{14}.42645$$
$$\left(\frac{1}{4}e^2\sin^2 1''\right) = \overline{14}.29856$$

We shall now test the precision of the above method by applying it to the solution of a numerical example taken

from *Clarke's Geodesy*. In that work, on pp. 109 and 110, the positions of two stations are given as follows, changing the designations of the stations:

	Lat.	Long.
Station A	53° 4′	4° 4′ W.
Station B	50 37	1 12 W.

from which the distance and mutual azimuths are computed as follows:

$$s = 1104377^{\text{ft}}.386$$
$$\alpha_1 = 142°\ 55'\ 50''.2183$$
$$\alpha_2 = 325°\ 11'\ 7''.4013$$

We take then as the data of our problem

$$q_1 = 53°\ 4'$$
$$\alpha_1 = 142°\ 55'\ 50''.2183$$
$$s = 1104377^{\text{ft}}.386$$

and use the other quantities given above as checks.

An outline only of the computation is given, showing the values of the various terms in the equations.

In the first place, as the distance s given above is the length of the section of the spheroid by the normal plane at A, we find the angle σ by the formula

$$\sigma = \frac{s}{N_1}\left(1 + \frac{1}{6}\,\frac{e^2}{1-e^2}\,\frac{s^2}{N_1^2}\cos^2\alpha_1\cos^2 q_1\right)$$

(JORDAN, Vol. 3, p. 371); or preferably we find $\log\sigma$ at once by expanding the logarithm of the terms in brackets, obtaining the expression

$$\log\sigma = \log\frac{s}{N_1} + \frac{M}{6}\,\frac{e^2}{1-e^2}\left(\frac{s}{N_1}\right)^2\cos^2\alpha_1\cos^2 q_1$$

which is sufficiently precise, as the omitted terms in the expansion do not amount to a unit in the twelfth decimal place. The spheroidal constants used in the problem are:

$$\log a = 7.32068747$$
$$\log e^2 = 3.830171 26$$
$$\log(1-e^2) = 1.998852531$$

whence we find

$$\log\sigma = 2.72148928$$

Then, to find q_2:

$$
\begin{array}{llrl}
\text{Eq. (1)} & u & = & -8666.94857 \\
 & & & -\quad 2.91101 \\
 & & & -\quad 0.00014 \\
 & & = & -8669.85972 \\
 & & = & -\ 2°\ 24'\ 29''.85972 \\
\therefore & q' & = & 50°\ 39'\ 30''.14028 \\
\text{Eq. (2)} & d & = & -\ 126.77585 \\
 & & & +\quad 0.07461 \\
 & & & +\quad 0.01064 \\
 & & & +\quad 0.04753 \\
 & & = & -\ 126.64307 \\
\therefore\ \text{Eq. (3)} & \Delta q' & = & -\ 8669.85972 \\
 & & & -\quad 126.64307 \\
 & & = & -\ 8796.50279
\end{array}
$$

$$
\begin{array}{llrl}
\text{Eq. (4)} & \Delta q & = & -8820.00317 \\
 & & & +\quad 0.00658 \\
 & & & -\quad 0.00323 \\
 & & = & -\ 8819.99982 \\
 & & = & -\ 2°\ 26'\ 59''.99982 \\
\therefore & q_2 & = & 50°\ 37'\ 00''.00018
\end{array}
$$

To find L_2:

$$
\begin{array}{llrl}
\text{Eq. (5)} & \Delta L & = & 10320.46388 \\
 & & & -0.46387 \\
 & & & -0.00046 \\
 & & = & 10319.99955 \\
 & & = & 2°\ 51'\ 59''.99955 \\
\therefore & L_2 & = & 1°\ 12'\ 00''.00045
\end{array}
$$

To find α_2: —

$$
\begin{array}{llrl}
\text{Eq. (6)} & q & = & 51°\ 50'\ 41''.7486 \\
 & \tfrac{1}{2}\Delta q & = & 1°\ 13'\ 18''.2514 \\
\text{Eq. (6)} & \Delta\alpha' & = & 8116.89050 \\
 & & & +1.69324 \\
 & & & -1.04746 \\
 & & = & 8117.53628 \\
 & & = & 2°\ 15'\ 17''.5363 \\
\therefore\ \text{Eq. (7)} & \alpha_2' & = & 325°\ 11'\ 07''.7546 \\
 & \delta_2 & = & -23''.4970 \\
\text{Eq. (9)} & \alpha_2-\alpha_2' & = & 0''.3532 \\
\therefore & \alpha_2 & = & 325°\ 11'\ 07''.4014
\end{array}
$$

The differences between the results thus found and those given above are therefore,

in latitude	0.0002
longitude	0.0004
azimuth	0.0001

The following table contains the values of $\log N$ and $\log R$ for latitudes between 40° and 60° north, computed by the formulas

$$\log N = \log a + \log(1+n) - Mn\cos 2q + \tfrac{1}{2}Mn^2\cos 4q - \tfrac{1}{3}Mn^3\cos 6q + \ldots$$

$$\log R = \log a + 2\log(1-n) + \log(1+n) - 3M\left(n\cos 2q - \tfrac{3}{4}n^2\cos 4q + \tfrac{2}{3}n^3\cos 6q\right)$$

in which $n = \dfrac{a-b}{a+b}$

a and b being the semi-axes. Adopting the CLARKE spheroid of 1866, in which

$$a = 6378206.4$$
$$b = 6356583.8$$

and computing the logarithms of the constant terms in the above equations, they may be written

$$\log N = 6.8054355339 - [4.8677004]\cos 2q + [7.79659]\cos 4q - [10.85044]\cos 6q$$

$$\log R = 6.8039592965 - [3.3448216]\cos 2q + [6.27374]\cos 4q - [9.32753]\cos 6q$$

which are the equations which were used in computing the table.

φ	log N	Diff.	log R	Diff.
40° 00′	6.80530670	423	6.80357339	1267
10′	1093	423	58606	1269
20′	1516	423	59875	1270
30′	1939	424	61145	1271
40′	2363	424	62416	1272
50′	2787	424	63688	1273
41° 00′	3211	425	64961	1275
10′	3636	425	66236	1275
20′	4061	426	67511	1277
30′	4487	425	68788	1277
40′	4912	426	70065	1278
50′	5338	427	71343	1279
42° 00′	5765	426	72622	1280
10′	6191	427	73902	1281
20′	6618	427	75183	1281
30′	7045	428	76464	1282
40′	7473	427	77746	1283
50′	7900	428	79029	1283
43° 00′	8328	428	80312	1284
10′	8756	428	81596	1284
20′	9184	428	82880	1285
30′	6.80539612	429	84165	1285
40′	6.80540041	428	85450	1286
50′	0469	429	86736	1286
44° 00′	0898	429	88022	1286
10′	1327	429	89308	1286
20′	1756	428	90594	1287
30′	2184	429	91881	1287
40′	2613	429	93168	1287
50′	3042	429	94455	1287
45° 00′	3471	429	95742	1287
10′	3900	429	97029	1287
20′	4329	429	98316	1287
30′	4758	429	6.80399603	1286
40′	5187	429	6.80400889	1287
50′	5616	429	02176	1287
46° 00′	6045	429	03463	1286
10′	6474	428	04749	1286
20′	6902	429	06035	1285
30′	7331	428	07320	1286
40′	7759	429	08606	1284
50′	8188	428	09890	1285
47° 00′	8616	428	11175	1284
10′	9044	428	12459	1283
20′	9472	427	13742	1283
30′	6.80549899	427	15025	1282
40′	6.80550326	428	16307	1282
50′	0754	427	17589	1280
48° 00′	1181	426	18869	1280
10′	1607	427	20149	1280
20′	2034	426	21429	1278
30′	2460	426	22707	1277
40′	2886	425	23984	1277
50′	3311	425	25261	1276
49° 00′	3736	425	26537	1274
10′	4161	425	27811	1274
20′	4586	424	29085	1272
30′	5010	424	30357	1271
40′	5434	423	31628	1270
50′	5857	423	32898	1269
50° 00′	6.80556280		6.80434167	
50° 00′	6.80556280	422	6.80434167	1268
10′	6702	422	35435	1266
20′	7124	422	36701	1265
30′	7546	421	37966	1263
40′	7967	421	39229	1262
50′	8388	420	40491	1261
51° 00′	8808	420	41752	1259
10′	9228	419	43011	1257
20′	6.80559647	418	44268	1256
30′	6.80560065	418	45524	1254
40′	0483	418	46778	1253
50′	0901	417	48031	1250
52° 00′	1318	416	49281	1249
10′	1734	416	50530	1247
20′	2150	415	51777	1245
30′	2565	414	53022	1244
40′	2979	414	54266	1241
50′	3393	413	55507	1239
53° 00′	3806	413	56746	1237
10′	4219	411	57983	1236
20′	4630	411	59219	1233
30′	5041	411	60452	1231
40′	5452	409	61683	1228
50′	5861	409	62911	1227
54° 00′	6270	408	64138	1224
10′	6678	407	65362	1222
20′	7085	407	66584	1219
30′	7492	405	67803	1217
40′	7897	405	69020	1215
50′	8302	404	70235	1212
55° 00′	8706	404	71447	1209
10′	9110	402	72656	1207
20′	9512	401	73863	1204
30′	6.80569913	401	75067	1202
40′	6.80570314	399	76269	1199
50′	0713	399	77468	1196
56° 00′	1112	398	78664	1194
10′	1510	397	79858	1190
20′	1907	396	81048	1188
30′	2303	395	82236	1185
40′	2698	394	83421	1181
50′	3092	393	84602	1179
57° 00′	3485	391	85781	1176
10′	3876	391	86957	1173
20′	4267	390	88130	1170
30′	4657	389	89300	1166
40′	5046	388	90466	1163
50′	5434	387	91629	1160
58° 00′	5821	385	92789	1157
10′	6206	385	93946	1154
20′	6591	383	95100	1150
30′	6974	382	96250	1147
40′	7356	381	97397	1143
50′	7737	380	98540	1140
59° 00′	8117	379	6.80499680	1136
10′	8496	378	6.80500816	1133
20′	8874	376	01949	1130
30′	9250	376	03079	1125
40′	6.80579626	374	04204	1122
50′	6.80580000	372	05326	1119
60° 00′	6.80580372		6.80506445	

University of Toronto.

OBSERVATIONS OF MINOR PLANETS AND COMETS,

MADE AT THE VASSAR COLLEGE OBSERVATORY,

BY MARY W. WHITNEY AND CAROLINE E. FURNESS.

1902–3–4 Gr. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log p Δ		Red. to App. Pl.	
					(57) *Mnemosyne.*					
1902 d h m s			m s	′ ″	h m s	° ′ ″			s	″
May 12 11 21 11	1	* 5 , 6	0 22.07	− 1 1.3	15 36 11.60	− 7 16 15.2	n9.481	0.808	+3.09	1.9
13 15 45 31	2	8 , 6	1 19.11	− 1 20.8	15 35 59.26	− 7 10 57.6	n9.193	0.818	+3.10	− 1.8
14 15 27 41	3	*10 , 12	−0 2.81	− 6 28.2	15 35 15.56	− 7 5 37.0	n9.255	0.817	+3.11	− 5.0
15 14 14 2	3	10 , 8	0 42.84	− 1 25.2	15 34 35.51	− 7 0 33.9	n9.466	0.807	+3.12	− 1.9
16 14 15 11	1	*10 , 8	+0 11.77	6 10.1	15 33 52.28	6 55 20.0	n9.453	0.808	+3.12	− 5.0
					(113) *Amalthea.*					
Nov. 3 16 21 25	5	12 , 6	−1 37.68	−10 23.8	3 7 52.28	+ 9 15 18.5	n9.004	0.676	+1.47	+12.6
4 15 6 39	6	12 , 6	+1 51.10	− 8 8.3	3 6 56.41	+ 9 11 22.8	n9.356	0.687	+1.48	+13.0
7 13 23 22	7	12 , 8	+0 55.10	+ 3 0.7	3 4 1.59	+ 8 59 32.3	n9.551	0.712	+1.51	+13.3
					(48) *Doris.*					
Nov. 19 14 23 27	8	* 8 , 8	−0 11.93	− 5 28.9	5 1 58.79	+13 59 56.1	n9.573	0.679	+1.76	− 0.6
20 14 43 1	9	* 8 , 8	−0 0.79	− 0 31.2	5 1 14.37	+13 56 51.1	n9.537	0.666	+1.78	− 0.5
					(46) *Hestia.*					
Jan. 6 13 58 30	10	* 8 , 8	−0 13.97	+ 1 6.1	5 11 40.57	+19 43 57.0	n9.296	0.512	+1.63	− 8.8
					(386) *Siegena.*					
Jan. 8 14 58 28	11	* 8 , 8	−0 32.21	+ 3 12.7	6 4 20.14	− 5 38 9.1	n8.995	0.810	+1.74	12.2
					(123) *Diotima.*					
Jan. 23 14 18 30	12	* 6 , 6	−0 1.24	3 11.8	6 19 26.21	+31 54 2.3	n9.029	0.190	+3.41	− 8.5
					(178) *Tergiste.*					
Jan. 28 14 27 40	13	* 7 , 8	+0 5.40	− 1 31.8	6 29 38.51	+ 7 31 8.7	n8.718	0.691	+1.89	13.1
					(79) *Eurynome.*					
Feb. 23 15 5 35	14	10 , 6	+0 52.68	1 21.3	10 16 46.30	+ 1 16 32.6	n9.330	0.733	+2.12	−16.2
24 14 11 11	15	* 8 , 8	− 0 5.91	3 2.6	10 15 50.95	+ 1 23 11.5	n9.382	0.733	+2.12	16.2
25 14 25 27	15	8 , 8	− 1 1.00	+ 3 15.7	10 14 55.90	+ 1 30 2.7	n9.120	0.731	+2.13	16.3
					COMET *a* 1903.					
1903 Jan. 30 12 25 13	16	6 , 4	+2 22.37	5 6.2	23 10 12.34	+ 5 7 37.5	9.629	0.753	0.28	+ 2.5
Feb. 14 12 1 45	17	8 , 4	2 31.13	+ 0 25.1	23 33 1.25	+ 9 52 12.2	9.640	0.711	0.22	+ 1.2
17 12 6 26	18	* 8 , 5	0 11.88	6 11.3	23 38 16.17	+10 57 7.6	9.646	0.712	0.23	+ 1.0
19 12 4 51	19	* 8 , 10	+0 16.65	+ 7 10.8	23 41 53.71	+11 41 30.6	9.648	0.711	0.22	+ 0.9
20 12 1 17	20	8 , 7	0 13.20	1 36.7	23 43 44.63	+12 1 0.5	9.648	0.710	0.21	+ 0.9
23 12 13 9	21	8 , 4	+3 3.89	1 16.1	23 49 30.28	+13 12 21.2	9.654	0.715	− 0.20	+ 0.8
25 12 21 19	22	* 8 , 8	+0 26.69	2 19.8	23 53 28.10	+13 57 57.3	9.657	0.750	0.18	+ 0.6
26 12 2 17	23	* 8 , 8	0 16.56	9 44.2	23 55 27.30	+14 20 12.1	9.656	0.759	0.18	+ 0.5
Mar. 3 11 56 51	24	† – , 2	. . . . .	1 0.4	. . . . .	+16 1 29.9	. . .	0.732	0.17	0.4
12 12 21 9	25	10 , 6	0 59.90	6 34.8	0 21 59.44	+17 14 44.6	9.663	0.761	0.12	0.8
13 12 15 2	26	10 , 4	1 24.13	5 23.9	0 23 16.03	+16 59 3.4	9.665	0.760	0.13	1.0
					(24) *Themis.*					
Mar. 2 14 13 13	27	* 6 , 6	+0 6.29	+ 9 14.3	11 23 58.80	+ 1 51 34.4	9.190	0.734	+2.08	15.0
17 14 51 3	28	* 8 , 8	− 0 21.13	− 6 10.1	11 12 33.01	+ 6 3 43.5	9.269	0.715	+2.16	15.6
18 15 38 36	28	10 , 8	1 6.69	4 10.7	11 11 47.45	+ 6 8 42.8	n8.953	0.710	+2.16	15.7

1903–4 Gr. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(29) Amphitrite.					
1903 Mar. 18 16 17 25	29	10 , 8	−1 18.03	− 2 27.5	13 8 24.15	− 9 53 56.7	n9.564	0.829	+2.28	−10.4[2]
					Comet 1903 *c*.					
July 17 14 38 4	30	10 , 7	−1 43.28	+10 21.2	18 35 1.17	+62 36 54.3	n9.481	n0.460	+3.36	+17.0[1]
23 15 30 17	31	12 , 8	−1 12.01	+ 7 51.8	13 52 32.86	+65 58 8.0	0.011	9.643	−0.15	+13.1[1]
					(324) Bamberga.					
Oct. 3 15 14 14	32	10 , 6	−1 12.74	− 8 14.9	21 42 17.59	− 5 59 2.1	9.194	0.811	+3.32	+24.5[1]
					Comet 1904 *a*.					
1904 May 2 15 48 42	33	12 , 8	+1 56.10	+ 3 18.6	16 4 4.81	+53 3 0.9	n9.646	n9.841	+2.11	− 2.6[1]
3 15 18 3	34	* 7 , 8	−0 52.78	− 0 29.5	16 0 0.32	+53 28 15.6	n9.701	n9.628	+2.11	− 2.3[1]
5 13 47 55	35	12 , 8	−1 17.24	− 3 26.0	15 51 44.82	+54 15 6.1	n9.821	9.927	+2.18	− 1.5[1]
7 15 15 39	36	12 , 8	+2 25.21	+10 10.8	15 42 40.66	+55 0 31.7	n9.616	n0.033	+2.21	− 0.5[1]
9 16 12 57	37	6 , 4	+2 30.60	+ 3 10.8	15 33 29.91	+55 10 27.5	n9.406	n0.271	+2.24	+ 0.4[1]
16 15 1 12	38	6 , 4	−1 51.00	− 8 14.9	15 1 8.95	+57 20 50.0	n9.436	n0.321	+2.23	+ 3.5[1]

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
1	15 37 3.58	− 7 20 44.6	Munich I. 11637	20	23 43 58.04	+12 5 36.3	A.G. Leipzig I, 9451
2	15 37 45.30	− 7 9 32.0	Schj. 5566 [5797]	21	23 46 26.59	+13 13 36.5	" " I, 9461
3	15 35 15.26	− 6 59 3.8	½ [Mü. I, 11612+ Mü. II,	22	23 53 1.59	+14 0 16.5	" " I, 9505
4	15 33 37.39	− 6 49 4.9	Munich I. 11592	23	23 55 14.04	+14 29 55.8	" " I, 9522
5	3 9 25.49	+ 9 25 29.7	A.G. Leipzig II, 1208	24	0 8 18.86	+16 2 30.1	A.G. Berlin A, 34
6	3 5 0.83	+ 9 19 18.1	" " II, 1184	25	0 22 59.16	+17 21 20.2	" " A, 109
7	3 3 1.98	+ 8 56 18.3	" " II, 1169	26	0 24 40.29	+17 1 28.0	" " A, 119
8	5 2 5.96	+14 5 25.6	" " I, 1512	27	11 23 50.43	+ 4 45 35.1	Toulouse 1778
9	5 1 10.38	+13 57 25.8	" " I, 1506	28	11 12 51.98	+ 6 10 9.2	A.G. Leipzig II, 5756
10	5 41 52.91	+19 39 59.7	A.G. Berlin A, 1716	29	13 9 39.90	− 9 51 18.8	Radcliffe '90, 3443
11	6 4 50.61	− 5 41 39.6	Radcliffe 1890, no. 1519	30	18 36 11.09	+62 26 16.1	A.G. Hels.-Gotha 9906
12	6 19 27.04	+31 57 55.6	A.G. Leiden 2597	31	13 53 45.02	+65 50 3.1	A.G. Christiania 2079
13	6 29 31.22	+ 7 38 53.6	A.G. Leipzig II, 3037	32	21 43 57.01	− 5 51 11.7	½ [Radcliffe '90, 5880 + Schjellerup 8856]
14	10 15 51.50	+ 4 21 10.1	A.G. Albany 4009	33	16 2 6.60	+52 59 14.9	A.G. Camb.(U.S.) 4911
15	10 15 54.77	+ 4 26 33.3	A.G. Albany 4010	34	16 0 50.96	+53 28 47.4	" " " 4903
16	23 8 20.25	+ 5 12 41.4	A.G. Leipzig II, 11563	35	15 52 59.88	+54 18 33.6	" " " 4869
17	23 35 35.90	+ 9 51 45.9	" " II, 11726	36	15 40 13.24	+54 50 21.4	½ [A.G. Cambridge (U.S.) 4822 + Hels.-Gotha 8474]
18	23 38 31.28	+11 3 50.9	" " II, 9412	37	15 30 57.10	+55 37 16.3	A.G. Hels.-Gotha 8408
19	23 41 37.28	+11 34 48.9	" " I, 9436	38	15 2 57.72	+57 29 1.4	" " " 8227

* Δα measured directly. † Clouds prevented complete observation.

[1] Observer, Mary W. Whitney. [2] Observer, Caroline E. Furness. [3] Observer, Elise C. Whitney.

CONTENTS.

Published at 16 Craigie St., Cambridge (Boston Postal District), Mass., semi-monthly, by S. C. Chandler. Address, Cambridge, Mass.
Associate Editors, Asaph Hall and Lewis Boss. Price, $5.00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. Closed January 8.

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 571.

VOL. XXIV. BOSTON, 1905 MARCH 28. NO. 19

THE CONSTANT OF ABERRATION.

By C. L. DOOLITTLE.

When observation with the zenith telescope was begun at the Flower Observatory in 1896, it was proposed to carry on the series according to a uniform plan for seven years, this constituting a complete cycle of the twelve and fourteen-month periods. The plan was carried out with but slight deviation from the original program. This particular series was closed 1903 Dec. 7.

It seems now to be in order to assemble the various determinations of the aberration-constant which have resulted from this series and from the similar work done at the Sayre Observatory. These will be combined into a mean value, which, in my judgement, should be regarded as the definitive value of this constant, as shown by the data here given.

The following are the individual values:

SAYRE OBSERVATORY.

		Observation.	Pairs.	Wt.
1	1889 Dec. 1–1890 Dec. 13	20.448 ± .011	1179	0
2	1892 Oct. 10–1893 Dec. 27	20.551 ± .009	2900	1
3	1894 Jan. 19–1895 May 16	20.537 ± .014	1989	1

FLOWER OBSERVATORY.

		Observation.	Pairs.	Wt.
4	1896 Oct. 19–1898 Aug. 16	20.580 ± .008	2009	1
5	1898 Oct. 8–1899 Nov. 27	20.540 ± .010	1503	2
6	1900 Mar. 5–1901 Aug. 30	20.561 ± .008	1994	2
7	1901 Oct. 3–1902 Dec. 1	20.513 ± .009	1935	2
8	1903 Jan. 22–1903 Dec. 7	20.521 ± .009	1554	2

The details of the observations on which these values depend have been published in full with the exception of the last two. For (1), (2) and (3), see Transactions of the American Philosophical Society, Vol. XX. For (4), (5) and (6), Publications of the University of Pennsylvania, Series in Astronomy Vol. I, Part II, and Vol. II, Part I. A brief statement as to the weight here assigned seems to be called for.

First, the instrument employed at the Sayre Observatory was one with which no one would now be likely to undertake such an investigation. The building was a brick structure with thick walls, calculated to absorb a great amount of heat during the day and to radiate the same during a considerable part of the night. The different values will be taken up in order.

Series (1). The method employed was such as to make it impossible to separate the aberrations from the latitude variation without introducing some assumption in regard to one or the other. In this case it was assumed that the latitude-variation could be represented by two periodic terms of twelve and fourteen months respectively. This was presumably admissible, and would doubtless have given a result worthy of some confidence if observation had extended over a full period of fourteen months, or if the data had been distributed symmetrically with respect to the maximum and minimum of the latitude. Neither condition was fulfilled. A maximum occurred not very far from the middle of the series which began soon after a minimum had been passed and ended before reaching the next one.

Series (2). Essentially the same method was followed, as in the previous case, but here the data were so distributed with respect to the time of maximum and minimum, and with reference to the right-ascensions of the stars employed, that nearly the same value for the aberration-constant is obtained whether the latitude-variation is considered or not. The latter is practically eliminated.

Series (3). In this and all of the following series the method was that of KÜSTNER, sometimes called the polygon method, by which the aberration is completely separated from the latitude-variation.

Series (4). This formed the first result obtained at the Flower Observatory. Some peculiarities in the behavior of the micrometer were developed in the process of reduction which it was suspected might be due, in part at least, to the effect of temperature. The computation has, therefore, been revised, after a [illegible] investigation of the temperature coefficient. In this examination 797 separate measures of differences of [illegible] were used, the range of temperature being from [illegible] to [illegible]87. T

Some slight changes in individual values resulted, but the value obtained for the aberration-constant remained precisely as before.

Series (5), (6), (7), (8). These values call for no especial comment. They have been assigned the wt. 2 each.

The resulting mean value of the aberration-constant is

20".540 ± .0055

It will be observed that no admissible assumption as to weight can change this result to the extent of 0".01.

A double series of observations is now in progress here, employing both the zenith telescope and the reflex zenith tube. It is hoped that this may be continued for two or three years at least.

Flower Observatory, Jan. 4, 1905.

OBSERVATIONS OF COMET *d* 1904 (BORRELLY),

MADE AT THE GOODSELL OBSERVATORY, NORTHFIELD, MINN., WITH THE 16-INCH REFRACTOR,

BY H. C. WILSON.

1905 Northfield M.T.	*	Comp	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	' "	h m s	° ' "			s	"
Jan. 2 8 47 39	1	12 , 6	−0 49.43	− 2 27.8	1 19 8.22	−6 31 16.7	9.375	0.843	−0.13	−8.9
2 8 47 39	2	12 , 6	−1 5.03	− 4 39.3	1 19 8.11	−6 31 16.3	9.375	0.843	−0.12	−8.9
3 8 3 32	3	17 , 6	+0 1.88	−12 32.8	1 20 34.	−5 14 −	9.230	0.832	−0.13	−8.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ' "			h m s	° ' "	
1	1 19 57.76	−6 28 10.0	Yarnall 698	3	1 20 32.	−5 31 −	S.DM. −5°254
2	1 20 13.27	−6 26 28.1	Yarnall 702				

NEW COMET, *a* 1905 (GIACOBINI).

A faint comet was discovered by GIACOBINI, at Nice, in the following position:

1905 March 26.3213 R.A. = $5^h\ 44^m\ 14^s.0$, Decl. = +10° 56' 56". Motion in α $+3^m$; in δ N. 1° 15'.

A second position has been observed by AITKEN, at Lick Observatory, as follows:

March 27.6692 R.A. = $5^h\ 48^m\ 54^s.8$, Decl. = +12° 35' 13".

NOTATION OF NEWLY DISCOVERED VARIABLE STARS.

The Committee of the *Astronomishe Gesellschaft*, undersigned, which has recently unfortunately suffered a grave loss in the retirement of Professor OUDEMANS from its membership on account of illness, herewith publishes a list of those new variables whose light-variations appear certain enough for the usual notation to be supplied. The number of new variables has recently extraordinarily increased, but the observation of them has unfortunately not kept pace correspondingly, so that for many objects nothing further has become known than is contained in the brief announcement of their variability, by the discoverers. The Committee has proceeded very carefully in the matter of the notation, regarding it as better to withhold for a time stars not completely certain, than to insert them in the list of known variables prematurely, with the liability of necessity for their subsequent withdrawal. The numerous variables which have been recently found in groups in the *Orion*-nebula, in the Magellanic clouds, and some other parts of the heavens, should not be denoted with letters in the usual way, but included in groups, and provided with current numbers for each special group, similarly as is intended for the variables in stellar clusters.

The arrangement of the following list is the same as for the earlier similar lists published by the Committee. But in the present list short notes are added at the end of the list containing essentially all that is at present known of the individual objects.

The Committee earnestly request observers of variables to devote especial attention to the new variables in this list, and to publish their observations in detail without undue delay, so that the Committee may be in position to include in the new Catalogue of Variables more accurate data on the light-variations of these stars.

The Committee for the A.G. Catalogue of Variable Stars.

DUNÉR, HARTWIG, MÜLLER.

Cur. No.	Provis. Notation A.N.	Name	Position for 1900 R.A.	Position for 1900 Decl.	Prec. 1900 R.A.	Prec. 1900 Decl.	Chart-Place R.A.	Chart-Place Decl.	Magn. Max.	Magn. Min.
			h m s	° ′	s	′	h m s	° ′		
1	60.1903	*V Piscium*	0 17 15	+ 6 7.2	+3.08	+0.33	0 14 57	+ 5 52.2	9.5	12
2	156.1904	*Z Ceti*	1 1 39	− 2 1.0	+3.06	+0.32	0 59 21	− 2 15.5	9	12
3	23.1903	*RS Sculptoris*	1 22 31	−33 25.6	+2.76	+0.31	1 21 22	−33 33.4	10	<11
4	17.1904	*RV Andromedae*	2 4 34	+48 27.6	+3.85	+0.29	2 1 41	+48 14.7	8	10
5	16.1904	*RS Persei*	2 15 20	+56 39.1	+4.20	+0.28	2 12 12	+56 26.6	8	10
6	1.1904	*RR Persei*	2 21 11	+50 49.1	+4.02	+0.27	2 18 13	+50 37.1	9	<13
7	155.1904	*RT Persei*	3 16 11	+46 12.2	+4.13	+0.22	3 13 39	+46 2.3	9.5	11
8	2.1904	*RT Tauri*	4 58 10	+23 30.4	+3.63	+0.09	4 55 27	+23 26.3	9	10
9	11.1904	*X Orionis*	5 32 36	− 1 19.9	+3.03	+0.01	5 30 19	− 1 51.8	11	<14
10	20.1904	*RU Tauri*	5 46.9	+15 57	+3.45	+0.02	5 44.3	+15 56	12	14.5
11	157.1904	*RS Aurigae*	5 56 27	+46 16.1	+4.47	+0.01	5 53 6	+46 15.8	9.2	10.5
12	133.1904	*RR Aurigae*	6 1.8	+13 11	+4.33	−0.01	6 1.5	+13 11	″	″
13	12.1904	*RT Geminorum*	6 40.7	+18 44	+3.52	−0.06	6 38.1	+18 47	10	<15
14	10.1904	*RS Monocerotis*	7 2 11	+ 5 8.7	+3.19	−0.09	6 59 47	+ 5 12.7	9	<10 11
15	141.1904	*RV Geminorum*	7 11 56	+24 6.1	+3.61	−0.10	7 9 12	+24 10.6	10	<15
16	24.1903	*T Can. majoris*	7 17 18	−25 15.6	+2.48	−0.11	7 16 16	−25 12.9	9	10.5
17	15.1904	*RU Geminorum*	7 21 0	+21 38.1	+3.57	−0.12	7 18 19	+21 43.5	12 13	14
18	3.1904	*Y Cancri*	7 58 37	+20 21.7	+3.50	−0.17	7 55 59	+20 32.1	12	14
19	. . .	*Z Camelopardalis*	8 14 5	+73 25.6	+6.82	−0.18	8 8 56	+73 33.8	10	13
20	. . .	*X Ursae majoris*	8 33.9	+50 29	+4.31	−0.21	8 30.7	+50 38	9	<12
21	19.1904	*S Leonis minoris*	9 47 46	+35 23.9	+3.59	−0.28	9 45 1	+35 36.5	8	9
22	24.1904	*RZ Carinae*	10 32.8	−70 12	+1.69	−0.31	10 32.1	−70 1	9	<13
23	25.1904	*Z Ursae majoris*	11 51 17	+58 25.7	+3.16	−0.33	11 48 56	+58 40.3	″	″
24	134.1904	*Y Ursae majoris*	12 35 17	+56 23.7	+2.76	−0.33	12 33 12	+56 38.6	8	9
25	113.1904	*U Ursae minoris*	14 14.9	+67 10	+1.31	−0.28	14 14.0	+67 23	8.5	12
26	135.1904	*RY Centauri*	14 43.3	−42 5	+3.86	−0.25	14 41.7	−41 59	[illegible]	
27	26.1903	*Y Normae*	16 25 39	−46 13.7	+4.37	−0.14	16 23 50	−46 10.3	8.8	10
28	18.1904	*RU Ophiuchi*	17 28 8	+ 9 29.9	+2.85	−0.05	17 25 59	+ 9 32.0	9	12
29	136.1904	*RV Ophiuchi*	17 29 15	+ 7 18.9	+2.90	−0.01	17 27 35	+ 7 20.7	9	12
30	158.1904	*RW Ophiuchi*	17 50 30	+ 7 51	+2.89	−0.01	17 48 20	+ 7 52	10	<12.13
31	30.1903	*SV Sagittarii*	17 57 11	−24 29.9	+3.68	0.00	17 55 39	−24 29.8	11	14
32	165.1904	*V Serpentis*	18 11 4	−15 33.4	+3.44	+0.02	18 8 29	−15 34.0	9.5	10.5
33	162.1904	*SV Herculis*	18 22 18	+24 58.0	+2.45	+0.03	18 20 27	+24 56.6	9.5	<15
34	140.1904	*V Scuti*	18 42 32	−12 14.1	+3.36	+0.06	18 40 0	−12 17.0	12	14
35	109.1904	*ST Lyrae*	19 6 39	+43 29	+1.86	+0.10	19 5 11	+43 25	10	<12
36	63.1903	*SS Lyrae*	19 10 24	+46 18.6	+1.72	+0.10	19 9 8	+46 14.0	9	13
37	137.1904	*SW Sagittarii*	19 13 26	−31 54.2	+3.86	+0.11	19 11 19	−31 56.8	9 10	11
38	71.1903	*RX Aquilae*	19 40 22	+ 8 12.2	+2.90	+0.11	19 38 11	+ 8 5.9	11	<15
39	77.1903	*RY Aquilae*	19 43 40	+11 16.6	+2.83	+0.15	19 41 32	+11 10.0	10	12
40	83.1903	*RZ Aquilae*	19 49 6	+ 9 21.0	+2.88	+0.15	19 46 57	+ 9 17.2	11	15
41	161.1904	*X Vulpeculae*	19 53 19	+26 17.3	+2.49	+0.16	19 51 27	+26 10.2	9.5	10.5
42	154.1904	*WW Cygni*	20 0 56	+41 18.2	+2.06	+0.17	19 59 3	+41 10.7	9.3	12
43	5.1904	*W Vulpeculae*	20 5 53	+25 59.1	+2.52	+0.17	20 3 59	+25 51.6	9	10
44	59.1903	*WX Cygni*	20 14 50	+37 8.2	+2.23	+0.19	20 13 10	+36 59.9		
45	1.1904	*V Vulpeculae*	20 32 17	+26 15.4	+2.55	+0.21	20 30 22	+26 6.2	8	10
46	61.1903	*UY Cygni*	21 0 26	+39 31.3	+2.29	+0.24	20 58 43	+39 23.7	9	10
47	138.1904	*U Microscopii*	21 17.5	−41 7	+3.83	+0.25	21 15.9	−41 13		
48	164.1904	*WY Cygni*	21 44 13	+43 46	+2.36	+0.28	21 42 57	+43 33	8 9	12 14
49	7.1904	*VZ Cygni*	21 47 41	+42 39.9	+2.40	+0.28	21 45 53	+42 27.3	8	9
50	163.1904	*W Lacertae*	22 3 11	+37 15	+2.58	+0.29	22 1 15	+37 2	9 10	12 13
51	142.1904	*RU Pegasi*	22 9 9	+12 12.4	+2.94	+0.30	22 6 57	+11 59.1	9.4	11.5
52	159.1904	*RV Pegasi*	22 21 2	+29 57.9	+2.75	+0.30	22 18 58	+29 44.3	9	11
53	86.1903	*T Tucanae*	22 33 58	−62 1.5	+4.00	+0.31	22 32 18	−62 12.2	8	14
54	110.1904	*V Lacertae*	22 44 33	+55 47.6	+2.44	+0.32	22 42 44	+55 33.4	8.5	[illegible]
55	31.1904	*RW Pegasi*	22 59.2	+14 45	+2.98	+0.32	22 57.0	+14 31	9	[illegible]
56	108.1904	*RS Cassiopeiae*	23 32 33	+61 52.6	+2.77	+0.33	23 30 29	+61 37.7	9	11
57	160.1904	*RT Cassiopeiae*	23 41 26	+53 57	+2.92	+0.33	23 39 15	+53 42	10	12
58	28.1903	*Y Ceti*	23 54 27	−24 59.1	+3.09	+0.33	23 53 10	−25 7.5	9.8	11.5

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By W. WALTER DINWIDDIE.

(Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.)

1904 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(156) *Xanthippe.*					
Jan. 25 11h 35m 26s	1	t30 , 6	−0m 39.68s	+ 1′ 51.7″	7h 50m 5.34s	+ 7° 54′ 32.3″	7.508	0.653	+1.60s	−13.6″
25 11 49 40	2	t29 , 6	−0 46.85	+ 4 19.5	7 50 4.69	+ 7 54 30.1	8.491	0.654	+1.60	−13.6
Feb. 4 10 44 23	3	t27 , 6	−4 5.46	1 20.5	7 40 51.48	+ 8 10 49.2	n7.266	0.650	+1.63	−14.3
					(388) *Charybdis.*					
Feb. 11 9 53 35	4	t28 , 6	+1 49.61	− 4 2.3	8 40 36.57	+25 5 2.6*	n9.257	0.355	+1.71	−13.3
11 10 11 5	5	t29 , 6	+0 29.77	− 2 30.6	8 40 35.75	+25 5 4.0	n9.159	0.311	+1.72	−13.3
11 10 32 14	6	t30 , 6	+2 27.64	− 3 23.4	8 40 34.73	+25 5 6.0	n8.991	0.328	+1.72	−13.3
15 10 40 40	7	t30 , 6	+3 46.37	− 4 33.3	8 37 14.11	+25 9 8.7	n8.589	0.346	+1.72	−12.9
17 12 49 21	8	t30 , 6	+2 4.53	− 7 12.5	8 35 35.40	+25 10 34.4	9.290	0.359	+1.72	−12.7
					(454) *Mathesis.*					
Feb. 17 9 54 25	9	t28 , 6	+2 45.11	− 9 22.9	10 40 52.32	+18 17 14.9	n9.532	0.568	+1.61	−14.0
20 10 20 53	10	t30 , 6	+3 12.76	− 2 40.1	10 38 3.25	+18 30 54.5	n9.438	0.534	+1.65	−13.9
22 10 15 13	11	t30 , 6	−1 14.79	− 3 38.4	10 36 8.94	+18 39 29.8	n9.427	0.529	+1.66	−13.8
22 10 34 22	12	t30 , 6	−2 33.12	− 9 15.6	10 35 8.13	+18 39 35.1	n9.363	0.516	+1.66	−13.8
					(386) *Siegena.*					
Feb. 24 14 26 4	13	t30 , 6	+1 11.38	+ 0 30.3	12 1 29.04	+ 2 43 27.3	8.896	0.714	+1.61	−11.0
Mar. 8 13 16 3	14	10 , 10	−0 18.53	− 8 1.0	11 53 18.47	+ 4 44 22.4	8.760	0.692	+1.76	−12.3
8 13 39 28	15	10 , 10	+0 8.32	− 8 33.9	11 53 17.79	+ 4 44 31.2	9.016	0.693	+1.76	−12.3
13 13 22 24	16	t29 , 6	+0 32.10	− 0 44.9	11 49 48.50	+ 5 32 18.0	9.063	0.684	+1.80	−12.6
13 13 35 41	17	t30 , 6	+0 37.17	+ 7 50.0	11 49 48.09	+ 5 32 23.6	9.150	0.685	+1.80	−12.6
					(334) *Chicago.*					
Mar. 9 9 55 9	18	10 , 10	−0 8.57	+ 4 19.5	10 45 21.56	+10 54 26.1	n9.297	0.626	+1.76	−13.9
9 10 38 2	19	t30 , 6	+2 34.84	− 2 12.0	10 45 20.66	+10 54 31.6	n9.065	0.617	+1.76	−13.9
13 11 38 14	20	t40 , 8	−1 17.66	+ 8 14.3	10 42 56.85	+11 11 12.4	8.626	0.610	+1.76	−13.8
					(47) *Aglaja.*					
Mar. 9 11 47 1	21	t29 , 6	−3 35.48	+ 4 22.3	10 53 59.09	+ 9 45 30.8	7.751	0.629	+1.76	−13.7
13 10 6 8	22	t30 , 6	−2 10.47	−11 16.0	10 50 46.01	+ 9 59 36.3	n9.197	0.633	+1.77	−13.7
13 10 32 34	23	t30 , 6	−2 27.88	−12 59.6	10 50 45.10	+ 9 59 38.5	n9.026	0.629	+1.77	−13.7
					(423) *Diotima.*					
Mar. 15 12 20 31	24	t30 , 6	−1 12.45	+ 0 31.0	12 20 27.98	+15 6 33.2	n8.736	0.548	+1.72	−11.9
15 12 36 10	25	t28 , 6	+0 50.32	+ 4 41.7	12 20 27.61	+15 6 39.3	n8.346	0.547	+1.72	−11.9
					(84) *Klio.*					
Mar. 16 12 28 23	26	t30 , 6	+2 21.64	+ 2 25.3	12 9 8.05	− 9 59 57.0	n7.823	0.820	+1.98	−11.4
					(46) *Hestia.*					
Mar. 16 13 41 39	27	t30 , 6	−4 3.41	+ 4 54.0	12 4 40.25	− 0 37 27.4	9.165	0.746	+1.86	−11.8
Apr. 3 11 25 41	28	t30 , 6	−2 29.29	+ 3 19.5	11 49 40.52	+ 1 12 39.9	8.684	0.729	+1.89	−12.6
3 11 47 58	29	t30 , 6	+0 42.40	+ 7 47.3	11 49 39.82	+ 1 12 44.7	8.965	0.729	+1.89	−12.7
					(317) *Roxane.*					
Apr. 16 10 16 56	30	t35 , 7	−1 13.60	+ 6 22.6	12 56 56.81	− 3 39 57.3	n9.079	0.772	+2.05	−10.1
18 12 4 47	31	t30 , 6	+1 5.27	− 2 36.0	12 55 4.78	− 3 27 37.3	9.054	0.770	+2.04	−10.2
18 12 22 23	32	t30 , 6	−2 27.58	+ 1 40.0	12 55 4.18	− 3 27 32.7	9.167	0.769	+2.05	−10.1
18 12 42 41	33	t18 , 4	−3 2.33	+ 2 27.3	12 55 3.27	− 3 27 27.2	9.267	0.768	+2.05	−10.1
20 13 44 58	34	t40 , 8	−1 28.47	+ 2 4.8	12 53 16.34	− 3 15 44.8	9.485	0.762	+2.04	−10.2
					(403) *Cyane.*					
Apr. 16 11 29 49	35	t30 , 6	+1 50.86	+ 6 24.1	12 54 25.21	−16 46 15.0	8.492	0.860	+2.26	−11.2
16 11 44 51	36	t25 , 5	+0 59.06	+ 2 50.8	12 54 24.51	−16 46 10.7	8.795	0.859	+2.25	−11.2
19 11 36 51	37	t34 , 7	+1 9.21	+ 0 19.3	12 52 14.85	−16 22 11.1	8.872	0.856	+2.24	−11.5
21 11 15 53	38	t36 , 6	−0 44.19	+ 6 25.4	12 50 52.77	−16 6 8.7	8.700	0.856	+2.24	−11.7
21 11 35 57	39	t30 , 6	−2 16.39	+ 9 17.0	12 50 52.23	−16 6 3.0	8.961	0.854	+2.24	−11.6

1904 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(178) *Belisana.*					
h m s			m s	′ ″	h m s	° ′ ″			s	″
Apr. 16 12 24 51	40	10 , 10	−0 11.59	+ 2 0.6	13 8 9.64	−18 17 8.4	9.071	0.864	+2.30	−10.2
21 9 44 54	41	t30 , 6	+4 3.58	+ 8 12.5	13 4 45.47	−17 36 11.9	n9.220	0.856	+2.29	−11.2
21 10 12 11	42	t30 , 6	+1 59.20	− 6 21.2	13 4 14.59	−17 36 0.8	n9.045	0.861	+2.29	−11.0
May 4 11 30 32	43	t30 , 6	−1 31.86	+ 3 41.4	12 57 5.38	−15 45 42.2	9.231	0.847	+2.24	−11.8
					(49) *Pales.*†					
Apr. 21 12 55 4	44	t29 , 6	+3 36.51	+ 3 42.1	13 52 50.32	−16 3 18.4	9.102	0.852	+2.33	−12.4
					(455) *Bruchsalia.*					
Apr. 21 13 43 1	45	t34 , 7	+0 42.00	− 2 15.1	11 10 40.21	+ 0 39 46.1	9.252	0.731	+2.07	− 6.5
21 14 7 58	46	t30 , 6	+2 40.55	− 7 44.9	11 10 39.28	+ 0 39 48.4	9.349	0.735	+2.07	− 6.6
May 4 12 54 22	47	t29 , 6	+2 26.48	+ 0 55.6	13 58 50.08	+ 1 10 31.0	9.311	0.730	+2.12	− 6.3
12 10 16 37	48	t30 , 6	+0 32.56	− 1 35.0	13 52 7.01	+ 1 18 12.0	n8.435	0.728	+2.12	− 5.8
					(108) *Hecuba.*					
May 7 11 41 8	49	t30 , 6	−2 19.46	+ 7 50.3	15 25 20.02	−24 34 8.5	n8.960	0.892	+2.67	− 1.1
					(241) *Germania.*					
May 7 12 26 24	51	t30 , 6	+2 2.31	− 3 34.9	15 34 49.86	−24 23 58.0	n8.177	0.894	+2.66	− 1.7
12 11 57 54	52	t30 , 6	+3 8.48	+ 4 3.1	15 30 39.23	−24 5 15.8	n8.366	0.893	+2.71	− 1.4
					(153) *Hilda.*					
May 12 12 45 39	53	t30 , 6	−1 8.93	− 5 51.3	16 12 54.58	−18 37 21.3	n8.022	0.869	+2.54	+ 1.9
					(90) *Antiope.*					
May 12 13 37 51	54	10 , 10	−0 21.05	− 7 27.9	15 52 53.90	−19 23 31.3	9.147	0.867	+2.58	+ 0.4

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	7 50 43.39	+ 7 52 54.2	Leipzig II, A.G. 4230	28	11 52 7.92	+ 1 9 35.0	Albany, A.G. 4382
2	7 50 49.91	+ 7 50 24.5	Leipzig II, A.G. 4234	29	11 48 55.53	+ 1 5 10.1	Albany, A.G. 4369
3	7 44 54.71	+ 8 12 21.0	Leipzig II, A.G. 4146	30	12 58 8.36	3 46 9.8	Strassburg, A.G. Zones
4	8 38 45.25	+25 9 18.2	Camb. Eng., A.G. 4674	31	12 53 57.47	3 24 51.1	Strassburg, A.G. Zones
5	8 40 4.26	+25 7 47.9	Camb. Eng., A.G. 4680	32	12 57 29.71	3 29 2.6	Strassburg, A.G. Zones
6	8 38 5.37	+25 8 12.7	Camb. Eng., A.G. 4669	33	12 58 3.55	− 3 29 44.4	Strassburg, A.G. Zones
7	8 33 26.02	+25 13 54.9	Camb. Eng., A.G. 4628	34	12 54 42.77	3 17 29.4	Strassburg, A.G. Zones
8	8 33 29.15	+25 17 59.6	Camb. Eng., A.G. 4629	35	12 52 32.09	16 52 27.9	Washington, A.G. Zones
9	10 38 5.60	+18 26 51.8	Berlin A., A.G. 4238	36	12 53 23.20	−16 48 50.3	Washington, A.G. Zones
10	10 34 48.84	+18 33 18.5	Berlin A., A.G. 4223	37	12 51 3.40	16 22 18.0	Washington, A.G. Zones
11	10 37 22.07	+18 13 22.0	Berlin A., A.G. 4234	38	12 51 34.72	−16 12 22.4	Washington, A.G. Zones
12	10 37 39.59	+18 49 4.5	Berlin A., A.G. 4240	39	12 53 6.38	−16 15 8.4	Washington, A.G. Zones
13	12 0 16.05	+ 2 43 8.0	Albany, A.G. 4419	40‡	13 8 18.93	−18 18 58.8	Washington, A.G. Zones
14	11 53 35.24	+ 1 52 35.7	½ (Leipzig II, A.G. [illegible]) + ½ (Albany, A.G. [illegible])	41	13 0 39.60	−17 44 13.2	Washington, A.G. Zones
15	11 53 7.71	+ 1 53 17.4	½ (Leipzig II, A.G. [illegible]) + ½ (Albany, A.G. [illegible])	42	13 2 13.10	−17 29 28.6	Washington, A.G. Zones
16	11 49 14.60	+ 5 31 45.7	Leipzig II, A.G. 5952	43	12 58 35.00	−15 49 11.8	Washington, A.G. Zones
17	11 49 9.12	+ 5 24 46.2	Leipzig II, A.G. 5950	44	13 48 51.48	−16 6 18.1	Washington, A.G. Zones
18	10 45 28.37	+10 50 20.5	Leipzig I, A.G. 4131	45	11 9 56.11	+ 0 42 7.7	[illegible]
19	10 42 41.06	+10 56 57.5	Leipzig I, A.G. 4115	46	11 7 56.66	+ 0 47 39.9	[illegible]
20	10 44 12.75	+11 3 11.9	Newcomb's Fund. Cat.	47	13 56 21.48	+ 1 9 41.7	Albany, A.G. 4849
21	10 57 32.81	+ 9 11 22.2	Leipzig II, A.G. 5666	48	13 51 32.33	+ 1 19 52.8	Albany, A.G. 4824
22	10 52 51.71	+10 14 6.0	Leipzig I, A.G. 4161	49	15 27 36.81	−24 41 57.7	[illegible]
23	10 53 11.21	+10 12 51.8	Leipzig I, A.G. 4162	50	15 28 12.69	−24 47 10.9	Cordoba G.C. 21077
24	12 21 38.71	+15 6 11.1	Berlin A., A.G. 4674	51	15 32 39.89	24 20 24.1	Cordoba G.C. 21184
25	12 19 35.57	+15 2 9.5	Berlin A., A.G. 4656	52	15 27 28.04	24 9 47.5	Cordoba G.C. 21062
26	12 6 11.13	−10 2 10.9	Wien, A.G. Zones	53	16 14 0.96	−18 31 31.9	Radcliffe 1890, 1250
27	12 8 11.80	− 0 12 9.6	Nicolajew, A.G. 3376	54	15 53 12.37	−19 16 3.9	[illegible]

The star places from the Strassburg Zones were furnished through the courtesy of the Director of the Observatory at that place. The above planets, with the exception of (46) *Hestia* and (47) *Aglaja*, were found photographically by Mr. G. H. Peters.

* Changed from 12′.6, one revolution of the micrometer, to agree with the two other observations on the same day.

† The position of (49) *Pales* determined from [illegible] of Apr 20, at 12h 30m W. M. T., is [illegible].

‡ Star No. 40 is double, of which each [illegible] in the Washington A.G. Zones. The mean of the two stars has been used.

MINIMA OF THE *ALGOL*-TYPE VARIABLE 7096 *SY CYGNI*,

R.A. $19^h\ 42^m\ 43^s$, Decl. $+32°\ 27'.6$ (1900),

BY J. A. PARKHURST.

In the course of investigations carried out under a grant from the Carnegie Institution, a number of observations, visual and photographic, have been secured of this variable, which proves to be of especial interest on account of its unusual range of variation. Table I gives the observed magnitudes, together with a comparison with the elements of CERASKI (*A.N.*, *151*, 223), J.D. 241 5001.971 + 6^d.006528 E; also with the elements given by PICKERING (*A.N.*, *152*, 89), who uses the same zero-epoch, but makes the period 6.006111 days. The columns headed *C* and *P* refer respectively to these two sets of elements. In the last column the visual observations are indicated by "vis.," the photographic by "phot." The plates for the latter were taken with 12 and 18-inches aperture, on the 24-inch reflecting telescope, with both ordinary and isochromatic plates.

TABLE I.

	G.M.T.	J.D.	Mag.	Epoch	Calculated Min. J.D. *C*	*P*	Phase *C*	*P*	
	h						d	d	
1901 Dec. 18	13.4	5737.56	13.5	122	37.77	37.72	−0.21	−0.16	phot.
Dec. 19	12.8	5738.53	10.8	122	37.77	37.72	+0.76	+0.81	phot.
1902 July 28	17.7	5959.74	13.6	159	60.01	59.94	−0.27	−0.20	phot.
1904 July 15	17.1	6677.71	10.9	278	74.79	74.67	+2.92	+3.04	phot.
July 15	17.4	6677.73	10.9	278	74.79	74.67	+2.94	+3.06	phot.
Nov. 22	13.3	6807.55	10.8	300	6.93	6.81	+0.62	+0.74	vis.
–	13.9	6807.58	10.8	300	6.93	6.81	+0.65	+0.77	phot.
–	14.1	6807.59	10.8	300	6.93	6.81	+0.66	+0.78	vis.
–	15.1	6807.63	10.7	300	6.93	6.81	+0.70	+0.82	vis.
Nov. 25	11.9	6810.49	10.7	301	12.93	12.81	−2.44	−2.32	phot.
Dec. 3	14.3	6818.60	12.8	302	18.94	18.82	−0.34	−0.22	vis.
Dec. 4	13	6819.54	10.5	302	18.94	18.82	+0.60	+0.72	vis.
Dec. 5	12.0	6820.50	10.8	302	18.94	18.82	+1.56	+1.68	vis.
Dec. 6	12.0	6821.50	10.8	302	18.94	18.82	+2.56	+2.68	vis.

Table II gives the coordinates from the variable and the magnitudes of the comparison-stars used. The coordinates were measured from the photographs. The magnitudes were obtained by a combination of measures with the equalizing wedge photometer on the 12-inch refractor, and of measures of diameters on the images on the photographs. The latter were reduced to the visual scale by the aid of negatives taken on isochromatic plates; which, used with the reflecting telescope, furnish means for correcting for star colors.

The color correction to the yellow star *g* amounts to 0.6 magnitude. The variable photographs 0^M.52 brighter than the visual magnitude.

The following conclusions are drawn from Table I: *First*, observations fell near three minima, epochs 122, 159 and 302. *Second*, if these observations are quite near minimum, they indicate a correction of $-0^d.2$ to PICKERING's ephemeris. If, however, the elements are correct, the range of variation must be considerably greater than three magnitudes.

TABLE II.

Star	Coordinates from V R.A.		Decl.	Mag.
	s	″	″	
m	− 3.4	− 43	− 25	12.6
h	− 1.7	− 21	+103	10.8
g	+ 0.3	+ 4	+ 58	10.1
f	+ 4.3	+ 55	− 96	11.2
+32°3559	+ 6.5	+ 82	−153	9.8
+32°3560	+12.3	+156	+ 35	9.0
n	+15.1	+191	+ 38	12.3
q	+17.0	+215	+138	13.2
k	+18.4	+233	− 65	11.1
l	+22.3	+282	−159	12.1

Yerkes Observatory, 1904 *December*.

DISCOVERY OF A SIXTH SATELLITE OF *JUPITER*.*

A dispatch from Prof. CAMPBELL to Harvard College Observatory, on Jan. 5, states that a sixth satellite of *Jupiter* has been discovered by PERRINE. It was suspected in December, and confirmed last night. Position angle with reference to *Jupiter*, 269°, distance 49′, decreasing 45″ daily. Apparent motion retrograde; magnitude 14. Crossley reflector, Dec. 3, 8, 9, 10; Jan. 2, 3, 4.

* *From Supplement to No. 570.*

COMET *e* 1904 (*BORRELLY*).*

[From RITCHIE's Circular, No. 137, of January 3.]

A message via Harvard College Observatory from Kiel, received on December 30, announces the discovery of a comet by BORRELLY at Marseilles, on December 28. A second position, by HAMMOND, came the following day through the courtesy of Admiral CHESTER, Director of the U.S. Naval Observatory, which was distributed to American astronomers, and later the same day, the Konigsberg position, by COHEN, was received from Kiel. Admiral CHESTER has also telegraphed the hereunder given Elements and Ephemeris, computed at the Observatory by Messrs. MORGAN and LAMSON, from observations of Dec. 30, 31 and Jan. 1. These positions are given below.

POSITIONS.

Gr. M.T. 1904	R.A. h	m	s	Decl. °	′	″	Observer
Dec. 29.365	1	13	10.	−10	..		Marseilles
30.6620	1	15	14.2	− 8	56	24	Hammond
31.2085	1	15	56.5	− 8	29	59	Cohen
31.5926	1	16	26.1	− 8	11	6	Seares

ELEMENTS.

T = 1905 Jan. 13.47 Greenw. M.T.

ω = 319 59
☊ = 72 57 Mean Eq. 1904.0
i = 32 17

q = 1.4899

EPHEMERIS.

Gr. Midnight 1905	R.A. h	m	s	Decl. °	′	Light
Jan. 5	1	23	12	−1	9	0.97
9	1	29	12	−0	51	
13	1	35	40	+2	18	
17	1	42	32	+5	25	0.87

Light, Dec. 29 = 1.

* From Supplement to No. 570.

ORBIT OF COMET *d* 1904 (*GIACOBINI*).*

[From RITCHIE's Circular, No. 137, of January 3].

The orbit of GIACOBINI's comet, which was circulated by telegraph to American astronomers, having run out, and the comet being exceedingly faint, the following Elements and Ephemeris are here published. They have been kindly communicated by Admiral COLBY M. CHESTER, having been computed by Messrs. MORGAN and LAMSON, at the U.S. Naval Observatory, from observations of Dec. 18, 20 and 21.

ELEMENTS.

T = 1904 Oct. 19.10 Greenw. M.T.

ω = 31 30
☊ = 217 1 Mean Eq. 1904.0
i = 98 55

q = 1.8091

EPHEMERIS.

Gr. Midnight 1905	R.A. h	m	s	Decl. °	′	Light
Jan. 6	17	9	8	+31	18	0.96
10	17	21	36	39	24	
14	17	34	36	41	32	
18	17	48	12	+43	10	0.87

Light, Dec. 17 = 1.

* From Supplement to No. 570.

OBSERVATIONS OF COMET *e* 1904 (*BORRELLY*).*

MADE WITH THE 16-INCH EQUATORIAL OF THE CINCINNATI OBSERVATORY.

BY J. G. PORTER.

1905 Cin. M.T. d h m s	*	Comp.	$\Delta\alpha$ m s	$\Delta\delta$ ′ ″	App. α h m s	App. δ ° ′ ″	log $p\Delta$		Red. to app. place s	″
Jan. 1 7 56 17	1	12 , 12	−1 51.01	+1 28.9	1 17 42.71	−7 23 17.4	9.207	0.799	0.13	−9.1
3 8 0 10	2	12 , 12	+1 7.10	+3 35.5	1 20 25.86	−5 45 7.2	9.253	0.787	0.11	−8.7

Mean Places of Comparison-Stars for 1905.0.

*	α h m s	δ ° ′ ″	Authority
1	1 19 33.85	7 24 37.2	Rad. 1890, 327; Gr 10 yr., 216; 2d Gr 10-yr., 516
2	1 19 18.90	5 48 34.0	DM 6 266; equatorial comparison with
	1 14 18.65	5 49 30.4	Ll. 2394, W 1[h], 486; Rum. 593; Par 1674

In last star, LALANDE's R.A., and WEISSE's Decl., rejected.

* From Supplement to No. 570.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR, BY A. W. QUIMBY.

1904	Time	New Grs.	Total Grs	Total Spots	Fac Grs	Def.
July 1	8	..	3	25	2	fair
2	8	..	2	18	2	fair
3	8	..	2	20	2	fair
4	8	..	1	10	1	fair
5	7	2	3	24	2	fair
6	7	..	2	10	..	poor
7	7	..	3	20	1	fair
8	7	..	2	3	..	poor
9	5	..	2	13	2	good
10	4	..	2	15	3	fair
11	7	..	1	20	1	fair
*12	7	..	2	15	2	fair
*13	7	1	2	11	1	fair
*14	7	..	2	20	..	fair
*15	7	2	4	21	1	fair
*16	7	2	6	22	1	fair
*17	7	..	6	32	2	fair
*18	7	..	6	34	2	fair
*19	7	..	6	28	2	fair
*20	8	..	3	21	2	poor
*21	7	1	3	12	2	poor
*22	7	..	1	6	..	v.poor
*23	3	3	6	22	1	fair
24	7	..	3	11	0	poor
25	1	..	3	13	..	poor
26	4	..	3	23	3	poor
27	8	..	3	11	2	fair
28	8	..	3	21	2	fair
29	8	..	3	50	3	good
30	7	..	3	25	2	fair
31	8	..	3	18	1	fair
Aug. 1	8	..	3	18	2	fair
2	8	1	3	12	2	poor
3	8	..	3	14	2	poor
4	8	2	5	17	3	poor
5	4	..	5	18	3	poor
6	10	..	5	24	3	good
7	8	..	3	12	2	good
8	8	1	3	12	2	fair
9	8	..	3	10	2	poor
10	4	1	4	13	2	poor
11	4	1	5	30	2	fair
12	8	1	6	32	2	fair
13	10	..	4	20	2	poor
14	9	..	3	26	3	good
15	8	1	4	17	3	poor
16	8	..	3	6	2	poor
17	7	..	2	4	2	poor
18	8	1	3	7	2	fair
19	8	..	3	6	2	fair
20	3	..	3	4	3	fair
21	8	..	2	3	3	fair
22	8	1	3	9	2	fair
23	8	..	3	20	2	fair
24	8	1	4	64	2	good
25	8	1	5	62	4	good
26	8	1	6	95	3	good
27	8	..	4	73	2	fair

1904	Time	New Grs	Total Grs	Total Spots	Fac Grs	Def
Aug. 28	8	..	4	61	2	poor
29	7	1	5	97	3	good
30	8	..	4	61	3	poor
31	7	..	5	63	3	fair
Sept. 1	8	..	4	54	3	fair
2	8	..	2	11	2	poor
3	8	1	3	12	4	fair
4	8	..	2	10	3	fair
5	3	..	1	7	2	fair
6	8	..	1	6	2	fair
7	8	..	1	4	2	fair
8	4	..	1	7	1	fair
9	3	1	2	11	1	fair
10	5	..	1	17	..	poor
11	2	..	1	8	1	poor
12	7	..	1	6	2	fair
13	7	..	1	6	1	fair
15	7	..	1	2	..	poor
16	7	..	1	14	..	fair
17	7	..	1	10	..	fair
18	8	1	2	7	1	poor
19	4	..	2	17	3	good
20	3	..	1	14	..	poor
21	7	2	3	16	3	poor
22	8	..	3	18	2	poor
23	8	..	3	40	3	good
24	7	..	3	12	2	fair
25	8	1	3	40	3	good
26	8	..	3	20	3	fair
27	8	..	3	17	2	fair
28	8	..	2	14	2	fair
29	8	1	2	6	1	poor
30	8	..	2	7	2	fair
Oct. 1	2	1	3	8	1	poor
2	9	..	2	3	..	v.poor
3	7	..	2	5	1	poor
*4	7	1	2	15	2	poor
*5	8	..	2	34	2	fair
6	11	1	3	36	1	fair
7	10	2	5	30	3	fair
8	8	..	5	30	2	poor
9	8	..	5	34	3	fair
10	10	..	4	26	1	poor
11	8	1	5	35	3	fair
13	10	1	5	20	1	poor
14	8	..	5	15	2	fair
15	8	..	4	9	3	poor
16	8	1	4	6	2	fair
17	8	1	5	14	4	fair
18	8	1	5	9	2	poor
19	9	1	5	14	4	fair
20	8	..	5	12	1	poor
21	12	..	3	14	2	poor
22	10	..	2	22	2	poor
23	8	..	3	22	2	poor
24	8	1	4	21	2	poor
25	8	..	4	20	2	poor
26	10	1	4	66	4	fair

1904	Time	New Grs.	Total Grs.	Total Spots	Fac Grs.	Def.
Oct. 27	7	1	3	65	4	fair
28	7	1	3	62	3	fair
29	8	..	3	79	3	fair
30	8	..	3	53	4	fair
31	8	..	3	37	3	fair
Nov. 1	8	1	3	14	3	poor
2	9	..	2	10	1	poor
3	8	..	2	8	..	poor
4	11	..	2	5	..	poor
5	9	1	3	6	..	poor
6	8	..	3	6	0	poor
7	7	..	3	5	1	poor
8	8	1	3	7	3	fair
9	8	..	1	3	3	fair
10	11	1	2	3	..	poor
11	2	..	2	5	2	fair
12	8	..	1	3	3	fair
14	9	1	2	6	..	poor
15	9	..	2	12	3	fair
16	9	1	3	14	1	fair
17	9	1	4	14	2	fair
18	8	..	4	10	3	poor
19	8	1	4	17	3	fair
20	8	..	4	14	3	fair
21	8	..	4	14	2	fair
22	8	1	4	21	3	fair
23	8	..	3	13	..	poor
24	1	..	4	28	3	fair
25	9	..	3	13	3	poor
26	8	..	4	40	4	fair
27	8	..	2	22	3	poor
28	8	..	2	20	2	poor
29	2	1	3	37	3	fair
30	10	1	3	18	2	fair
Dec. 1	8	..	3	8	1	poor
2	8	..	1	4	1	fair
3	8	..	1	3	..	poor
5	9	..	1	4	..	poor
6	8	3	4	14	2	fair
7	10	2	6	22	2	fair
8	12	1	6	36	2	poor
9	11	1	7	51	3	fair
11	8	1	7	72	3	fair
13	8	2	8	42	4	poor
14	8	1	8	43	5	fair
15	8	..	7	34	2	poor
16	10	..	4	21	1	poor
17	9	..	1	5	..	v.poor
18	8	..	3	22	2	poor
19	8	..	2	8	1	poor
20	8	..	2	10	2	poor
21	8	1	2	12	3	fair
22	10	1	1	3	4	fair
23	12	2	2	7	4	fair
28	8	1	1	10	3	fair
29	8	..	1	8	2	poor
30	7	1	1	6	..	v.poor
31	9	1	2	11	3	fair

*$2\frac{1}{2}$-in h refractor.

SUNSPOT OBSERVATIONS,

MADE AT THE AMHERST COLLEGE OBSERVATORY,

BY ROBERT H. BAKER.

1904		New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots	Def
	d h									
Oct.	2 22	1	6		–	1	6	2	7	3
	4 20	1	21	–	–	–		2	28	3
	5 1	–	6	–		–	–	2	32	3
	5 22	2	11	–		2	8	1	15	5
	6 3	–	–	–		–	–	1	10	4
	7 0	1	5	–		1	1	5	28	5
	10 5	–	–	–		–	–	4	11	1
	11 3	–	16	1	1	–	–	3	36	3
	13 22	1	7	–	–	1	1	4	20	3
	14 20	–	1	–	–			4	14	3
	15 2	–	–	–	–	–	–	4	8	3
	15 21	1	2	–	–	–	–	4	9	4
	16 4	1	6	–		1	1	5	15	5
	16 21	–	2	1	1	–		4	9	3
	17 4	–	3	–	–	–	–	4	9	4
	17 21	–	2	–	–	–	–	4	16	3
	18 3	–	1	–	–	–	–	4	10	4
	18 22	–	1	–	–	–	–	4	9	3
	19 4	1	15	–	–	–	–	5	24	5
	21 21	–	–	1	1		–	3	18	3
	22 4	–	–	–	–	–	–	2	17	4
	23 0	1	3	–	–	–	–	3	17	3
	23 22	–	4	–	–	–	–	2	11	3
	24 4	–	13	–	–		–	2	27	5
	24 22	–	–	–	–	–	–	1	9	1
	25 4	1	10	–	–	1	3	3	28	4
	26 21	1	34	1	3	1	3	3	59	3
	27 3	–	–	–	–			3	55	4
	27 22	–	5	–	–		–	3	17	3
	28 22	–	8	–				2	16	4
	29 22	–	1	–				2	16	3
	30 4	–	–	–			–	2	34	3
	30 21	–	–	–	–	–		2	19	3
	31 23	1	1	–	–	1	1	3	12	2
Nov.	1 3	1	3	–	–			4	12	4
	1 22	–		1	2			2	8	2
	2 4	1	5	–	–		1	3	12	1
	3 22	–	–	–				2	4	1
	5 2	–	–	–				3	7	3
	5 23	–	1	–		–		3	8	3
	6 4	–	–	–		–		3	6	3
	7 4	–	–	1	1			2	2	2
	7 21	–	–	–				1	1	4
	8 22	–	–			–	–	4	1	2
	9 22	1	3	–	–	1	3	2	1	4
	10 3	–	–	–				2	1	5
	11 3	–	–	–				2	1	5
	11 20	–	–					2	3	3
	12 3	2	9		–	1	1	1	12	3
	14 21	–	10	1	1	–		2	11	2
Nov.	15 23	–				–		2	4	1
	16 22	1	2		–	1	2	3	10	3
	17 3	–	3		–			3	11	5
	17 22		1				–	3	9	3
	18 3		3	–		–		3	12	4
	18 21	1	5	–	–	1	1	3	14	4
	19 3		1	–	–	–	–	3	15	5
	20 0	–		–	–			3	11	3
	21 0		1					3	10	3
	21 21		3	1	1			2	16	5
	22 3	–	4	–				2	20	5
	22 21	–		–			–	2	19	5
	23 3		–	–	–	–		2	10	2
	25 3	–	–		–		–	2	10	2
	27 21	–		–	–		–	1	6	1
	28 4	1	14	–	–		–	2	20	3
	29 3	–	8	–	–		–	2	28	1
	30 3	–		–	–			2	3	2
	30 21			–	–			2	4	2
Dec.	1 3	–	–	–	1			2	4	4
	1 21		1	1	1		–	1	4	3
	2 3	–	–					1	2	1
	3 22	2	3			1	1	3	4	4
	4 3		–	–				3	4	3
	5 22	–	14	1	1	–	11	2	17	5
	6 3		9	–			–	2	26	5
	6 21	1	12	–	5	1	1	3	32	3
	8 22		6	1	1		3	2	37	1
	10 22		3					2	26	1
	11 3		–		–	–		2	24	3
	11 21	–	7		3	–		2	27	3
	13 2	2	23	–		1	7	1	46	5
	13 21	–	6	–	–			3	13	3
	14 3		1	–	–			3	14	1
	15 22	–	–	–	3			3	34	1
	16 3	–	8	–	–	–		3	54	5
	16 21		–	1	3			2	15	2
	17 22			–	–			2	24	5
	18 3	–						2	19	5
	19 22				7			2	9	3
	20 22	–		–				2	9	5
	21 3	–	–	–				2	6	2
	24 22	1	3	1	1			1	3	1
	22 3	–	–	–				1	2	3
	27 23		2					1	5	1
	28 21							1	5	3
	29 22	1	14	–				2	17	4
	30 3		9		–			2	28	1
	31 0							2	14	1
	31 20	–		–				2	10	4

Observed with 6-inch Reflector.

OBSERVATION OF COMET *e* 1901 (*BORRELLY*).*

By E. E. BARNARD.

Time 6h slow of Gr.	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	Red. to app. place	
1904 Dec. 31 $7^h\ 0^m\ 0^s$	16.8	$+2^m\ 12^s.36$	$-1'\ 7''.5$	$1^h\ 16^m\ 22^s.34$	$-8°\ 13'\ 35''.5$	$+2^s.85$	$+9''.9$

	α	δ	Authority
Comp.-Star 1901.0	$1^h\ 14^m\ 7^s.13$	$-8°\ 12'\ 37''.9$	Wien-Ottakring, A.G.C. 263

The comet is rather large, 2′ 3′ in diameter, of the 11th magnitude, very much brighter in the middle, to a nucleus or very strong condensation. The measures were made with the 10-inch. The time is 6^h slow of Greenwich M.T.

Yerkes Observatory, 1905 *January* 1.

* *From Supplement to No.* 570.

NOTE ON THE MASSES OF *MERCURY*, *VENUS* AND THE *EARTH*, AND ON THE SOLAR PARALLAX.

By A. HALL.

In his investigations on the motions of the four principal inner planets LEVERRIER computed equations of condition between the perturbations of these planets and his assumed values of their masses. The mass of a planet was assumed to be of the form $m(1+\nu)$; the quantities with no accent standing for *Mercury*, those with one accent for *Venus*, with two for the *Earth*, &c. Prof. WILLIAM HARKNESS in his work on the *Solar Parallax, and its Related Constants*, has collected nineteen of these equations, and solved them for corrections to the masses of the three inner planets. The reciprocals of the masses adopted by LEVERRIER are

3 000 000 : 401 847 : 354 936.

The values found by HARKNESS are

8 704 559 : 404 681 : 332 768.

The mass of *Mercury* is very uncertain, and in view of several recent estimates it seemed to me worth while to assume a value of ν, or the mass of *Mercury*, and to solve LEVERRIER'S nineteen equations for corrections to the adopted masses. The following table gives my assumed value of ν, the corresponding mass of *Mercury*, and the resulting masses of *Venus* and the *Earth*. I add the value of $[nn, 2]$, from the least square solutions, since it gives an idea of the accuracy of the different solutions.

ν	$\frac{1}{m}$	$\frac{1}{m'}$	$\frac{1}{m''}$	$nn, 2$	π
				$''$	$''$
$-\frac{3}{4}$	12 000 000	404 557	332 935	4.510	8.758
$-\frac{2}{3}$	9 000 000	404 669	332 785	4.256	8.762
$-\frac{2}{5}$	5 000 000	405 015	332 320	6.085	8.764
0	3 000 000	405 529	331 624	16.251	8.770

A value $\frac{1}{m} = 10\,000\,000$ will satisfy the equations very well. The reciprocal 5 000 000 is nearly the value found from the motions of the ENCKE and WINNECKE comets.

At the Transits of *Venus*, in 1874 and 1882, great efforts were made to determine the solar parallax by photographic methods, and we hoped to fix its value within 0″.01. The German astronomers applied the heliometer for the same purpose. The mean result of the photographic and heliometer methods is about 8″.86; and these attempts must be considered practical failures.

When we compare the fall of a body on the *Earth* with the fall of the *Earth* toward the *Sun*, we find the following equation:

$$\pi = 609''.50 \times (m'')^{\frac{1}{3}}.$$

π being the solar parallax, and m'' the mass of the *Earth*. The numerical coefficient in this equation was computed several years ago from BESSEL'S dimensions of the *Earth*, and probably it will not be changed much in the future. If the mass of the *Earth* were accurately known we should have the solar parallax by this simple equation. The secular perturbations produced by the *Earth* will at length give its mass, and this method will give, I think, the standard value of the solar parallax. Taking the mass of the *Moon* $\frac{1}{81}$ that of the *Earth*, I have computed the value of π, in the last column of the table, by the above formula and the corresponding mass of the *Earth*.

1905 *February* 3.

OBSERVATIONS OF *PHOEBE*, THE NINTH SATELLITE OF *SATURN*.

By E. E. BARNARD.

In 1898 Professor WM. H. PICKERING announced the discovery, by photography, of a very faint ninth satellite to *Saturn*, which he named *Phoebe*. I was anxious to observe this object visually and wrote to Professor PICKERING for an ephemeris. He was unable to give me one because the observations were not enough to determine the orbit. Matters lay thus for six years, but as there was no ephemeris it was impossible to look for the satellite visually, especially as it was an extremely faint object and at a very great distance from *Saturn*.

In May, 1904, I learned that Professor PICKERING had secured more photographs of the satellite and was getting out an orbit. I wrote again for an ephemeris. Fortunately, however, Professor PICKERING was himself able to come to the Yerkes Observatory and make a search for the satellite, but, unfortunately, an urgent telegram called him away after but one night's search.

Professor PICKERING left with me some photographs and later sent an ephemeris. This ephemeris he afterward found was somewhat in error because of the unusual motion of the satellite. I was requested to compare one of the photographs with the sky so as to form some idea of the probable visual magnitude of *Phoebe*, as the satellite was as bright as the fainter stars shown on the photograph. Some of these stars were marked 16th, 17th and $17\frac{1}{2}$ magnitude. The position was $\alpha = 17^h 50^m$ $\delta = -22\frac{1}{2}°$. On July 4, with the position an hour west of the meridian, I was able to see all the stars in the region marked out by Professor PICKERING. From this it would appear that *Phoebe* ought to be seen visually with the 40-inch. I had already looked for it on the nights of June 17 and 25. It was also looked for on July 4 and 10, but the ephemeris was not exact enough for one thing and the seeing was not perfect enough for another, and the satellite was not found.

Finally, three ephemeris positions of ten-day intervals were issued in a Bulletin from Harvard College Observatory, but no favorable opportunity occurred to look for the satellite until the ephemeris had expired. The last of these positions was for Aug. 3. I waited for a continuation of the new ephemeris, which did not come until Aug. 18.

In the first of August we had the great pleasure of a visit from Professor TURNER, of Oxford, England, who had just visited Harvard College Observatory. From what he told me of his visit to Cambridge I was further convinced that the existence of the satellite was real, though I had never doubted it from the first announcement of its discovery. The night of Aug. 8 being favorable, Professor TURNER extrapolated a position of the satellite from the expired ephemeris and we gave [illegible] for the satellite. A small star was found exactly at the indicated place, but measures showed no motion. A fainter star was seen about one minute of arc north following, but its distance from the assigned place made it unlikely that it was the satellite. Both Professor TURNER and I estimated this star to be one magnitude fainter than the star just measured, which I had referred by the micrometer to an 11th magnitude star in

$$1904.0 \qquad \alpha = 21^h\ 23^m\ 0^s.2 \qquad \delta = -16°\ 37'\ 15''.5$$

(This position was obtained by comparison with *Saturn*.)

Measures of the small star with respect to the 11th star gave,

$$\text{Pos. Ang. } 300°.99\ (4) \qquad \text{Dist. } 53''.57\ (2)$$

$$\text{These give } \Delta\alpha - 0'\ 46''.4 = -0^m\ 3^s.22$$

$$\Delta\delta + 0'\ 26''.8$$

This faint star was, therefore, in

$$1904.0 \qquad \alpha = 21^h\ 22^m\ 57^s.0 \qquad \delta = -16°\ 36'\ 48''.7 \quad 15^M \pm$$

The position of the fainter star, which I had sketched on the diagram, can be quite closely determined. By measuring the diagram the object was found to be $4^s.8$ following and 39″ north of the above star. This would give it the position,

$$1904.0 \qquad \alpha = 21^h\ 23^m\ 1^s.8 \qquad \delta = -16°\ 36'\ 9''$$

with a probable error of about 5″.

Later, Prof. E. C. PICKERING informed me that no star existed in any of his photographs at the place of this object, and furthermore, that it nearly occupied very closely the ephemeris position of *Phoebe*. This position was examined several times subsequently, but the seeing was such as to leave one in doubt as to the existence of the object, but on Sept. 5 the conditions were good enough to be certain that it had disappeared. On this date I again searched for the satellite at the ephemeris place, but the seeing had become poor and it was not found. On Sept. 12 the conditions were better and the absence of the object was certain. On this night, which was [illegible], I again searched for *Phoebe*, and near its position found a faint star which measures soon showed was not the satellite (see star 2 of the list of positions later on). I then applied a higher power, which often brings out faint stars, and two excessively faint stars were seen near the ephemeris place. The following of the two, and which was slightly the brighter, was measured with reference to the 12^M star.

	$\Delta\delta$	
h m s	″	
10 8 16	0 45.33 (2)	Faint object north.
10 13 38	0 44.62 (2)	
10 10 57	0 44.97 (4)	

	$\Delta\alpha$	
h m s	′ ″	
10 23 6	1 48.74 (3)	Faint object following.
10 37 41	1 45.06 (3)	
10 47 46	1 43.03 (3)	
10 36 11	1 45.61 (9) = 0^m $7^s.40$	

Though this object was excessively faint and difficult (est. $16^M.7$) the measures in $\Delta\alpha$ clearly showed motion towards the west. The intervals in the $\Delta\delta$ measures were not great enough to make motion certain, but the slight difference would indicate a change towards the south. The theoretical motion of *Phoebe* was southwest.

The $\Delta\alpha$ consistently gave a westward motion of $0''.226$ per minute of time. Using this motion and reducing the two last measures to the epoch of the first one we have

h m s		′ ″
10 23 6	$\Delta\alpha$	1 48.74
10 37 41		1 48.12
10 47 46		1 48.71

I have lately determined the position of the 13^M comparison star with reference to Radcliffe 1890 Catalogue, No. 5764, whose place, brought up, is

$$1904.0 \quad \alpha = 21^h\ 16^m\ 54^s.05 \qquad \delta = -17^\circ\ 14'\ 36''.6$$

The observations gave (13^M star -5764)

$$\Delta\alpha = -4^m\ 35^s.07 \qquad \Delta\delta = -12'\ 24''.3$$

This gives for the comparison star

$$1904.0 \quad \alpha = 21^h\ 12^m\ 18^s.98 \qquad \delta = -17^\circ\ 27'\ 0''.9$$

Reduction to apparent place $+3^s.11 \quad +19''.8$

The place of the satellite was, therefore,

$$10^h\ 10^m\ 57^s \quad \delta \text{ apparent } = -17^\circ\ 25'\ 56''.1$$
$$10\ \ 36\ \ 11 \quad \alpha \text{ apparent } = \quad 21^h\ 12^m\ 29^s.49$$

Professor PICKERING informs me that the above position falls almost exactly in the ephemeris place.

I have searched a number of times since with an ephemeris kindly supplied me by Dr. F. E. Ross, of Washington, who is investigating the orbit of the satellite, but the conditions have never been such as would show such a faint object.

When *Saturn* gets further north, however, there will be no trouble in visually observing the satellite, but at the present low altitude the seeing is seldom good enough to see a very faint object like *Phoebe*.

In these observations five other stars were incidentally observed. Following are the positions:

No.	Mag.	α 1904.0	δ 1904.0
		h m s	° ′ ″
1	11	21 12 9.20	−17 23 39.2
2	16	21 12 29.74	−17 25 46.3
3	9	21 14 57.67	−17 24 58.2
4	10	21 15 5.14	−17 23 20.7
5	14	21 16 25.89	−17 19 1.8

The third star is S.D. 17°.6239.

The 13^M star (to which the satellite was referred) was compared also with this star. When the place of 6239 is accurately determined it will give a better position of the 13^M star, because in the comparison with Radcliffe 5764 three intermediate stars had to be used.

The measures are

13^M star $-$ S.D. 17°.6239

$$\Delta\alpha = -2^m\ 38^s.69 \ (12)$$
$$\Delta\delta = -2'\ \ 2''.6 \ \ (4)$$

At the observation of Aug. 8 (assuming the faint object to have been the satellite) *Phoebe* would not be fainter than 16^M. At the observation of Sept. 12 it would not be brighter than $16\frac{1}{2}^M$. The estimation on this last date made it $16^M.7$.

The observations have not been printed before because I had hoped to get more measures to go with them, but there is now no further chance this season.

NOTE.

Professor E. C. PICKERING has supplied me with the position of the star S.D. 17°.6239 which has been carefully observed at the Naval Observatory, Washington.

$$1900.0 \quad \alpha = 21^h\ 14^m\ 44^s.40 \qquad \delta = -17^\circ\ 25'\ 59''.2$$

Using this star, the position of the 13^M comparison star of Sept. 12 is

$$1904.0 \quad \alpha = 21^h\ 12^m\ 19^s.11 \qquad \delta = -17^\circ\ 27'\ 1''.6$$

The resulting place of *Phoebe* on Sept. 12 was therefore

$$10^h\ 10^m\ 57^s \quad \text{apparent } \delta = -17^\circ\ 25'\ 56''.8$$
$$10^h\ 36^m\ 11^s \quad \text{apparent } \alpha = \quad 21^h\ 12^m\ 29^s.60$$

The times are Central Standard time ($6^h\ 0^m$ slow of G.M.T.).

Yerkes Observatory, 1904 *November*.

CONTENTS.

PUBLISHED AT BOSTON, AND CUSHING ROAD, WELLESLEY HILLS, MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, BOX 216, WELLESLEY HILLS, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed March 27.*
Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 572.

VOL. XXIV. BOSTON, 1905 APRIL 12. NO. 20

INVESTIGATION OF THE PIVOTS OF THE OLCOTT MERIDIAN CIRCLE OF THE DUDLEY OBSERVATORY.

BY LEWIS BOSS.

In preparation for the series of observations of a fundamental character upon the positions of the principal stars contemplated at the Dudley Observatory it becomes almost indispensable that the pivots of the meridian circle should be investigated. The Olcott Meridian Circle of this Observatory, made in 1856 by Pistor and Martins of Berlin, has been in use for about forty-seven years. However perfect the form of the pivots may have been originally, it is natural to suppose that they may have suffered deterioration from the exposure and wear of nearly half a century. Some ground for this suspicion is offered in the discordance of the meridian described by this instrument in 1897–8 from the mean of the best modern determinations (*A.J.* 531 & 549).

Accordingly it was decided to refigure the pivots; and this was accomplished in October, 1904. In order to avoid a number of inconveniences and risks it was decided to perform the work of refiguring the pivots without dismounting the instrument. In 1891, Warner and Swasey, of Cleveland, Ohio, had shaped a cast-iron tool to serve as a grinder, or polisher, of these pivots. This tool consisted of two half segments of a hollow cylinder accurately fitting the pivots. The two halves are held together by bolts passing through flanges exterior to the cylinder. Thin brass wedges slightly separate the two halves, and in conjunction with the bolts permit of delicate adjustment. Convenient handles for operating the tool are provided. The process of grinding was carried out by the staff of the Observatory. The inside of the tool was lightly smeared with a mixture of olive oil and very finely powdered soapstone. The tool was then slipped on the pivot under an adjustment which permitted rather rapid rotation upon the pivot without excess of fatigue to the operator. The tool was frequently shifted from one pivot to the other under precisely the same adjustment. The tool was still more frequently reversed, so that a given face of it was alternately outward and inward, in order to provide against the formation of a conical surface, either of the tool or of the pivot. In placing the instrument on the reversing carriage the telescope was set successively at 90°, 270°, 45°, 225°, 315° and 135°, of zenith-distance. The process of grinding was continued until minute examination of both pivot and tool indicated that the surfaces were in equal contact throughout the entire circumference. The result was further tested near the end of the work by preliminary measurement of inequalities in the manner hereinafter described. When it was deemed probable that no decided improvements in the form of the pivots would be attained by further grinding, the work was completed by the use of rouge such as is employed in the polishing of lenses. After use of the rouge for five or six hours the pivots received a very fine polish.

The pivots of the Olcott Circle are unusually small for an instrument of its general dimensions. They are only 51.5 mm. in diameter; and the perforations for admitting illumination of the reticle are only 22 mm. in diameter. This condition seemed to prohibit the adoption of the axis-collimator method, which appears to offer fewest objections whenever the construction is such as to admit of the employment of a sufficiently large objective lens. It appeared at first sight, however, that virtually the same principle could be employed by the use of a reflector adjusted to the end of the pivot and perpendicular to the mean axis of rotation, from the surface of which the reflected images of parallel wires in the focus of a collimator, adjusted perpendicular to the surface of the mirror, could be observed. This method would have an advantage in the employment of a collimator with sufficient aperture for optically good results. It has the disadvantages, subsequently ascertained, of great sensitiveness to changes of temperature during observation, and of including with the variations of angle produced by inequalities in the rotating pivot, the full effect of flexure, or buckling, of the axis. This latter effect was speedily found to be very sensible, though remarkably conformable to a definite mathematical law, as will be seen further on.

An adjustable mirror, silver-on-glass, of 49 mm. in diameter, was provided. The plane surface of the mirror is of the high perfection naturally expected of Mr. Brashear, of Allegheny, who constructed the apparatus. The reflected images were so bright that no difficulty whatever was experienced in using the apparatus in full daylight. The mirror, about 6 mm. in thickness, is fastened to a metal disc of the same diameter by means of three adjustable spring clamps, which serve to hold the mirror in place during rotation, without undue pressure. It was difficult to realize, at first, how little pressure was needed, and how much the action of the mirror was impaired by excess of pressure beyond that needful. In the disc supporting the mirror are four adjusting screws abutting against the end of the pivot. A brass plug was inserted in the aperture of the pivot, and a screw passing through the center of the supporting disc into the plug served to hold the apparatus firmly in place. Its total height from the face of the pivot was only 15 mm. The entire arrangement worked well. The mirror could be speedily attached and adjusted, or transferred from one pivot to the other.

The collimator consisted of a small telescope of unknown make, 70 mm. in aperture, and 114 cm. in focal length. To this was adapted a spare micrometer like those employed on the circle microscopes. This micrometer proved to be exceptionally bad. The collimator was intended only for a preliminary experiment; but, later, in order to avoid delay, it was decided to complete with it the definite investigation. The periodic errors of the micrometer screw was very thoroughly investigated, and corrections which proved to be sensible were applied to all the observations. Successive revolutions of the screw proved to be sensibly equal and of the value 45″.29. The parallel threads were about 17″ asunder. The magnifying power was approximately 50 diameters.

The collimator was fastened in the central holes of the pier by means of cast iron rings, set in plaster, and provided with adjusting screws abutting against the tube of the collimator. This proved to be a very convenient and satisfactory arrangement.

The investigation was conducted in the following manner. After adjustment of the apparatus, the circle-telescope was pointed to the zenith, and the reflected wires in the collimator were read upon precisely as in ordinary observation of the nadir. Illumination was provided by a small oil light. The ray from this was reflected down the collimator tube by means of a small bit of plane glass inclined at an angle of 45° in front of the aperture of the eye-piece. The first reading completed, the telescope was moved to 15° in zenith-distance and a reading was taken as before. This process was continued at intervals of 15° until 0° was again reached, when the readings were repeated in inverse order of rotation. Thus it might be expected, that effects of temperature, etc., affecting the apparatus nearly in proportion to the elapsed time, would be eliminated.

If the discordance between the readings near the beginning and end of the series amounted to more than 0″.7, however, the entire series was discarded. Such rejection was rarely necessary in the investigation proper. At a more favorable season of the year (the present observations were in November and December) and by awaiting more patiently periods when the external temperature would be likely to be nearly constant, it is probable that a rejection on account of discordance would not have been necessary at all. About one hour was consumed in the readings for a series of 49 settings. Readings for the entire investigation, including rejected series, consumed 35 hours, on 9 days.

After a series of measurements of the horizontal component of the displacement of the images, the micrometer was rotated 90°, and a like series of measurements upon the vertical displacements was made. These two series together constitute one series in the meaning of this paper.

In the next series the order of progressive rotation was the reverse of that in the first series. In the third and fourth, otherwise like the first and second, the mirror was rotated 180° from its first position. These four series were then combined into one set of means. The clamp pivot is designated as A, the other, B. The first four series were read upon A, — collimator in west pier, giving AW. The meridian circle was then reversed, and this series is designated BW. With the telescope in the east pier we have in like manner AE and BE.

It was easy to adjust the mirror so that the inclination of its plane to the mean axis of rotation did not differ from 90° much more than 1″ in any case. But it was necessary, of course, to determine the inclination for each series. Let

I = Inclination of the mirror to the plane perpendicular to the mean axis of rotation.
ζ = Zenith-distance of the instrumental pointing.
ζ' = Zenith-distance of the line perpendicular to the node of the mirror-plane upon the meridian plane of the instrument when the pointing is zero.
$a = I\cos\zeta'$.
$b = I\sin\zeta'$.

Then we shall have for the horizontal displacement of the image on account of inclination of the mirror,

$$i' = b\cos\zeta + a\sin\zeta.$$

and for the vertical displacement,

$$i' = a\cos\zeta - b\sin\zeta.$$

The horizontal and vertical sub-series each give a determination of a and b which, theoretically, ought to agree

within what is to be expected from the unavoidable errors of measurement. In fact, this agreement was usually within 0″.2, but was not always so close as the precision of the measurements seemed to require. A sensible part of these discordances is attributed to slight movements of the telescope from one series to another, presumably to be attributed chiefly to the effects of varying temperature. The telescope projected beyond the pier unsupported for half its length, and was not protected from the heat of the observer's body. The effects attributed to small displacements of this kind were much more marked in the measurement of the vertical, than in that for the horizontal component. Since the temperature of the observing room was almost always lower than +2° C., and even lower than −5° at times, it may easily be imagined that the contracted heat of the observer's body for an hour might produce a sensible effect on the line of collimation of the collimator-telescope, and that this would be more marked in observation of the vertical component. Accordingly, a and b were determined and applied for each subseries separately.

Table I exhibits the final results of the measurements upon the horizontal and vertical components, corrected for inclinations of the mirror. As already stated, each set is the mean of four subseries for each of the components.

TABLE I. Observed Angular Deviations, Φ, of the Pivots during Rotation.

	Horizontal Component					Vertical Component				
ζ	AW	BE	AE	BW	p'	AW	BE	AE	BW	p
°	″	″	″	″	″	″	″	″	″	″
0	+.10	−.08	−.06	−.03	+.01	+.73	+.79	+.76	+.74	−.01
15	+.41	+.46	+.28	+.33	−.02	+.78	+.67	+.63	+.49	+.06
30	+.63	+.80	+.50	+.67	−.08	+.50	+.40	+.47	+.29	+.07
45	+.80	+1.01	+.64	+.90	−.12	+.04	+.01	+.08	−.16	+.06
60	+.58	+.83	+.63	+.80	−.11	−.42	−.38	−.37	−.47	+.02
75	+.27	+.54	+.34	+.49	−.10	−.79	−.68	−.67	−.68	−.02
90	−.07	+.05	+.10	+.03	−.01	−.79	−.82	−.78	−.86	+.03
105	−.38	−.42	−.37	−.39	+.02	−.72	−.76	−.75	−.61	−.03
120	−.66	−.83	−.55	−.73	+.18	−.38	−.46	−.42	−.22	−.03
135	−.71	−.96	−.72	−.83	+.18	−.04	.00	−.04	+.13	−.05
150	−.63	−.88	−.62	−.79	+.11	+.45	+.44	+.39	+.46	−.02
165	−.31	−.45	−.37	−.46	+.06	+.78	+.80	+.66	+.73	−.02
180	+.04	−.06	−.03	−.10	+.04	+.79	+.82	+.80	+.77	.00
195	+.45	+.43	+.33	+.36	−.01	+.70	+.74	+.68	+.65	.00
210	+.63	+.79	+.63	+.74	−.07	+.28	+.35	+.37	+.33	−.01
225	+.75	+.91	+.71	+.89	−.09	−.06	−.05	+.01	−.06	+.02
240	+.62	+.82	+.59	+.77	−.10	−.41	−.48	−.37	−.55	+.06
255	+.32	+.56	+.37	+.50	−.09	−.62	−.71	−.64	−.82	+.07
270	−.02	+.05	−.04	+.03	−.04	−.71	−.84	−.77	−.89	+.06
285	−.43	−.41	−.39	−.35	−.02	−.59	−.66	−.67	−.62	.00
300	−.69	−.71	−.56	−.71	+.06	−.48	−.35	−.40	−.29	.06
315	−.77	−1.01	−.66	−.92	+.12	−.01	+.01	−.02	+.25	.08
330	−.62	−.90	−.51	−.76	+.13	+.31	+.46	+.31	+.56	.10
345	−.29	.53	−.28	−.42	+.10	+.64	+.63	+.63	+.82	.04

The zenith-distances, ζ, are counted toward the south for sets AW and BE, and toward the north for BW and AE. For AW and BW, vertical component, a positive sign means that the axis points above the western horizon; for AE and BE, that the axis points above the eastern horizon. Similarly for the horizontal component, positive values of Φ indicate that for AW and BE the axis of the pivots points southward of the prime vertical, and, for AE and BW, northward. The effect of these conventions should be, that, if the measurements indicate the error of pivots alone, then AW and AE should give p with proper sign referred to the instrument, clamp west; and BW and BE should give *minus* p, though, otherwise, referred to clamp west and south zenith-distances. Consequently we should have:

$$p = \frac{AW - BE}{2} = \frac{AE - BW}{2}$$

The mean values of observed p computed in this manner are given in the sixth and eleventh columns of Table I. These are, in effect, the values finally adopted. Whatever the actual source of p, thus derived, it is evident that the pivot error must be exceedingly small. It is well toward the minimum that can be determined by instrumental means with absolute certainty as a real quantity. Yet it seems worth while to examine the composition of the quantities, Φ, for each of the sets in Table I.

The most casual inspection of the various determinations of Φ indicate that p forms a comparatively small part of them. For example, consider the values of Φ in the vertical component for $\zeta = 0°$. The positive values, +.73

for AW indicates that the west end of the axis, for that setting, points above the western horizon; while, simultaneously the result, $+''.79$, for BE indicates that the axis of the eastern pivot is pointing above the eastern horizon. This state of the case is confirmed by comparison of BW and AE; as well as for other settings of the instrument. Simultaneous values of Φ, having the same sign, also characterize the measurements of the horizontal component. The explanation obviously is, that there is buckling, or flexure, of the axis.

An attentive examination of the observed material leads to the hypothesis that this flexure takes place in a plane, parallel to the axis of rotation, and fixed with reference to the cube of the instrument; as if the axis were rigid in all planes except this in which the flexure is supposed to take place. Let the maximum buckling, or flexure, be designated, $2F$; and let the inclination of the flexure-plane to the prime vertical plane when the instrumental pointing is at the zenith be called ζ_1'. The maximum flexure of the axis will then occur when the instrumental pointing is $-\zeta_1'$; and it would become zero at $\zeta = 90^\circ - \zeta_1'$, *i.e.*, when the flexure-plane is in the horizon. Another maximum, identical with the first, would occur at $180^\circ - \zeta_1'$. Moreover the flexure, f, for any setting, would be proportional to the component of gravity resolved in the direction of the flexure-plane. We shall then have:

$$f = 2F \cos(\zeta + \zeta_1')$$

$$f' = \text{horizontal comp.} = 2F \cos(\zeta + \zeta_1') \sin(\zeta + \zeta_1') \qquad (1)$$

$$f'' = \text{vertical comp.} \quad = 2F \cos^2(\zeta + \zeta_1') \qquad (2)$$

Putting

$$\left.\begin{aligned} F \cos 2\zeta_1' &= b' \\ F \sin 2\zeta_1' &= a' \end{aligned}\right\} \qquad (3)$$

$$f' = a' \cos 2\zeta + b' \sin 2\zeta \qquad (4)$$

$$f'' = b' \cos 2\zeta - a' \sin 2\zeta \qquad (5)$$

In the expression for f'' a constant, F, is neglected. This is the mean inclination of the end of the axis to the mean axis of rotation, which, like the inclination due to difference of level of the two pivots, cannot be separated from the determination of the inclination of the mirror, I. This neglect obviously has no influence upon the effective determination; and it can subsequently be remedied and applied through equations (3).

Recurring to the values of Φ, Table I, it is evident that we have:

$$\text{for } AW \text{ and } AE, \quad \Phi = f' + p' \quad \text{and} \quad f'' + p''$$
$$\text{for } BE \text{ and } BW, \quad \Phi = f' - p' \quad \text{and} \quad f'' - p''$$

Hence, assuming for the moment that the flexure is the same for each pivot, we shall have:

$$f', \text{ or } f'' = \frac{AW + BE}{2} = \frac{AE + BW}{2}$$

Table II exhibits the result of these combinations, and, also, their mean, which is adopted as the observed mean flexure.

TABLE II. Observed Values of Flexure.

	Horizontal Component				Vertical Component			
ζ	$\frac{AW+BE}{2}$	$\frac{AE+BW}{2}$	Mean f'	C—O	$\frac{AW+BE}{2}$	$\frac{AE+BW}{2}$	Mean f''	C—O
	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$
0	+.01	−.05	−.02	+.02	+.76	+.75	+.75	+.05
15	+.43	+.31	+.37	+.03	+.72	+.56	+.64	+.05
30	+.72	+.58	+.65	+.05	+.45	+.38	+.41	−.01
45	+.90	+.77	+.84	−.04	+.02	−.04	−.01	+.01
60	+.70	+.72	+.71	−.02	−.40	−.42	−.41	+.01
75	+.40	+.41	+.41	−.01	−.73	−.68	−.70	.00
90	−.01	+.06	+.03	−.03	−.80	−.82	−.81	+.01
105	−.40	−.38	−.39	−.01	−.74	−.68	−.71	+.02
120	−.75	−.64	−.69	−.01	−.42	−.32	−.37	−.03
135	−.84	−.78	−.81	+.01	−.02	+.04	+.01	−.01
150	−.76	−.70	−.73	+.04	+.44	+.42	+.43	−.03
165	−.38	−.42	−.40	.00	+.79	+.70	+.74	−.04
180	−.01	−.06	−.04	+.04	+.81	+.79	+.80	.00
195	+.44	+.34	+.39	+.01	+.72	+.66	+.69	.00
210	+.71	+.68	+.70	.00	+.32	+.35	+.33	+.07
225	+.84	+.80	+.82	−.02	−.06	−.02	−.04	+.04
240	+.72	+.68	+.70	−.01	−.44	−.46	−.45	+.05
255	+.44	+.44	+.44	−.04	−.66	−.73	−.70	.00
270	+.02	.00	+.01	−.01	−.77	−.83	−.80	.00
285	−.42	−.37	−.40	.00	−.63	−.65	−.64	−.05
300	−.72	−.65	−.68	−.02	−.42	−.34	−.38	−.02
315	−.89	−.79	−.84	+.04	+.02	+.11	+.06	−.06
330	−.76	−.64	−.70	+.01	+.38	+.44	+.41	−.01
345	−.41	−.35	−.38	−.02	+.64	+.72	+.68	+.02

Through the employment of equations (4) and (5) the numbers in Table II lead to the following results for the coefficient of flexure:

	b'	a'
	″	″
Horizontal Comp.	+0.81	.02
Vertical Comp.	+0.80	+.03
Adopted mean	+0.80	.00

The discrepancies of the observed values from the formula are shown in Table II under the column C−O. They are surprisingly small, but they show slight indications of a systematic character at certain points. This naturally results from the slightly differing values of b' and a' resulting from the horizontal and vertical series separately. These may easily originate in the local, systematic, differences between $\frac{AW+BE}{2}$ and $\frac{AE+BW}{2}$ which, under any hypothesis, should, theoretically, be the same. In fact, of the 48 computed values of the flexure, 30 fall between the corresponding observed values that make up the means, 6 are identical with one of them, and only 12 are outside both values. Comparison in pairs of the separate observed values of Φ in Tables I and II indicate that the probable error of an observed mean in Table II should be ±0″.022. From C−O in Table II we have for this probable error, ±0″.020, (= ±0ˢ.0013). Thus the representation of the observations by the theoretical formula for flexure is slightly better than is required. This degree of agreement strongly confirms the hypothesis involved in formulas (4) and (5). It points to a very homogeneous and flawless construction of the axis in all its parts; and renders it probable that the flexure of the two ends of the axis is sensibly the same. When the telescope is pointed to the zenith the plane in which the flexure takes place is inclined only 8′ 34″ to the prime-vertical plane. The probable uncertainty of this angle is decidedly greater than its value; so that it may be said that the maxima of flexure are found when the telescope points either to the zenith or to the nadir. The most plausible explanation of this fact seems to be that the flexure takes place in the cube, and that the line of least resistance is determined by the perforations of 60 mm. in diameter through the cube at right angles to the line of collimation of the circle-telescope. The two halves of the telescope are provided with very strong flanges that are firmly bolted to the cube. The rigidity of the axis may thus be much greater in a direction perpendicular to the plane passing through the line of collimation and the axis of rotation than it is in the flexure-plane as found by these observations.

That the flexure, or buckling, occurs between the hangers of the counterpoise supports, rather than in the pivots, is shown by supplementary tests for the elucidation of this point. The adopted weight, normally resting upon each of the pivots of the Olcott Circle, is 29 pounds. This was the adjustment during investigation of the pivots. In the supplementary investigation this weight was made alternately, 12 and 50 pounds. A single series of measurements for AW and BE showed an increase of flexure of 0″.11 ± 0″.03 for an increase of pressure on each pivot from 12 to 50 pounds. We have the following results:

12 pounds	$F =$ 0″.77
29 pounds	0.80
50 pounds	0.88

The increase of weight upon the pivots increases the moment of inertia tending to produce flexure of the axis. The indications are that with no resultant weight on the pivots the value of F would still be fully 0″.70. The distance between the counterpoise hangers is about 70 cm. and between the pivot bearings 99 cm.

If it were still considered desirable to investigate the difference of flexure, $2k$, between the two ends of the axis, we may consider the differences $AW-BE$ and $AE-BW$ to be equal to $2p+2k'$, k' being computed from k. We might then consider the values of p (Table II) as containing an element expressed by the formula:

$$\frac{a'k}{b'}\cos 2\zeta + k\sin 2\zeta \quad \text{for horizontal component}$$

$$k\cos 2\zeta - \frac{a'k}{b'}\sin 2\zeta \quad \text{for vertical component}$$

Applying these formulas we have, $k = -0''.06$; $-0''.11$ from the horizontal component, and $-0''.00$ from the vertical component. The discordance between the results from the two components is noticeable, and may be attributed chiefly to the probability that there is a small relative ellipticity of the two pivots which, in the case of rectangular Y's would affect only the horizontal component. We shall assume that the flexure for the two ends of the axis are sensibly alike. The column headed p in Table I would, accordingly, represent the observed effects due to deviation of one, or both, of the pivots from an absolutely circular section. Furthermore, consulting the values of p and in Table II, it becomes evident that sensibly correct values of the collimation can be obtained through reversal on the collimators, or on the nadir. Therefore the inclusion in of some small effect of difference of flexure in the two ends of the axis would have no sensible effect upon the deduced right-ascensions of stars, provided the polar deviation is derived separately for each position of the instrument according to the usual custom.

To preclude misapprehension, it may be well to observe that axial flexure, of conceivable amount, does not sensibly affect the position of the virtual axis of rotation. Its only effect is to alter the vertical section of the pivot to an ellipse with axis having in the maxima the ratio, cos $2F$.

Adopting the value of flexure already found, we may

derive values of p' and p'' for each of the four series exhibited in Table I, — a procedure which will better enable us to understand what degree of confidence attaches to the final result and to any attempted formulation of it. These are found in Table III, which is formed from the expressions: Φ_A, f, and $f-\Phi_B$.

TABLE III. Individual and Mean Values of p' and p''.

	Horizontal Component						Vertical Component					
ζ	AW	BE	AE	BW	Mean p'	C—O	AW	BE	AE	BW	Mean p''	C—O
°	″	″	″	″	″	″	″	″	″	″	″	″
0	+.10	+.08	−.06	+.03	+.04	.00	−.07	+.01	−.01	+.06	−.01	+.01
15	+.01	−.06	.12	+.07	−.03	+.01	+.09	+.02	−.06	+.20	+.06	−.04
30	−.07	−.10	−.20	+.03	−.09	+.02	+.10	.00	+.07	+.11	+.07	−.03
45	.00	−.21	−.16	−.10	−.12	+.01	+.04	−.01	+.08	+.16	+.07	−.02
60	−.11	−.14	−.06	−.11	−.11	.00	−.02	−.02	+.03	+.07	+.02	+.03
75	−.13	−.14	−.06	−.09	−.10	+.01	−.09	−.02	+.03	−.02	−.02	+.05
90	−.07	−.05	+.10	−.03	−.01	−.03	+.01	+.02	+.02	+.06	+.03	−.03
105	+.02	+.02	+.03	−.01	+.02	.00	−.03	+.07	−.06	−.08	−.03	+.01
120	+.04	+.13	+.15	+.03	+.09	−.02	+.02	+.06	−.02	−.18	−.03	−.01
135	+.09	+.16	+.08	+.03	+.09	+.02	−.01	.00	−.04	−.13	−.05	.00
150	+.06	+.19	+.07	+.10	+.10	+.01	+.05	−.04	−.01	−.06	−.02	−.03
165	+.09	+.05	+.03	+.06	+.06	+.03	+.08	−.10	−.04	−.03	−.02	−.01
180	+.01	+.06	−.03	+.10	+.04	.00	−.01	−.02	.00	+.03	.00	.00
195	+.05	−.03	−.07	+.04	.00	−.02	+.01	−.05	−.01	+.04	.00	+.02
210	−.07	−.09	−.07	−.04	−.07	.00	−.12	+.05	−.03	+.07	−.01	+.05
225	−.05	−.14	−.09	−.09	−.09	−.02	−.06	+.05	+.01	+.06	+.02	+.03
240	−.07	−.13	−.10	−.08	−.10	−.01	−.01	+.08	+.03	+.15	+.06	−.01
255	−.08	−.16	−.03	−.10	−.09	.00	+.08	+.01	+.06	+.12	+.07	−.04
270	−.02*	−.05	−.01	−.03	−.04	.00	+.09	+.04	+.03	+.09	+.06	−.06
285	−.03	+.01	+.01	−.05	−.01	+.03	+.10	−.03	+.02	−.07	.00	−.02
300	+.01	+.04	+.11	+.04	+.06	+.01	−.08	−.05	.00	−.11	−.06	+.02
315	+.03	+.21	+.11	+.12	+.12	−.01	−.01	−.01	−.02	−.25	−.08	+.03
330	+.07	+.21	+.18	+.07	+.13	−.02	−.09	−.06	−.09	−.16	−.10	+.05
345	+.11	+.13	+.12	+.02	+.09	.00	−.06	+.07	−.07	−.12	−.04	+.01

Inspection of the individual values of p' and p'' in Table III shows at once that, minute as these quantities are, sensible systematic differences between the series are to be recognized. This is particularly true of p'', and it tempts one to the conclusion that, with the exception of two or three spots, $p''=0$ might be the safest deduction. During the observations much difficulty was experienced with the sub-series for the vertical component of BW, when the temperatures were not only low, as a rule, but also varied more rapidly than in the others; and this is precisely the series which shows the largest systematic discordance from the others. Nevertheless, it was thought best to attempt a representation of both p' and p'' by formulas involving $\sin 2\zeta$ and $\cos 2\zeta$. We have:

$$(6)\quad \left\{\begin{aligned} p' &= -0''.11\sin 2\zeta + ''.04\cos 2\zeta \\ p'' &= +0''.05\sin 2\zeta - ''.00\cos 2\zeta \end{aligned}\right.$$

The representation of observed p' by the above formula for p' seems to be extremely satisfactory. With rectangular Y's this would mean that there is a slight relative ellipticity between the two pivots, and that other irregularities are practically insensible. It would, therefore, also mean that p'' should be practically insensible. The Y-bearings do not, however, present a rectangular support. The virtual angle is approximately 77°; and the bearings of soft brass, somewhat worn, extend on each side of the pivot from about 46° to 57° of nadir-distance. The conditions are, therefore, sensibly different from that of rectangular supports, and pivots bearing along lines of no material breadth. Accordingly, after inspection of the observed values of p'', it was decided that they would more safely be represented by the formula (6) for p''. The residuals for p'', thus represented, are much larger than for p'; but it is noticeable that the rejection of only a single one of the four individual values which make up observed p'' would reduce every one of the six larger values of C—O materially, so that in the mean they would be very slightly more than half as large as now. From comparison with the adopted representation the probable error of each observed p' is $\pm 0''.011$, and of each observed p'', $\pm 0''.021$. The latter corresponds to the predicted probable error, $\pm 0''.022$, previously stated; while the former is much less. This unexpectedly close correspondence of the observed and computed values of p' must be attributed

partly to accident, and partly to the fact that the systematic discordances of the individual series which contribute materially to the deduction of probable errors for them are partially eliminated in the means.

It remains to show the effects, P, of observed and computed p' and p'' upon the line of collimation of the circle-telescope during rotation about the axis. These are given in Table IV. Under the column marked "Obs." are given the values of P, computed from the formula, $p' \sin\zeta + p'' \cos\zeta$, the values of p' and p'' being taken from Table III.

TABLE IV. Observed and Computed Corrections for Collimation, P, arising through Deviations of the Material Axis from the Mean Axis during Rotation of the Instrument.

ζ	Obs.	Comp.	C—O
°	s	s	s
0	+.004 sec δ	.000 sec δ	—.004
15	+.003	+.001	—.002
30	+.001	+.001	.000
45	—.002	—.002	.000
60	—.006	—.004	+.002
75	—.007	—.005	+.002
90	—.001	.000	+.001
105	+.002	+.002	.000
120	+.006	+.005	—.001
135	+.007	+.007	.000
150	+.004	+.006	+.002
165	+.002	+.003	+.001
180	.000	.000	.000
195	.000	—.001	—.001
210	+.003	—.001	—.004
225	+.003	+.002	—.001
240	+.001	+.001	.000
255	+.005	+.005	.000
270	+.003	.000	—.003
285	+.001	—.002	—.003
300	—.005	—.005	.000
315	—.009	—.007	+.002
330	—.010	—.006	+.004
315	—.004	—.003	+.001

Multiplying the first of equations (6) by $\sin\zeta$ and the second by $\cos\zeta$, and adding the results, we obtain the quantities given under "Comp." in Table IV. This column constitutes a table of corrections of transits when the instrument is in the position, clamp W, and ζ represents south zenith-distances. For convenience the quantities in Table IV are expressed in seconds of time.

The mirror-collimator method of investigating the pivots of a transit seems to lie under certain disadvantages which have been pointed out in the foregoing. Among the advantages of this method are its simplicity, its economy of expense in the provision of apparatus, facility in use, its ready adaptation to instruments not specially constructed with this point in view, its great precision for a given expenditure of labor, and the information which it gives of the behavior of the axis under the stress of gravity. The latter point is not ordinarily investigated. In case of suspected asymmetry of buckling the question of its existence can be determined by this method; and by a suitable combination of collimations by reversal on collimators, nadir, and polar stars, with comparison of transits, $W-E$, the effects of such buckling can be analyzed and referred to their original elements.

It may be pointed out that in all methods which involve the microscopic observation of a mark at the center of the end of the pivot a slight effect of a flexure of the axis may be present. The amount of this effect would be determined by the relative projection of the end of the pivot beyond the points of support. In all ordinary cases this effect would obviously be very minute.

The axis-collimator method under ordinary conditions would afford no information as to the amount of flexure of the axis, or as to the symmetry of that flexure. For the investigation of error of pivots alone that method would seem to be superior to all others. But it requires the use of a comparatively large objective in the axis (scarcely less than 50 mm.) and, consequently, very large pivots, — perhaps larger than would ordinarily be preferred.

ON THE VARIABILITY OF F. 32 *CASSIOPEAE*,

By PAUL S. YENDELL.

The variability of this star was announced in No. 569 of this *Journal*, by Mr. Barr.

I began the examination of the star late in November of 1904, and up to 1905 Jan. 26, have observed it on sixteen dates, the observations numbering one hundred and fifteen.

The comparison-stars used and the provisional light-scale resulting from my observations are,

	DM.	Mag.	Light	Mag.
d =	63° 99	5.8	7.4	5.80
a =	63 119	6.0	4.1	6.00
e =	65 115	6.0	0.0	6.25

The value of a step is nearly $0^M.06$.

The period deduced from my observed phases (on decreasing) is $7^h\ 58^m.4$, a quantity so nearly commensurate with the sidereal day that I was long very suspicious as to the reality of the changes observed by myself.

A provisional mean light-curve has been formed from the observations, which cover the star's entire period. The readings therefrom are as follows. The T's are dated from the time of passing the phase e on the decrease. The readings are step-values from the above light-scale.

T—t	St.	T—t	St.
0h 0m	1.1	4h 0m	5.2
20	3.3	20	6.1
40	2.6	40	6.5
1 0	2.1	5 0	6.8
20	1.7	20	6.9
40	1.5	40	6.9
2 0	1.5	6 0	6.9
20	1.7	20	6.8
40	2.1	40	6.6
3 0	2.6	7 0	6.3
20	3.3	20	5.8
40	4.1	40	5.0
		58	4.1

From these readings it appears that the star's light-curve does not differ in character from those of the ordinary stars of short period. The increase is slightly quicker than the decrease. There seems to be no halt either at maximum or minimum. The minimum phase occurs $1^h 50^m$ after the phase *ea* from which the readings are dated, and the maximum at $5^h 45^m$ from that point, so that the interval from minimum to maximum is $3^h 55^m$. It seems likely that it will prove, upon the accumulation of sufficient material, that the light-curve is sensibly symmetrical.

The changes remarked in the period and light-curve by Mr. Barr I have not observed. The star seems to me on the contrary to be very regular, like most of the stars of its type, and resembles *U Pegasi* in general character more closely than any other. The probable errors of the normals for the above curve are small, considering the few observations from which they are formed, and the mean departure of the curve from these normals is very small.

The light-range observed is from 1.9 to 7.1 of the above light-scale, from $5^M.8$ to $6^M.2$, very nearly four-tenths of a magnitude. This range is somewhat masked in the mean curve.

The observed phases are as follows, the phase *ea* being always on the decreasing branch of the curve.

By the Mean Curve

	Maxima Local M.T.	Obs.	ea Local M.T.	Obs.	Minima Local M.T.	Obs.
1904 Dec. 29.	7h 56m	2				
30.	7 45	5				
1905 Jan. 1.			10h 3m	2 obs.	11h 36m	4 obs.
7.			9 32	1		
14.			8 43	1	11 15	1
23.			8 20	2	10 30	2
26.			8 0	2		

By the Single Curve

Maximum Local M.T.	w	Minimum Local M.T.	w
1904 Dec. 28 8^h 43^m	2	1905 Jan. 26 9^h 41^m	5

As the phases occur a trifle earlier on each succeeding evening, and the star is observable all the year round, it is hoped that enough observations of minima and maxima will be secured within a reasonable time to enable us to get more exact elements.

Dorchester, 1905 *Jan.* 28.

ELLIPTIC ELEMENTS OF COMET *e* 1904 (*BORRELLY*).

The following elliptic elements, computed by Aitken of the Lick Observatory, have been received through Mr. Ritchie.

$$T = 1905 \text{ Jan. } 16.53$$

$$\left.\begin{aligned}\omega &= 352^\circ\ 10'\\ \Omega &= 76\ 35\\ i &= 30\ 36\end{aligned}\right\}\text{Eq. } 1905.0$$

$$q = 1.3994$$
$$e = 0.6282$$

Period 7.30 years.

The computation was based on observations taken on 1905 Jan. 0, Jan. 17, and Jan. 27.

OBSERVATIONS, ELEMENTS AND EPHEMERIS OF COMET *a* 1905 (*GIACOBINI*).

Additional observations to those printed in No. 571, p. 156, are.

1905		α	δ
Mar. 28d 7h 48m 8s	Wash. M.T.	5h 52m 0s.24	+13° 39′ 30″.1
30d.7185	Gr. M.T.	5 59 59.5	+16 19 11

The first was made at Naval Observatory, the second at Lick Observatory by Aitken.

Elements and ephemeris computed by Morgan and Lamson of the Naval Observatory, from Ritchie's Circular, follow:

$$T = 1905 \text{ April } 3.57 \text{ Greenwich M.T.}$$

$$\left.\begin{aligned}\omega &= 357^\circ\ 13'\\ \Omega &= 154\ \ 8\\ i &= 42\ 59\end{aligned}\right\}\text{Eq. } 1905.0$$

$$q = 1.1507$$

Ephemeris for Greenwich Midnight.

1905	App. α	App. δ	Brightness
April 1	6h 6m 4s	+18° 14′	1.00
5	21 28	22 45	
9	37 56	27 2	
13	6 55 52	+31 0	0.88

CONTENTS.

Published at Boston, and Cushing Road, Wellesley Hills, Mass., semi-monthly, by S. C. Chandler. Address, Box 216, Wellesley Hills, Mass.
Associate Editors, Asaph Hall and Lewis Boss. Price, $5.00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. *Closed April* 10

Entered February 2 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 573.

VOL. XXIV. BOSTON, 1905 APRIL 28. NO. 21

DEFINITIVE ORBIT OF COMET 1903 III.

BY HENRY A. PECK.

The Comet 1903 III was discovered by GRIGG, at Thames, New Zealand, April 17, in right-ascension $3^h 7^m$, and declination $-21° 6'$. It was ten days later that the news reached TEBBUTT, at Windsor, N.S.W. He immediately began a series of square-bar micrometer observations on the eight-inch equatorial, which continued for a month. The observations for the first few days were good. Later they were interfered with, and interrupted by moonlight and cloudy weather. May 4, W. H. COX, at the Cape, began to observe the comet with the seven-inch telescope. The comet by this time was an extremely faint, nebulous mass, with no visible nucleus. After May 21 the comet was barely visible in the instrument used, and the observations were rough. These two series are the only ones that have fallen under my notice.

No attempt has ever been made to determine the orbit aside from that of KREUTZ and EBELL in *A.N.* 3867. These elements are the usual ones founded on the earliest telegraphic dispatches, and are not of sufficient accuracy to form a basis for the reduction of the observations. It was therefore first necessary to form new elements. In this work it was discovered that the definitive elements would be uncertain to a considerable extent, and that many of the refinements often used in work of this character would be a useless waste of time.

The elements used as a basis of the work are

$$
\begin{aligned}
T &= \text{March } 25.42450 \text{ Gr. M.T.} \\
\omega &= 184°\ 18'\ 53'' \\
\Omega &= 213\ \ 7\ \ 31 \quad 1903.0 \\
i &= 66\ \ 29\ \ 32 \\
\log q &= 9.697270
\end{aligned}
$$

to which correspond the equatorial coordinates

$$
\begin{aligned}
x &= (9.937203)\, r \sin(109°\ 24'\ 14.5'' + v) \\
y &= (9.923205)\, r \sin(\ \ 11\ \ 33\ \ 49.8 + v) \\
z &= (9.869790)\, r \sin(167\ \ 44\ \ 14.5 + v)
\end{aligned}
$$

From these elements is derived the following ephemeris:

		App. α	App. δ	log Δ	Aberr. time
		h m s	° ′ ″		
April	26	3 58 8.36	−16 0 9.3	0.1392	0.00794
	27	4 3 52.38	27 32.0		
	28	9 36.28	54 5.6	.1406	796
	29	15 19.91	17 19 50.1		
	30	21 3.23	44 45.4	.1426	800
May	1	26 46.01	18 8 51.3		
	2	32 28.16	32 7.9	.1451	805
	3	38 9.54	54 35.6		
	4	43 50.01	19 16 14.9	.1481	810
	5	49 29.43	37 6.0		
	6	55 7.68	57 9.3	.1515	817
	7	5 0 44.60	20 16 25.1		
	8	6 20.04	34 54.1	.1554	824
	9	11 53.85	52 36.8		
	10	17 25.91	21 9 34.1	.1598	832
	11	22 56.07	25 46.5		
	12	28 24.18	41 14.9	.1647	842
	13	33 50.13	56 0.0		
	14	39 13.80	22 10 3.0	.1700	852
	15	44 35.06	23 24.8		
	16	49 53.79	36 6.7	.1756	863
	17	55 9.89	48 9.6		
	18	6 0 23.25	59 34.8	.1817	875
	19	5 33.77	23 10 23.4		
	20	10 41.34	20 36.8	.1881	888
	21	15 45.84	30 16.0		
	22	20 47.37	39 22.7	.1947	902
	23	25 45.72	47 58.1		
	24	30 40.86	56 3.3	.2016	916
	25	35 32.71	24 3 39.5		
	26	40 21.27	10 47.9	.2088	932
	27	45 6.47	17 29.6		
	28	49 48.29	23 46.1	.2162	948
	29	54 26.69	29 38.9		
	30	6 59 1.62	24 35 9.8	0.2238	0.00964

In making a preliminary computation of the orbit the star places were followed as they had been originally published with the observations. It was intended to correct them later, but as many of the catalogues were not in the library, and it was seen that only approximate results could be arrived at, it was not deemed worth the time and expense to make the necessary corrections. The places used are as follows:

No.	α 1903.0	δ 1903.0	Authorities
	h m s	° ′ ″	
1	4 2 33.15	−16 14 53.2	Lal. 7700; A.G.W.Z. 150,141
2	11 3.12	41 16.4	C.G.C. 4736; Rad.$_3$ 1013
3	19 15.09	17 46 26.0	AO$_L$ 3037; A.G.W.Z. 81,106
4	29 35.93	18 23 7.7	Rad.$_3$ 1090
5	33 26.68	31 24.0	Lal. 8773
6	46 16.00	19 13 24.7	AW$_e$ 2852; Cinc. Z. 696
7	5 12 21.76	21 12 26.9	Cinc. Z. 760 [1312
8	23 28.49	27 28.2	C.G.C. 6321; Cinc. Z. 797; R.$_3$
9	26 48.90	39 0.5	Cinc. Z. 806
10	28 48.40	49 13.9	Cinc. Z. 815
11	36 26.31	22 12 29.0	Cinc. Z. 842
12	52 9.71	51 16.7	C.G.C. 7004
13	57 11.95	15 16.4	Eq. diff. from No. 12 [1511
14	6 2 29.38	−23 5 57.4	Gr. 10-yr. 1041; Stone 2794; R.$_3$ Cape$_{80}$ 2794; Cape$_{85}$ 409
15	6 8 51.99	−23 14 1.6	Eq. diff. from No. 16
16	9 39.45	19 17.9	C.G.C. 7486
17	16 33.80	35 31.0	Y.$_3$ 2675; C.G.C. 7675
18	18 22.51	32 36.1	C.G.C. 7721
19	26 23.54	24 4 35.5	AO$_E$ 5202; C.G.C. 7945
20	29 15.00	23 40 11.3	AO$_E$ 5278; C.G.C. 8026
21	30 21.34	47 55.3	Six ep. with No. 20 [Rad.$_3$ 1642
22	32 4.22	24 2 21.0	AO$_E$ 5343-4; C.G.C. 8089;
23	38 6.13	8 50.4	C.G.C. 8276
24	42 36.54	2 22.7	AO$_E$ 5664; C.G.C. 8403
25	51 32.46	15 32.8	C.G.C. 8663 [Rad.$_3$ 1738
26	6 53 33.82	−24 30 16.5	C.G.C. 8724; Stone 3314;

The only two series of observations that have fallen under my notice are found in *A.N.*, Band 163, and are as follows:

Date	Place	App. α	π	App. δ	π	Δα	Δδ	*
		h m s	s	° ′ ″	″	s	″	
April 26.85372	Windsor	4 3 1.62	+0.35	−16 23 26.1	−3.0	−0.10	+ 5.6	1
27.85494	Windsor	8 45.63	.36	50 10.4	2.9	0.41	4.2	2
29.85626	Windsor	20 12.43	.36	17 41 3.2	2.9	1.12	7.3	3
May 1.84394	Windsor	31 32.89	.34	18 28 25.6	2.7	1.58	4.9	4
1.84394	Windsor	33.24	.34	16.3	2.7	(1.23)	(14.2)	5
4.24218	Cape	45 9.97	.36	19 21 13.9	2.9	1.98	5.5	6
9.85773	Windsor	5 16 37.33	.34	21 7 3.3	2.5	1.11	6.3	7
11.21755	Cape	24 4.88	.33	. . . . .	. .	2.42	. .	8
11.23470	Cape	. . . . .	. .	29 18.2	2.6	. .	7.5	8
11.85718	Windsor	27 36.75	.34	38 59.2	2.4	0.36	3.3	8
11.85718	Windsor	36.84	.34	53.5	2.4	0.29	9.0	9
12.23743	Cape	29 40.39	.35	. . . . .	. .	1.03	. .	10
12.24385	Cape	. . . . .	. .	44 59.1	2.7	. .	2.4	10
12.85374	Windsor	33 1.18	.33	53 41.9	2.4	1.08	8.9	10
13.85833	Windsor	38 25.89	.34	22 8 2.8	2.4	1.85	0.8	11
17.23857	Cape	56 22.61	.34	50 53.7	2.4	1.95	0.6	13
17.86302	Windsor	59 39.64	.33	57 48.8	2.3	0.51	+12.0	14
18.22410	Cape	6 1 31.36	.32	23 2 5.3	2.2	1.41	− 4.4	14
18.85975	Windsor	4 49.21	.32	8 39.5	2.2	0.86	+12.9	14
19.21904	Cape	6 40.42	.31	12 22.8	2.1	0.67	5.8	15
20.22521	Cape	11 48.38	.32	22 43.8	2.2	1.49	4.1	16
21.23737	Cape	16 56.94	.32	32 26.2	2.3	0.43	0.1	18
21.86873	Windsor	20 7.38	.32	37 57.3	2.2	0.26	13.2	17
22.86454	Windsor	25 4.81	.31	46 39.0	2.1	0.37	9.0	20
22.86454	Windsor	5.03	.31	39.5	2.1	0.15	+ 8.5	21
23.87156	Windsor	30 2.94	.32	55 7.7	2.2	−0.17	− 7.2	19
24.86453	Windsor	34 53.88	.31	24 2 34.9	2.1	+0.83	+ 5.3	22
25.23149	Cape	36 38.22	.31	5 31.7	2.3	−1.27	−12.9	23
25.88127	Windsor	39 48.36	.32	9 49.1	2.2	+1.51	+ 7.1	24
26.20937	Cape	41 22.35	.29	12 16.5	1.8	1.38	− 4.1	24
27.87860	Windsor	49 15.69	.31	23 0.1	2.2	+1.75	− 0.6	26
28.20861	Cape	6 50 45.97	+0.28	−24 24 57.8	−1.8	−0.40	+ 2.0	25

After rejecting the second observation on May 1, the others were given equal weight, and the following normal places formed

	Δα cos δ	Δδ		Δα cos δ	Δδ
	″	″		″	″
April 30.0	−14.8	+5.5	May 19.0	−13.1	+5.5
May 12.0	−16.2	+5.5	May 25.0	+ 4.8	+0.8

The Schönfeld equations are

$$\begin{array}{rrrrrrrl}
+9.5623\,\partial k & -9.7329\,k\sqrt{_2}\,\partial T & +8.3986\,\partial\lambda & +9.6671\,\partial v & +9.3110\,\frac{\partial e}{2} & -9.8647\,\partial q & +1.1703 & =0\\
9.7262 & 9.7051 & -8.7838 & 9.6495 & 9.1361 & 0.0130 & +1.2965 & \\
9.7918 & 9.6751 & 8.9112 & 9.5968 & +8.7718 & 0.0619 & +1.1173 & \\
+9.8389 & -9.6428 & -8.9814 & 9.5284 & -8.3274 & 0.0874 & -0.6812 & \\
-9.4727 & +9.7031 & +8.4251 & 9.6936 & 9.3673 & +9.7715 & 0.7404 & \\
9.4093 & 9.5797 & -8.9289 & 9.7946 & 9.4211 & 9.7184 & 0.7404 & \\
9.3291 & 9.4939 & 9.1755 & 9.8311 & 9.4537 & 9.6448 & -0.7404 & \\
-9.2271 & +9.4116 & -9.3048 & 9.8518 & -9.1831 & +9.5565 & -9.9031 &
\end{array}$$

In a direct solution of the equations of condition the coefficients of both $\frac{\partial e}{2}$ and ∂q practically vanish. As to the latter quantity it will be seen that its coefficients are very nearly a multiple of those of ∂k. In an attempt to ascertain how much uncertainty must attach itself to the final elements, the first four quantities were made functions of the remaining two in the normals as follows:

$$\begin{array}{rrrrrrrl}
+0.1789\,\partial k & -0.1534\,k\sqrt{_2}\,\partial T & -8.8062\,\partial\lambda & +9.4990\,\partial v & +9.6222\,\frac{\partial e}{2} & -0.4538\,\partial q & +1.3560 & =0\\
-0.1534 & +0.1850 & -8.2355 & +8.8407 & -9.7671 & +0.4362 & -1.4331 & \\
-8.8062 & -8.2355 & +8.9643 & -9.5617 & +9.0565 & +8.9799 & +7.9031 & \\
+9.4990 & +8.8407 & -9.5617 & +0.3583 & -9.7136 & -9.6950 & +0.8586 &
\end{array}$$

From these are derived the following:

$$\begin{array}{l}
\partial k -9.9745\,k\sqrt{_2}\,\partial T -8.6273\,\partial\lambda +9.3201\,\partial v +9.4433\,\frac{\partial e}{2} -0.2749\,\partial q +1.1771 = 0\\
\qquad k\sqrt{_2}\,\partial T -9.6143\,\partial\lambda +0.2891\,\partial v -0.0029\,\frac{\partial e}{2} +9.4179\,\partial q -1.4808\\
\qquad\qquad \partial\lambda -0.5471\,\partial v +9.9694\,\frac{\partial e}{2} -8.9170\,\partial q -1.3793\\
\qquad\qquad\qquad \partial v -8.7713\,\frac{\partial e}{2} -8.2519\,\partial q +1.0146
\end{array}$$

The solution of this system of equations results in

$$\begin{array}{rcrrr}
\partial k & = & +29''.6 & +0.240\,\frac{\partial e}{2} & +1.662\,\partial q\\
k\sqrt{_2}\,\partial T & = & +45.6 & +0.594 & -0.237\\
\partial\lambda & = & -15.1 & -0.721 & +0.146\\
\partial v & = & -11.1 & +0.059 & +0.018
\end{array}$$

As care was taken to control the coefficients of the original equations by arbitrary variations of the elements, we may substitute these values and obtain the following system, the coefficients being expressed in units of the third decimal place.

$$\begin{array}{rrrl}
+15\,\partial q & -20\,\frac{\partial e}{2} & -4''.6 & =0\\
-26 & +33 & +1.8 & \\
-11 & -14 & +6.9 & \\
+20 & -28 & -6.7 & \\
-10 & +6 & +2.9 & \\
+5 & -3 & -1.4 & \\
+2 & -2 & -2.8 & \\
+3 & -2 & +1.2 &
\end{array}$$

Any value of $\frac{\partial e}{2}$ that might be found could only be regarded as an accident. Therefore, making $\partial e = 0$ we have $\partial q = +283''$. Making the substitution in the above $[vv] = 31''.2$. If now successive values be substituted for ∂q, the following table may be formed:

∂q	$[vv]$		
$+50''$	116.1		
100	83.6	−32.5	
150	58.9	24.7	+7.8
200	42.1	16.8	7.9
250	33.0	9.1	7.7
300	31.7	1.3	7.8
350	38.2	+ 6.5	7.8
400	52.5	+14.3	+7.8

From this it is evident that ∂q can hardly be known within a minute of arc. Adopting $+283''\pm 60''$ as the best value, the remaining unknown quantities are

$$\begin{array}{rcrl}
\partial k & = & +50''.0 & \pm 10''.0\\
k\sqrt{_2}\,\partial T & = & 21 & \pm 11\\
\partial\lambda & = & +26 & \pm 9\\
\partial v & = & -6 & \pm 1
\end{array}$$

Substituting in the original equations the residuals are

$$\begin{array}{rr}
+0''.6 & -0.3\\
+2.9 & -0.3\\
-3.6 & +1.8\\
+1.1 & -1.6
\end{array}$$

and $[vv] = 27''.7$, thus proving the accuracy of the numerical work.

Transforming the functions into the corrections of the elements we have

$$T = \text{March } 25.12031 \pm 0.00279 \text{ G.M.T.}$$

$$\left.\begin{array}{rcl}
\omega & = & 184^\circ\ 57'\ 1'' \pm 1'\ 36''\\
\Omega & = & 213\ \ 8\ \ 3\ \pm\ \ 8\\
i & = & 66\ 29\ 36\ \pm\ \ 2
\end{array}\right\}\ 1900.0$$

$$\log q = 9.698466 \pm 0.000257$$

Syracuse University, 1904 Nov. 26.

MAXIMA AND MINIMA OF LONG-PERIOD VARIABLES,

BY J. A. PARKHURST.

The following summary of maxima and minima of long-period variable stars is offered, pending the publication, by the Carnegie Institution of complete details of the writer's photometric work on a selected list of variable star fields. Provisional determinations of part of the following data have already appeared in the *Astronomical Journal*, but the results here given were obtained by a new reduction of all the estimates and measures, based on photometric magnitudes of the comparison-stars. The observations of the variables were made with a 6-inch reflector and the 12 and 40-inch refractors, mostly by ARGELANDER'S method; a few, however, are photometer measures. The magnitudes of the comparison-stars were measured with the Pickering Equalizing Wedge Photometer, used on the same telescopes. These magnitudes are based on at least three standard stars in each field which are contained in both the Harvard and Potsdam catalogues. The differences between the two systems range, for these fields, from $+0^{M}.42$ to $-0^{M}.27$. No attempt has been made to reconcile the two, but the magnitudes are expressed in the Potsdam system, with which these measures accord better than with the Harvard. The comparison-stars, down to the faintest used, were carefully identified on photographs taken with the 24-inch reflector.

The table requires little explanation. The letters "mc" in the magnitude column indicate that the time of maximum or minimum was found by the aid of the mean-curve, the observations being too few to fix definitely the magnitude. The fifth column contains the corrections to the ephemeris computed from the present set of observations.

These results furnish material corrections to the period of two variables, viz.:

6894. *S Lyrae.*

CHANDLER'S revised elements in *A.J.* 553 give the period as 218 days; my observations show about double, or 438 days: the six minima falling close to the time of intermediate maxima as computed from the shorter period.

7269. *SV Cygni.*

In the *Vierteljahrsschrift* for 1904, p. 261, HARTWIG uses the maxima of 1899 Aug. 20 and 1904 Feb. 18, and dividing the interval, 1642 days, by 3, finds the period to be 548 days. My observations show that the divisor should be 4, giving 409 days.

Maxima						Minima					
Epoch	Date Calendar	J.D.	Mag.	Corr.	Weight	Epoch	Date Calendar	J.D.	Mag.	Corr.	Weight
				103. *T Andromedae.*							
50	1894 Mar. 2	2890	mc	+13	6	51	1894 July 26	3036	mc	+ 1	2
51	Dec. 3	3166	8.57	+ 5	24	52	1895 May 6	3320	mc	+ 1	7
52	1895 Sept. 9	3446	8.66	+ 1	28	53	1896 Feb. 14	3604	mc	+ 1	6
53	1896 June 13	3724	mc	− 5	2	54	1896 Nov. 24	3888	12.98	+ 1	15
54	1897 Mar. 21	4005	mc	− 8	3	55	1897 Aug. 16	4153	12.90	−18	17
55	Dec. 31	4290	8.38	− 7	13	56	1898 June 3	4444	mc	−11	1
56	1898 Oct. 18	4581	8.40	0	10	57	1899 Mar. 11	4725	13.0	−14	4
57	1899 July 22	4858	mc	− 7	4	58	Dec. 25	5014	12.98	− 9	20
58	1900 Apr. 29	5139	8.6	−10	7	59	1900 Sept. 28	5291	13.08	−16	10
59	1901 Feb. 2	5417	mc	−16	2	62	1903 Feb. 14	6160	13.3	+ 1	1
64	1905 Jan. 14	6860	mc	+ 7	2	64	1904 Sept. 6	6730	mc	+ 3	5
				267. *V Andromedae.*							
1	1897 July 26	4132	10.0	0	10	1	1897 Apr. 10	4025	mc	+ 4	3
2	1898 Apr. 15	4395	8.99	+ 4	12	2	Dec. 8	4267	mc	−13	4
3	Dec. 20	4644	9.39	− 6	20	3	1898 Sept. 1	4534	14.19	− 5	10
4	1899 Sept. 20	4918	9.69	+ 9	17	4	1899 May 10	4785	mc	−13	2
5	1900 May 17	5157	9.2	−11	8	5	1900 Feb. 5	5056	13.69	− 1	21
6	1901 Feb. 6	5422	9.6	− 5	1	6	Oct. 24	5317	13.96	+ 1	5
7	Oct. 22	5680	9.3	− 6	1	7	1901 June 24	5560	mc	−15	1
10	1903 Dec. 1	6450	mc	−13	1	8	1902 Mar. 9	5818	13.9	−16	2
11	1904 Sept. 9	6733	10.39	+11	6	9	Dec. 4	6088	14.3	− 5	3
						12	1905 Jan. 11	6857	13.61	−13	4
				787. *W Andromedae.*							
0	1899 Dec. 7	4996	7.83	− 9	27	0	1899 June 16	4819	13.4	+ 6	20
1	1901 Jan. 15	5400	mc	− 1	1	1	1900 July 23	5224	13.77	+15	9
2	1902 Feb. 28	5809	7.8	+12	4	2	1901 Aug. 13	5610	mc	+ 5	1
3	1903 Mar. 16	6190	mc	− 3	1	3	1902 Aug. 31	5993	13.4	− 8	2
4	1904 Apr. 17	6588	mc	− 1	1	4	1903 Sept. 25	6383	13.4	−14	5
5						5	1904 Nov. 15	6800	13.75	+ 7	6

Maxima						Minima					
Epoch	Date Calendar	J.D.	Mag.	Corr.	Weight	Epoch	Date Calendar	J.D.	Mag.	Corr.	Weight
					5798. *RU Herculis.*						
1	1898 Mar. 13	4362	9.12	− 7	13	1	1897 Aug. 30	4167	14.17	+29	10
2	1899 June 23	4829	7.67	− 9	16	2	1898 Nov. 9	4603	13.97	18	25
3	1900 Nov. 2	5326	10.01	+ 5	7	3	1900 Apr. 15	5125	14.13	+21	23
4	1902 Mar. 11	5820	me	+10	2	5	1902 Nov. 26	6080	me	+10	1
5	1903 June 28	6294	me	+ 7	3	6	1904 Feb. 28	6540	14.0	−13	6
6	1904 Oct. 16	6770	7.97	0	12						
					6100. *RV Herculis.*						
−1	1897 Aug. 23	4160	me	− 1	10	0	1897 Dec. 1	4260	ne	−20	1
0	1898 Mar. 15	4364	10.75	+ 3	12	1	1898 July 2	4473	15.8	7	8
1	Sept. 28	4561	10.8	0	20	2	1899 Jan. 15	4670	me	−10	1
2	1899 Apr. 6	4751	9.93	−10	17	3	Aug. 11	4878	me	− 2	1
3	Nov. 1	4960	10.50	− 1	12	4	1900 Feb. 17	5068	15.60	−12	12
4	1900 May 21	5161	11.55	0	12	5	Sept. 11	5274	15.22	− 6	8
5	Dec. 6	5360	me	− 1	1	6	1901 Mar. 26	5470	ne	−10	1
6	1901 June 29	5565	me	+ 4	1	7	Oct. 29	5687	me	+ 7	2
7	1902 Jan. 22	5772	me	+11	2	8	1902 May 20	5890	me	+10	0
9	1903 Feb. 24	6170	me	+ 9	2	9	Dec. 16	6100	me	+20	0
10	Sept. 22	6380	me	+19	1	10	1903 July 14	6310	me	+30	1
11	1904 Apr. 2	6572	me	+11	1	12	1904 Aug. 3	6696	14.2	+16	2
12	Oct. 18	6772	me	+11	3						
					6891. *S Lyrae.*						
3	1897 Mar. 4	3988	me	+ 1	7	4	1897 Dec. 19	4278	ne	+10	6
4	1898 May 12	4422	10.37	0	20	5	1899 Feb. 13	4699	me	− 7	6
5	1899 July 7	4843	10.42	−17	27	6	1900 Apr. 25	5135	15.32	− 9	16
6	1900 Sept. 29	5292	10.46	6	9	7	1901 July 14	5580	me	− 2	1
7	1901 Dec. 26	5745	me	+ 9	3	8	1902 Sept. 25	6018	15.32	− 2	9
8	1903 Mar. 12	6186	me	+12	1	9	1903 Dec. 13	6462	ne	+ 4	2
9	1904 May 23	6621	me	−12	3	10					
					7220. *S Cygni.*						
32	1893 Oct. 26	2763	10.35	− 2	33	32	1893 May 19	2603	ne	0	1
33	1894 Sept. 20	3092	me	+ 1	5	33	1894 Apr. 6	2925	me	− 1	2
34	1895 July 25	3400	me	17	2	34	1895 Feb. 15	3240	me	15	1
37	1898 Apr. 16	4396	me	+ 1	1	38	1898 Sept. 25	4558	me	1	4
40	1900 Dec. 27	5381	me	+ 8	6	40	1900 July 29	5230	me	+19	8
41	1901 Nov. 1	5690	10.6	− 9	1	42	1902 Apr. 20	5860	ne	3	1
42	1902 Oct. 17	6040	10.5	+15	1	43	1903 Mar. 26	6200	15.8	+13	1
44	1904 July 18	6680	10.3	+ 3	1	45	1904 Dec. 25	6840	15.4	+ 1	13
					7269. *SX Cygni.*						
0	1900 Oct. 2	5295	9.58	0	20	0	1900 Apr. 16	5126	13.65	+ 4	33
1	1901 Nov. 1	5690	9.4	14	2	1	1901 May 15	5520	me	−11	1
2	1903 Jan. 5	6120	9.2	+ 7	1	2	1902 July 9	5940	me	0	7
3	1904 Feb. 19	6530	me	+ 8	0	3	1903 Aug. 23	6350	ne	+ 1	2
						4	1904 Oct. 7	6761	13.95	+ 5	7
					7458. *U Delphini.*						
3	1895 May 15	3329	10.76	10	6	4	1896 May 30	3710	ne	+ 3	1
4	1896 Nov. 16	3880	9.66	+12	19	5	1897 Nov. 18	4247	ne	+11	1
5	1898 May 5	4115	me	+18	10	7	1900 Sept. 27	5290	17.3	1	8
6	1899 Oct. 1	4929	8.08	+ 3	24	8	1902 Feb. 26	5807	ne	16	2
7	1901 Mar. 16	5455	me	+ 5	1	9	1903 Aug. 23	6350	ne	2	2
8	1902 Aug. 22	5984	10.3	0	1						
9	1904 Jan. 20	6500	me	13	1						
					8629. *Y Cassiopeiae.*						
0	1898 Mar. 11	4360	9.44	+ 6	7	1	1898 Nov. 4	4598	[illegible]	8	17
1	1899 Apr. 8	4753	8.83	11	11	2	1899 Dec. 13	5012	[illegible]	4	19
2	1900 June 9	5180	10.30	+ 6	7	3	1901 Feb. 16	5432	me	+ 6	4
3	1901 July 24	5590	me	+ 6	1	4	1902 Mar. 31	5840	ne	+ 2	1
4	1902 Sept. 7	6000	me	+ 6	2	5	1903 May 15	6250	ne	+ 4	2
5	1903 Oct. 10	6398	8.43	6	2	6	1904 June 8	6640	[illegible]	16	5
6	1904 Nov. 27	6812	9.36	2	9						

Yerkes Observatory, 1905 *March* 25.

OBSERVATIONS OF BROOKS'S PERIODIC COMET,

MADE WITH THE 40-INCH REFRACTOR.

By E. E. BARNARD.

On account of bad weather only eight observations of this comet were secured. Every effort was made to get more measures, but it was impossible. For the same reason, coupled with the low altitude, earlier measures were not possible, though the comet was looked for frequently. With poor seeing there is always present in a large telescope a milkiness of the field that greatly interferes with the visibility of a faint object, such as a comet or nebula — even more perhaps than in the case of a faint star.

Following are a few notes on the appearance of the comet:

October 19. Very faint from bad seeing. Very gradually brighter in the middle. Twelfth or thirteenth magnitude, and ¾′ in diameter.

December 14. Large.

December 22. Excessively faint; somewhat elongated east and west. About ½′ diameter, with possibly a faint nucleus.

January 4. Very faint; 16 magnitude. Very small and indefinite.

On four dates, it was necessary to compare the comet with faint stars which were compared by transits and $\Delta\delta$ with known stars.

1903–4 Cen. St. Time	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ
1903 d h m s			m s	′ ″	h m s	° ′ ″
Oct. 19 7 48 46	1	4	. . . .	−0 53.5	. . . . .	−21 41 37.5
7 56 17	1	6	+0 14.73	. . . .	21 7 49.12	. . . . .
8 3 34	1	4	. . . .	−0 46.3	. . . . .	−21 41 30.3
8 9 23	1	4	+0 15.20	. . . .	21 7 49.59	. . . . .
20 8 23 44	2	5, 8	+0 2.09	−3 17.8	21 8 53.9	−21 32.1
26 7 35 52	3	5, 8	+2 5.37	−2 8.6	21 15 14.8	−20 32.7
7 45 6	3	2	. . . .	−2 3.9	. . . . .	−20 32.6
Nov. 10 6 30 37	4	6, 6	−1 57.48	+0 43.2	21 36 15.99	−17 48 47.5
Dec. 14 6 43 20	5	4, 8	−2 38.31	+2 9.4	22 34 49.05	−10 23 6.0
21 6 42 38	6	7, 10	+0 7.09	−2 6.2	22 48 6.56	− 8 41 3.9
22 6 26 34	7	6, 9	+0 12.77	+4 18.1	22 50 0.62	− 8 26 28.0
1904 Jan. 4 6 16 33	8	8, 6	−0 59.54	+1 17.8	23 15 25.50	− 5 10 34.2

Mean Places of Comparison-Stars, 1903, 1904.

*	α	Red. to app. place	δ	Red. to app. place	Authority
	h m s	s	° ′ ″	″	
1	21 7 31.24	+3.15	−21 41 2.5	+18.5	Algiers, A.G.C.
2	21 8 48.8	+3.05	−21 29.1	+18.0	SD. 21°5969
3	21 13 36.4	+2.96	−20 30.9	+18.3	SD. 20°6182
4	21 38 10.67	+2.80	−17 49 48.9	+18.2	Washington Zones 67–142, Vol. II
5	22 37 24.75	+2.61	−10 25 34.5	+19.1	H.C.O., A.G. Catal.
6	22 47 56.89	+2.58	− 8 39 16.7	+19.0	L. de Ball, A.G.C.
7	22 49 45.27	+2.58	− 8 31 35.1	+19.0	L. de Ball, A.G.C.
8	23 16 24.62	+0.42	− 5 11 52.5	+ 0.5	Strassburg, A.G.C.

Following are the micrometer measures of the comet with the small stars, which were afterwards compared with known stars:

COMET — STAR.

Time	Cps.	$\Delta\alpha$	$\Delta\delta$
h m s		′ ″	″
Oct. 26 7 35 10	5	−2 41.5	. . .
7 35 52	10	. . .	−0 28.2
Nov. 10 6 31 30	6	−1 4.0	. . .
Dec. 14 6 43 37	4	+1 32.8	. . .
6 43 20	8	. . .	−0 40.7
Jan. 4 6 17 40	6	−2 6.2	. . .
6 16 33	8	. . .	+0 51.2

These small stars were then compared with the final comparison-stars (small star — comp. star).

	$\Delta\alpha$	Comp.	$\Delta\delta$	Mag.
	m s		″	
Oct. 26	+2 16.83	6, 3	−1 40.2	12
Nov. 10	−1 52.84	16	. . . .	12
Dec. 14	−2 44.58	8, 3	+3 0.0	12
Jan. 4	−0 51.00	18, 4	+0 26.6	13

On Nov. 10, the comet was compared directly in $\Delta\delta$ with the comparison-star, and the $\Delta\alpha$ was measured with a 12^M star, which was then compared in $\Delta\alpha$ with comp.-star by transits. The small star was s.f. the comp.-star.

I am greatly indebted to M. TREPIED of Algiers, Admiral CHESTER of Washington, Professor PICKERING of Cambridge, Mass.; Dr. DE BALL of Vienna, and Dr. BECKER of Strassburg, for star places in advance of publication of their catalogues.

Yerkes Observatory.

NOTES ON SOME LONG-PERIOD VARIABLE STARS,

By A. STANLEY WILLIAMS

The introductory remarks to the notes published in the *A.J.*, No. 559, p. 62, will apply generally to those which follow. The observations on which the present notes are based were all made with a $6\frac{1}{2}$-inch reflector, a power of 73 being usually employed. Last year was on the whole a favorable one for observation.

RV Andromedae.

R.A. = 1h 32m 47s , Decl. = +38° 9′.5 (1900).

Observations were made on 19 nights, between 1904 Aug. 2 and Nov. 12. When first observed the star appears to have been just about at maximum brightness, and from this time it at first very slowly, but later somewhat rapidly, declined to 12M.3 on Nov. 12. With the help of the previously obtained light-curves, a maximum was deduced for 1904 Aug. 2 (9M.8), but this is necessarily a little uncertain, owing to the absence of observations during the increase. The computed date of maximum according to the elements in *A.J.* 559, p. 62, is Aug. 17.

562. *Y Andromedae*.

The variable rose rapidly from 10M.7 on 1904 Oct. 28 to a sharply defined maximum (8M.5) on Nov. 18. The decrease was slower than the increase. Fifteen observations were obtained up to 1905 Jan. 21, when the star was 10M.0.

RW Andromedae.

R.A. = 2h 4m 34s , Decl. = +48° 27′.6 (1900).

According to the elements published in the *A.N.* 3911, a minimum of this star should have occurred on July 29 of last year, and a maximum on Oct. 28, assuming that the latter occurs midway between two minima. Observations were made on 21 nights, between July 24 and 1905 Jan. 7. These give 1904 Aug. 1 and Oct. 13 as the observed dates of minimum and maximum respectively. It would seem from this that the rise from minimum to maximum takes place rather more quickly than the decline, the interval $M-m$ being 73 days. The brightness at maximum was 8M.3 (assuming DM. +48 614 to be 8M.7), and at minimum 10M.2. Hartwig observed the star 8M.5 on 1904 Sept. 30 (*A.N.* 3981, col. 371), in good agreement with the observations made here.

1205. *Y Persei*.

Observations made on 15 nights, between 1904 Aug. 2 and 1905 Jan. 12, indicate a maximum for 1904 Oct. 22 (8M.4), but this is not very satisfactory, the star being a very difficult one to observe, owing to its peculiar orange-red color. The observations of August 1904 are discordant, and seem to suggest that a secondary maximum may have occurred at about the end of July. The date given in the *V.J.S. Ephemerides* for the maximum is Sept. 30. By Jan. 12 of this year the star had decreased to 9M.7

RY Lyrae.

R.A. = 18h 41m 15s , Decl. = +[illegible]4° 34′.6 (1900).

This star was observed 11M.4 on 1904 May 14, and rose to a well-defined maximum (10M.6) on June 10. By Sept. 3 it had declined to 12M.4, and on Oct. 3 it was invisible in a $6\frac{1}{2}$-inch reflector. Observations were made on 18 nights, between the above limiting dates, but there is a gap between June 23 and July 21, and for this reason the above date of maximum is not perfectly satisfactory, though it is not likely to be in error by more than 2 or 3 days.

RH Lyrae.

R.A. = 18h 42m 7s , Decl. = +43° 31′.9 (1900).

Observations were made on 26 nights, between 1904 May 14 and Dec. 13. The star was invisible in a $6\frac{1}{2}$-inch reflector prior to Aug. 28, when it was 13M, and just visible in this telescope. From the last-mentioned date it rose to a well-defined maximum on 1904 Nov. 2 (9M.6), and by Dec. 13 had declined to 10M.8.* The form of the light-curve differs somewhat from that of the maximum of 1903, being more rounded and the decrease more rapid than it was in that year, and it appears to resemble more closely the photographic light-curve of 1900. Comparison of the visually observed maximum of 1904 Nov. 2 with that of 1903 May 25 gives 527 days for the length of the period; whilst comparison of the former maximum with the photographic maximum of 1900 Oct. 6 would make the period 496 days. Having regard to the similarity in the form of the light-curve, it seems the more likely that the latter represents the average period of the variable, so that the following are the revised elements of variation.

$$\text{Maximum} = 1900 \text{ Oct. } 6 \text{ (J.D. } 2415299) + 496 E$$

and these satisfy all the published photographic and visual observations, including the two published by Hartwig in the *V.J.S. Ephemerides* for 1904, p. 244. The next maximum will be due 1906 Mar. 13.

6783. *RX Lyrae*.

Observations were made on 21 nights, between 1904 May 14 and Oct. 13. The star was invisible in a $6\frac{1}{2}$-inch reflector up to Aug. 8, on which date it was just visible distinctly, and about 13M. It then increased rapidly to a well-defined maximum on 1904 Sept. 2, when it was 11M.1 (0M.2 brighter than the star marked (2) on the chart in the

* For light scale and magnitudes of comparison stars, see *A.J.* 559, p. 62.

A.N. 3857). The decline was not so rapid as the rise, but by Oct. 13 the star had decreased to $12^M.5$. The predicted date of maximum in the *VJS. Ephemerides*, 1904, is Sept. 9.

6816. *Z Lyrae.*

Observations on 36 nights, between 1904 May 14 and Oct. 29, enable a good determination of maximum to be made. The star rose from $12^M.7$ on May 14 to a well-defined maximum on 1904 Aug. 30 ($9^M.3$), and had declined to $11^M.1$ by Oct. 29.

6827. *RT Lyrae.*

In 1903 this star was observed $11^M.5$ on May 9, and from this date it decreased in brightness, until by June 21 it was no longer visible in a 6½-inch reflector, and it had not re-appeared by Sept. 11.

In 1904 observations were made on 21 nights, between the under-mentioned limiting dates. The variable was invisible in the 6½-inch reflector on May 14, but was observed $13^M.1$ on June 12, and from this date it rose rapidly to a well-defined maximum on 1904 Aug. 6, when it was $9^M.3$, or $0^M.5$ fainter than DM. $+37°3306$ ($8^M.8$). The decline was equally rapid with the rise, and by Oct. 3 the star had diminished to $12^M.6$.

Comparing the above maximum with the one that was well observed here on 1902 July 22 (*A.J.* 529),* we have the following elements of variation:

$$\text{Maximum} = 1902 \text{ July } 22 \text{ (J.D. } 2415953) + 248^d.7\ E$$

and these satisfy all the published observations of this star. The next maxima should occur on 1905 April 12 and Dec. 16.

6895. *RU Lyrae.*

The variable was invisible in a 6½-inch reflector between 1904 May 14 and June 20. It was observed $12^M.5$ on July 21, and from this date it rose rapidly to a sharply-defined maximum on 1904 Aug. 28, and by Oct. 3 had decreased to $12^M.0$. This maximum seems to have been an unusually faint one, the greatest brightness being only $10^M.9$. The date, too, is 22 days earlier than the computed date of maximum according to the elements in the *A.J.* 559, p. 63, so that the period of 380 days is a little too long. Observations were made on 18 nights between the above limiting dates.

7019. *TY Cygni.*

This star was invisible in a 6½-inch reflector on 1904 May 14, but it was observed $12^M.2$ on June 8, and from this date it rose at first slowly, but after July rapidly, to a well-defined and well-observed maximum ($9^M.1$) 1904 on Oct. 10. By Nov. 27 the variable had declined to $10^M.6$. Nineteen observations were made between the above limiting dates.

7505. *UX Cygni.*

Between 1904 May 14 and 1905 Jan. 7 observations were made on 15 nights. On the former of the above dates the star was 13^M, being just visible in a 6½-inch reflector, and it remained faint, but slowly brightening, until Aug. 15, when it was $12^M.4$. A very rapid increase in brightness then set in, and by Sept. 21 the star had risen 4.5 magnitudes, or at the rate of a magnitude in 8 days. A well-defined and well-observed maximum ($7^M.4$) occurred on 1904 Oct. 9. The decline was rapid from Oct. 20 to Nov. 20, but then became slower, the magnitude on 1905 Jan. 7 being 10.0.

Comparison of the above maximum with the photographic one of 1901 Oct. 23 (see *A.J.* 529, p. 7) would give 511 days for the period of variation, but this cannot be exactly right, since carried back it would imply a maximum on 1895 Nov. 19; whereas the faintness of the star on a photograph obtained by Max Wolf at Heidelberg two days later (see *A.N.* 3752, col. 116) shows that it was then at some distance from that epoch. It seems likely that the recent maximum was abnormally late, and that the period of 518 days suggested in the *A.J.* 559, p. 63, is nearly correct.

7571*a*. *TW Cygni.*

This variable rose rapidly from $11^M.4$ on May 14 to a very sharply-defined maximum ($9^M.2$) at about 1904 June 28, and by Aug. 15 it had decreased to $11^M.3$. This maximum appears to have been a very sharply-defined one, but the exact date is not well ascertained, as, owing to my absence from home, there are no observations between June 23 and July 21. The form of the light-curve of this variable differs considerably at different maxima.

* Hartwig gives July 9 for the date of maximum in 1902 from his own observations, but does not mention any particulars (*VJS. Ephemerides* for 1903, p. 283).

20 *Hove Park Villas, Hove*, 1905 *Jan.* 31.

CONTENTS.

Published at Boston, and Cushing Road, Wellesley Hills, Mass., semi-monthly, by S. C. Chandler. Address, Box 216, Wellesley Hills, Mass.
Associate Editors, Asaph Hall and Lewis Boss. Price, $5.00 the Volume. Press of Thos. P. Nichols, Lynn, Mass. *Closed April 27*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD

No. 574.

VOL. XXIV. BOSTON, 1905 JUNE 16. NO. 22

ON SYSTEMATIC ERRORS IN DETERMINING VARIATIONS OF LATITUDE.

BY FRANK SCHLESINGER.

Observations for determining the variation of latitude, however carefully they may be made, seem to be subject to considerable systematic discordances. It is doubtful whether these have their origin in external causes (such as meteorological), or whether their explanation is to be sought in the instrument or in the observer.

It is obvious that the question can be decided by setting up two instruments side by side, and having two observers make simultaneous observations with them. It will occasionally happen with each instrument that a night's observations will deviate largely from those of the preceding and succeeding nights. If these deviations follow the same course for both instruments, we must conclude that they arise from some external cause, probably beyond the control of the observer.

The conditions for such a test happen to be well fulfilled by certain observations made before the present subject was in mind. I refer to the two independent series by MARCUSE and PRESTON, at Waikiki, near Honolulu, in the Hawaiian Islands, in 1891 and 1892. The former of these observers represented the International Geodetic Association, the latter the United States Coast and Geodetic Survey. In that day the reality of latitude-variations was still doubted by some, and Hawaii was selected as a site for an observing station because the latitude-variations at that place should be (and in fact proved to be) the reverse of those at European stations, the difference in longitude being about 180°. Two observers were sent, because "previous experience has shown that a single series may easily suffer interruption because of the illness of the observer, or the failure of the instrument." Waikiki is on the south side of the island of Oahu, about two miles southeast of Honolulu. PRESTON's station was within 400 feet of the shore, and MARCUSE's was 31 feet north and 18 feet west of PRESTON's. To the north and west rise the [illegible] mountains of the island, so that the character of the country surrounding the stations was [illegible] as diverse as possible. This circumstance is unfavorable as regards the determination of the latitude itself, but for present purposes it is rather fortunate than otherwise. The instruments employed were of the same general type, [illegible] telescopes broken near the centre, and they differed only in detail; it would have been better for the purposes of this paper if they had been radically different instruments, if, for example, one of them had used some other means than a level for determining the zenith point.

The observers used the same star-list, but reduced their observations with different systems of declinations. As is well understood, this has no influence upon the derived latitude-variations, the method of observation being such as to eliminate errors in the assumed declinations. However, the use of different systems of declinations makes impossible a direct comparison, and I have therefore altered PRESTON's results by reducing his observations with MARCUSE's declinations.

In the accompanying table are collected all the necessary data. They are based on PRESTON's paper in the report of the Superintendent of the U. S. Coast and Geodetic Survey for 1892, Appendix No. 2, and on ALBRECHT's *Resultate der Beobachtungsreihen in Honolulu 1892* [illegible] *der Internationalen Erdmessung*.

Column 1 gives the date of observation. Only those dates have been used on which both observers secured complete groups simultaneously. [illegible] isolated pairs might also have been [illegible] would have involved a [illegible] work, without adding [illegible]. [illegible] the restriction to complete groups [illegible] for any [illegible] correction [illegible] has been observed.

Column 2 gives the [illegible] served at the end of the series [illegible] it contains seven pairs [illegible] eight pairs. The material [illegible] groups and 777 pairs [illegible].

Column 3 contains MARCUSE's [illegible] 92 to 94 of ALBRECHT's paper [illegible] errors." These [illegible] by MARCUSE and PRESTON [illegible] 185.

1	2	3	4	5	6	7	8	9
					First Solution		Second Solution	
Date	Group	Observed Latitudes		Marcuse minus Preston	Marcuse minus Albrecht	Preston minus Albrecht	Marcuse minus Curve	Preston minus Curve
		Marcuse	Preston					
		″	″	″	″	″	″	″
1891 June 8	I	5.13	4.83	+0.30	+0.07	–0.23	+0.12	–0.23
13		5.23	5.09	+ .14	– .20	+ .06	+ .25	+ .06
15		4.90	4.85	+ .05	– .12	– .17	– .07	– .17
17		5.07	5.09	– .02	+ .06	+ .08	+ .11	+ .08
22		4.84	4.94	– .10	– .15	– .05	– .10	– .04
June 27	II	4.95	5.03	– .08	– .02	+ .06	+ .04	+ .06
30		4.88	5.05	– .17	– .07	+ .10	– .02	+ .09
July 18		4.71	4.65	+ .06	– .16	– .22	– .11	– .21
26		4.70	4.82	– .12	– .13	– .01	– .07	+ .01
Aug. 5		4.74	4.76	– .02	– .05	– .03	– .01	– .02
6		4.71	4.71	.00	– .08	– .08	– .04	– .06
10		4.98	4.75	+ .23	+ .20	– .03	+ .25	.00
13		4.68	4.62	+ .06	– .09	– .15	– .05	– .12
14		4.48	4.63	– .15	– .28	– .13	– .24	– .10
July 17	III	4.74	4.69	+ .05	– .13	– .18	– .08	– .17
18		4.93	4.80	+ .13	+ .06	– .07	+ .11	– .06
27		4.65	4.65	.00	– .18	– .18	– .13	– .16
Aug. 5		4.77	4.72	+ .05	– .02	– .07	+ .02	– .06
13		4.40	4.67	– .27	– .37	– .10	– .33	– .07
14		4.51	4.38	+ .13	– .25	– .38	– .21	– .35
26		4.40	4.81	– .41	– .33	+ .08	– .29	+ .13
Sept. 2		4.47	4.66	– .19	– .24	– .05	– .21	+ .01
19		4.32	4.70	– .38	– .36	+ .02	– .33	+ .09
26		4.42	4.68	– .26	– .25	+ .01	– .23	+ .08
28		4.76	4.78	– .02	+ .09	+ .11	+ .11	+ .18
Aug. 20	IV	4.64	4.71	– .07	– .11	– .04	– .06	– .01
23		4.76	4.87	– .11	+ .02	+ .13	+ .06	+ .18
Sept. 2		4.80	4.91	– .11	+ .09	+ .20	+ .12	+ .26
9		4.81	4.66	+ .15	+ .12	– .03	+ .14	+ .02
12		4.69	4.65	+ .04	.00	– .04	+ .03	+ .02
26		4.62	4.58	+ .04	– .05	– .09	– .03	– .02
28		4.63	4.42	+ .21	– .04	– .25	– .02	– .18
Oct. 5		4.60	4.61	– .01	– .07	– .06	– .05	+ .02
28		4.88	4.63	– .25	+ .20	– .05	+ .21	+ .01
30		4.73	4.96	– .23	+ .05	+ .28	+ .05	+ .33
31		4.53	4.69	– .16	– .15	+ .01	– .15	+ .06
Nov. 4		4.58	4.59	– .01	– .11	– .10	– .10	– .05
6		4.66	4.81	– .15	– .03	+ .12	– .03	+ .17
14		4.81	4.69	+ .12	+ .10	– .02	+ .10	+ .04
17		4.52	4.79	– .27	– .19	+ .08	– .20	+ .12
Oct. 30	V	4.90	4.62	+ .28	+ .22	– .06	+ .22	– .01
31		4.63	4.57	+ .06	– .05	– .11	– .05	– .06
Nov. 6		4.77	4.64	+ .13	+ .08	– .05	+ .08	.00
14		4.68	4.69	– .01	– .03	– .02	– .03	+ .04
17		4.77	4.77	.00	+ .06	+ .06	+ .05	+ .10
19		4.78	4.68	+ .10	+ .06	– .04	+ .06	+ .01
23		4.88	4.76	+ .12	+ .15	+ .03	+ .15	+ .08
Dec. 10		4.83	4.89	– .06	+ .05	+ .11	+ .05	+ .14
12		4.77	4.91	– .14	– .02	+ .12	– .03	+ .15
15		4.72	4.61	+ .11	– .08	– .19	– .08	– .16
21		5.06	4.73	+ .33	+ .24	– .09	+ .24	– .06
23		4.83	4.82	+ .01	+ .01	.00	.00	+ .03
26		4.84	4.81	+ .03	.00	– .03	– .01	.00
30		4.77	4.73	+0.04	–0.08	–0.12	–0.09	–0.10

1 Date	2 Group	3 Observed Latitudes Marcuse	4 Observed Latitudes Preston	5 Marcuse minus Preston	6 First Solution Marcuse minus Albrecht	7 First Solution Preston minus Albrecht	8 Second Solution Marcuse minus Curve	9 Second Solution Preston minus Curve
1891 Dec. 12	VI	4.83	4.88	−0.05	+0.04	+0.09	+0.03	+0.12
15		4.75	4.77	− .02	− .05	− .03	− .05	.00
23		4.71	4.93	− .22	− .11	+ .11	− .12	+ .14
26		4.71	4.95	− .24	− .13	+ .11	− .14	+ .14
29		4.81	4.92	− .11	− .03	+ .08	− .05	+ .09
1892 Jan. 6		4.98	4.71	+ .27	+ .11	− .16	+ .09	− .16
12		5.00	5.16	− .16	+ .10	+ .26	+ .08	+ .26
18		4.88	5.03	− .15	− .04	+ .11	− .07	+ .10
20		4.89	4.85	+ .04	− .04	− .08	− .06	− .10
27		5.07	5.49	− .42	+ .11	+ .53	+ .09	+ .52
Feb. 3		4.79	4.85	− .06	− .20	− .14	− .22	− .16
8		4.99	4.92	+ .07	− .02	− .09	− .03	− .12
13		4.95	4.73	+ .22	− .09	− .31	− .10	− .33
Jan. 20	VII	4.97	4.54	+ .43	+ .04	− .39	+ .02	− .41
Feb. 3		5.13	5.03	+ .10	+ .14	+ .04	+ .12	+ .02
4		5.11	4.92	+ .19	+ .11	− .08	+ .10	− .09
8		5.03	4.92	+ .11	+ .02	− .09	+ .01	− .12
12		5.06	4.84	+ .22	+ .03	− .19	+ .01	− .22
13		4.92	4.92	.00	− .12	− .12	− .13	− .14
24		5.24	5.23	+ .01	+ .16	+ .15	+ .15	+ .12
27		5.12	5.16	− .04	+ .03	+ .07	+ .02	+ .03
29		5.05	5.06	− .01	− .05	− .04	− .06	− .08
Mar. 2		5.07	5.09	− .02	− .04	− .02	− .04	− .06
5		5.18	5.36	− .18	+ .06	+ .24	+ .06	+ .20
9		5.09	4.86	+ .23	− .05	− .28	− .04	− .31
10		4.98	5.20	− .22	− .16	+ .06	− .15	+ .02
24		5.21	5.34	− .13	+ .03	+ .16	+ .04	+ .15
Mar. 2	VIII	5.22	5.25	− .03	+ .11	+ .14	+ .11	+ .10
5		5.14	5.54	− .40	+ .02	+ .42	+ .02	+ .38
9		5.07	5.26	− .19	− .07	+ .12	.06	+ .09
10		5.15	5.13	+ .02	+ .01	− .01	+ .02	− .05
21		5.10	5.19	− .09	− .08	+ .01	− .07	− .02
30		5.12	5.18	− .06	− .07	− .01	− .05	− .05
31		5.13	5.28	− .15	− .06	+ .09	.04	+ .05
April 9		5.29	5.45	− .16	+ .08	+ .24	+ .11	+ .22
19		5.16	5.15	+ .01	− .06	− .07	.02	.08
May 2		5.47	5.22	+ .25	+ .25	.00	+ .30	+ .01
5		5.19	5.18	+ .01	− .03	.04	+ .02	− .02
May 4	I	5.27	5.19	+ .08	+ .05	− .03	+ .10	.03
5		5.32	5.27	+ .05	+ .10	+ .05	+ .15	+ .07
9		5.13	4.91	+ .22	.09	.31	− .03	.27
14		5.11	4.95	+ .16	.10	− .26	.04	.25
15		4.94	4.90	+ .04	.27	.31	.21	.27
18		5.21	5.30	0.09	0.00	+0.09	+0.08	+0.15

	Marcuse	Preston	Mean
Group 1	0.00	0.00	0.00
Group 2	+0.01	−0.01	0.01
Group 3	−0.10	0.23	0.16
Group 4	+0.37	+0.39	+0.38
Group 5	+0.15	+0.17	+0.16
Group 6	+0.19	+0.10	+0.15
Group 7	+0.01	+0.02	+0.01
Group 8	+0.08	+0.07	+0.08

PRESTON's group-reductions have of course [illegible] rected for differences in assumed declinations, [illegible] presently. MARCUSE's reductions should perhaps be entitled to greater weight, but even if we assigned to them double the weight of PRESTON's, the resulting corrections would differ by insignificant quantities (never more than 0″.02) from the simple means. I have therefore adopted the latter as given above, and have applied them to [illegible]

MARCUSE's and PRESTON's observations. To save space and printing, 21° 16′ 20″ has been subtracted from each latitude.

Column 4 contains PRESTON's latitudes. In order to make them directly comparable with MARCUSE's, the following corrections have been applied to the latitudes given by PRESTON on pages 148 to 150 of his paper: (1) The group reductions as given above; (2) A constant correction of +0″.30, PRESTON's instrument having been so much south of MARCUSE's; (3) The difference in the assumed declinations, the means by groups being,

(1)	+0″.20	(2)	+0″.31	(3)	+0″.16	(4)	+0″.09
(5)	+0.15	(6)	+0.28	(7)	+0.20	(8)	+0.09

Column 5 contains the differences between corresponding numbers in the preceding columns. The mean of these 98 differences, having regard to signs, is −0″.008. If we denote by E_M the mean error of MARCUSE's determination of the latitude from one group, and by E_P a similar quantity for PRESTON, then by squaring and adding the numbers in column 5 we have

$$E^2_M + E^2_P = \frac{2.737}{98} = 0.0279 \qquad (1)$$

It is now in order to determine how much the two observers differ separately from the true latitude, and to compare these residuals with column 5. Several ways of doing this are open to us, depending upon what we adopt for the variation of latitude. Besides the observations made at Waikiki during the period under discussion, similar series were being carried on at six other stations well distributed in longitude; and the results have been assembled by Dr. ALBRECHT in his "*Bericht uber der Stand der Erforschung der Breitenvariation im Dez, 1897.*" Instead of using these results, some might prefer, for the present purpose, to use only the determinations of the variation of latitude made at Waikiki itself; and if this latter course were decided upon there would still be a choice in the manner of combining the two separate series. In order to see how much these different alternatives would affect our conclusions, I have made two extreme solutions. In the first MARCUSE's observations have been compared with ALBRECHT's latitude-curve (as given in the *Bericht*) and PRESTON's with the same curve. In the second solution MARCUSE's separate observations were compared with his own latitude-curve, and PRESTON's observations with PRESTON's latitude-curve. The former solution may be regarded as yielding the upper limit for the systematic error, and the latter the lower limit.

From the data on pages 6 and 7 of ALBRECHT's *Bericht* a curve showing the variation of latitude at Waikiki was computed and plotted on a generous scale. The numbers in columns 3 and 4 were then subtracted from the corresponding curve numbers with the results shown in columns 6 and 7. Squaring, adding and taking the mean for column 6 we get 0.0169; and for column 7, 0.0238. If there were no common error present we should expect the sum of these two numbers (0.0407) to be about equal to $E^2_M + E^2_P$; but we see that this sum is considerably greater than what we found in Equation (1). We must therefore conclude that some systematic error is present which tends to displace both series in the same direction, so that the simultaneous observations agree better than we should infer from their divergencies from the true latitude. We may arrive at the size of this systematic tendency in this way; let E be its mean value (in a sense corresponding to mean error). Then columns 6 and 7 give respectively,

$$E^2 + E^2_M = 0.0169 \qquad (2)$$
$$E^2 + E^2_P = 0.0238 \qquad (3)$$

Combining with Equation (1) we get

$$\left.\begin{aligned} E &= 0''.080 \\ E_M &= 0.102 \\ E_P &= 0.132 \end{aligned}\right\} \text{First Solution}$$

This value of E includes the uncertainties in ALBRECHT's determination of the latitude-variation; a consideration of the data leads us to conclude that when the most liberal allowance is made for this the above value of E cannot be decreased by more than 0″.01.* But this point is entirely avoided in the Second Solution, details for which appear in columns 8 and 9. The numbers in the former column were obtained by subtracting those in column 3 from the corresponding values of the latitude as given by MARCUSE's curve in the "*Resultate der Beobachtungsreihe in Honolulu.*" Those in column 9 were obtained by subtracting the numbers in column 4 from the corresponding latitudes as given by the diagram opposite page 156 of PRESTON's paper. To the scaled numbers in the latter case 0″.51 has been added in order to allow for the difference of latitude (+0″.30) between the two instruments, and for the mean difference of the declinations (0″.21) upon which the diagrams are based. Squaring, adding and taking the means, we get from columns 8 and 9,

$$E^2 + E^2_M = 0.0155 \qquad (4)$$
$$E^2 + E^2_P = 0.0235 \qquad (5)$$

Combining as before with Equation (1) we get

$$\left.\begin{aligned} E &= 0''.075 \\ E_M &= 0.100 \\ E_P &= 0.134 \end{aligned}\right\} \text{Second Solution}$$

These are in very satisfactory accord with the results of the first solution. There is therefore no doubt as to the reality of a common error affecting the two series.

This investigation obviously tells us nothing of the *cause* of the systematic error. The writer hopes to throw some light upon this question in a future paper.

Yerkes Observatory, 1904 *December* 24.

* This estimate includes the effect of KIMURA's recently-discovered term.

THE SECULAR PERTURBATIONS OF *MARS* ARISING FROM THE ACTION OF *SATURN*.

BY ERIC DOOLITTLE.

The elements employed in the following computation are from Dr. G. W. Hill's "*New Theory of Jupiter and Saturn*," pages 19, 192, and 558:

Mars.	*Saturn.*
$\pi = 333^\circ\ 17'\ 51''.74$	$\pi' = 90^\circ\ 6'\ 41''.37$
$i = 1\ 51\ 2.24$	$i' = 2\ 29\ 40.19$
$\Omega = 48\ 23\ 54.59$	$\Omega' = 112\ 20\ 49.05$
$e = 0.093\ 26803$	$e' = 0.056\ 06025$
$n = 689050''.784$	$n' = 43996''.21506$
$\log a = 0.182\ 8971$	$\log a' = 0.979\ 4956$
$m = 1 \div 3093500$	$m' = 1 \div 3501.6$

Epoch 1850.0 G.M.T.

The orbit of *Mars* was divided into twelve parts with regard to the eccentric anomaly. This was found to be fully sufficient, for although the approximate tests furnished by comparing the sums of the functions corresponding respectively to the odd and even points of division were in many cases inapplicable, yet those sums from which the final values of the differential coefficients are derived were in almost perfect agreement, thus furnishing two separate determinations of the coefficients which are in some degree independent. All known test equations were also applied, and the work was duplicated throughout; in the duplication the form of the formulas was changed when possible, and addition and subtraction tables of logarithms were employed. The equation,

$$\sin q \ldots A_1 + \cos q \ldots B \ldots = 0$$

was found to give the residual, +0.000 000 000 000 [illegible]

If m' is left indefinite, the resulting values of the differential coefficients are as follows:

$$\left[\frac{de}{dt}\right]_{00} = +22022.051 \qquad 4.342\ 8578$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +23387.60 \qquad [illegible]$$

$$\left[\frac{di}{dt}\right]_{00} = -86.444370 \qquad [illegible]$$

$$\left[\frac{d\Omega}{dt}\right]_{00} = -920.858.04 \qquad [illegible]$$

$$\left[\frac{d\pi}{dt}\right]_{00} = +2888.2557 \qquad [illegible]$$

$$\left[\frac{dL}{dt}\right]_{00} = -2935.3283 \qquad 3.467\ 6567$$

If the value 1 : 3501.6 is adopted [illegible] then obtained the following results:

$$\left[\frac{de}{dt}\right] = +0.006289\ 1406$$

$$\left[\frac{d\chi}{dt}\right] = +6.679\ 0508$$

$$\left[\frac{di}{dt}\right] = -0.024\ 687281$$

$$\left[\frac{d\Omega}{dt}\right] = -0.262\ 9825$$

$$\left[\frac{d\pi}{dt}\right] = +0.[illegible]776785$$

$$\left[\frac{dL}{dt}\right] = -0.838\ 28212$$

The values found by LeVerrier are [illegible] *Annales* [illegible] *Paris*, Tome II, page 7[illegible], and [illegible] page 189; those of Newcomb are in [illegible] [illegible]. If the results are [illegible] they will compare with those here [illegible]:

	LeVerrier	Newcomb	Method of Gauss
$\frac{de}{dt}$	+0.00627	+0.00629	+0.006 2891
[illegible]	+0.00622[illegible]	+0.006226	+0.006228 14
$\sin i \frac{d\Omega}{dt}$	0.00852	0.00849	0.008 4927
[illegible]	0.02467	0.02468	0.024 6873
$\frac{dL}{dt}$	0.838		0.838 2821

The Flower Observatory, 1905 J[illegible]

OBSERVATIONS OF COMETS,

MADE WITH THE 11-INCH EQUATORIAL AT THE SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.

BY MARY E. BYRD.

Greenwich M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			Comet *c* 1903 (Borrelly).							
			m s	′ ″	h m s	° ′ ″			s	″
*July 24 16 0 9	1	9, 6	−5 36.76	−6 21.4	13 23 26.39	+64 10 6.8	9.999	0.378	−0.25	+11.3
Aug. 1 14 59 45	2	9	. . . .	+3 59.8	. . . . . .	+51 10 8.4	. .	0.708	. .	+ 0.8
8 13 19 22	3	10, 10	−0 28.81	−5 15.1	11 10 57.65	+43 43 6.4	9.777	0.671	+0.24	− 3.4
12 13 47 33	4	9	−3 18.85†	. . . .	10 57 24.41	. . . . .	9.728	. .	+0.32	. .
			Encke's Comet, 1904 *b*.							
Dec. 7 13 1 18	5	13, 9	+0 51.96	− 1.0	20 49 19.28	+ 5 41 33.4	9.603	0.749	+1.72	+21.8
8 11 48 11	6	12, 10	+0 28.36	−5 35.4	20 45 42.72	+ 5 6 0.7	9.526	0.740	+1.70	+21.5
8 13 28 47	7	15, 4	+0 22.15	−3 38.0	20 45 27.23	+ 5 3 19.7	9.626	0.758	+1.70	+21.4
9 12 32 40	8	12, 10	+1 44.17	−6 29.5	20 41 43.64	+ 4 26 34.6	9.591	0.752	+1.68	+21.0
11 12 3 15	9	12, 9	−1 35.22	+2 32.8	20 33 54.36	+ 3 8 56.9	9.576	0.756	+1.66	+20.2
11 12 3 15	10	12	−1 33.35	. . . .	20 33 54.51	. . . . . .	9.576	. .	+1.66	+20.2
13 12 9 7	11	12, 9	−1 51.41	−1 18.7	20 25 38.83	+ 1 46 57.7	9.597	0.764	+1.61	+19.3
13 12 9 24	12	11	−1 54.50	. . . .	20 25 38.96	. . . . . .	9.597	. .	+1.61	+19.3
14 11 51 3	13	12, 10	+1 35.38	+1 11.5	20 21 25.83	+ 1 4 59.1	9.588	0.766	+1.58	+18.7
‡14 12 53 52	14	12, 4	+0 15.83	+3 57.7	20 21 14.22	+ 1 3 4.7	9.630	0.769	+1.58	+18.7
16 12 6 8	15	12, 9	−0 35.92	+ 29.5	20 12 31.08	− 0 23 32.4	9.614	0.771	+1.54	+17.9
18 11 6 47	16	12, 10	−1 6.38	+ 8.5	20 3 20.79	− 1 54 42.1	9.577	0.778	+1.52	+17.0

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	13 29 3.40	+64 16 16.9	Hels.-Gotha, A.G. 7613	9	20 35 27.92	+3 6 3.9	Albany, A.G. 7223
2	11 40 59.31	+51 6 7.8	Camb., U.S., A.G. 3921	10	20 35 26.20	+3 7 3.5	Albany, A.G. 7222
3	11 11 26.22	+43 48 24.9	Bonn, A.G. 7965	11	20 27 28.63	+1 48 27.1	Albany, A.G. 7158
4	11 0 42.94	+40 19 8.9	Bonn, A.G. 7898	12	20 27 31.85	+1 48 18.5	Albany, A.G. 7159
5	20 48 25.60	+ 5 41 12.6	Leipzig II, A.G. 10436	13	20 19 48.87	+1 3 28.9	Nicolajew, A.G. 5161
6	20 45 12.66	+ 5 11 14.6	Leipzig II, A.G. 10391	14	20 20 56.81	+0 58 48.3	Nicolajew, A.G. 5165
7	20 45 3.08	+ 5 6 36.3	Leipzig II, A.G. 10387	15	20 13 5.46	−0 24 19.8	Nicolajew, A.G. 5122
8	20 39 57.79	+ 4 32 43.1	Albany, A.G. 7250	16	20 4 25.65	−1 55 7.6	Nicolajew, A.G. 5071

* It is possible that this time is fast from two to ten seconds. † Refraction is not included.

‡ Observation by HARRIET W. BIGELOW.

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE* AT THE OPPOSITION OF 1904–5.

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY.

BY H. L. RICE AND J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

In the following observations, each position-angle and distance is the mean of 8–10 settings of the micrometer. The settings in position-angle were made, half before and half after the measurements in distance. Each printed time is the mean of the times corresponding to the settings in position-angle. The corresponding mean of the times for the measurements in distance never differs by more than $2\frac{1}{2}$ minutes, and, in this interval, the change in distance of the satellite never exceeds 0″.01. No correction for this change has been applied.

The position-angle of the micrometer in measuring the distance was the last setting of the first set of position-angles. This may differ by as much as 2° from the mean, but the correction to be applied to the distance for an error of this amount in the position-angle is inappreciable.

In measuring position-angles, a single wire was placed

so as to pass through the satellite and bisect the disc of the planet. The disc was also bisected in measuring the distance. A magnifying power of 400 with bright wire illumination was always used.

The computed positions, with which the comparisons are made, were derived from data given in the *Connaissance des Temps*.

Washington Mean Time	Observer	Position-Angle p_o	Position-Angle p_c	Distance s_o	Distance s_c	O−C δp	O−C δs	Seeing
1904 Nov. 21 15h 37m 15s	H.	65.05	65.32	14.86	15.00	−0.27	−0.14	Excellent
30 11 35 45	H.	235.91	237.73	14.40	14.24	−1.82	+0.16	Fair
Dec. 14 11 16 1	H.	93.70	94.63	16.16	16.83	−0.93	−0.67	Fair
16 10 47 25	H.	325.04	325.10	12.53	12.29	−0.06	+0.24	Excellent
18 11 21 16	H.	221.68	223.16	13.03	12.90	−1.48	+0.13	Excellent
19 10 50 20	H.	141.52	141.43	12.79	12.57	+0.09	+0.22	Bad
29 9 42 52	H.	261.50	261.68	17.33	16.82	−0.18	+0.51	Fair
1905 Jan. 1 9 22 9	H.	81.53	82.72	16.69	16.74	−1.19	−0.05	Excellent
16 9 31 16	R.	247.77	249.01	16.01	15.67	−1.24	+0.34	Poor
27 9 25 39	R.	280.36	280.48	16.90	16.40	−0.12	+0.50	Fair
Feb. 7 8 31 54	R.	324.89	324.58	12.28	12.1[illegible]	+0.31	+0.18	Fair
10 8 20 32	R.	139.54	140.58	12.67	12.37	−1.04	+0.30	Fair
21 8 21 45	R.	21.76	21.52	11.36	11.39	+0.24	−0.03	Good
Mar 10 9 2 48	R.	242.92	244.79	15.21	15.00	−1.87	+0.21	Fair
13 7 43 17	H.	63.57	64.49	14.84	14.95	−0.92	0.11	Good
25 8 20 51	H.	50.26	49.80	13.36	13.29	+0.46	+0.07	Excellent
31 8 51 30	H.	41.26	40.44	11.96	12.38	+0.82	−0.42	Fair

THE SECULAR PERTURBATIONS OF THE *EARTH*.

By ERIC DOOLITTLE.

The secular perturbations of the *Earth* arising from each of the other planets, except *Neptune*, have been published in the *Astronomical Journal*, Nos. 473, 493, 506, 518, 564 and 567. In these computations, Dr. G. W. HILL's first development of GAUSS's method was used, and after completing the work it was in each case duplicated from the beginning, the form of the equations being modified in the duplication when this was possible.

The perturbations arising from the action of *Neptune* were computed from the following elements ("*New Theory of Jupiter and Saturn*," pages 192, 554 and 161):

Earth.	*Neptune.*
$\pi = 100^\circ\ 21'\ 39''.73$	$\pi' = 43^\circ\ 17'\ 30''.30$
$i = 0\ \ 0\ \ 0.00$	$i' = 1\ \ 47\ \ 1.68$
$\Omega = \ldots\ldots$	$\Omega' = 130\ \ 7\ \ 31.83$
$e = 0.01677114$	$e' = 0.0084962$
$n = 1295977''.416$	$n' = 7861''.935$
$\log a = 0.0000000$	$\log a' = 1.4781414$
$m = 1 \div 327000$	$m' = 1 \div 19700$

Epoch 1850.0, G. M. T.

The orbit of the *Earth* was divided into eight parts with regard to the eccentric anomaly; in those cases in which the sums of the functions corresponding respectively to the odd and even points of division should be in substantial agreement, this test was satisfied very exactly. The computation was also duplicated from the beginning, and all known test equations were applied. The equation of Mr. INNES, $\sin q \cdot \frac{1}{2} A_1 + \cos q \cdot B_1 = 0$, was found to give the residual +0.000,000,000,000,0014.

The resulting values of the differential coefficients were as follows:

		log coeff.
$\left[\frac{de}{dt}\right]_{00} =$	$-0.011831221\ n$	8.0730296
$\left[\frac{d\chi}{dt}\right]_{00} = \left[\frac{d\pi}{dt}\right]_{00} =$	$+35.402545\ n$	1.5490345
$\left[\frac{dp}{dt}\right]_{00} =$	-0.7189583	9.8567037
$\left[\frac{dq}{dt}\right]_{00} =$	-0.8520049	9.9304398
$\left[\frac{dL}{dt}\right]_{00} =$	-17.671428	1.2472582

When the above value is substituted for [illegible], the following results are obtained:

$$\left[\frac{de}{dt}\right]_{00} = -0.000000600056972$$

$$\left[\frac{d\chi}{dt}\right]_{00} = \left[\frac{d\pi}{dt}\right]_{00} = +0.0017970888$$

$$\left[\frac{dp}{dt}\right]_{00} = -0.000026495344$$

$$\left[\frac{dq}{dt}\right]_{00} = -0.000043248757$$

$$\left[\frac{dL}{dt}\right]_{00} = -0.002119869 8$$

The values found by LEVERRIER are given in the *Annales de l'Observatoire de Paris*, Vol. II, page 59, and Vol. IV, pages 11 and 12; those obtained by NEWCOMB are in the "*Secular Variations of the Orbits of the Four Inner Planets*," pages 336 and 377; the values of $\left[\frac{dp}{dt}\right]_{00}$ and $\left[\frac{dq}{dt}\right]_{00}$ computed by Dr. HILL are given in the "*New Theory*," pages 511 and 512. If the various results are reduced to the above value of m', they will compare with those here obtained as follows:

	LEVERRIER	NEWCOMB	HILL	Method of GAUSS
	$''$	$''$		$''$
$\left[\frac{de}{dt}\right]_{00}$	0,000 00	0,000 00	. . . . .	−0,000 000 06
$e\left[\frac{d\pi}{dt}\right]_{00}$	+0,000 03	+0,000 03	. . . . .	+0,000 030 14
$\left[\frac{dp}{dt}\right]_{00}$	−0,000 04	−0,000 04	−0,0000 366	−0,000 036 49
$\left[\frac{dq}{dt}\right]_{00}$	−0,000 04	−0,000 04	−0,0000 435	−0,000 043 25

We may now express the variations which arise from the action of all the disturbing planets by the following equations:

$$\left[\frac{de}{dt}\right]_{00} = -\,0''.08565518 - 0''.0011613570\,\omega + 0''.013483339\,\omega' - 0''.01571893\,\omega'''$$
$$-\,0''.081841849\,\omega^{\text{IV}} - 0''.00013305713\,\omega^{\text{V}} + 0''.000017278801\,\omega^{\text{VI}} - 0''.0000000600056972\,\omega^{\text{VII}}$$

$$\left[\frac{d\chi}{dt}\right]_{00} = \left[\frac{d\pi}{dt}\right]_{00} = +11''.4790620 - 0''.10990815\,\omega + 3''.4537341\,\omega' + 0''.97534889\,\omega'''$$
$$+\,6''.9652565\,\omega^{\text{IV}} + 0''.18725994\,\omega^{\text{V}} + 0''.0056636605\,\omega^{\text{VI}} + 0''.0017970838\,\omega^{\text{VII}}$$

$$\left[\frac{dp}{dt}\right]_{00} = +\,0''.052747507 + 0''.0025085775\,\omega + 0''.074457966\,\omega' + 0''.0063317401\,\omega'''$$
$$-\,0''.025114405\,\omega^{\text{IV}} - 0''.0054235259\,\omega^{\text{V}} + 0''.000023679306\,\omega^{\text{VI}} - 0''.0000036495344\,\omega^{\text{VII}}$$

$$\left[\frac{dq}{dt}\right]_{00} = -\,0''.46768416 - 0''.0020986842\,\omega - 0''.28462399\,\omega' - 0''.0071872066\,\omega'''$$
$$-\,0''.16046146\,\omega^{\text{IV}} - 0''.013188086\,\omega^{\text{V}} - 0''.000078487295\,\omega^{\text{VI}} - 0''.000043248757\,\omega^{\text{VII}}$$
$$-\,0$$

$$\left[\frac{dL}{dt}\right]_{00} = +\,1''.756333 + 0''.39309355\,\omega + 11''.232473\,\omega' - 0''.23457335\,\omega'''$$
$$-\,9''.1916336\,\omega^{\text{IV}} - 0''.43251400\,\omega^{\text{V}} - 0''.0080929604\,\omega^{\text{VI}} - 0''.0024198698\,\omega^{\text{VII}}$$

The quantities $\omega, \omega', \omega'''$, etc., are corrections to the masses adopted for *Mercury*, *Venus*, *Mars*, etc., respectively, and are connected with the true masses, m, m', m''', etc., by the equations

$$m = m_0(1+\omega)\,,\quad m' = m'_0(1+\omega')\,,\quad m''' = m_0'''(1+\omega'''),\ \text{etc.}$$

LEVERRIER has stated a similar system of equations in the *Annales*, Vol. IV, page 12. By introducing the corrections necessary to bring the masses employed by LEVERRIER into accordance with those here adopted, and by treating the results of NEWCOMB and HILL in a similar manner, the four determinations will be found to compare as follows:

	LEVERRIER	NEWCOMB	HILL	Method of GAUSS
	$''$	$''$	$''$	$''$
$\left[\frac{de}{dt}\right]_{00}$	−0,08569	−0,08563	. . . . .	−0,085655
$e\left[\frac{d\pi}{dt}\right]_{00}$	+0,19254	+0,19248	. . . . .	+0,192516
$\left[\frac{dp}{dt}\right]_{00}$	+0,05290	+0,05276	+0,0527225	+0,052748
$\left[\frac{dq}{dt}\right]_{00}$	−0,46754	−0,46768	−0,4676079**	−0,467684
$\left[\frac{dL}{dt}\right]_{00}$	+1,7570*	. . . .	. . . . .	+1,756333

* The action of *Neptune* is not included.

** The large difference arises almost wholly from the action of *Venus*. Dr. HILL obtains $-0''.2845280$, and NEWCOMB $-0''.28462$, while my result is $-0''.284624$. I have duplicated the computation of δp and δq by Dr. HILL's methods as outlined in chapter 26 of the "*New Theory*," and arrive at the same results as he obtained. It is to be noticed that these are merely the results obtained by substituting the proper values in LEVERRIER's expansion (*Annales*, Vol. II, pages 94 to 96), and were regarded by Dr. HILL as provisional only.

The Flower Observatory, 1905 April 19.

CONTENTS.

PUBLISHED AT BOSTON, AND CUSHING ROAD, WELLESLEY HILLS, MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, BOX 216, WELLESLEY HILLS, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, Lynn, Mass. *Closed June 15.*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 575.

VOL. XXIV. BOSTON, 1905 JULY 10. NO. 23.

OBSERVATIONS OF THE SATELLITES OF *JUPITER* IN 1903,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

BY T. I. KING AND J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

Of the following observations, those extending to Nov. 10, inclusive (79 in number), were made by Mr. KING; those from Nov. 22 on (71 in number), by Mr. HAMMOND. Mr. KING's observations consist of six settings in position-angle, three before and three after the distances — the latter comprising four measures made on one side of coincidence. Mr. HAMMOND's observations were similarly made as regards position-angle; but they involve six measurements in distance — three on each side of coincidence.

The position-angles were always taken about the inner satellite of each pair. Bright wire illumination was employed throughout the series.

I — II.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1903	h m s	°	h m s	"		1903	h m s	°	h m s	"
1	June 30	15 59 11	245.14	15 59 1	264.96	21	Nov. 4	9 16 11	76.17	9 16 14	45.69
2	July 20	15 11 16	230.17	15 11 58	19.31	22	7	7 28 40	66.31	7 28 58	108.58
3	20	15 30 56	229.47	15 32 0	17.75	23	9	8 9 10	47.93	8 9 16	56.51
4	24	15 29 15	169.18	15 29 26	6.68	24	10	8 23 55	66.18	8 24 5	92.74
5	Aug. 11	15 8 6	239.91	15 8 19	102.61	25	22	7 7 15	237.70	7 7 15	68.71
6	17	13 18 40	72.90	13 18 24	49.46	26	25	6 26 3	66.65	6 26 21	15[illegible].56
7	21	12 6 13	74.10	12 6 33	27.83	27	28	7 9 27	65.83	7 9 51	79.89
8	21	12 28 24	68.96	12 28 54	90.83	28	30	6 43 33	247.31	6 44 23	195.36
9	Sept. 22	10 53 18	67.88	10 54 8	83.18	29	Dec. 5	6 36 21	65.19	6 36 37	73.93
10	25	9 13 8	68.04	9 13 30	89.96	30	6	6 46 16	73.20	6 46 29	58.81
11	29	12 36 31	67.80	12 37 0	68.15	31	7	6 29 21	246.46	6 29 39	245.45
12	Oct. 2	10 38 34	67.58	10 39 8	95.25	32	14	6 58 44	245.90	6 59 1	268.02
13	6	8 50 1	67.98	8 50 18	68.02	33	15	6 38 43	63.75	6 39 20	145.71
14	12	10 54 38	64.23	10 55 9	258.30	34	20	7 0 52	63.34	7 1 6	151.09
15	13	11 35 9	67.52	11 35 36	78.85	35	22	7 10 11	63.20	7 10 19	134.49
16	20	8 59 35	67.73	8 59 55	67.01	36	23	6 22 38	64.75	6 22 59	117.64
17	27	8 51 50	67.19	8 52 19	71.75	37	30	6 32 18	64.79	6 32 33	101.39
18	28	11 4 9	234.75	11 4 34	55.64	38	31	6 32 28	231.59	6 32 28	78.55
19	Nov. 3	7 33 16	66.54	7 34 0	86.63		1904				
20	3	9 38 51	66.86	9 39 11	75.73	39	Jan. 3	7 5 38	66.08	7 6 9	180.25

I — III.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1903						1903				
1	May 8	16 52 45	243.29	16 53 1	217.74	9	Sept. 25	9 1 4	57.68	9 1 13	68.91
2	June 2	15 57 48	273.46	15 56 18	24.87	10	Oct. 2	10 27 4	55.15	10 28 5	40.47
3	10	16 31 54	64.15	16 32 11	346.60	11	12	11 7 1	70.56	11 7 28	149.68
4	14	16 16 36	243.78	16 17 5	380.13	12	Nov. 6	8 19 18	301.83	8 20 3	6.21
5	July 8	15 51 53	61.78	15 52 1	255.94	13	25	6 38 17	243.00	6 38 17	119.50
6	19	15 53 2	238.67	15 51 25	130.82	14	27	7 37 11	250.55	7 37 8	68.67
7	Sept. 21	9 4 50	174.46	9 5 7	18.30	15	29	7 1 42	64.78	7 5 19	275.99
8	24	10 28 26	38.27	10 28 44	33.39	16	Nov. 30	6 57 3	65.01	6 57 21	251.6

I – III. (*Continued*).

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1903	h m s	°	h m s	"		1903	h m s	°	h m s	"
17	Dec. 7	6 41 16	64.62	6 41 52	224.96	24	Dec. 23	6 33 32	240.60	6 33 48	76.26
18	11	7 7 45	247.51	7 7 40	144.35	25	28	4 36 23	64.08	4 36 17	163.81
19	13	5 20 44	63.81	5 20 56	314.99	26	31	6 54 34	241.89	6 54 36	180.10
20	14	6 35 59	64.47	6 36 23	201.30		1904				
21	15	7 2 31	67.48	7 2 56	223.39	27	Jan. 3	7 17 28	61.84	7 17 30	186.26
22	18	6 32 5	246.70	6 32 21	191.64	28	4	7 18 0	63.72	7 18 16	153.43
23	22	6 36 49	66.50	6 37 1	287.51						

I – IV.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1903						1903				
1	June 2	16 36 31	239.87	16 36 31	119.33	10	Dec. 5	6 49 15	241.35	6 49 15	543.73
2	10	16 5 46	53.50	16 6 1	51.73	11	6	7 11 55	244.63	7 12 13	444.17
3	Aug. 17	13 36 53	53.01	13 37 30	82.63	12	11	6 55 55	58.76	6 56 27	126.25
4	21	12 44 17	162.53	12 44 2	22.08	13	14	7 10 24	64.25	7 10 18	395.22
5	Sept. 25	12 21 10	68.47	12 21 41	238.85	14	18	6 42 40	66.50	6 43 8	323.83
6	Oct. 6	9 0 49	57.76	9 1 13	150.56	15	22	6 48 36	243.61	6 49 15	314.12
7	13	11 24 12	121.41	11 24 20	15.34	16	28	4 15 35	254.25	4 15 49	63.94
8	21	8 51 36	256.18	8 52 1	64.42		1904				
9	Dec. 3	6 26 59	240.36	6 27 47	173.29	17	Jan. 4	6 55 1	71.73	6 55 17	96.88

II – III.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1903						1903				
1	May 22	16 34 22	72.70	16 34 23	90.99	14	Oct. 26	10 6 15	78.61	10 6 41	76.45
2	28	16 22 1	64.15	16 22 30	314.16	15	Nov. 7	6 57 55	48.85	6 58 11	51.55
3	June 10	16 18 44	61.38	16 18 39	72.14	16	24	6 55 59	239.15	6 56 30	122.43
4	14	16 25 2	244.94	16 27 54	147.53	17	29	7 13 11	63.40	7 18 10	365.66
5	30	15 41 50	60.22	15 43 9	68.84	18	Dec. 1	6 53 4	231.78	6 52 59	55.10
6	July 8	15 36 30	51.32	15 36 30	16.20	19	6	6 58 43	62.65	6 59 8	293.75
7	21	15 0 5	273.16	15 0 26	16.55	20	11	7 19 17	214.41	7 19 43	196.53
8	Aug. 11	14 53 2	249.07	14 52 48	127.88	21	15	6 50 10	73.51	6 50 20	85.01
9	Sept. 23	10 41 37	247.66	10 41 50	103.05	22	17	6 31 33	245.08	6 34 12	130.92
10	25	9 29 31	284.13	9 29 55	21.62	23	18	6 22 50	244.01	6 23 17	175.60
11	Oct. 2	10 50 17	262.10	10 50 40	45.32	24	20	6 35 55	59.44	6 36 19	140.27
12	12	10 40 34	234.55	10 40 47	100.12	25	22	7 1 3	69.73	7 1 18	145.96
13	19	9 13 19	108.21	9 13 33	28.03	26	31	6 43 2	244.74	6 43 20	133.35

II – IV.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1903						1903				
1	May 4	16 42 46	245.10	16 43 28	311.16	13	Oct. 21	8 41 3	299.55	8 41 30	26.91
2	8	16 25 7	63.52	16 25 13	125.99	14	Nov. 22	7 24 12	248.43	7 23 42	276.07
3	22	16 11 55	245.35	16 13 23	272.18	15	25	6 53 52	56.13	6 53 48	113.97
4	July 7	15 58 7	238.07	15 58 22	143.09	16	28	6 57 16	64.79	6 57 37	357.84
5	19	16 11 9	66.09	16 12 1	314.38	17	Dec. 1	7 6 35	71.19	7 7 5	148.96
6	Aug. 17	13 59 41	36.30	14 0 8	50.00	18	3	6 38 47	70.88	6 39 17	95.00
7	21	12 44 49	236.19	12 44 43	100.37	19	6	7 24 33	245.73	7 24 18	459.80
8	Sept. 25	12 36 25	68.29	12 36 58	179.12	20	7	6 53 9	241.96	6 53 22	360.92
9	Oct. 6	9 12 2	50.29	9 12 21	88.56	21	11	6 44 52	61.07	6 44 57	79.80
10	12	10 29 45	72.45	10 29 50	176.28	22	20	6 47 2	247.59	6 47 16	142.29
11	13	11 47 8	237.85	11 47 19	68.60	23	28	5 4 0	61.62	5 4 19	219.93
12	14	9 40 23	80.59	9 41 10	38.47						

III – IV.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1903						1903				
1	May 8	16 38 51	60.05	16 38 52	180.31	10	Nov. 30	7 9 19	67.17	7 9 39	160.30
2	28	16 9 58	63.47	16 9 51	261.37	11	Dec. 1	7 18 22	65.85	7 18 38	207.10
3	June 2	16 19 27	234.10	16 20 29	110.70	12	13	5 7 18	64.88	5 7 45	170.22
4	July 7	16 9 16	238.01	16 9 18	194.84	13	14	6 46 54	63.91	6 47 25	185.94
5	Sept. 25	12 48 34	75.51	12 48 55	139.70	14	15	7 13 23	64.15	7 13 43	357.86
6	Oct. 12	10 17 31	65.89	10 17 43	267.17	15	28	4 54 34	246.97	4 54 57	226.17
7	Nov. 9	6 48 4	55.15	6 48 31	97.42		1904				
8	24	6 39 7	259.08	6 39 27	88.47	16	Jan. 3	7 29 58	68.57	7 30 22	257.07
9	29	6 49 30	67.15	6 50 2	262.32	17	4	7 6 31	230.93	7 6 43	60.68

OBSERVATIONS OF MINOR PLANETS,

[illegible]

BY W. WALTER DINWIDDIE.

[illegible] Rear-Admiral C. M. Chester, U. S. N. [illegible]

1904 Wash. M. T.	★	Comp.	Δα	Δδ	App. α	App. δ				
				(57) M						
Oct. 21 13 5 28	1	[illegible]	+1 27.[illegible]	−2 2[illegible]	[illegible]	+7 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24 12 9 57	2	[illegible]	0 20[illegible]	−1 24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24 12 21 10	3	30 , 6	+2 22.92	−7 43.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27 10 4 57	4	[illegible] , 6	[illegible] 10.97	−7 27.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27 10 31 56	5	[illegible] , 6	+2 18.61	−1 21.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
				[illegible]						
Oct. 27 11 51 52	6	[illegible] , 6	+2 [illegible]	+1 2[illegible]	2 [illegible]	+3 50 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27 12 16 1	7	[illegible] , 6	2 [illegible].64	+5 [illegible]	2 [illegible].72	+3 5[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28 11 48 9	8	10 , 10	−0 [illegible].55	−[illegible] 42.5	2 [illegible] 21.00	+3 51 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30 10 32 51	8	10 , 7	−1 45.17	+0 51.5	2 [illegible]	+3 42 50.1	[illegible]	[illegible]	[illegible]	[illegible]
30 10 54 12	9	30 , 6	−2 [illegible] 47	+[illegible] 7.[illegible]	2 [illegible]	+3 42 56.5	[illegible]	[illegible]	[illegible]	[illegible]
				(170) M						
Nov. 1 11 11 14	10	30 , 6	−3 [illegible]	+[illegible] 55.7	2 44 25.04	+[illegible] 54 [illegible]	[illegible]	[illegible]	+1 [illegible]	[illegible]
6 10 8 11	11	30 , 6	+0 40.96	+1 45.1	2 38 58.88	+[illegible] 31 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6 10 25 4	12	30 , 6	−1 11.64	+5 5.7	2 38 58.27	+30 31 37.1	0.252	[illegible]	[illegible]	[illegible]
6 10 41 59	13	30 , 6	−2 29.53	+0 35.7	2 38 52.27	+[illegible] 31 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
				(451) [illegible]						
Nov. 6 11 41 57	14	[illegible] , 8	−0 21.[illegible]	−[illegible] 57.1	2 [illegible] 71	4 2[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11 11 22 10	15	30 , 6	−0 11.48	−[illegible] 51.8	2 27 50.44	4 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11 11 36 20	16	30 , 6	+0 59.58	+1 55.7	2 27 50.03	4 19 29.8	[illegible]	[illegible]	[illegible]	[illegible]
14 10 31 59	17	30 , 6	+3 47.73	[illegible] 22.2	2 25 25.85	4 21 45.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
				[illegible]						
Nov. 11 12 16 51	18	[illegible] , 6	+1 22.42	+2 11.9	2 40 41.76	+11 56 47.2	[illegible]	[illegible]	[illegible]	[illegible]
11 12 30 16	19	30 , 6	1 15.24	+2 20.7	2 40 41.2[illegible]	+11 56 44.2	[illegible]	[illegible]	[illegible]	[illegible]
14 9 20 15	20	30 , 6	0 16.83	0 26.[illegible]	[illegible]	+11 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
				[illegible]						
Nov. 15 11 24 27	21	[illegible] , 6	+[illegible]	1 56.8	2 18 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15 11 40 29	22	17 , 4	2 2[illegible]	12 25.6	2 18 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21 10 31 7	23	[illegible]	[illegible]	[illegible]	2 14 2[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
				[illegible]						
Nov. 16 12 [illegible] 28	24	20 , 6	+[illegible]	+2 48.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
				(151) [illegible]						
Dec. 1 11 0 22	25	[illegible]	[illegible]	[illegible] 16.2	1 [illegible]	+[illegible] 10 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1 9 53 10	26	20 , 6	+1 [illegible]	+6 [illegible]	1 [illegible] 24.41	+[illegible] 11 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1 10 9 8	27	14 , 5	+2 [illegible]	+7 30.3	1 15 25.61	+[illegible] 11 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
				(208) [illegible]						
Dec. 1 12 5 32	28	30 , 6	[illegible] 13.01	+2 5[illegible]	1 48 [illegible]	+21 56 12.7	[illegible]	[illegible]	[illegible]	[illegible]
				(270) [illegible]						
Jan. 30 9 44 43	29	30 , 6	+2 12.[illegible]	+6 29.9	8 28 [illegible]	+15 51 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Feb. 1 11 52 6	30	30 , 6	+0 55.76	−2 11.6	8 22 57.[illegible]	+16 9 [illegible]	[illegible]	[illegible]	+1 2[illegible]	[illegible]
7 11 31 42	31	30 , 6	2 42.74	0 15.6	8 19 49.61	+16 19 5[illegible]	[illegible]	[illegible]	+1 2[illegible]	[illegible]
				(65) [illegible]						
Feb. 4 13 5 10	32	30 , 6	+[illegible] 26.55	[illegible] 5 11.2	10 2[illegible] 42.53	+[illegible] 10 [illegible]	[illegible]	[illegible]	+1 [illegible]	[illegible]
				[illegible]						
Feb. 21 13 0 29	33	30 , 6	+1 19.09	−1 7.2	10 8 2.22	1 58 [illegible]	[illegible]	[illegible]	+1 [illegible]	[illegible]

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	1 46 55.83	+ 7 35 30.0	Leipzig II, A.G. 704	18	2 39 15.42	+14 54 20.1	Leipzig I. A.G. 801
2	1 46 41.28	+ 7 6 2.1	" " 703	*19	2 41 52.51	+14 54 11.9	Veröff., Bonn, No. 4*
3	1 43 37.89	+ 7 12 23.1	" " 676	20	2 38 52.73	+14 39 51.7	Leipzig I. A.G. 800
4	1 44 53.99	+ 6 44 51.4	" " 685	21	2 18 6.12	+ 5 41 12.4	Leipzig II. A.G. 888
5	1 41 23.78	+ 6 35 51.4	" " 667	22	2 21 12.60	+ 5 51 40.9	" " 908
6	2 33 24.20	+ 5 52 14.0	" " 975	23	2 15 20.36	+ 5 26 33.5	" " 870
7	2 38 17.77	+ 5 51 1.4	" " 1000	24	3 13 19.31	+ 5 35 37.2	" " 1226
8	2 35 11.86	+ 5 41 55.8	" " 984	25	4 21 51.94	+38 13 27.5	Lund. " 2249
9	2 36 4.34	+ 5 39 37.4	" " 989	26	4 13 40.68	+38 7 59.3	" " 2207
10	2 47 30.12	+39 52 55.3	Lund. A.G. 1474	27	4 12 39.52	+38 7 0.8	" " 2198
11	2 37 59.09	+39 29 43.8	" " 1360	28	4 52 16.75	+24 51 9.0	Berlin B. " 1581
12	2 40 0.08	+39 26 20.0	" " 1381	29	8 26 19.68	+15 45 16.7	" A. " 3378
13	2 41 16.97	+39 30 46.3	" " 1403	30	8 22 22.34	+16 12 4.6	" " " 3342
14	2 32 22.23	− 4 19 29.1	Strassburg. A.G. Zones	31	8 22 31.07	+16 20 50.9	" " " 3345
15	2 28 28.43	− 4 15 47.2	" " "	32	10 20 14.83	+ 9 16 5.6	Leipzig II. " 5470
16	2 26 47.17	− 4 24 17.0	" " "	33	10 6 11.69	− 1 56 49.7	Nicolajew. " 3011
17	2 21 34.63	− 4 18 34.5	" " "				

* The declination of Star No. 19 in Leipzig I, A.G. is wrong by about 6′.

The star places from the Strassburg Zones were furnished through the courtesy of the Director of the Observatory at that place. The above planets, with the exception of (57) *Mnemosyne* and (17) *Thetis*, were found photographically by Mr. G. H. Peters.

OBSERVATIONS OF COMET 1904 *e* (BORRELLY),

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY, BY J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. Chester, U.S.N., Superintendent.]

1904–05 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
1904 h m s			m s	′ ″	h m s	° ′ ″			s	″
Dec. 30 10 14 58	1	30 , 6	−0 37.23	− 1 52.7	1 15 14.19	− 8 56 48.4	9.610	0.774	+2.86	+9.6
31 7 54 20	2	30 , 6	+2 12.36	− 1 2.0	1 16 22.35	− 8 13 30.1	9.188	0.804	+2.86	+9.9
1905 Jan. 1 7 12 44	3	25 , 5	+1 24.62	+ 1 41.1	1 17 38.71	− 7 25 45.1	8.892	0.801	−0.15	−9.0
1 7 27 16	4	25 , 5	−1 54.19	− 0 29.2	1 17 39.56	− 7 25 15.6	9.028	0.800	−0.12	−9.2
2 7 3 6	5	30 , 6	+2 28.61	+ 2 29.8	1 18 58.62	− 6 37 1.8	8.806	0.795	−0.15	−8.8
4 6 49 33	6	30 , 6	−1 16.73	− 1 53.8	1 21 14.07	− 4 59 41.6	8.671	0.784	−0.12	−8.6
8 7 12 36	7	20 , 4	−5 21.97	+ 2 30.1	1 27 39.01	− 1 44 54.3	9.047	0.756	−0.08	−8.0
20 7 13 18	8	20 , 4	+0 56.12	− 0 12.0	1 47 55.03	+ 7 35 29.8	9.219	0.663	−0.07	−6.1
26 7 11 21	9	20 , 4	+1 26.56	+ 5 25.2	1 59 26.71	+11 58 42.7	9.272	0.610	−0.06	−5.4
27 7 7 29	10	25 , 5	+0 51.99	+ 3 7.8	2 1 27.47	+12 41 9.9	9.264	0.600	−0.05	−5.2
28 7 17 34	11	24 , 5	−2 13.06	+ 9 33.6	2 3 29.86	+13 23 43.2	9.316	0.594	−0.03	−5.3
30 7 2 6	12	12 , 12	−0 14.51	− 3 35.2	2 7 37.49	+14 46 25.1	9.270	0.569	−0.03	−4.9
Feb. 2 7 16 11	13	25 , 5	+2 13.61	− 3 56.1	2 14 3.99	+16 48 31.9	9.351	0.546	−0.04	−4.4
4 7 21 56	14	25 , 5	+2 38.41	− 1 7.1	2 18 29.15	+18 7 43.5	9.395	0.531	−0.04	−4.2
7 7 8 40	15	20 , 4	+2 16.69	− 2 9.9	2 25 15.72	+20 2 38.7	9.363	0.491	−0.04	−3.9
10 7 18 58	16	14 , 2	+5 19.44	− 0 35.9	2 32 18.77	+21 54 10.4	9.418	0.468	−0.05	−3.7
14 7 11 26	17	8 , 8	−0 14.09	− 1 52.4	2 42 2.37	+24 16 1.3	9.421	0.417	−0.02	−3.6
24 7 24 30	18	6 , 6	+2 0.13	+10 32.1	3 8 12.51	+29 38 51.5	9.513	0.326	−0.04	−2.9
26 7 30 43	19	34 , 7	+1 11.15	− 3 24.5	3 13 45.44	+30 37 50.8	9.537	0.316	−0.04	−2.8
Mar. 2 7 14 20	20	24 , 5	+2 51.73	+ 0 37.1	3 25 6.42	+32 29 26.3	9.518	0.234	−0.06	−2.5
25 7 42 29	21	10 , 10	+0 28.35	+ 6 58.9	4 38 6.40	+40 43 26.3	9.658	0.089	−0.15	−2.0
28 8 29 52	22	8 , 8	+0 29.25	− 6 47.4	4 48 32.55	+41 29 26.8	9.728	0.284	−0.16	−1.9
29 8 4 37	23	25 , 5	−1 4.58	− 0 12.5	4 51 57.45	+41 43 24.9	9.700	0.171	−0.16	−1.9
30 9 11 56	24	25 , 5	+0 55.85	+ 5 56.4	4 55 36.37	+41 57 47.1	9.767	0.425	−0.18	−1.9
31 8 20 6	25	8 , 8	+0 30.02	− 0 11.5	4 58 58.92	+42 10 31.8	9.123	0.232	−0.18	−1.8
Apr. 2 8 26 34	26	8 , 8	+0 10.12	− 7 9.5	5 6 2.67	+42 35 45.1	9.734	0.252	−0.20	−1.7
8 9 58 45	27	25 , 5	+1 53.74	+ 1 38.2	5 27 41.30	+43 40 26.6	9.799	0.546	−0.24	−1.8
22 8 49 11	28	8 , 8	+0 41.16	− 6 12.4	6 18 14.80	+45 6 13.9	9.779	0.317	−0.31	−1.6
23 8 27 8	29	8 , 8	−0 21.90	+12 31.7	6 21 48.40	+45 9 41.9	9.761	0.218	−0.30	−1.6
May 1 9 19 18	30	10 , 12	−0 2.35	+ 6 29.8	6 50 38.58	+45 19 33.0	9.801	0.436	−0.34	−1.3
2 9 35 48	31	10 , 9	−2 57.18	+ 7 0.4	6 54 14.25	+45 19 9.7	9.807	0.489	−0.33	−1.2
21 8 57 10	32	8 , 6	+1 26.93	+ 2 49.9	7 58 46.38	+44 9 6.0	9.786	0.415	−0.37	−1.0
23 9 13 53	33	6 , 8	+0 19.27	+14 29.1	8 5 14.26	+43 55 46.4	9.793	0.475	−0.36	−0.9
24 10 0 24	34	8 , 6	−2 38.88	+ 0 57.0	8 8 31.04	+43 48 30.7	9.802	0.599	−0.35	−0.7

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	1 15 18.56	− 8 55 5.3	Wien, A.G. 271	18	3 6 12.12	+29 28 22.3	Camb., Eng., A.G. 1595
2	1 11 7.13	− 8 12 38.0	" " 263	19	3 12 34.33	+30 41 18.1	Leiden, A.G. 1245
3	1 16 14.24	− 7 27 17.2	" " 272	20	3 22 11.75	+32 28 51.7	Leiden, A.G. 1318
4	1 19 33.87	− 7 24 37.2	" " 284	21	4 37 38.20	+40 36 29.4	Bonn, " 3797
5	1 16 30.16	− 6 39 22.8	" " 274	22	4 18 3.16	+41 36 16.1	" " 3940
6	1 23 0.92	− 1 57 39.2	Strassburg, A.G. Zones	23	4 53 2.19	+41 43 39.3	" " 4011
7	1 33 4.06	− 1 47 46.1	Nicolajew, " 317	24	4 51 10.70	+41 51 52.6	" " 4040
8	1 46 58.98	+ 7 35 47.9	Leipzig II. " 704	25	4 58 29.08	+42 10 45.1	" " 4103
9	1 58 0.21	+11 53 22.9	" I. " 612	26	5 5 52.75	+42 42 56.3	" " 4216
10	2 0 35.53	+12 38 7.3	" " " 622	27†	5 25 47.80	+43 38 50.2	" " 4521
11*	2 5 42.95	+13 14 14.9	" " " 644	28	6 17 33.95	+45 12 57.9	" " 5214
12	2 7 52.03	+14 50 5.2	" " " 654	29	6 22 13.60	+44 57 11.8	" " 5269
13	2 11 50.42	+16 52 32.4	Berlin A. " 634	30	6 50 11.27	+45 13 4.5	" " 5622
14	2 15 50.78	+18 8 54.8	" " " 648	31	6 57 12.06	+45 12 10.5	" " 5717
15	2 22 29.07	+20 1 52.5	" " " 677	32	7 57 19.82	+44 6 17.1	" " 6366
16	2 26 59.38	+21 51 50.0	Berlin B. " 768	33	8 4 55.35	+43 41 18.2	" " 6436
17	2 42 16.48	+24 17 57.3	" " " 834	34	8 11 10.27	+43 47 31.4	" " 6497

*Star No. 11 is double. The n.f. component was used in observing, while the position of the mean is given in the catalogue. A correction of $+0^s.17$ in α, and $+1''.2$ in δ, was applied to this mean.

†The declination of this star, No. 27, is one minute wrong as published in the Bonn A.G. Catalogue. The observations of May 21, 23 and 24 were exceedingly difficult.

The star place from the Strassburg Zones was furnished through the courtesy of the Director of the Observatory at this place.

OBSERVATIONS OF COMET 1905 *a* (GIACOBINI),

MADE WITH THE 26-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY, BY J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1905 Washington M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Mar. 28 7 48 18	1	8 , 8	−0 10.84	− 2 4.3	5 52 0.24	+13 39 30.0	9.429	0.604	+0.12	−11.5
29 7 36 16	2	8 , 8	+0 20.64	− 4 15.6	5 55 36.15	+14 52 12.7	9.309	0.582	+0.12	−11.1
30 8 16 18	3	8 , 8	−0 18.45	− 6 36.5	5 59 24.06	+16 7 28.2	9.501	0.588	+0.13	10.6
31 7 46 44	4	8 , 8	−0 6.59	5 50.4	6 3 6.30	+17 18 57.8	9.435	0.553	+0.13	10.2
Apr. 1 8 44 12	5	20 , 4	−0 51.61	− 7 34.8	6 7 6.72	+18 31 31.6	9.561	0.577	+0.13	9.7
2 7 16 0	6	8 , 8	+0 13.71	+ 6 9.4	6 10 53.89	+19 44 2.2	9.439	0.514	+0.13	− 9.4
3 8 39 41	7	20 , 4	1 39.17	+ 3 22.5	6 15 3.52	+20 58 33.1	9.560	0.543	+0.14	− 8.9
7 8 50 27	8	25 , 5	1 32.82	+ 2 30.6	6 32 0.12	+25 39 29.8	9.591	0.486	+0.14	7.3
8 9 9 56	9	30 , 6	+1 22.01	+ 2 10.5	6 36 29.69	+26 18 19.7	9.623	0.496	+0.13	6.9
22 8 4 26	10	25 , 5	+3 13.12	+11 18.9	7 47 0.30	+40 12 22.1	9.533	9.751	+0.15	1.8
23 9 10 48	11	25 , 5	−0 55.98	− 3 44.4	7 52 52.86	+40 59 22.6	9.672	0.119	+0.17	1.1
24 9 6 20	12	18 , 4	+3 29.90	+ 3 17.8	7 58 33.40	+41 42 21.3	9.667	0.055	+0.15	− 1.1
May 1 8 25 1	13	10 , 10	+2 13.63	+ 2 0.7	8 39 37.95	+45 49 18.1	9.582	9.213	+0.19	+ 1.1
2 8 13 35	14	9 , 10	+0 50.85	+ 0 48.0	8 45 44.46	+46 18 7.5	9.626	8.053	+0.21	+ 1.4
21 9 36 12	15	6 , 8	−0 9.56	− 1 39.9	10 38 9.33	+49 53 45.4	9.689	9.409	+0.40	+ 5.4
23 10 31 37	16	6 , 8	+0 47.57	+ 2 44.6	10 49 7.00	+49 47 34.2	9.772	9.829	+0.43	+ 5.7
June 1 10 29 18	17	6 , 8	− 1 44.94	−11 58.9	11 33 36.39	+48 33 55.2	9.746	9.784	+0.55	+ 7.3

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	5 52 10.96	+13 41 45.8	Leipzig I. A.G. 1908	10	7 43 17.03	+40 0 35.0	[illegible]
2	5 55 15.39	+14 57 9.4	" " " 1935	11	7 53 48.67	+41 3 8.4	Bonn, A.G. 6354
3	5 59 42.38	+16 14 15.3	Berlin A. A.G. 1890	12	7 55 3.35	+41 39 4.6	" " 6344
4	6 3 12.76	+17 24 58.4	" " " 1931	13	8 37 24.13	+45 17 16.3	" " 6747
5	6 7 58.23	+18 42 19.4	" " " 1994	14	8 44 53.40	+46 17 18.1	" " 6809
6	6 10 40.05	+19 38 2.2	" " " 2032	15	10 38 18.49	+49 55 19.9	[illegible]
7	6 16 12.85	+20 55 19.5	Berlin B. A.G. 2334	16	10 48 19.00	+49 44 43.9	[illegible]
8	6 33 32.80	+25 37 6.5	Camb., Eng., " 3395	17*	11 35 17.78	+48 45 46.8	Bonn, A.G. 8150
9	6 35 7.55	+26 16 16.1	" " " 3422				

* Comparison-star No. 17 is given as B.D. +48 2076 in the Bonn A.G. Catalogue. It should be B.D. +49 2076.

OBSERVATIONS OF THE SATELLITES OF *JUPITER* IN 1904,

MADE WITH THE 12-INCH EQUATORIAL AT THE U. S. NAVAL OBSERVATORY,

By H. L. RICE.

Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.

Position-angles were always taken about the inner satellite of each pair. In general, the observations in p consist of eight settings, half before and half after the measurements in distance. The latter also comprise eight measurements, four on each side of coincidence. Bright wire illumination was employed throughout the entire series.

I — II.

No.	Date	Wash. M.T. (h m s)	p	Wash. M.T. (h m s)	s
1	1904 Sept. 4	10 15 43	77.29	10 15 46	92.11
2	7	12 51 52	68.76	12 52 9	319.52
3	11	10 37 10	73.61	10 37 13	115.41
4	15	10 27 16	234.60	10 27 22	10.81
5	16	11 22 27	245.15	11 22 12	154.11
6	21	17 12 1	68.18	17 12 31	316.78
7	22	11 11 28	228.97	11 11 53	30.25
8	23	11 30 21	213.68	11 30 25	121.91
9	26	11 31 28	201.15	11 31 18	9.07
10	Oct. 1	11 22 12	255.68	11 22 41	118.49
11	3	11 31 25	182.69	11 31 25	6.96
12	4	10 11 28	246.97	10 11 41	288.93
13	7	11 10 34	234.80	11 10 23	14.81
14	10	9 15 59	217.17	9 16 15	13.16
15	14	11 15 56	211.35	11 15 49	17.34
16	15	9 30 38	249.66	9 30 19	302.63
17	16	10 13 56	67.33	10 14 0	325.97
18	17	8 51 26	202.78	8 51 12	10.53
19	18	8 35 27	241.20	8 35 3	183.08
20	1904 Oct. 19	10 17 18	55.39	10 17 16	102.41
21	21	10 22 14	92.19	10 22 33	20.07
22	22	9 19 29	248.53	9 19 36	330.03
23	24	9 23 22	152.95	9 23 30	8.01
24	27	15 7 13	76.62	15 7 57	65.55
25	Nov. 1	8 40 10	239.95	8 40 5	91.10
26	5	8 5 11	246.57	8 4 54	321.17
27	7	7 44 31	79.12	7 44 52	14.71
28	9	11 50 42	260.79	11 50 38	86.00
29	14	13 1 16	83.07	. . . .	. .
30	15	10 38 12	232.55	10 38 12	47.94
31	19	10 35 52	245.76	10 35 17	301.13
32	21	7 26 46	71.58	7 27 4	103.51
33	28	9 4 38	71.00	9 4 44	106.44
34	30	11 23 11	218.97	11 22 56	275.21
35	Dec. 16	8 45 2	72.03	8 45 34	51.92
36	20	8 49 56	69.67	8 50 4	96.81
37	30	7 13 13	70.23	7 13 23	78.37

I — III.

No.	Date	Wash. M.T. (h m s)	p	Wash. M.T. (h m s)	s
1	1904 Sept. 4	11 34 12	68.15	11 31 51	231.51
2	7	12 17 45	240.64	12 17 11	116.84
3	11	10 56 44	67.65	10 56 57	259.25
4	15	10 2 12	248.16	10 2 28	487.11
5	16	11 2 56	251.05	11 3 13	163.00
6	21	13 38 59	232.81	13 39 38	78.00
7	22	11 23 5	247.83	11 23 33	494.23
8	23	11 16 24	250.64	11 16 32	170.98
9	26	11 10 38	67.38	11 10 44	120.29
10	27	10 17 24	81.32	10 17 59	88.23
11	Oct. 1	10 30 35	258.66	10 30 18	143.92
12	4	9 52 11	76.08	9 52 35	168.63
13	7	10 32 11	249.83	10 31 26	188.39
14	10	9 36 19	65.60	9 36 49	325.99
15	14	10 47 35	249.41	10 48 5	203.73
16	15	9 48 59	251.68	9 49 22	282.33
17	16	10 26 6	64.70	10 26 10	300.23
18	17	9 9 19	64.87	9 9 15	276.17
19	18	8 53 34	70.77	8 53 57	335.25
20	19	11 21 2	83.20	11 21 11	53.15
21	1904 Oct. 21	10 11 6	248.87	10 10 59	223.65
22	22	9 37 16	250.24	9 38 4	325.66
23	24	8 59 50	61.09	9 0 0	234.71
24	27	14 36 46	244.83	14 37 31	132.30
25	31	9 3 55	63.17	9 3 59	198.78
26	Nov. 3	7 18 53	237.80	7 18 47	166.29
27	5	8 21 10	248.15	8 21 1	363.26
28	6	12 38 24	62.20	12 38 19	229.97
29	7	8 1 43	62.24	8 1 50	156.05
30	9	11 38 13	75.66	11 38 21	75.72
31	15	10 14 26	67.51	10 11 12	478.61
32	16	10 53 25	75.25	10 52 45	76.96
33	19	10 11 29	247.19	10 15 2	366.96
34	21	7 41 23	60.76	7 41 9	112.72
35	28	9 19 30	59.52	9 19 44	93.76
36	30	11 35 50	72.30	11 35 42	101.30
37	Dec. 14	5 52 54	69.42	5 53 18	217.38
38	16	8 26 17	244.42	8 25 49	335.28
39	20	8 35 8	64.01	8 35 2	345.97
40	30	7 6 40	242.07	7 6 26	230.80

II — III.

No.	Date	Wash. M.T. (h m s)	p	Wash. M.T. (h m s)	s
1	1904 Sept. 2	11 27 42	55.69	11 28 16	62.57
2	4	11 9 34	62.80	11 10 18	157.92
3	7	13 32 0	246.36	13 31 24	485.13
4	10	10 20 47	60.48	10 21 9	53.80
5	11	10 0 10	60.05	10 0 47	112.94
6	15	10 50 6	249.55	10 50 4	138.20
7	1904 Sept. 16	10 46 33	288.84	10 46 50	25.23
8	21	17 37 9	245.73	17 37 52	475.92
9	22	11 3 19	248.98	11 2 52	472.31
10	23	11 0 50	264.45	11 1 11	59.89
11	25	11 48 24	53.98	11 48 40	70.49
12	30	11 5 11	257.62	11 5 6	102.05

II – III (*Concluded*).

No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1904	h m s	°	h m s	"
13	Oct. 7	10 51 18	254.12	10 52 10	119.31
14	10	9 57 16	61.60	9 57 7	342.11
15	14	11 1 58	252.37	11 2 10	190.58
16	15	9 13 7	39.12	9 13 20	18.23
17	16	10 51 35	281.98	10 51 18	25.19
18	17	9 27 18	63.52	9 27 30	290.21
19	18	9 13 53	68.55	9 13 43	523.97
20	19	11 1 22	216.59	11 1 16	69.34
21	21	10 59 52	250.94	10 59 57	234.65
22	21	8 44 36	62.02	8 45 12	229.52
23	31	8 47 1	60.06	8 47 21	180.37
24	Nov. 5	8 44 9	261.05	8 44 5	37.28
25	7	8 18 38	56.35	8 18 3	121.28
26	Nov. 9	12 19 0	79.31	12 19 27	149.29
27	16	11 12 23	73.57	11 11 33	254.90
28	19	9 53 41	253.26	9 54 12	74.22
29	21	7 57 1	15.86	7 57 1	26.18
30	30	11 9 40	69.86	11 10 16	280.54
31	Dec. 12	8 4 59	255.25	8 4 57	101.98
32	16	8 9 6	245.39	8 9 18	[illegible]
33	20	7 16 32	61.52	7 16 12	[illegible]
34	30	7 25 2	241.22	7 25 5	[illegible]
	1905				
35	Jan. 2	8 7 4	249.03	8 11 12	[illegible]
36	4	6 28 29	66.01	6 28 35	178.14

III – IV

No.	Date	Wash. M.T.	p	Wash. M.T.	s
	1904				
1	Sept. 2	10 59 30	247.62	11 0 56	571.25
2	10	11 2 15	71.68	11 2 27	180.33
3	11	10 17 55	79.79	10 17 51	147.92
4	15	11 12 31	219.46	11 12 53	51.22
5	16	11 12 34	242.32	11 12 44	326.89
6	21	14 5 41	49.07	14 5 50	155.53
7	22	10 13 49	63.67	10 14 25	197.17
8	25	11 31 56	68.61	11 32 6	340.45
9	26	10 46 22	67.59	10 45 56	284.90
10	30	11 32 34	78.21	11 33 13	249.44
11	Oct. 1	10 12 28	235.21	10 12 37	241.40
12	6	13 27 17	269.13	13 27 39	84.91
13	7	10 14 29	53.31	10 14 26	85.61
14	10	11 6 13	49.41	11 6 21	77.79
15	15	10 11 22	73.12	10 9 56	174.88
16	16	10 11 32	218.02	10 11 58	106.01
17	21	11 21 38	247.21	11 22 10	277.15
18	22	9 55 6	246.67	9 55 10	374.07
19	24	9 50 10	249.79	9 50 2	514.13
20	Oct. 31	9 25 5	81.50	9 25 14	124.92
21	Nov. 1	8 17 37	232.39	8 18 1	85.77
22	5	7 47 3	241.47	7 47 12	[illegible]
23	9	12 42 10	254.16	12 42 8	[illegible]
24	14	13 29 47	67.74	[illegible]	[illegible]
25	15	9 53 25	66.97	9 54 6	[illegible]
26	16	11 35 50	66.54	11 36 14	[illegible]
27	19	9 55 29	77.02	9 56 8	216.78
28	28	9 37 19	53.00	9 37 18	44.25
29	30	10 54 58	62.20	10 54 58	346.80
30	Dec. 12	8 24 56	216.92	8 25 4	[illegible]
31	13	9 26 18	248.83	9 26 14	[illegible]
32	14	6 9 29	254.21	6 9 57	[illegible]
33	19	9 50 43	69.89	9 52 39	[illegible]
34	20	7 30 26	78.90	7 30 22	[illegible]
35	30	8 2 52	53.02	8 1 5	[illegible]
	1905				
36	Jan. 2	7 49 40	60.69	7 49 58	[illegible]
37	4	6 18 29	67.18	6 18 47	[illegible]

MAXIMA OF LONG-PERIOD VARIABLES,

BY IDA WHITESIDE.

The times of maxima of the following sixteen stars were determined from the single light-curves deduced from observations made at the Vassar College Observatory.

103. *T Andromedae*.

The last maximum of this star occurred Jan. 23, 1905, at which time the star's magnitude was 8^{M}.4. This date and value were deduced from nine observations, beginning Dec. 9, 1904, and ending March 15, 1905. The date of maximum predicted in CHANDLER's "Ephemerides of Long-Period Variables" was Dec. 17, 1904. The observations indicate that the maximum did not occur at that time, and the true date is probably not far from that given (Jan. 23, 1905).

782. *R Arietis*.

From seventeen observations, the maximum of this star was found to be 8^{M}.0 on December 19, 1904. The observations extended from Oct. 16, 1904, to March 6, 1905, and were well scattered along the curve. [illegible] exactly with that of the predicted maximum, Dec. 19, 1904.

1623. *T* [illegible]

Twelve observations of this star were [illegible] period from Nov. 26, 1904, to Apr. 15, 1905. The [illegible] but 0^{M}.7 during that time. The curve [illegible] well-determined maximum of 8^{M}.2 on Feb. 27, 1905. The predicted maximum was February 24. [illegible] in the curve would bring the observed [illegible] same date.

7045. *R* [illegible]

From twelve observations, the maximum of this star was found to be 7^{M}.4 on Dec. 15, 1904. The observations extended from Oct. 15, 1904, to Dec. 21, 1904. The [illegible]

rose quite sharply until Nov. 20. The magnitude then remained nearly stationary until Dec. 1. From that time it again rose abruptly to the maximum. But one observation occurs after the observed maximum, as the star was approaching the horizon. The predicted maximum was Dec. 21, 1904. If more observations had been made the maximum might have been found nearer the predicted time.

7085. *RT Cygni.*

From ten observations the maximum of this star was determined as $6^{M}.6$ on Nov. 7, 1904. The observations extended from Oct. 1, 1904, to Dec. 21, 1904. This date of maximum agrees fairly well with the predicted time (Nov. 6, 1904).

7120. *χ Cygni.*

The maximum of this star as deduced from ten observations was $4^{M}.4$ on Dec. 17, 1904. The observations covered the time from Nov. 21, 1904, to Jan. 2, 1905, and were well distributed along the curve. The adopted curve leaves little doubt regarding the above date, although it differs thirteen days from that predicted (Dec. 4, 1904).

8068. *S Lacertae.*

Six observations of this star determine the maximum as $8^{M}.1$ on Feb. 13, 1905. The predicted time was Dec. 29, 1904. Though there were only a few observations, they were evenly distributed from Oct. 28, 1904, to Feb. 24, 1905. The curve rose very slightly before Jan. 14, 1905, and then more rapidly to the maximum.

1222. *R Persei.*

The maximum of this star, $8^{M}.2$, occurred on Nov. 18, 1904. There were thirteen observations, extending from Oct. 29, 1904, to Feb. 3, 1905. The maximum predicted was for Nov. 26, 1904. The observations were so situated as to give a well-determined curve.

1717. *V Tauri.*

This star had a maximum of $8^{M}.8$ on Nov. 10, 1904. The observations, eleven in number, extended from Nov. 3, 1904, to Feb. 3, 1905, and gave a well-determined maximum, which agrees fairly well with that predicted (Nov. 12, 1904).

4557. *S Ursae Majoris.*

From thirteen observations of this star, the maximum was found to be $7^{M}.4$ on Jan. 14, 1905. The observations extended from Oct. 15, 1904, to March 13, 1905. The predicted date of maximum was Jan. 9, 1905. As there were no observations immediately preceding maximum, the date determined is somewhat doubtful, and the predicted date is very possibly correct.

4511. *T Ursae Majoris.*

A maximum of this star occurred on March 2, 1905. The star rose to $7^{M}.1$. This maximum was determined from seven observations quite evenly distributed between Jan. 28, 1905, and April 15, 1905. The predicted time of maximum was Feb. 15, 1905, at which time its magnitude was about 7.5.

5601. *S Ursae Minoris.*

Six observations of this star were made between Nov. 28, 1904, and Feb. 27, 1905, only one of which preceded maximum. The curve showed a maximum of $7^{M}.8$ on Dec. 5, 1904, instead of on the predicted date (Dec. 8, 1904).

The maxima of the three following stars were determined by the subtangent method used by Mr. PARKHURST (*A.J.*, No. 400), as the observations were all on one side of maximum. The results are, therefore, subject to considerable error.

2625. *V Geminorum.*

Eight observations of this star, made between Jan. 14, 1905, and March 11, 1905, all followed maximum. The subtangent method gave a maximum of $9^{M}.1$ on Jan. 4, 1905. The predicted time of maximum was Jan. 16, 1905.

8373. *S Pegasi.*

The observations of this star, extending from Sept. 26, 1904, to Dec. 21, 1904, all preceded maximum. After the last date, the star was so near the horizon that no more observations could be made. The deduced maximum was $8^{M}.4$ on Dec. 22, 1904. The predicted maximum was for Nov. 30, 1904.

1577. *R Tauri.*

All the observations of this star were made after the maximum was past. They extended from Dec. 9, 1904, to Feb. 24, 1905, and were ten in number. They gave a maximum of $8^{M}.1$ on Nov. 29, 1904. The predicted maximum of Jan. 16, 1905, was apparently wrong, as the curve slopes continuously from the first to the last observation.

1805. *V Orionis.*

Six observations, extending from Dec. 18, 1904, to March 14, 1905, showed that the maximum probably occurred near the predicted time, Dec. 17, 1904. On Dec. 18, the observed magnitude was 9.5. On the last date it was below 11.5 magnitude.

Vassar College Observatory, 1905 *April* 29.

CONTENTS.

PUBLISHED AT BOSTON, AND CUSHING ROAD, WELLESLEY HILLS, MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, BOX 210, WELLESLEY HILLS, MASS.
ASSOCIATE EDITORS, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME. PRESS OF THOS. P. NICHOLS, LYNN, MASS. *Closed July 9.*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

THE

ASTRONOMICAL JOURNAL.

FOUNDED BY B. A. GOULD.

No. 576.

VOL. XXIV. BOSTON, 1905 SEPTEMBER 5. NO. 24

ON THE LIGHT-VARIATIONS OF 2279 *T MONOCEROTIS.*

By PAUL S. YENDELL.

The variability of this star was discovered at Cordoba in 1871, by GOULD, the first notice of it being published in the *American Journal of Science* for December, 1872. It appears in SCHÖNFELD's *Second Catalogue*, in 1875.

In the notes to the latter, p. 33, SCHÖNFELD gives a mean light-curve from seventy-four observations, which is the only published one that has come to my knowledge. He remarks: "Greatest and least brightness fluctuate about a couple of steps." He also notes the color as "gold-yellow." CHANDLER, in his *First Catalogue of Variable Stars*, gives it an estimated coloration of 2. I have not observed it for color.

The period of this star, 27ᵈ.0122, places it on the limit of the stars that can be distinctly classed as of short period. I have found no evidence of irregularity or periodic inequality in this period.

The star, however, as will be seen from the evidence presented below, is hardly of the usual short-period type of variable, its light-changes more nearly resembling those of 7137 *X Cygni*, as its light-curve is not constant in different periods.

I began observing *T Monocerotis* in the late winter of 1888–89, and followed it up as closely as circumstances permitted until the season of 1897, when the observations became less continuous. There has been no year, however, during which it has been entirely neglected.

In the Spring of 1904 the observations had accumulated to the number of rather more than five hundred, and it seemed probable that they would furnish material for a mean light-curve of a somewhat definitive character.

They were accordingly assembled in the order of their $T-t$, as deduced from comparison with the computed times of maxima from my elements adopted by CHANDLER in his *Third Catalogue*, and from which I found no reason for departing. Grouping them in twenties, there were found to be five hundred and thirty observations in all, from which were formed twenty-six normals of twenty observations each, and one of ten.

The comparison-stars and light-scale used are shown in the following table. The adopted magnitudes are those of the Potsdam Photometric Durchmusterung for the stars *a*, *b* and *d*, and for the fainter stars *e* and *f*, as also the star *g*, used by myself for a few comparisons, values found from the step-value indicated by the first three, whose intervals corresponded closely with their *Potsdam Durchmusterung* magnitudes, which step-value was 0ᵐ.089. The magnitudes found by extrapolation from the use of this step-value with their places in the step-scale are in fair accordance with those of the *Bonn Durchmusterung*, and have been retained.

The stars *e* and *g* are both coarse pairs, which appear as single stars in the field-glass used in the observations.

COMPARISON-STARS.

		1855 α		1855 δ			P.DM.	Y.	L.-S.
		h	m s	°	′	M	M	M	
a	DM 5 1168	6	9 36.6	5	8.6	6.5	5.98	5.98	24.0
b	6 1172		7 54.0	6	6.1	7.3	6.10	6.40	19.2
d	7 1216		9 9.2	7	6.2	7.5	6.79	6.90	13.7
e	7 1312		22 0.2	7	13.7	8.5		7.45	7.4
	1314		22 7.0		12.1	8.2			
f	8 1367		22 10.1	8	0.5	7.8	. .	7.73	1.0
g	6 1253		20 18.1	6	0.5	7.8		7.05	11.9
	1254		20 21.7		2.4	7.9			

The normals found are displayed in the subjoined table, where the heading $T-t$ signifies the time from the previous computed maximum, St. the step-value of each normal, e its probable error, and v its departure from the curve as finally drawn.

TABLE OF NORMAL LIGHTS.

$T-t$	St.	e	v	$T-t$	St.	e	v
0.262	20.97	0.37	+0.03	15.250	7.58	0.28	+0.27
1.042	20.54	0.32	−0.36	16.126	6.52	0.31	−0.34
1.984	20.27	0.40	+0.07	17.094	6.53	0.40	−0.07
2.936	18.03	0.56	−1.33	18.036	6.99	0.38	+0.39
3.893	17.52	0.38	−0.82	19.096	6.61	0.31	−0.29
5.104	18.07	0.36	+0.96	19.954	7.69	0.39	+0.35
6.264	16.56	0.27	+0.78	20.842	8.14	0.39	+0.16
7.404	14.63	0.53	+0.03	21.832	8.58	0.40	−0.35
8.592	13.65	0.53	+0.14	23.040	9.49	0.39	−1.11
9.801	11.97	0.49	−0.23	23.884	12.79	0.45	+0.19
10.953	11.61	0.47	+0.56	25.095	17.13	0.43	+0.54
12.196	9.76	0.32	−0.17	26.206	19.38	0.43	−0.32
13.394	8.90	0.42	+0.10	26.865	20.63	0.39	−0.25 10 obs.
14.429	8.52	0.30	+0.42				

The mean departure of the normals from the curve is $0.\overset{\text{St.}}{39}$. The mean probable error of a single normal is found to be $0.\overset{\text{St.}}{40}$, and of a single observation as found from the normals, $1.\overset{\text{St.}}{79}$. These quantities are unsatisfactorily large, and seem to indicate that the light-curve is, as remarked above, not constant in different periods, and that in consequence no mean light-curve of the star can be of great value. This impression is distinctly confirmed upon plotting the single light-curves together, by which mean differences between the successive curves, far too great to be entirely attributable to observational errors, are strongly brought out. This inference is also confirmed by the fact that the probable error of a single observation of two comparisons (which is about the mean used by me) found from the observations by themselves, is $0.\overset{\text{St.}}{4}$.

The observed brightness of the star at its maxima ranges from $5^{\text{M}}.80$ to $6^{\text{M}}.36$; at minimum, from $7^{\text{M}}.43$ to $7^{\text{M}}.84$.

The curve plotted from the above normals shows a maximum of $6^{\text{M}}.24$ at $0^{\text{d}}.138$ after the computed time; in view of the facts just stated, however, this would not seem to indicate any certain correction to the elements used.

The decrease is regular and nearly uniform to a minimum of $7^{\text{M}}.54$, which falls $17^{\text{d}}.10$ after the maximum, making the interval from minimum to maximum $9^{\text{d}}.912$. This agrees with my observations, according to which the minima generally fall early by comparison with the computed times. The first half of the increase is slow, the last half quite rapid, and the maximum pretty sharply marked. This latter phase, however, in the single curves, is quite irregular, the maxima being often very flat, as indicated in SCHÖNFELD's mean curve, above referred to.

The readings from the mean light-curve are shown in the following table.

The data are from the time of maximum shown by the plotted curve. The column headed St. contains the step-values, that headed M the magnitudes on the adopted scale.

READINGS FROM MEAN LIGHT CURVE.

d	St.	M	d	St.	M
0	21.00	6.24	15	7.33	7.46
1	20.58	6.28	16	6.77	7.50
2	19.80	6.35	17	6.55	7.53
3	18.81	6.43	17.1	6.50	7.54
4	17.73	6.53	18	6.74	7.51
5	16.65	6.63	19	7.09	7.48
6	15.58	6.71	20	7.72	7.42
7	14.55	6.82	21	8.60	7.34
8	13.55	6.91	22	9.73	7.24
9	12.55	7.01	23	11.68	7.07
10	11.55	7.09	24	14.43	6.84
11	10.59	7.17	25	17.70	6.53
12	9.72	7.24	26	20.08	6.32
13	8.85	7.32	27	21.00	
14	8.09	7.39			

Dorchester, 1905 April 8.

THE VARIABLE STAR *Z GEMINORUM* = 9.1903.

BY J. A. PARKHURST.

The identification of this star has been disputed, the position variously given, and the observations to prove the variation not published; hence the following observations of position and magnitude are offered towards the solution of the problem.

The positions were measured with a Gærtner machine

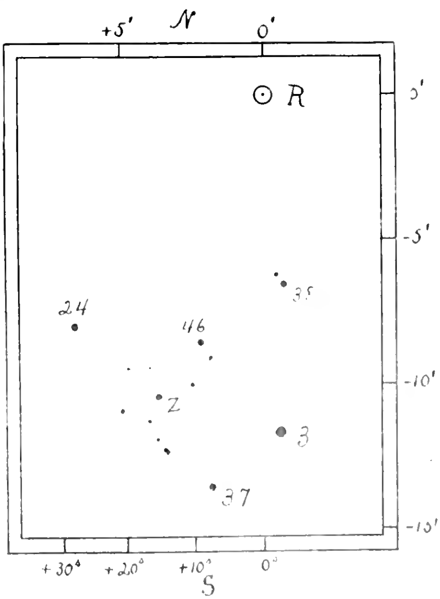
on a photograph taken 1904 December 30, with 30 cm. aperture on the 60 cm. reflector, focal length 2.35 meters. They are based on the A.G. Catalogue places of Bonn B. 2775, 2777, 2778 and 2792.

Star	B.D.	A.G.	Mag.	1900 R.A.	1900 Decl.	Coord. from *R Geminorum* R.A.	Coord. from *R Geminorum* Decl.
				7^h ^{m} ^{s}	$+22°$ $'$ $''$	^{m} ^{s}	$'$ $''$
H 3	+22°1576	2775	8.40	1 17.32	39 17.7	−0 2.87	−11 10.8
H 5	+22°1575	2777	–	1 18.50	31 13.2	−0 1.65	−19 15.1
R	+22°1577	2778	–	1 20.17	51 28.9	0	0
H 37	–	–	11.59	1 27.59	37 51.7	+0 7.11	−13 36.8
H 41 = *Z*	+22°1579	2788*a*	11.6±	1 35.74	40 59.5	+0 15.59	−10 29.0
H 21	–	2792	–	1 18.36	43 21.8	+0 28.27	− 8 3.9
P 1	–	–	13.73	1 31.17	39 4.5	+0 11.02	−12 21.0
P 2	–	–	13.96	1 34.62	39 9.6	+0 14.47	−12 18.9
P 3	–	–	14.19	1 35.73	39 30.0	+0 15.58	−11 58.5
P 4	–	–	14.31	1 36.84	40 10.2	+0 16.69	−11 18.3
P 5	–	–	12.97	1 41.09	40 30.8	+0 20.94	−10 57.7

In the first column H stands for Hagen, and P for Parkhurst. The magnitudes are the result of photometer measures based on the following four stars found in both the Potsdam and Harvard photometric catalogues.

		H.C.O.45	Potsdam	Color
A	+22°1558	7.28	7.59	WG
B	+22°1566	5.94	6.33	W+
C	+21°1531	6.90	7.04	WG
D	+21°1528	6.46	6.62	WG

The magnitudes in the table are given on the Potsdam scale.



R AND *Z* GEMINORUM

The star Hagen 44 was selected at Harvard in 1900 as one of the 12th magnitude stars in the *R Geminorum* field, for the work of determining standards for faint stellar magnitude participated in by Harvard, McCormick, Lick, Yerkes and Halsted (Princeton) observatories. In accordance with the scheme for cooperation directed by Professor Pickering, this star has been measured since 1900 at all these observatories, in addition to the estimates made by Hagen in preparing his *Atlas Stellarum Variabilium* for the *R Geminorum* field. In the catalogue sheet Hagen has the footnote:

"BD. +22°1579, 9m.5, = +16s, −8'.9 *nunquam visa.*"

He also expressed to the writer the opinion that his star 44 was not identical with the variable *Z Geminorum*, but when the substance of this note was communicated to him, he concluded that the two stars were probably identical. The doubt arose from the difference in the declination coordinates. The B.D. place

+22°1579	6^h 58^m $52^s.7$	+22°	46'.5	1855
corr. for precession, gives	7 1 35.3	+22	42.6	1900
while the place of *R*,	7 1 20.2	+22	51.5	1900
gives coordinates of *Z* from *R*,	+0 15.1		8.9	1900
The plate coordinates of *Z* are	+0 15.6		10.5	
Hagen's coordinates of 44 are	+0 15		11.4	

so that the Hagen place of 44 agrees [illegible] B.D. place of +22°1579.

[illegible] the star [illegible] should not prove to be the [illegible] of the stars *Z* [illegible] *Z5*, [illegible] only [illegible] the [illegible]

The following table gives [illegible] 44 [illegible]. In column 7, [illegible] photographic measures, [illegible]

OBSERVED MAGNITUDES OF *Z Geminorum*.

	1900	J.D.	Aperture	Mag.		Seeing
			cm			
1	Oct. 23	5315.83	102	11.63	p	fair
2	24	5317.79	102	11.97	p	fair
3	1902 Mar. 25	5831.59	60	12.4	ph	fair
4	1903 Jan. 16	6131.59	102	11.79	p	fair
5	Feb. 18	6164.74	102	11.66	p	good
6	1904 Mar. 8	6518.64	30	11.95	p	good
7	19	6559.63	30	11.91	p	fair
8	May 2	6602.61	30	11.98	p	fair
9	3	6603.60	30	11.97	p	fair
10	Dec. 4	6819.67	30	11.69	v	fair
11	30	6845.67	30	12.2	ph	fair
12	1905 Jan. 3	6849.70	102	11.72	p	good
13	Apr. 3	6939.67	30	11.71	v'	clouds
14	7	6943.58	15	11.74	v	good
15	8	6944.58	102	11.84	v	fair
16	11	6947.56	102	11.74	v	moon
17	21	6957.69	15	11.74	v	good
18	22	6958.63	102	11.85	p	dull

No confirmation of variability can be found in the above table, the range, 11.63 to 11.98, being scarcely greater than can reasonably be attributed to extreme conditions of seeing. The color of the star is slight, the magnitudes found on the photographs, observations 11, being

	R	*Z*
Ordinary plate,	12.10	12.25
Isochromatic plate,	11.55	12.13
Color effect,	0.55	0.12

In view of the use which has been made of this star as a standard, it is especially desirable that all observations should be promptly published, so that its behavior since 1900 can be known. As far as can be concluded from the observations so far published, it seems to belong to the rather numerous class of 9.5 magnitude B.D. stars which are now between 11 and 12 magnitude, and not sensibly varying in light.

Yerkes Observatory, 1905 April.

NOTES ON VARIABLE STARS. — No. 41.

By HENRY M. PARKHURST.

RESULTS OF OBSERVATIONS.

No.	Star	Phase	Observed Date Julian	Observed Date Calendar	E	Corr.	Wt.	Mag.	Factors		d	Remarks
103	*T Andromedae*	Max.	6551	Mar. 11	63	–	E	–	–	–	–	1904 from last obsn., —9.
103	*T Andromedae*	Max.	6832	Dec. 17	64	–	E	–	–	–	–	Probably later.
976	*T Arietis*	Min.	4199	Oct. 1	29	–	E	–	–	–	–	1897 disappearance.
976	*T Arietis*	Max.	4326	Feb. 5	29	–	E	–	–	–	–	1898 only one obsn.
976	*T Arietis*	Max.	6527	Feb. 16	36	+10	1	–	–	–	–	1904 mean of two highest obs.
976	*T Arietis*	Max.	6830	Dec. 15	27	–	E	–	–	–	–	Probably later.
1166	*X Ceti*	Max.	6497	Jan. 17	28	–	E	–	–	–	–	+20 to 50 days probably.
1577	*R Tauri*	Max.	6534	Feb. 23	47	– 3	5	–	–	–	–	
1717	*V Tauri*	Max.	6441	Nov. 21	67	–16	9	8.83	0.44	0.54	11	1903.
1717	*V Tauri*	Min.	6704	Aug. 11	–	–	–	–	–	–	–	1904.
1717	*V Tauri*	Max.	6802	Nov. 17	69	+ 5	9	9.13	0.43	0.37	16	
1761	*R Orionis*	Max.	6485.9	Jan. 6	47	+20	5	–	–	–	–	Period 214 days.
2690	*Z Puppis*	Max.	6552	Mar. 12	17	0	9	7.97	2.67	1.26	23	
2690	*Z Puppis*	Min.	6861	Jan. 15	19	– 9	–	10.1	–	–	–	
2690	*Z Puppis*	Max.	6980	May 14	19	–	E	–	–	–	–	Lost in the west.
3425	*X Hydrae*	Max.	6620	May 19	15	–	E	–	–	–	–	Obsns indicate min.
4315	*R Com. Ber.*	Max.	6685.2	July 23	74	+ 5.8	8	–	–	–	–	2389906.3 *A.J.* 556.
4573	*RU Virginis*	Min.	6708	Aug. 15	7	–	E	–	–	–	–	*A.J.* 415.
4596	*U Virginis*	Max.	6632.6	May 31	67	+11.9	9	8.02	0.50	0.54	27	
4605	*RT Virginis*	Max.	6635	June 3	5	–	–	–	–	–	–	Highest obs. – changes slight.
6132	*R Ophiuchi*	Max.	6738	Sept. 14	57	+ 6	9	6.32	1.78	0.98	14	
6207	*Z Ophiuchi*	Max.	6748	Sept. 24	12	–18	9	8.66	1.38	2.86	32	
6452	*RY Herculis*	Max.	6662:	June 30	6	–	1	–	–	–	–	Period 224 days.
6682	*X Ophiuchi*	Max.	6736	Sept. 12	20	–25	9	6.16	1.23	1.47	29	
6892	*RX Sagittarii*	Max.	6750	Sept. 26	11	– 4	9	9.42	1.30	1.13	30	
6923	*Z Sagittarii*	Min.	5611	Aug. 14	11	–	E	–	–	–	–	1901.
6923	*Z Sagittarii*	Max.	6737	Sept. 13	13	– 4	9	9.35	1.64	2.57	32	1904.
7040	*RT Aquilae*	Max.	6766	Oct. 12	16	–64	6	7.5	–	–	–	Period 328 days.
7056	*RV Aquilae*	Max.	6844	Dec. 29	15	–	E	–	–	–	–	Not seen Oct. to Dec. 9.

Individual Observations.

Including observations by Arthur C. Perry.

103 *T Andromedae.*

(Cont. from 540. Comp. Stars 346)

Julian	Calendar	Mag.
	1903	
6406	Oct. 18	11.8 to
	1904	
6498	Jan. 18	11.0]
	5 obsns.	
6520	Feb. 9	10.7
6522	11	10.2
6524	13	9.51_2
6768	Oct. 14	11.2]
6783	29	11.2]
6793	Nov. 8	11.2]
6823	Dec. 8	8.90_2
6844	29	8.3
	1905	
6869	Jan. 23	7.8

976 *T Arietis.*

(Cont. from 540. Comp. Stars 403)

Julian	Calendar	Mag.
	1903	
6429	Nov. 10	9.06_2
6455	Dec. 6	9.0
	1904	
6498	Jan. 18	8.0
6522	Feb. 11	9.7
6556	Mar. 16	8.0
6784	Oct. 30	8.11_2
6793	Nov. 8	8.27_2
6816	Dec. 31	8.31_2
	1905	
6907	Mar. 2	8.3

1166 *X Ceti.*

(Cont. from 540. Comp. Stars 468)

Julian	Calendar	Mag.
	1904	
6498	Jan. 18	11.:]
6499	19	11.4
6522	Feb. 11	12.]
6792	Nov. 7	9.2
6793	8	9.02_2
	1905	
6847	Jan. 1	10.64_2
6908	Mar. 3	9.6

1577 *R Tauri.*

(Continued from 540.)

Julian	Calendar	Mag.
	1903	
6455	Dec. 6	11.]
6463	14	10.51_2
6464	15	10.54_2
	1904	
6522	Feb. 11	9.1
6524	13	8.15_2
6556	Mar. 16	10.2

1717 *V Tauri.*

(Cont. from 540. Comp. Stars 513)

Julian	Calendar	Mag.
	1903	
6428	Nov. 9	9.81_2
6433	14	8.95_2
6437	18	9.52_2
6438	19	8.65_2
6439	20	8.48_2
6443	24	9.19_2
6445	26	9.17_2
6460	Dec. 11	8.80_2
6461	12	9.21_2
6462	Dec. 13	9.32_2
	1904	
6501	Jan. 24	12.4
6522	Feb. 11	to
6583	Apr. 12	12.]
	4 obsns.	
6784	Oct. 30	9.05_2
6785	31	9.40_2
6792	Nov. 7	9.13_2
6796	11	9.33_2
6799	14	9.21_2
6800	15	9.07_2
6801	16	8.90_2
6812	27	9.47_2
6819	Dec. 4	9.43_2
6823	8	10.49_2
6846	31	11.4

1761 *R Orionis.*

(Continued from 540.)

Julian	Calendar	Mag.
	1903	
6466	Dec. 17	9.44_2
6467	18	9.34_2
6471	22	9.05_2
	1904	
6522	Feb. 11	9.5
6556	Mar. 16	10.]

2690 *Z Puppis.*

(Cont. from 540. Comp. Stars 403)

Julian	Calendar	Mag.
	1904	
6524	Feb. 13	8.3
6528	17	8.37_2
6531	20	7.97_2 r
6536	25	8.24_2
6538	27	8.11_2
6541	Mar. 1	8.03_2 r
6548	8	7.91_3
6549	9	8.07_2 r
6553	13	7.74_4
6558	18	8.06_3
6559	19	8.16 r
6560	20	8.16_2
6574	Apr. 3	8.6
6575	4	8.76_2
6576	5	8.23_2
6583	12	8.6 r
6591	20	8.4: r
6823	Dec. 8	10.5 r
	1905	
6847	Jan. 1	10.0
6853	7	10.0
6854	8	9.8
6854	8	10.0 r
6860	14	9.9
6860	14	10.5 r
6861	15	10.0
6863	17	9.9:
6872	26	10.0] r
6874	28	10.4
6876	30	11.0 r
6884	Feb. 7	10.4 r
6888	Feb. 11	9.9
6898	21	10.0
6903	26	9.85
6904	27	10.4 r
6908	Mar. 3	10.0
6916	11	10.0
6928	23	10.3
6928	23	10.3 r
6932	27	9.7
6933	28	9.6
6934	29	9.7
6936	31	9.7
6936	31	10.2 r
6937	Apr. 1	9.8
6938	2	9.6
6938	2	10.4 r
6942	6	9.9
6952	16	9.8
6954	18	10.0
6955	19	10.1
6958	22	9.8
6958	22	10.3 r
6959	23	9.9
6961	25	9.9
6964	25	10.2 r

3425 *X Hydrae.*

(Cont. from 440. Comp. Stars 350)

Julian	Calendar	Mag.
	1904	
6583	Apr. 12	11.7
6584	13	11.9
6593	22	11.7
6609	May 8	11.7
6623	22	11.2]

1315 *R Comae Ber.*

(Cont. from 556. Comp. Stars 415)

Julian	Calendar	Mag.
	1904	
6642.6	June 10	10.42_2
6646.6	11	10.11
6647.6	15	10.11
6649.6	17	10.75_2
6654.6	22	10.94_2
6655.6	23	10.49_4
6657.6	25	10.21_2
6661.6	29	10.33_2
6663.6	July 1	10.07_2
6664.6	2	10.80
6665.6	3	10.02_{12}
6670.6	8	9.75_4
6671.6	9	10.13_4
6675.6	13	9.12_4
6676.6	14	9.08_4
6678.6	16	9.28_4
6682.6	20	8.69_4
6689.6	27	9.15_4
6691.6	29	9.11_4
6693.6	31	8.93_4

1573 *RU Virginis.*

(Continued from 556.)

Julian	Calendar	Mag.
	1904	
6587	Apr. 16	10.9
6603	May 2	11.2
6612	May 11	10.5
6622	21	10.9
6671	July 9	10.9

1596 *U Virginis.*

(Cont. from 556. Comp. Stars 444)

Julian	Calendar	Mag.
	1904	
6587	Apr. 16	10.3
6593	22	10.30
6603	May 2	9.71_2
6612	11	9.38_2
6622	21	8.56_2
6628	27	8.44_2
6629	28	7.89_2
6635	June 3	8.15_2
6642	10	8.58_2
6665	July 3	9.03_2
6670	8	9.27_2

1665 *RT Virginis.*

(Continued from 556.)

Julian	Calendar	Mag.
	1904	
6587	Apr. 16	8.8
6612	May 11	8.6
6622	21	9.0
6629	28	9.12_2
6635	June 3	8.55
6671	July 9	8.91_2

6132 *R Ophiuchi.*

(Cont. from 490. Comp. Stars 476)

Julian	Calendar	Mag.
	1904	
6693	July 31	8.26_2
6702	Aug. 9	8.06_2
6707	14	7.49_2
6714	21	7.11_2
6719	26	6.77_2
6728	Sept. 4	7.04
6732	8	6.35_2
6736	12	6.46_2
6740	16	5.94_2
6741	17	6.36_2
6746	22	7.51_2
6747	23	7.26_2

6207 *Z Ophiuchi.*

(Cont. from 563. Comp. Stars 558)

Julian	Calendar	Mag.
	1904	
6707	Aug. 14	10.25_2
6716	23	9.65_2
6719	26	9.28
6728	Sept. 4	9.74
6739	15	8.58
6742	18	8.40
6754	30	8.84
6761	Oct. 7	8.68
6770	16	8.86_2
6773	19	8.93

6452 *RY H[illegible]*

(Cont. from 563. Comp. Stars [illegible])

Julian	Calendar	Mag.
	1904	
6708	Aug. 15	10.45
6717	24	11
6723	30	11.13_2

6682 *X O[illegible]*

(Cont. from [illegible] Comp. Stars 421)

Julian	Calendar	Mag.
	1904	
6708	Aug. 15	7.22
6717	24	6.90
6723	30	6.70
6729	Sept. 5	6.49
6739	15	5.41
6742	18	6.52
6752	28	6.42
6759	Oct. 5	6.65
6769	15	7.65

6892 *RX S[illegible]*

(Cont. from 56[illegible] Comp. Stars 421)

Julian	Calendar	Mag.
	1904	
6708	Aug. 15	10.84
6717	24	10.
6718	25	10.51
6723	30	10.51
6729	Sept. 5	10.39
6739	15	9.41
6752	28	9.10
6755	Oct. 1	9.38
6761	7	9.79
6770	16	10.30
6775	21	10.62

6923 *Z S[illegible]*

(Cont. from 482. Comp. Stars 311)

Julian	Calendar	Mag.
	1901	
5609	Aug. 12	to
5664	Oct. 6	12.]
	6 obsns.	
	1904	
6708	Aug. 15	9.5
6714	18	10.04
6718	25	9.83
6723	30	9.56
6729	Sept. 5	9.57
6739	15	8.60
6752	28	10.13
6755	Oct. 1	9.23
6761	7	10.07

7010 *RT A[illegible]*

(Cont. from 4[illegible] Comp. Stars 482)

Julian	Calendar	Mag.
	1904	
6757	Oct. 3	7.76
6758	4	7.57
6769	15	7.48
6781	27	7.62
6791	Nov. 6	8.55
6800	15	8.13
6813	28	8.28
6824	Dec. 9	9.22

7056 *R[illegible]*

Julian	Calendar	Mag.
	1904	
6757	Oct. 3	[illegible]
6813	Nov. 28	[illegible]
6824	Dec. 9	9.77
6826	11	9.66
6828	13	9.63
6843	28	9.5

Comparison-Stars, 1893–1904.

1717 *V Tauri.*

Star	DM.	Mag.	*n*
W	+17°793	9.08	13
Y	+17°801	9.73	55
Z	+17°799	9.72	56
a	3*n*2*f* *W*	10.26	35
d	2*n*2*f* *Y*	11.62	10
e	2*n*2*p* *Y*	11.61	10
l	1*s*3*f* *V*	11.31	13
k	1*s*8*f* *V*	11.52	16
j	5*s*1*p* *Y*	11.36	28
m	1*s*2*f* *Z*	11.90	17

2690 *Z Puppis.*

Star	DM.	Mag.	*n*
L	−20°2003	7.77	32
P	−20°2009	8.30	35
Q	−20°2015	8.33	35
V	−20°2025	9.00	25
1*W*	−20°2021	9.38	11
W	−20°2011	9.79	36
X	−20°2016	9.87	41
Y	−20°2006	9.96	56
p	*s* *f* *V*	10.77	10
q	*n* *f* *V*	10.81	7

6207 *Z Ophiuchi.*

Star	DM.	Mag.	*n*
1*E*	+1°3121	7.03	5
L	+1°3127	8.38	8
X	+1°3118	10.50	30
Z	+1°3420	11.07	21
1*Z*	+1°3119	11.10	13
a	8*s* 1*E*	9.96	15
b	3*f* *a*	10.61	2
c	3*s* 1*f* *V*	11.19	15
d	1*n* 3*p* *V*	11.70	26
g	2*s* 1*f* *V*	12.60	18

6682 *X Ophiuchi.*

Star	DM.	Mag.	*n*
E	+8°3797	6.67	43
F	+8°3799	6.88	42
G	+8°3791	6.73	50
1*X*	+9°3789	8.12	2
T	+9°3793	9.16	2
1*T*	+9°3791	9.28	3
V	+8°3798	8.83	42
1*W*	+8°3787		
1*Z*	+9°3798	9.32	17
3*Z*	+8°3783	9.66	16

SUNSPOT OBSERVATIONS,

MADE AT THE AMHERST COLLEGE OBSERVATORY,

By ROBERT H. BAKER.

1905		d h	New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots	Def.
Jan.	1	3	–	–	–	–	–	–	1	8	3
	4	3	–	–	–	–	–	–	1	6	2
	4	20	–	–	–	–	–	–	1	2	2
	5	3	1	1	–	–	–	–	2	3	4
	7	21	3	24	1	2	1	2	4	25	3
	8	3	–	6	–	–	–	–	4	30	2
	8	21	–	–	–	–	–	–	4	21	3
	9	21	1	7	–	–	1	3	4	26	4
	10	3	–	5	–	–	–	–	4	28	2
	10	21	–	5	–	–	–	–	4	25	5
	12	22	–	16	–	–	–	–	3	29	5
	13	3	–	–	–	–	–	–	3	23	4
	13	22	2	3	1	3	2	3	4	30	5
	14	4	–	3	–	–	–	–	4	31	3
	14	22	–	11	–	–	–	–	4	41	4
	15	4	1	3	–	–	–	–	5	36	5
	15	21	–	1	–	–	–	–	5	31	5
	16	3	–	–	–	–	–	–	5	28	5
	16	22	–	5	1	3	–	–	4	29	4
	17	22	–	6	–	–	–	–	4	35	5
	19	2	–	7	1	1	–	–	3	35	5
	19	22	–	19	–	–	–	–	3	51	5
	20	4	1	1	–	–	1	1	4	44	3
	22	3	–	–	–	13	–	–	3	10	2
	22	21	–	4	–	–	–	–	3	20	3
	21	2	–		1	4	–	–	2	3	1
	26	23	–		1	2	–	–	1	2	1
	28	21	1	6	–	–	1	3	2	11	4
	30	0	–	12		–	–	–	2	21	5
	31	1	–	14	–	–	–	–	2	29	3
Feb.	1	22	1	31	–	–	1	2	3	58	5
	2	4	–	5	–	–	–	–	3	63	1
	2	22	1	1	1	1	1	1	3	45	3
	3	1	–	1	–	–	–	–	3	13	2
	3	20	1	7	–	–	1	5	4	59	4
	4	4	–	28	–	–	–	–	4	99	5
	4	22	1	1		–	1	1	5	60	1
	5	21	1	5	–	–	–	–	6	42	5
	7	0	–	–	–	–	–	–	5	31	5
	10	0	–	1	1	20	–	–	4	12	2
	13	20	1	16	–	–	1	7	5	27	1
	14	4	–	3	–	–	–	–	5	28	4
	15	21	–	–	1	2	–	–	3	11	4
Feb.	16	23	–	–	–	–	–	–	2	5	2
	17	21	2	5	–	–	1	1	4	9	3
	19	0	–	11	–	–	–	–	3	17	5
	20	21	–	–	1	1	–	–	2	5	2
	22	20	–	7	–	–	–	–	1	8	2
	23	4	–	–	–	–	–	–	1	7	2
	23	21	–	6	–	–	–	–	1	14	3
	24	4	–	1	–	–	–	–	1	15	4
	26	5	1	12	–	–	1	12	2	19	4
	26	20	–	–	–	–	–	–	2	17	4
	28	4	–	–	–	–	–	–	2	12	4
	28	21	1	9	–	–	1	1	3	21	5
Mar.	1	5	–	2	–	–	–	2	3	23	5
	1	22	–	9	–	–	–	7	3	29	4
	2	4	–	10	–	–	–	–	3	42	5
	2	21	–	1	–	–	–	–	3	30	3
	4	5	1	36	1	4	–	–	3	62	4
	4	22	–	–	–	–	–	–	3	57	3
	5	21	–	2	–	–	–	–	3	53	5
	6	5	–	–	–	–	–	–	3	41	5
	6	22	–	–	–	–	–	–	3	40	4
	9	0	–	–	1	11	–	–	1	12	1
	10	0	–	–	–	–	–	–	1	27	4
	12	5	–	–	–	–	–	–	1	10	4
	12	22	–	–	–	–	–	–	1	6	4
	13	5	–	–	–	–	–	–	1	4	5
	13	20	–	–	1	4	–	–	–	–	2
	14	4	1	1	–	–	1	1	1	1	4
	11	20	–	–	–	–	–	–	1	1	3
	16	21	–	5	–	–	–	–	1	6	4
	17	5	–	3	–	–	–	–	1	9	4
	17	22	–	–	–	–	–	–	1	8	3
	22	5	2	7	–	–	1	1	3	11	5
	25	21	1	13	–	–	1	12	2	19	3
	27	5	1	1	–	–	–	–	3	12	3
	27	21	–	14	–	–	–	–	3	31	5
	28	5	–	–	–	–	–	–	3	25	4
	28	21	–	–	–	–	–	–	2	14	2
	29	5	1	1	–	–	–	–	3	13	3
	29	22	–	–	–	–	–	–	3	10	3
	30	6	–	–	–	–	–	–	3	13	5
	30	21	–	7	1	1	–	–	1	18	2
	31	5	–	3	–	–	–	–	2	22	3

Observed with 6-inch Reflector.

OBSERVATIONS OF COMETS,

MADE AT THE CINCINNATI OBSERVATORY,

By J. G. PORTER and DeLISLE STEWART.

Cincinnati M.T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.		Obs.
					Comet 1904 I.						
1904 d h m s			m s	′ ″	h m s	° ′ ″			s	″	
Apr. 29 9 9 46	1	4 , 8	+ 0 6.64	+4 41.5	16 16 14.44	+51 38 25.5	n9.836	0.178	+2.03	− 3.8	P
30 9 6 58	2	8 , 8	+ 0 32.41	−5 15.9	16 12 20.41	+52 7 10.5	n9.837	0.130	+2.07	− 3.4	P
May 4 8 14 5	3	12 , 8	+ 1 38.29	+0 36.9	15 56 2.12	+53 51 28.7	n9.869	0.214	+2.16	− 1.8	P
5 8 0 17	6	10 , 8	− 1 15.90	−3 36.1	15 51 16.16	+54 14 56.0	n9.876	0.239	+2.18	− 1.5	P
9 8 8 18	7	6 , 5	+ 2 57.42	+1 26.5	15 33 56.76	+55 38 43.1	n9.861	9.808	+2.24	+ 0.3	P
10 7 56 31	8	10 , 8	− 0 42.14	+2 14.5	15 29 24.59	+55 56 35.1	n9.867	9.834	+2.25	+ 0.7	P
24 8 45 41	9	8 , 6	+ 2 21.55	+5 15.5	14 24 39.81	+58 9 19.0	n9.520	n0.389	+2.00	+ 6.6	P
28 9 50 19	10	4 , 4	+12 34.37	−1 25.2	14 7 24.18	+58 10 23.0	8.468	n0.461	+1.76	+ 7.9	S
June 3 8 47 38	11	12 , 14	+ 0 18.12	+1 47.4	13 44 20.14	+57 19 5.4	n8.490	n0.453	+1.55	+ 9.1	S
8 10 38 31	12	12 , 10	− 1 9.77[1]	−3 27.5	13 27 12.50	+57 14 2.7	9.684	n0.419	+1.32	+10.0	S
17 9 26 36	13	12 , 12	+ 3 50.05	−3 22.8	13 2 47.73	+55 48 14.7	9.630	n0.225	+0.89	+10.2	S
20 8 51 35	15	10 , 6	+ 1 52.31	−0 14.1	12 56 13.56	+55 15 23.6	9.571	n0.254	+0.79	+10.3	P
July 2 10 42 58	16	8 , 8	− 1 34.67	+0 36.7	12 36 6.15	+52 54 27.6	9.858	0.229	+0.46	+ 9.7	S
6 10 30 12	17	3 , 8	− 0 22.31	+4 45.8	12 31 24.54	+52 7 34.8	9.855	0.290	+0.31	+ 9.2	P
13 9 49 5	18	10 , 10	+ 0 41.44	−3 11.5	12 25 4.11	+50 47 35.6	9.837	0.279	+0.17	+ 8.1	S
18 9 28 15	19	13 , 10	+ 0 34.82	−5 8.3	12 21 45.72	+49 52 53.1	9.831	0.312	+0.07	+ 7.6	S
Aug. 2 10 6 36	20	6 , 8	− 4 0.65[1]	−3 10.1	12 16 27.97	+47 24 11.0	9.824	0.654	−0.12	+ 5.6	S
16 9 3 45	21	10 , 8	+ 1 12.33	+6 27.2	12 15 58.91	+45 31 23.5	9.808	0.661	−0.23	+ 2.8	S
Sept. 13 8 2 4	22	8 , 8	+ 2 57.88	+2 14.6	12 22 1.36	+43 6 39.2	9.775	0.737	−0.32	2.9	S
Oct. 1 7 32 2	23	7 , 10	+ 3 1.25	+4 45.5[2]	12 28 2.52	+12 36 1.4	9.745	0.786	−0.27	7.0	S
13 16 11 10	24	8 , 9	− 1 17.65	+3 0.1	12 32 13.72	+12 46 49.3	n9.782	0.706	−0.32	9.8	S
Nov. 7 16 32 19	25	10 , 6	− 1 28.17	+1 31.6	12 38 27.38	+11 38 55.7	n9.791	0.423	+0.05	17.3	S
14 16 25 7	26	10 , 8	− 1 14.85	+8 55.6	12 39 6.95	+15 34 17.2	n9.786	0.334	+0.21	20.0	S
Dec. 5 16 30 30	27	10 , 14	+ 0 28.24	+5 11.0	12 35 54.58	+49 26 36.3	n9.708	8.278	+0.75	−27.4	S
					Encke's Comet.						
Nov. 29 8 1 1	28	10 , 9	− 0 18.28	+8 6.7	21 18 13.72	+10 25 55.1	9.514	0.665	+1.96	+21.7	P
30 7 11 52	29	12 , 8	+ 1 28.09	+0 11.0	21 15 5.63	+ 9 51 13.4	9.524	0.665	+1.91	+21.3	P
Dec. 5 9 0 55	30	8 , 15	− 0 13.59	+4 51.3	20 56 33.88	+ 6 52 36.1	9.647	0.723	+1.76	+22.5	S
14 6 41 11	31	14 , 13	+ 1 30.61	+0 16.7	20 21 21.07	+ 1 4 4.9	9.590	0.737	+1.58	+18.7	S
20 6 38 36	32	8 , 13	+ 0 4.03	−5 57.0	19 53 13.31	3 35 11.0	9.634	0.752	+1.50	+16.4	S

[1] It was assumed that the position-circle was set wrong by 10°, and Δα corrected accordingly.

[2] It was assumed that the micrometer was read wrong by four turns, and Δδ corrected accordingly. One turn = [illegible]79.

Mean Places of Comparison-Stars for the beginning of the year.

✱	α	δ	Authority
	h m s	° ′ ″	
1	16 16 5.77	+51 33 44.8	Harvard, A.G. 4960
2	16 11 15.93	+52 12 29.8	" A.G. 4947
3	15 54 21.67	+53 50 53.6	B.D. [illegible] compared with [illegible]
4	15 41 28.57	+53 55 4.4	Harvard, A.G. 4839
5	15 46 13.34	+53 58 19.3	" A.G. 4847
6	15 52 59.88	+54 18 33.6	" A.G. 4869
7	15 30 57.10	+55 37 16.3	Hels. Gotha, A.G. 8408
8	15 30 4.48	+55 54 19.9	" A.G. 8402
9	14 22 16.29	+58 3 56.9	" A.G. 7966
10	13 54 48.05	+58 11 40.3	" A.G. 7774
11	13 44 0.44	+57 14 8.9	" A.G. 7709
12	13 28 20.95	+57 17 20.2	" A.G. 7608
13	12 58 56.79	+55 51 27.3	B.D. +56° 1637 comp. with
14	13 11 23.77	+55 19 25.4	Hels. Gotha, A.G. 7496
15	12 54 20.46	+55 15 27.4	" A.G. 7388
16	12 40 40.36	+52 53 41.2	Harvard, A.G. 4434
17	12 31 46.54	+52 2 39.8	" A.G. 4404
18	12 24 22.50	+50 50 39.0	Harvard, A.G. 4085
19	12 21 10.83	+49 57 53.8	" A.G. 4072
20	12 20 28.74	+47 27 15.5	Bonn, A.G. 8494
21	12 14 46.81	+45 24 53.5	" A.G. 8450
22	12 19 3.80	+43 4 27.5	" A.G. 8482 & [illegible]
23	12 24 58.54	+12 31 22.9	" A.G. 8528
24	12 33 31.69	+12 43 59.0	" A.G. 8594
25	12 39 55.54	+11 37 41.4	" A.G. 8642
26	12 40 21.59	+15 25 11.6	" A.G. 8647
27	12 35 25.59	+19 21 22.7	" A.G. 8610
28	21 19 30.01	+10 17 23.7	Leipzig I, A.G. 8476
29	21 13 35.63	+ 9 50 58.1	" II, A.G. 10665
30	20 56 45.71	+ 6 47 22.3	[illegible]
31	20 19 48.88	+ 1 3 29.5	[illegible]
32	19 53 7.78	3 30 0.1	B.D. −3° 4755 comp. with
33	19 57 36.16	3 36 33.8	Karls. & Rad. 1890, 5364

OCCULTATIONS OF STARS BY THE *MOON*,

OBSERVED AT THE U.S. NAVAL OBSERVATORY,

BY H. L. RICE, J. C. HAMMOND AND MATT FREDERICKSON.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

No.	Date	Star	Phe-nom.	Obs.	Sid. Time h m s	Wash. M.T. h m s
1	1904 May 21	B.A.C. 3398	DD	H.	14 47 11.7	10 49 47.0
2	Aug. 5	γ *Tauri*	DB	R.	22 51 28.6	13 56 54.8
3	5	γ *Tauri*	RD	R.	23 53 21.6	14 55 38.2
4	23	W.B.XX,1293	DD	R.	1 26 56.9	15 18 11.9
5	Oct. 11	η *Librae*	DD	R.	20 20 23.4	6 59 49.1
6	26	75 *Tauri*	RD	F.	6 48 14.1	16 26 58.4
7	26	B.A.C. 1391	RD	F.	7 46 17.9	17 21 52.7
8	26	B.A.C. 1406	DB	F.	8 40 25.2	18 18 51.1
9	26	α *Tauri*	DB	F.	9 47 33.9	19 25 48.8
10	27	115 *Tauri*	DB	F.	8 47 42.3	18 22 11.1
11	Nov. 23	B.A.C. 1526	DB	F	1 14 4.8	9 3 38.4
12	26	5 *Cancri*	DB	F.	9 53 13.1	17 29 33.9
13	26	5 *Cancri*	RD	F.	10 52 14.7	18 28 25.8
14	Dec. 1	*Mars I*	DB	H.	6 49 51	14 7 5
15	1	*Mars II*	DB	H.	6 50 4	14 7 15
16	1	*Mars I*	DB	F.	6 49 40.8	14 6 52.2
17	1	*Mars II*	DB	F.	6 49 50.3	14 7 1.6
18	1	*Mars*	RD	H.	7 51 21.4	15 8 22.6
19	1	*Mars*	RD	F.	7 51 20.9	15 8 22.2
20	14	20 *Piscium*	DD	F.	4 22 6.6	10 48 35.3
21	20	B.A.C. 1391	DD	R.	23 18 2.7	5 21 45.7
22	20	B.A.C. 1394	DD	R.	23 23 26.7	5 27 8.9
23	20	B.A.C. 1406	DD	R.	0 53 17.5	6 56 44.9
24	20	α *Tauri*	DD	H.	2 14 31.8	8 17 45.9
25	20	α *Tauri*	DD	R.	2 14 31.6	8 17 45.7
26	20	α *Tauri*	RB	H.	3 35 50.8	9 38 51.6
27	20	α *Tauri*	RB	R.	3 35 50.8	9 38 51.6
28	1905 Jan. 14	B.A.C. 764	DD	F.	1 57 22.9	6 22 22.0
29	14	B.A.C. 764	RB	F.	2 55 38.6	7 20 28.1
30	16	γ *Tauri*	DD	F.	8 38 6.8	12 54 8.4
31	16	γ *Tauri*	RB	F.	9 4 36.7	13 20 34.0
32	20	5 *Cancri*	DD	F.	3 34 12.4	7 35 20.2
33	20	5 *Cancri*	RB	F.	4 34 26.4	8 35 24.3
34	25	*k Virginis*	DB	F.	9 46 54.2	13 27 21.4
35	25	*k Virginis*	RD	F.	10 53 52.9	14 34 9.1
36	Feb. 14	130 *Tauri*	DD	F.	10 43 15.7	13 4 55.5
37	17	B.A.C. 2888	DD	F.	6 46 48.3	8 57 19.1
38	20	β *Virginis*	DB	F.	16 32 41.4	18 29 48.5
39	24	γ *Librae*	DB	R.	15 1 33.1	16 43 11.5
40	24	γ *Librae*	RD	R.	16 22 36.6	18 4 1.7
41	26	DM.$-18°$4516	RD	R.	13 56 27.4	15 30 24.6
42	Mar. 12	70 *Tauri*	DD	F.	6 48 53.7	7 28 58.3
43	12	θ^1 *Tauri*	DD	F.	8 39 29.1	9 19 15.6
44	12	θ^2 *Tauri*	DD	F.	8 52 57.1	9 32 41.4
45	27	Lal. 35499	DB	F.	16 44 20.9	16 23 49.4
46	27	Lal. 35499	RD	F.	18 10 54.2	17 50 8.5
47	Apr. 18	l^2 *Virginis*	DD	F.	10 13 57.0	8 27 59.5
48	18	l^2 *Virginis*	RB	F.	11 20 6.6	9 33 58.2
49	20	γ *Librae*	DB	F.	12 37 2.1	10 42 49.3
50	20	γ *Librae*	RD	F.	13 44 22.0	11 49 58.2
51	24	Mayer 811	DB	F.	16 43 41.9	14 33 5.1
52	24	Mayer 814	RD	F.	17 38 27.5	15 27 41.7
53	May 16	94 *Virginis*	DD	F.	13 14 14.2	9 37 41.7
54	June 14	η *Librae*	DD	F.	17 6 53.3	11 35 41.3
55	14	η *Librae*	RB	F.	18 10 35.7	12 39 13.3

Nos. 11, 14, 15, 24, 26, 47, 48, were observed with the 26-inch equatorial; No. 8, with the 5-inch finder of the 26-inch equatorial; the remainder with the 12-inch equatorial. Nos. 14 and 15 were observed eye-and-ear.

In the fourth column DD signifies that the star disappeared at the dark limb of the Moon:
" " " DB " " " " " bright " " " :
" " " RD " " " reappeared " dark " " " :
" " " RB " " " " " bright " " " :

Nos. 5, 10, 30, 31, 32, 33, 37, 42, were observed with a power 240; Nos. 6, 7, 8, 9, 12, 13, 26, 27, with a power 165; Nos. 11, 14, 15, 24, 26, 47, 48, with a power of 180; No. 18 with a power 30; Nos. 54, 55 with a power 335; the remainder were observed with a power 116.

Nos. 32, 33, full moon about $6\frac{1}{2}$ hours after emersion. Nos. 47, 48, full moon about 11 hours after emersion.

Nos. 4, 5, 22, 38, uncertain on account of moon's nearness to horizon. No. 45 uncertain on account of daylight. Nos. 3, 29, probably late.

CONTENTS.

PUBLISHED AT BOSTON, AND CUSHING ROAD, WELLESLEY HILLS, MASS., SEMI-MONTHLY, BY S. C. CHANDLER. ADDRESS, BOX 216, WELLESLEY HILLS, MASS.
Associate Editors, ASAPH HALL and LEWIS BOSS. PRICE, $5.00 THE VOLUME PRESS OF THOS. P. NICHOLS, LYNN, Mass. *Closed Sept. 4*

Entered February 2, 1903, at Boston, Mass., as second-class matter, under Act of Congress of March 3, 1879.

INDEX

TO THE TWENTY-FOURTH VOLUME.

D.

E.

F.

G.

H.

I.

J.